Exercices

Exercice 1 - Rips vs Cech

Let $\mathbf{x} = \{x_1, \ldots, x_n\}$ be a set of points on $\mathbb{R}^2$ (the results actually holds for any metric space).

Prove that for any $r > 0$,

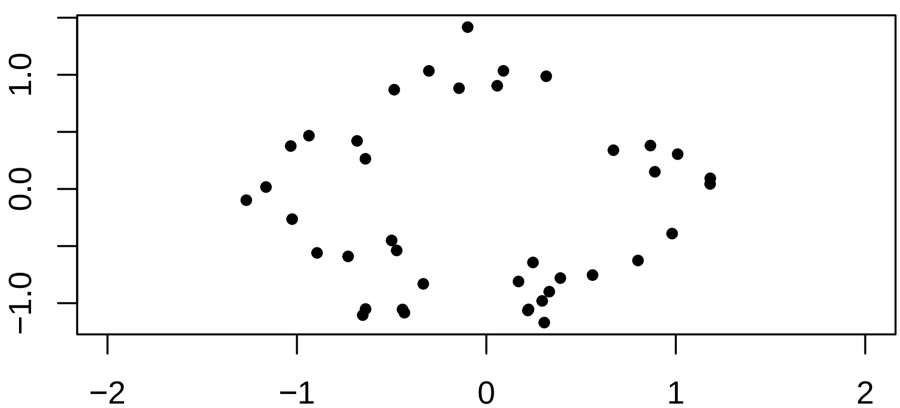
$$R_{\frac{r}{2}}(\mathbf{x}) \subset C_r(\mathbf{x}) \subset R_r(\mathbf{x}).$$

Exercice 2 - General questions on TDA

- With respect to standard summary statistics in spatial statistics (K, g …), what possible advantages do you see in TDA techniques?
- Compute the Ripley's K function of a Poisson point process with intensity 500 on the unit square.
- Do the same for a Baddeley Silverman point process.
- Compare their estimated Ripley's K function.
- Compute the persistence diagram for the connected components and loops using the α-complex and Rips complex.
- Do you observe any differences?

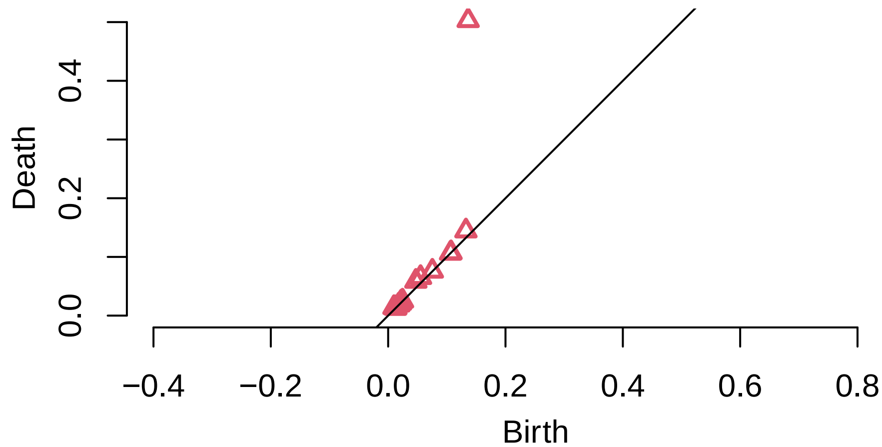
Exercice 3

Here is a plot of a point pattern



and the corresponding persistence diagram for the loops (topological feature of dimension 1).

```
plot(diagr, dimension=1, asp=1)
```



- Can you guess which kind of complexes have been used?
- How would you draw, with respect to the bottleneck distance, a confidence interval of length 0.05 centered at this persistence diagram?
- What would you expect with another kind of complex used?