Assigment TDA/spatial statistics

Test for complete spatial randomness

The aim of this assignment is to use TDA to assess how likely an observed point pattern follows a Poisson point process distribution. For simplicity we will assume the intensity to be known.

Initialising simulated data

- Simulate one cell (Baddeley-Silverman) point process with intensity $\rho = 2$ on the window $W = [0, 10]^2$. We denote this realisation by x_0.
- Compute its persistence diagram using one simplicial complex from the course. Is there any simplicial complex that should be preferred in this situation?

Computing the statistic for the null model.

- Simulate 1000 realisations of Poisson point processes with intensity $\rho = 2$ on the windows $W = [0, 10]^2$. If you dispose of enough computational power, use 10000 realisations.
- Compute the scaled accumulated persistence function, from the slides of the course, for each of realisation.
- Compute the persistence diagram of each point pattern, using the same simplicial complex as above.
- Estimate the mean and variance of the test statistic: $\frac{1}{|W|\sqrt{\rho}}\mathrm{APF}_1(\frac{r}{\sqrt{2}})$ for $r = 0.5$.
- (optional) Using results from the course, argue why the accumulated persistence function verifies a central limit theorem.

Statistical tests

- Visual inspection: do you observe any differences between the persistence diagrams? You may compute various exploratory statistics from the persistence diagram, such as boxplots, density plots, histogram of the birth times ...
- Test the assumption $\mathscr{H}_0$ that x_0 follows a Poisson point process distribution of intensity 2.
- (optional) Repeat the study over 100 realisations to estimate the power of the statistical test used above.

Global rank envelopes (optional)

- Assess the CSR assumption using a global rank envelope test.
- Compare the power of the global rank envelope test with the one obtained in the above section.