# SPH simulation of water falling in bucket

Soumyadeep Paul*
Department of Mechanical Engineering, The University of Tokyo

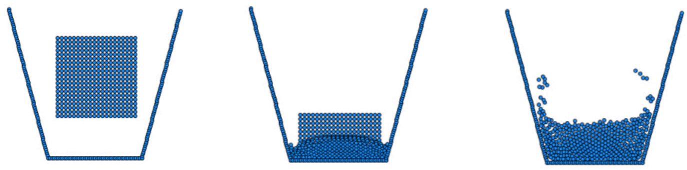

**Figure 1:** *Smoothed Particle Hydrodynamics of water falling in bucket*

## Abstract

Particle based fluid simulation offers an attractive option to animate fluid flow with free surfaces. In this work, we have implemeted a smoothed particle hydyrodynamics approach to solve this problem. By choosing appropiate kernels we have solved the Navier-Stokes equation for the marker particles. Special attention has been given to capture the effect of surface tension in this study. We have performed 2D SPH animation of water falling in a bucket by considering the water phase to consist of 625 marker particles.

**Keywords:** smoothed particle hydrodynamics, animation, kernel, surface tension

## 1 Introduction

Fluid flow is ubiquitous in nature. Physicists have been studying various aspects of fluids for the past century. Many aspects of fluid flow like bubble coalescence and jet breaup are still not clearly understood. Most of these interesting fluid phenomena trace back to the interface between two phases. Analytically it is rather difficult to model the complicated physics behind such behaviour. Thus over the years many numerical techniques have been developed to model these effects in multiphase flows: volume of fluid, level set technique and phase field models. Although these techniques are quite robust from a physics point of view, in terms of interactive applications like computer animations, they are not very attractive.

Smoothed Particle Hydrodynamics offers a unique platform to animate fluid flows for interactive applications. Originally proposed by [Lucy 1977] for astrophysical problems, later this method was employed to solve Navier Stokes equations for both Newtonian and non-Newtonian fluids by using mobile interpolation points [Nair and Tomar 2014]. Basically the entire fluid region is represented by marker particles. The flow variables like velocity, density, pressure etc are centered at the marker particles. For the region in between fluid properties are interpolated using kernel functions. By choosing appropiate kernels, the spacial derivatives are obtained at the maker points. Thus the Navier Stokes equations are formulated in a Lagrangian framework. The velocity of each marker particles are thus obtained and they are advected accordingly.

Apart applications in computer games and medical simulators [Müller et al. 2003], SPH has also caught significant attention in the physics community. In a recent paper [Nair and Tomar 2014] implemented SPH for a buoyant cylinder entering a liquid and showed excellant agreement with experimental results for the same problem. In another recent work, [Nair and Poeschel 2018] showed dynamic cappillary phenomenon using incompressible SPH. This method superimposes molecular dynamics behaviour with continuum fluid. Thus it is able to capture phenomena of complex fluids in real engineering problems, where mesh based simulation approaches fail.

In this study, we have implemented SPH of water falling inside a bucket under gravity. By tuning the simulation parameters we have showed a realistic animation of water splashing.

## 2 Related work

One of the most prominent work on SPH of pouring water was done by Muller and coworkers [Müller et al. 2003]. For the pressure term, they used the "Spiky kernel". For the viscous terms, they developed their own kernel which has a positive laplacian throughout. Thus the flow is always dissipative. This offers a higher stability. For the density and surface tension terms, they designed a polynomial kernel. The surface tension force was implemented following Bracbill's formula. As this force only exists at the fluid surface, a colour field was defined to identify the fluid interface. Later this approach was challenged by other researchers owing to the error associated in calculating the laplacian of colour field near the surface where there are only a few molecules [Becker and Teschner 2007]. [Becker and Teschner 2007] showed an alternative way to model the surface tension force. They showed that by adding an attractive force between two particles, the surface tension can be implemented for high curvature surfaces. In this study we have studied both the models and compared the corresponding results.

*e-mail:soumyadeep.paul@thml.t.u-tokyo.ac.jp

## 3 Methods

The two-dimensional Navier-Stokes equation was solved to obtian the velocity field.

$$\frac{Du_j}{Dt} = -\frac{1}{\rho}(\frac{\partial P}{\partial x})_j + \gamma[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}]_j + f_x \tag{1}$$

$$\frac{Dv_j}{Dt} = -\frac{1}{\rho}(\frac{\partial P}{\partial y})_j + \gamma[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}]_j + f_y - g \tag{2}$$

Here $j$ is the index of the marker particle. $f_x$ and $f_y$ are the surface tension forces and $g$ is the accelartion due to gravity.

SPH is fundamentally based on interpolating the fluid properties which are defined only at the discrete particle locations. To obtain the derivative of fluid properties, we have first interpolated them over suitable kernels and then took the derivative. The general formula for interpolating over a kernel is:

$$A_s(r) = \sum_j m_j \frac{A_j}{\rho_j} W(r - r_j, h) \tag{3}$$

When calculating the derivative of variable

$$\nabla A_s(r) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(r - r_j, h) \tag{4}$$

where $W$ is the kernel and $h$ is the kernel size. For density we can write;

$$\rho_s(r) = \sum_j m_j W(r - r_j, h) \tag{5}$$

We have used a polynomial kernel for density here following [Müller et al. 2003].

$$W = \frac{3}{\pi h^6}(h^2 - r^2)^2 \tag{6}$$

[Müller et al. 2003] polynomial formula has been modified for 2D simulations such that

$$\int_0^h W 2\pi r dr = 1 \tag{7}$$

Equation 5 has been now used to obtain the pressure. The pressure at marker points have been calculated using Tait's equation:

$$P_j = \frac{\rho_j {c_s}^2}{7}((\frac{\rho_s(r_j)}{\rho_j})^7 - 1) \tag{8}$$

Next we have assumed the pressure to be smoothed by the "spiky" kernel

$$P(r) = \sum_j m_j \frac{P_j}{\rho_j} \frac{10}{3\pi h^5}(h - |r - r_j|)^3 \tag{9}$$

By taking spacial gradient of Equation 9 at the marker points, we get the pressure gradient terms in the Navier-Stokes equations. To obtain the laplacian of velocity for the vicsous dissipation term, we have used the same kernel as [Müller et al. 2003].

$$\nabla^2 u = \sum_j m_j \frac{u_j - u_i}{\rho_j}(\frac{45}{\pi h^6}(h - r_j)) \tag{10}$$

For the surface tension term we have showed results for both the brackbill model and the one used by [Becker and Teschner 2007]. For the bracbill model, we defined a colour function which follows

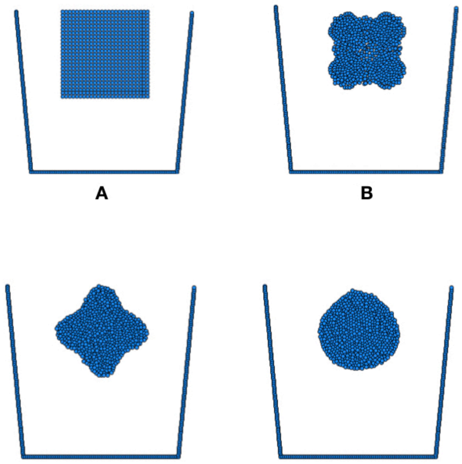


**Figure 2:** *SPH of drop deformation under surface tension following Becker's model*

the polynomial kernel and the surface tension force was calculated using the following formula

$$f = -\sigma \nabla^2 c \frac{n}{|n|} \tag{11}$$

However for the other model, the force was modelled as an inter particle force. We have used a similar kernel as [Tartakovsky and Meakin 2005] for this model.

$$f_i = -\sigma \sum_j (r_i - r_j) W(r_i - r_j, h) \tag{12}$$

The kernel for above equation for $r < \frac{h}{3}$

$$W = \frac{63}{478\pi h^2}(66 - ((3 - \frac{3r}{h})^5 - 6(2 - \frac{3r}{h})^5 + 15(1 - \frac{3r}{h})^5)) \tag{13}$$

For $\frac{h}{3} <= r < \frac{2h}{3}$

$$W = \frac{63}{478\pi h^2}(66 - ((3 - \frac{3r}{h})^5 - 6(2 - \frac{3r}{h})^5)) \tag{14}$$

For $\frac{2h}{3} <= r < h$

$$W = \frac{63}{478\pi h^2}(66 - ((3 - \frac{3r}{h})^5)) \tag{15}$$

Thus the entire Navier-Stokes was solved using SPH interpolations.

## 4 Results and Discussion

First we studied the effect of surface tension without considering gravity. Initially the system has 625 markers arranged in a square fashion. According to physics, the drop would try to become circular to reduce its surface energy. First we use Equation 12 to model $f$. The results of drop deformation is shown in Figure 2. After many fluctuations, eventually the drop stabilizes into a circular one.

The same problem was also solved using Brackbill's surface tension force, Equation 11. The SPH results for this case is shown in Figure 3. In this case we see that the solution diverges. This is

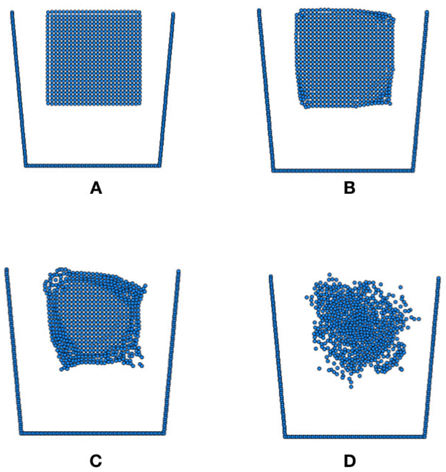


**Figure 3:** *SPH of drop deformation under surface tension following Brackbill model using colour field*

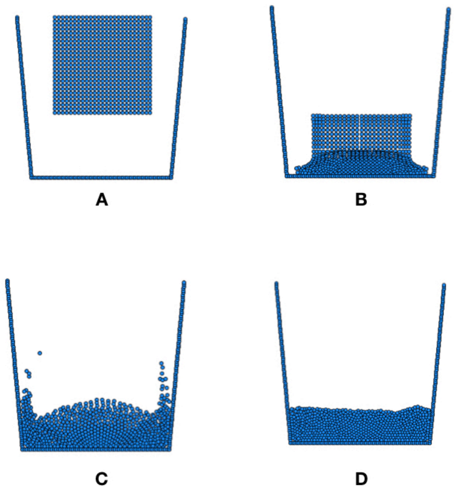


**Figure 4:** *SPH of drop falling under gravity*

probably due to the error associated with calculating the laplacian of the colour field. Following these results, we find that Becker's model is more robust. So for the water falling in bucket problem we used Becker's model.

We simulated water falling inside bucket using the model as described in Section 3. We used a grid based algorithm to reduce computation. To calculate the fluid variables for each marker particle we search for particles in that grid itself and 8 neighbouring grids. If we find a particle with the prescribed kernel size, we calculate the effect of that particle using the kernel functions. The SPH results are shown in Figure 4. Due to limitation of computation power, we could not use more than 625 particles.

## 5 Conclusion

In this paper we used SPH to animate the falling of a liquid drop into a bucket under gravity. We found that the Bracbill surface tension model is difficult to implement in SPH. The model given by [Becker and Teschner 2007] is more robust. To demonstrate this, we showed that the square drop finally converges to a circular one for this model however for the Bracbill model, the results diverge.

Overall SPH is very attractive simulation technique both for interactive and scientific applications. However critical issues remain. Surface tension modelling is one of them. Also reduction of computation cost is necessary for more widespread applications.

## 6 Contact Information

If you have questions or suggestions regarding this document, please contact Soumyadeep Paul at "soumyadeep.paul@thml.t.u-tokyo.ac.jp".

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