Written Exam Economics winter 2025-26

Programming for Economists

January 17 to January 19

This exam question consists of 7 pages in total

Answers only in English.

A home assignment exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

You should hand-in a single zip-file named with your groupname only. The zip-file should contain:

1. A general README.md for your portfolio
2. Your data analysis project (in the folder 01_dataproject)
3. Your model analysis project (in the folder 02_modelproject)
4. Your exam project (in the folder 03_examproject)

Use of GAI tools is permitted. You must explain how you have used the tools. You are required to complete the GAI declaration and upload it as an attachment when submitting your assignment. If you do not submit the GAI declaration, your exam submission will be rejected, and you will have used an exam attempt.

When text is solely or mainly generated by a GAI tool, the tool used must be quoted as a source.

1 Danish House Prices

1.1 Question 1

Use an API call to download data on house prices from table `EJ56` at Denmark Statistics. Drop all provinces ["landsdele" in Danish] with any missing observations. Plot the evolution of house prices across Danish provinces indexed to 100 in the first quarter of 1992. Rank the provinces in terms of total house price growth in the sample period.

1.2 Question 2

Download data on the Consumer Price Index (CPI) from table `PRIS113` at Denmark Statistics. Plot the evolution of *real* house prices across Danish provinces.

1.3 Question 3

Import data on house prices per m^2 across municipalities from the provided Excel file `BM010_houses.xlsx`.[1] Drop all municipalities with any missing observations. Plot the total house price growth across municipalities against the initial level of the house price. Is the correlation between the initial level and total growth positive? Label the five municipalities with the highest total house price growth and the five municipalities with the five largest initial house price levels (some of which might be the same).

1.4 Question 4

Compute a four-quarter backward-looking rolling average of house prices per m^2 in each municipality. Compute the change in this rolling average of house prices in each municipality from its peak (maximum value) before the 2008 Financial Crisis until the latest observation in the data set. Which municipalities are still below the peak? Illustrate your answer.

[1] The data has been downloaded from Finans Danmark's Boligmarkedsstatistikken.

2 Exchange economy

We consider an *exchange economy* with two consumers, A and B, and two goods, x_1 and x_2. The total endowment of each good is always 1. Consumer A's endowment is $\boldsymbol{\omega}^A = (\omega_1^A, \omega_2^A)$. Consumer B's endowment is $\boldsymbol{\omega}^B = (\omega_1^B, \omega_2^B) = (1 - \omega_1^A, 1 - \omega_2^A)$. Prices are $\boldsymbol{p} = (p_1, p_2)$. The numeraire is $p_2 = 1$.

Both consumers have utility functions on the Constant Elasticity of Substitution (CES) form

$$u(x_1, x_2; \alpha, \beta) = \left(\alpha x_1^\rho + \beta x_2^\rho\right)^{1/\rho}, \qquad \rho \neq 0. \tag{1}$$

The indifference curve for utility level u can be written

$$x_2(u; \alpha, \beta) = \left(\frac{u^\rho - \alpha x_1^\rho}{\beta}\right)^{\frac{1}{\rho}}. \tag{2}$$

Defining $\sigma = \frac{1}{1-\rho}$ and $m = p_1\omega_1 + \omega_2$. The demand functions are

$$x_1(p_1, m) = \alpha^\sigma p_1^{-\sigma} \frac{m}{\alpha^\sigma p_1^{1-\sigma} + \beta^\sigma} \tag{3}$$

$$x_2(p_1, m) = \beta^\sigma \frac{m}{\alpha^\sigma p_1^{1-\sigma} + \beta^\sigma}. \tag{4}$$

Consumer A has preference parameters α^A, β^A and ρ^A, and income

$$m^A = p_1\omega_1^A + \omega_2^A.$$

Consumer B has preference parameters α^B, β^B and ρ^B, and income

$$m^B = p_1\omega_1^B + \omega_2^B.$$

In equilibrium excess demand is zero. This implies that all equilibrium prices is a solution to

$$\epsilon_1(p_1) \equiv x_1^{A\star}(p_1, m^A) + x_1^{B\star}(p_1, m^B) - 1 = 0. \tag{5}$$

Calibration. The preference parameters are $\alpha^A = \beta^B = 1$, $\beta^A = \alpha^B = (12/37)^3$ and $\rho^A = \rho^B = -2$. The endowments are $\omega_1^A = 1.0 - 10^{-8}$ and $\omega_2^A = 10^{-8}$.

Code. You can use the code in `ExchangeEconomyModel.py` uploaded together with the exam as your starting point.

2.1 Question 1

Create a figure with two plots of respectively

1. demand for good 1, $x_1^{A\star}(p_1, m^A)$, $x_1^{B\star}(p_1, m^B)$
2. excess demand for good 1, $\epsilon_1(p_1)$

where $p_1 \in$ `np.linspace(0.25,5,100)`.

When is $\epsilon_1(p_1) \approx 0$? How many equilibria are there?

2.2 Question 2

Consider a tâtonnement algorithm for $\tau = 10^{-8}$, $\nu = 50$ and $K = 5000$:

1. Guess p_1^0 and set $k = 0$
2. Calculate $\epsilon_1^k = \epsilon_1(p_1^k)$
3. If $\left|\epsilon_1^k\right| < \tau$ stop and return $p_1^* = p_1^k$ as the solution
4. If $k > K$ stop and return error
5. Set $p_1^{k+1} = p_1^k + \nu\epsilon_1^k$
6. Increment k and go to step 2

Implement this algorithm and apply it for $p_1^0 = 0.9$ and $p_1^0 = 1.1$.

Plot the implied sequences for p_1^k and ϵ_1^k for $k = 0, 1, 2, \ldots$.

Does the choice of initial guess determine which equilibrium is found?

2.3 Question 3

Apply the tâtonnement algorithm for $p_1^0 \in$ `np.linspace(0.25,5,50)`.

Plot p_1^0 against the resulting p_1^*. Do you find all equilibria?

2.4 Question 4

Consider a so-called dampened Newton-Rhapson algorithm for $\tau = 10^{-8}$, $\varphi = 0.1$, $\iota = 0.99$ and $K = 5000$:

1. Guess p_1^0 and set $k = 0$
2. Calculate $\epsilon_1^k = \epsilon_1(p_1^k)$

3. If $\left|\epsilon_1^k\right| < \tau$ stop and return $p_1^* = p_1^k$ as the solution
4. If $k > K$ stop and return error
5. Calculate $\Delta = \frac{\epsilon_1(p_1^k+h)-\epsilon_1^k}{h}$ for $h = 10^{-6}$
6. Set $p_1^{k+1} = p_1^k - \varphi\frac{\epsilon_1^k}{\Delta}$
7. If $p_1^{k+1} < 0$ set $p_1^k = \iota p_1^k$
8. Increment k and go to step 2

Apply this algorithm for $p_1^0 \in$ `np.linspace(0.25,5,50)`.

Plot p_1^0 against the resulting p_1^*. Do you find all equilibria?

2.5 Question 5

Illustrate all the equilibria you have found, preferably in an Edgeworth box (see the method `create_edgeworthbox`) with the indifference curves for each consumer.

3 The AS-AD Model

Consider an AS–AD model for a closed economy. Output in period t is denoted y_t and the long-run level of output is $\bar{y}$. Inflation in period t is denoted π_t and the inflation target is π^*.

The central bank sets the nominal interest rate, i_t, according to a Taylor-rule

$$i_t = \bar{r} + \pi_t^e + \alpha_1(\pi_t - \pi^*) + \alpha_2(y_t - \bar{y}), \tag{6}$$

where $\bar{r}$ is the long-run real interest rate, π_t^e are inflation expectations, $\alpha_1 > 0$ measures the response to the inflation gap, and $\alpha_2 > 0$ measures the response to the output gap.

Market clearing in the goods market requires

$$y_t - \bar{y} = -b(i_t - \pi_t^e - \bar{r}) + v_t, \tag{7}$$

where $b > 0$ is the real-rate sensitivity, and v_t is a *demand shock*.

Together these two equation imply the **Aggregate Demand (AD) curve**

$$\pi_t = \pi^* - \frac{1}{\alpha}\Big[(y_t - \bar{y}) - z_t\Big], \tag{8}$$

where

$$\alpha \equiv \frac{b\alpha_1}{1+b\alpha_2} > 0$$
$$z_t \equiv \frac{v_t}{1+b\alpha_2}.$$

The **Short-Run Aggregate Supply (SRAS) curve** is

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}), \tag{9}$$

where $\gamma > 0$ is the slope.

Inflation expectation are adaptive and given by

$$\pi_t^e = \phi\pi_{t-1}^e + (1-\phi)\pi_{t-1},$$

where $\phi \in (0,1)$ is the weight on the expectation in the previous period, and $1-\phi$ is weight on inflation in the previous period.

In each period, the model is in equilibrium with $AD = AS$. This implies

$$y_t^\star = \bar{y} + \frac{1}{\frac{1}{\alpha}+\gamma}\left[\pi^* - \pi_t^e + \frac{1}{\alpha}z_t\right] \tag{10}$$
$$\pi_t^\star = \pi_t^e + \frac{\gamma}{\frac{1}{\alpha}+\gamma}\left[\pi^* - \pi_t^e + \frac{1}{\alpha}z_t\right]. \tag{11}$$

Calibration. The parameters are $\bar{y} = 1$, $\pi^* = 0.02$, $b = 0.6$, $\alpha_1 = 1.5$, $\alpha_2 = 0.10$, $\gamma = 4$ and $\phi = 0.6$.

Code. You can use the code in `ASADModel.py` uploaded together with the exam as your starting point.

3.1 Question 1

Create a figure containing:

1. The AD-curve
2. The SRAS-curve for $\pi_t^e = \pi^*$
3. The long-run equilibrium of $\pi_t = \pi^*$ and $y_t = \bar{y}$.

Illustrate how the SRAS-curve moves if π_t^e jumps from $\pi^* = 0.02$ to 0.08. Confirm the intersection with the AD-curve is given by $y_t^\star$ and $\pi_t^\star$ calculated without a demand shock, i.e. $z_t = 0$.

3.2 Question 2

Consider the effect of a temporary demand shock. Let $v_0 = 0.1$ while for $t > 0$ we have $v_t = 0$. Initially, $\pi_0^e = \pi^*$.

Simulating forward for $t = 0, 1, \ldots,$ can now be done as:

1. If $t > 0$: Compute $\pi_t^e = \phi \pi_{t-1}^e + (1-\phi)\pi_{t-1}^\star$
2. Compute $y_t^\star$ and $\pi_t^\star$ given π_t^e

Run the simulation for 5 periods, and for each period plot the new position of the AD and AS curves together with the equilibrium in a single figure.

3.3 Question 3

Consider the case where v_t is given by

$$v_t = \rho v_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon),$$

where $\rho = 0.8$ and $\sigma_\epsilon = 0.01$. Initially, $\pi_0^e = \pi^*$.

Simulating forward for $t = 0, 1, \ldots,$ can now be down as:

1. If $t > 0$: Compute $\pi_t^e = \phi \pi_{t-1}^e + (1-\phi)\pi_{t-1}^\star$
2. Draw random number $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon)$
3. Compute $v_t = \rho v_{t-1} + \epsilon_t$ (where $v_{-1} = 0$)
4. Compute $y_t^\star$ and $\pi_t^\star$ given π_t^e and v_t

Run this simulation for 500 periods using the seed 123, and plot the time series of output, $y_t^\star$, inflation, $\pi_t^\star$, and the demand shock v_t. Compute the standard deviation of output, $y_t^\star$, and inflation, $\pi_t^\star$, and their correlation.

Repeat this exercise for $\rho = 0.50$ using the same random draws.