An Overview on Description Logics, and Axiom Pinpointing in $\mathcal{EL}$

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Outline

I am working on...

- Description Logics:
  - logics for representing conceptual knowledge.
  - describe the world with a logical language, classify the descriptions
- Formal Concept Analysis:
  - field of lattice theory for conceptual data analysis
  - derive a classification from the given samples
- combining them, using methods from one field in the other

Knowledge Representation

Develop formalisms that have

- a well-defined syntax,
- formal, unambigious semantics,
- practical methods for reasoning, and efficient implementations

Conceptual knowledge

- Classes: country, coastal country, ...
- Relations: has border to, has neighbour, ...
- Individuals: Portugal, Mediterranean, Atlantic, ...

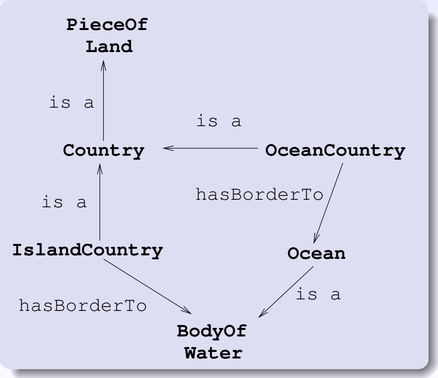
Early days

Semantic networks [Quillian(1967)]

- nodes represent classes
- links represent relations
- `hasBorderTo`: has a border to sth., or only has border to sth.?
- ambigious semantics!

KL-ONE [Brachman & Levesque(1985)]

- logic-based semantics



Description Logics (DLs)

- family of logic-based knowledge representation formalisms
- describe an application domain in terms of
  - concepts (classes) like Country, Ocean, ...
  - individuals like Portugal, Atlantic, ...
  - roles (relations) like hasBorderTo, hasNeighbour,...
- well-defined, formal semantics
- decidable fragments of First Order Logic

- $\text{Sea} \sqcup \text{Ocean}$,
- $\text{Country} \sqcap \exists \text{hasBorderTo}.\text{Ocean}$,
- $Ocean(Atlantic)$,
- $OceanCountry(Portugal)$

The DL $\mathcal{ALC}$

$\mathcal{ALC}$: The smallest propositionally closed DL

- Atomic concepts: $A, B, \ldots$ *(unary predicates)*
- Atomic roles: $r, s, \ldots$ *(binary predicates)*
- Constructors:
  - $\neg C$ *(negation)*
  - $C \sqcap D$ *(conjunction)*
  - $C \sqcup D$ *(disjunction)*
  - $\exists r.C$ *(existential restriction)*
  - $\forall r.C$ *(value restriction)*

Examples:

- Country $\sqcap$ $\neg$Island
- Country $\sqcap$ $\forall$hasBorderTo.LandMass
- Country $\sqcap$ $\exists$hasNeighbour.($\exists$hasBorderTo.Ocean)

Semantics of $\mathcal{ALC}$

based on interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consisting of
a domain $\Delta^{\mathcal{I}}$, and an interpretation function $\cdot^{\mathcal{I}}$

Concept and role names:

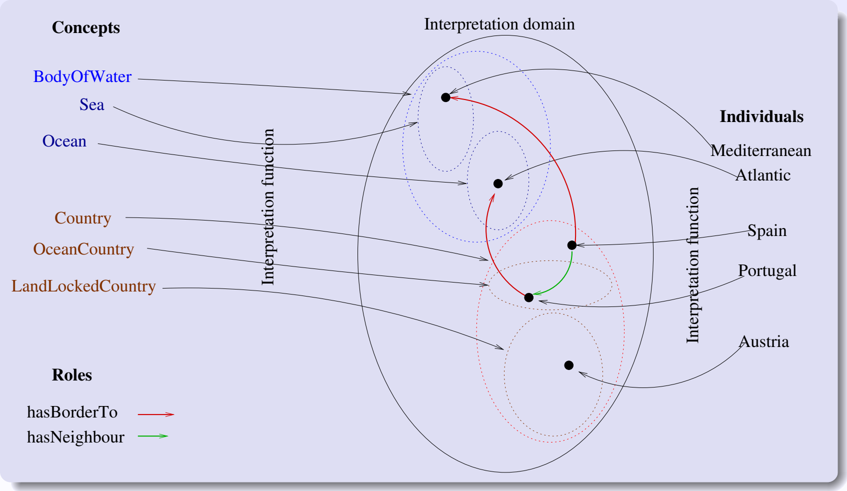
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ *(concepts interpreted as sets)*
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ *(roles interpreted as binary relations)*

Complex concept descriptions:

- $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}, \quad (C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}, \quad (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \text{ there is } e \in \Delta^{\mathcal{I}} : (d,e) \in r^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\}$
- $(\forall r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \text{ for all } e \in \Delta^{\mathcal{I}} : (d,e) \in r^{\mathcal{I}} \text{ implies } e \in C^{\mathcal{I}}\}$

$\mathcal{I}$ is a model of C if $C^{\mathcal{I}} \neq \emptyset$

Example of an interpretation



$$(\text{Country} \sqcap \exists \text{hasNeighbour}.(\exists \text{hasBorderTo}.\text{Ocean}))^{\mathcal{I}} = \{\mathit{Spain}\}$$

Relation to FOL

- $\mathcal{ALC}$ is a fragment of FOL
- Concept names are unary predicates, role names are binary predicates
- Concept descriptions yield formulae with one free variable
- $\mathcal{ALC}$ concept desriptions can be written in the two-variable fragment ($\mathcal{L}^2$) of FOL

$$\begin{array}{rcl} \forall \mathrm{r}.(\exists \mathrm{r}.\mathrm{A}) & \Rightarrow & \forall y.(r(x,y) \rightarrow (\exists z.(r(y,z) \wedge A(z))) \\ & \Rightarrow & \forall y.(r(x,y) \rightarrow (\exists x.(r(y,x) \wedge A(x)))) \end{array}$$

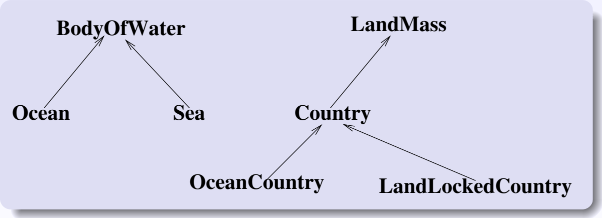
- $\mathcal{L}^2$ is decidable [Mortimer(1975)]

Reasoning tasks

Two main reasoning tasks:

- concept satisfiability: Is there a model of C?
- concept subsumption: Does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold for all $\mathcal{I}$? (written $C \sqsubseteq D$?)

Concept subsumption for computing the subsumption hierarchy:



For $\mathcal{ALC}$, satisfiability and subsumption are mutually reducible:
$C \sqsubseteq D$ iff $C \sqcap \neg D$ is unsatisfiable

DL Knowledge Bases (Ontologies)

DL Knowledge Base (Ontology) = TBox + ABox

- TBox ($\mathcal{T}$) defines the terminology of the application domain
- ABox ($\mathcal{A}$) states facts about a specific "world"

TBox: set of concept definitions

$$\mathsf{LockedCountry} \equiv \mathsf{Country} \sqcap \forall \mathsf{hasBorderTo.LandMass}$$
$$\mathsf{OceanCountry} \equiv \mathsf{Country} \sqcap \exists \mathsf{hasBorderTo.Ocean}$$

ABox: set of concept and role assertions

Ocean(*Atlantic*), Sea(*Mediterrenaean*), hasBorderTo(*Portugal*, *Atlantic*),

General TBox: set of GCIs (general concept inclusion axiom)

$$\mathsf{Ocean} \sqsubseteq \mathsf{BodyOfWater}$$
$$\exists \mathsf{hasBorderTo.BodyOfWater} \sqsubseteq \mathsf{CoastalCountry}$$

DL Knowledge Bases

An interpretation $\mathcal{I}$ is a model of

- a TBox $\mathcal{T}$ if it satisfies all its concept definitions: $A^{\mathcal{I}} = C^{\mathcal{I}}$ for all $A \equiv C \in \mathcal{T}$
- a general TBox $\mathcal{T}$ if it satisfies all its concept inclusions: $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all $C \sqsubseteq D \in \mathcal{T}$
- an ABox $\mathcal{A}$ if it satisfies all its assertions:
  - $a^{\mathcal{I}} \in C^{\mathcal{I}}$ for all $C(a) \in \mathcal{A}$
  - $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$ for all $r(a, b) \in \mathcal{A}$
- a knowledge base $(\mathcal{T}, \mathcal{A})$ if it is a common model of $\mathcal{T}$ and $\mathcal{A}$.

Reasoning with TBox and ABox

$\mathcal{T}$ a TBox, $\mathcal{A}$ an ABox, C and D concept descriptions, a an individual name.

TBox Reasoning

- Satisfiability: Is there a common model of C and $\mathcal{T}$?
- Subsumption: Does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold in all models $\mathcal{I}$ of $\mathcal{T}$? (written $C \sqsubseteq_{\mathcal{T}} D$)

ABox Reasoning

- Consistency: Is there a common model of $\mathcal{A}$ and $\mathcal{T}$?
- Instance: Does $a^{\mathcal{I}} \in C^{\mathcal{I}}$ hold in all models $\mathcal{I}$ of $\mathcal{A}$ and $\mathcal{T}$? (written $\mathcal{T}, \mathcal{A} \models C(a)$)

Tableau Algorithms

Satisfiability ⇔ *Subsumption*
Consistency ⇔ *Instance*

Satisfiability ⇒ *Consistency*

Tableau Algorithm:

- decision procedure for checking ABox consistency
- tries to generate a finite model for the input ABox
- extends the ABox by applying tableau rules
- checks for contradictions
- if no rule applies and no contradiction is found, the ABox is consistent

Complexity of Reasoning in $\mathcal{ALC}$

- with empty TBox PSPACE-complete [Schmidt-Schauß & Smolka(1991)]
  - in PSPACE: the tableau algorithm
  - PSPACE-hard: reduction from satisfiability in QBF (quantified boolean formulae)
- with general TBoxes: EXPTIME-complete
  - in EXPTIME: tableau algorithm in [Donini & Massacci(2000)]
  - EXPTIME-hard: [Schild(1991)]

PSPACE: solvable by a deterministic machine with polynomial space
EXPTIME: solvable by a deterministic machine in exponential time

P	$\subseteq$	NP	$\subseteq$	PSPACE	$\subseteq$	EXPTIME
Horn		Boolean		$\mathcal{ALC}$ (empty TBox)		$\mathcal{ALC}$ (general TBox)

tradeoff between expressivity and computational complexity!

More expressive DLs $\mathcal{ALCN}$ and $\mathcal{ALCQ}$

$\mathcal{ALC}$ cannot express countries that have at most 3 neighbours

Unqualified number restrictions: Country $\sqcap$ ($\leq$ 3 hasNeighbour)

- at most: $(\leq n\ r)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid card(\{e \mid (d,e) \in r^{\mathcal{I}}\}) \leq n\}$
- at least: $(\geq n\ r)^{\mathcal{I}} = \ldots$

$\mathcal{ALCN}$ cannot express countries that have at least 1 neighbour that is an ocean country

Qualified nr. restrns: Country $\sqcap$ ($\geq$ 1 hasNeighbour.OceanCountry)

- at least:
  $(\geq n\ r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid card(\{e \mid (d,e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\}) \geq n\}$
- at most: $(\leq n\ r.C)^{\mathcal{I}} = \ldots$

More expressive DLs $\mathcal{ALCQI}$, $\mathcal{ALCQO}$

$\mathcal{ALCQI}$: $\mathcal{ALCQ}$ + inverse roles

Inverse roles: $(r^-)^\mathcal{I} = \{(e, d) \mid (d, e) \in r^\mathcal{I}\}$

countries recognized by an EU member

$\mathsf{Country} \sqcap \exists \mathsf{recognizes}^-.\mathsf{EUmember}$

$\mathcal{ALCQO}$: $\mathcal{ALCQ}$ + nominals

Nominals: $\{a_1, \ldots, a_n\}^\mathcal{I} = \{a_1^\mathcal{I}, \ldots, a_n^\mathcal{I}\}$

countries that have territories in Europe or Asia

$\mathsf{Country} \sqcap \exists \mathsf{hasTerritoryIn}.\{\mathit{Asia}, \mathit{Europe}\}$

Very expressive DLs $\mathcal{SHIF}$, $\mathcal{SHOIN}$

- transitive roles (R_+): $\mathsf{hasGoodRelationsWith}_+$
- role hierarchy ($\mathcal{H}$): hasNeighbour $\sqsubseteq$ hasBorderTo
- functional restriction($\mathcal{F}$): hasCapitalCity
- Concrete domains (D): natural numbers, reals, time, ...
- $\mathcal{S}$: $\mathcal{ALC}$ with transitive roles

OWL: the standard ontology language for The Semantic Web!

- OWL Lite $= \mathcal{SHIF}$ (D) (reasoning ExpTime)
- OWL DL $= \mathcal{SHOIN}$ (D) (reasoning NExpTime)
- OWL Full $=$ beyond DLs (reasoning undecidable!)

(ExpTime $\subseteq$ NExpTime: solvable in exp. time by a nond. mach.)

DL Reasoners

- very high worst-case complexity: ExpTime, NExpTime
- almost 20 years of research
- highly optimized tableau algorithms
- efficient implementations
- Reasoners: FaCT++, RacerPro, Pellet, KAON2, Hermit ...
- can classify large real world ontologies written in very expressive DLs!

Leightweigt DLs:The $\mathcal{EL}$ family

- DLs with polynomial time reasoning!
- still expressive enough for widely used bio-medial ontologies
  - SNOMED (Systematized Nomenclature of Medicine): medical terminology for diseases, treatments, ...
  - Gene Ontology: controlled vocabulary to describe gene and gene product attributes
- $\mathcal{EL}^+$ constructors:
  - $\top$ (top)
  - $C \sqcap D$ (conjunction)
  - $\exists r.C$ (existensial restriction)
  - $C \sqsubseteq D$ (GCI)
  - $r_1 \circ \cdots \circ r_n \sqsubseteq s$ (role inclusion)

Axiom pinpointing in $\mathcal{EL}^+$

- SNOMED contains more than 350.000 axioms
- ontology development is an error prone task
- in SNOMED Amputation-of-Finger is classified as a subconcept of Amputation-of-Arm
- obviously a modelling error
- what is the reason, which axioms out of 350.000 are responsible?
- Axiom pinpointing: finding small subsets of the knowledge base that have given consequence
- Minimal explanation of $C \sqsubseteq D$ in $\mathcal{T}$ is a $\mathcal{T}' \subseteq \mathcal{T}$ s.t. $\mathcal{T}' \models C \sqsubseteq D$ and $\mathcal{T}'$ minimal

Complexity of Axiom pinpointing in $\mathcal{EL}^+$

- finding a minimum cardinality explanation is NP-complete

  [Baader *et al.*(2007)Baader, Peñaloza, & Suntisrivaraporn]
- finding one minimal explanation is polynomial: remove axioms one by one and test
- what about finding all minimal explanations?
- there can be exponentially many of them!
- clearly not computable in polynomial time

Enumaration complexity

- polynomial delay: polynomial time between each consequetive solution
- output-polynomial: polynomial in the size of the input and the total number of solutions

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We have considered the following:

Can we enumerate min. exps. in a given order with poly. delay?

- NO! cannot be done with polynomial delay unless $\mathrm{P} = \mathrm{NP}$
- checking whether a given minimal explanation is is the first one is coNP-complete

Can we compute all min. exps. in output-polynomial time?

- We do not know :(
- the corresponding decision problem is in coNP
- it is TRANS-HYP-hard (*hypergraph problem open for 20 years*)

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Complexity of Axiom pinpointing in $\mathcal{EL}^+$

We have considered the following:

- Is there a minimal explanation that contains a given axiom?
  - NP-complete
- Is there a min. exp. that does not contain any of the given subsets of the original KB?
  - NP-complete
- Determine the number of minimal explanations
  - #P-complete

Summary

- DLs are logic-based KR formalisms
- decidable fragments of FOL
- theoretical background of OWL: the W3C standard ontology language for The Semantic Web
- Lightweight DLs, polynomial time reasoning
- Axiom pinpointing: finding explanations

References

Baader, F., Peñaloza, R., & Suntisrivaraporn, B. (2007).
Pinpointing in the description logic $\mathcal{EL}^+$.
In J. Hertzberg, M. Beetz, & R. Englert (Eds.), *Proceedings of the 30th German Conference on Artificial Intelligence (KI2007)*. Springer-Verlag, vol. 4667 of *Lecture Notes in Artificial Intelligence*, 52–67.

Brachman, R. J. & Levesque, H. J. (Eds.) (1985).
Readings in Knowledge Representation.
Los Altos (CA), USA: Morgan-Kaufmann.

Donini, F. M. & Massacci, F. (2000).
$\mathrm{EXPTIME}$ tableaux for $\mathcal{ALC}$.
Artificial Intelligence, *124*(1), 87–138.

Mortimer, M. (1975).
On languages with two variables.
Zeitschrift für mathematische Logik und Grundlagen der Mathematik, *21*, 135–140.

Quillian, M. R. (1967).
Word concepts: A theory and simulation of some basic capabilities.
Behavioral Science, *12*, 410–430.
Republished in [Brachman & Levesque(1985)].

Schild, K. (1991).
A correspondence theory for terminological logics: preliminary report.
In J. Mylopoulos & R. Reiter (Eds.), *Proceedings of the 12th International Joint Conference on Artificial Intelligence (IJCAI'91)*. Morgan Kaufmann, 466–471.

Schmidt-Schauß, M. & Smolka, G. (1991).
Attributive concept descriptions with complements.
Artificial Intelligence, *48*(1), 1–26.