First Steps in the Computation of Root Justifications

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1 INTRODUCTION

Description Logics (DLs) are widely accepted as an appropriate class of knowledge representation languages to represent and reason about ontologies [2]. Tools for performing standard reasoning tasks such as satisfiability and consequence checking have grown increasingly powerful and sophisticated in the last decade [3, 7]. In section 2 we review the standard approach for the resolution of modelling errors encountered in ontologies and propose the first steps in a new method for resolving such errors in section 3. The method is based on the notion of *root justifications* which we define and discuss. The approach we describe is applicable to a wide class of DLs. We don't provide a comprehensive formal introduction to DLs, but rather point the reader to the book by Baader et al. [2]. For our purposes a DL *TBox* consists of a finite set of *axioms* specifying the terminological part of an ontology. The Tbox includes (but need not be limited to) *subsumption statements* of the form $E \sqsubseteq F$ where E and F are (possibly complex) concept descriptions, built up from basic concepts. The semantics of DLs is based on the classical model theory for first-order logic. A DL interpretation I contains a non-empty *domain* Δ^I of elements and a mapping which interprets a basic concept A as a subset A^I of Δ^I. For purposes of illustration we shall assume that complex concepts can be constructed using negation ($\neg E$) and conjunction ($E \sqcap F$), where $\neg E$ is interpreted as $\Delta^I \setminus E^I$, and $E \sqcap F$ is interpreted as $E^I \cap F^I$. However, the inclusion of negation and conjunction is not a requirement. In addition, there may be other ways of constructing complex concepts.

An interpretation I is a *model* of a Tbox axiom $E \sqsubseteq F$ if and only if $E^I \subseteq F^I$. Given a Tbox Γ, a subsumption statement $E \sqsubseteq F$, and a basic concept A, *(i)* Γ is *A-unsatisfiable* if and only if for all models I of Γ, $A^I = \emptyset$, and *(ii)* $E \sqsubseteq F$ is a *consequence* of Γ if and only if every model of all axioms in Γ is also a model of $E \sqsubseteq F$.

2 ONTOLOGY DEBUGGING AND REPAIR

The A-unsatisfiability of Γ may be an indication that Γ contains a modelling error. Concept unsatisfiability is a special case of identifying *unwanted axioms* to eliminate modelling errors in ontologies. In general, ontology construction is an iterative process. During each iteration a potential ontology is constructed, a domain expert identifies *unwanted consequences* of the ontology, and (minimal) modifications are made to the ontology to ensure that unwanted consequences are eliminated. Formally, we are provided with a Tbox Γ and an unwanted axiom U with the requirement that $\Gamma \not\models U$.

In order to eliminate an unwanted axiom it is useful to determine the possible *causes* of the axiom being a consequence of Γ. A subset J of Γ is a *U-justification* for Γ if and only if $J \models U$ and for every $J' \subset J$, $J' \not\models U$ [1]. The notion of a U-justification is a generalisation of a minimal unsatisfiability preserving sub-Tbox (MUPS) [6], where the latter applies to unsatisfiable concepts. We denote by $\mathcal{J}_\Gamma(U)$ the set of all U-justifications for Γ. As a running example in this paper, consider the following four Tbox axioms:

1. $C \sqsubseteq A$ 2. $C \sqsubseteq \neg A$ 3. $F \sqsubseteq C \sqcap \neg A$ 4. $F \sqsubseteq C$

Let Γ be the set containing the four axioms above. We represent Γ as the set $\{1, 2, 3, 4\}$ with the understanding that the natural numbers contained in the set are indices, representing the four axioms. Using the same notation, and taking the axiom $F \sqsubseteq \bot$ to be an unwanted axiom, it can be verified that $\mathcal{J}_\Gamma(F \sqsubseteq \bot) = \{\{1, 3\}, \{1, 2, 4\}\}$.

Justifications are useful for a number of reasons. They allow for the *pinpointing* of the causes of modelling errors. In our example they show that axioms 1 and 3 may not both occur in the ontology without having $F \sqsubseteq \bot$ as a consequence. Similarly axioms 1, 2, and 4 may not all occur in the ontology without having $F \sqsubseteq \bot$ as a consequence. In practice it is frequently the case that justifications are significantly smaller than the Tbox as a whole.

Justifications can also be used to perform *ontology repair*. A subset R of a Tbox Γ is a *U-repair* for Γ if and only if $R \not\models U$, and for every R' such that $R \subset R' \subseteq \Gamma$, $R' \models U$. We denote by $\mathcal{R}_\Gamma(U)$ the set of U-repairs for Γ. For our example it can be verified that $\mathcal{R}_\Gamma(F \sqsubseteq \bot) = \{\{2, 3, 4\}, \{1, 4\}, \{1, 2\}\}$.

A subset D of Γ is a *U-diagnosis* for Γ if and only if $D \cap J \neq \emptyset$ for every $J \in \mathcal{J}_\Gamma(U)$. D is a *minimal* U-diagnosis for Γ if and only if there is no U-diagnosis D' for Γ such that $D' \subset D$. The set of minimal U-diagnoses for Γ, denoted by $\mathcal{D}_\Gamma(U)$, can be used to generate all the U-repairs for Γ as follows [5, 1, 6]:

Theorem 1 $\mathcal{R}_\Gamma(U) = \{\Gamma \setminus D \mid D \in \mathcal{D}_\Gamma(U)\}$.

For our example $\mathcal{D}_\Gamma(F \sqsubseteq \bot) = \{\{1\}, \{2, 3\}, \{3, 4\}\}$ from which it can be verified, as we have seen, that $\mathcal{R}_\Gamma(F \sqsubseteq \bot) = \{\{2, 3, 4\}, \{1, 4\}, \{1, 2\}\}$. There are efficient methods for generating U-repairs from the U-justifications, with Reiter's *hitting set algorithm* [5], and variants of it, probably being the best known.

So far we have dealt with a single unwanted axiom, but as the discussion above indicates, it may well be that a domain expert identifies a *set* $\mathcal{U}$ of unwanted consequences. We are interested in *(i)* finding the causes of the unwanted axioms, and *(ii)* repairing the Tbox Γ by replacing it with a Tbox Γ' with the requirement that $\Gamma' \not\models U$ for every $U \in \mathcal{U}$. This is a generalisation of the idea of finding minimal incoherence-preserving sub-TBoxes (MIPS) as a way of eliminating all unsatisfiable concepts in a Tbox [6]. Finding the causes of the unwanted axioms is a matter of generating *all* U-justifications for every $U \in \mathcal{U}$. We denote the set of all such U-justifications by $\mathcal{J}_\Gamma(\mathcal{U})$. That is, $\mathcal{J}_\Gamma(\mathcal{U}) = \bigcup_{U \in \mathcal{U}} \mathcal{J}_\Gamma(U)$.

For our example, let $\mathcal{U} = \{F \sqsubseteq \bot, C \sqsubseteq \bot\}$. We have already seen that $\mathcal{J}_\Gamma(F \sqsubseteq \bot) = \{\{1, 3\}, \{1, 2, 4\}\}$. It is easily seen that $\mathcal{J}_\Gamma(C \sqsubseteq \bot) = \{\{1, 2\}\}$ and therefore that $\mathcal{J}_\Gamma(\mathcal{U}) =$

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$\{\{1,3\},\{1,2,4\},\{1,2\}\}$.

For Tbox repair our goal is to find the $\mathcal{U}$-repairs for Γ. A subset R of Γ is a *$\mathcal{U}$-repair* for Γ if and only if $R \not\models U$ for every $U \in \mathcal{U}$, and for every R' for which $R \subset R' \subseteq \Gamma$, $R' \models U$ for some $U \in \mathcal{U}$. We denote the set of $\mathcal{U}$-repairs for Γ by $\mathcal{R}_\Gamma(\mathcal{U})$. For our example it can be verified that $\mathcal{R}_\Gamma(\mathcal{U}) = \{\{2,3,4\},\{1,4\}\}$.

An obvious method for computing Tbox repair is to eliminate unwanted axioms sequentially using existing methods for repair appropriate for dealing with a single unwanted axiom. However, the naïve approach to do so is not guaranteed to generate *only* U-repairs (i.e., elements of $\mathcal{R}_\Gamma(\mathcal{U})$): Suppose, in our example, that we decide to eliminate $F \sqsubseteq \bot$ first (followed by the elimination of $C \sqsubseteq \bot$). As we have seen on the previous page, the $(F \sqsubseteq \bot)$-repairs for Γ are $\Gamma_1 = \{2,3,4\}$, $\Gamma_2 = \{1,4\}$, and $\Gamma_3 = \{1,2\}$. Having eliminated $F \sqsubseteq \bot$, we then move on to eliminating $C \sqsubseteq \bot$ from each Γ_i, for $i = 1,2,3$. It is easy to see that $\Gamma_1 \not\models C \sqsubseteq \bot$ and $\Gamma_2 \not\models C \sqsubseteq \bot$, but that $\Gamma_3 \models C \sqsubseteq \bot$. We therefore leave Γ_1 and Γ_2 unchanged, but we need to obtain the $(C \sqsubseteq \bot)$-repairs for Γ_3. It can be verified that the $(C \sqsubseteq \bot)$-repairs for Γ_3 are $\{\{2\}\}$ and $\{\{1\}\}$. We thus have, as candidates for the $\mathcal{U}$-repairs of Γ, the sets $\Gamma_1 = \{2,3,4\}$ and $\Gamma_2 = \{1,4\}$, as well as the two $(C \sqsubseteq \bot)$-repairs of Γ_3: $\{2\}$ and $\{1\}$. But observe that the two $(C \sqsubseteq \bot)$-repairs of Γ_3 are not $\mathcal{U}$-repairs for Γ (we will refer to such sets as *false* repairs).

It can be shown that the process described above *(i)* will generate subsets of $\mathcal{U}$-repairs for Γ only, and *(ii)* will generate at least all $\mathcal{U}$-repairs for Γ. From this it follows that false $\mathcal{U}$-repairs can be identified and removed: they will all be strict subsets of the $\mathcal{U}$-repairs for Γ. Nevertheless, it would be useful, for the sake of efficiency, to eliminate the *generation* of such false $\mathcal{U}$-repairs altogether.

It is possible to do better than the naïve approach described above by making an *informed* choice about which unwanted axioms to eliminate first. Suppose that, in our example, and in contrast to what we did above, we choose to eliminate $C \sqsubseteq \bot$ first (followed by the elimination of $F \sqsubseteq \bot$). It can be verified that one of the $(C \sqsubseteq \bot)$-repairs for Γ is the set $\{2,3,4\}$, which also turns out to be a $(F \sqsubseteq \bot)$-repair for Γ. The reason for it being a $(F \sqsubseteq \bot)$-repair for Γ as well, is that one of the $(C \sqsubseteq \bot)$-justifications for Γ ($\{1,2\}$) is a strict subset of one of the $(F \sqsubseteq \bot)$-justifications for Γ ($\{1,2,4\}$). In this case it will thus be more efficient to choose $C \sqsubseteq \bot$ as the unwanted axiom to be eliminated first, since we get the elimination of $F \sqsubseteq \bot$ for free.

This heuristic can be formalised by drawing a distinction between *root* and *derived* unwanted axioms [4]. Formally, an unwanted axiom U is a *derived* unwanted axiom for Γ if and only if there exists a U-justification J for Γ and a U'-justification J' for Γ such that $J' \subset J$. U is a *root* unwanted axiom for Γ if and only if it is *not* a derived unwanted axiom for Γ. The goal is to eliminate root unwanted axioms first with the expectation that in the process of doing so, other unwanted axioms may be eliminated as well. In our example $F \sqsubseteq \bot$ is a derived unwanted axiom for Γ since there is a $(F \sqsubseteq \bot)$-justification for Γ ($\{1,2,4\}$) which is a strict superset of a $(C \sqsubseteq \bot)$-justification for Γ ($\{1,2\}$, while $C \sqsubseteq \bot$ is a root unwanted axiom for Γ. According to this heuristic we should therefore choose to eliminate the unwanted axiom $C \sqsubseteq \bot$ first.

Unfortunately the use of root unwanted axioms does not eliminate the possibility of generating false U-repairs. Suppose that, in our example, we decide to eliminate $C \sqsubseteq \bot$ first because it is a root unwanted axiom. It is easily verified that the $(C \sqsubseteq \bot)$-repairs for Γ are $\Gamma_3 = \{2,3,4\}$ and $\Gamma_4 = \{1,3,4\}$. Having eliminated $C \sqsubseteq \bot$, we then proceed to eliminate the remaining unwanted axiom $F \sqsubseteq \bot$ from both Γ_3 and Γ_4. It is easily verified that $\Gamma_3 \not\models F \sqsubseteq \bot$, but that $\Gamma_4 \models F \sqsubseteq \bot$. So we leave Γ_3 unchanged, but we need to generate the $(F \sqsubseteq \bot)$-repairs of Γ_4. They are $\{3,4\}$ and $\{1,4\}$. The candidate $\mathcal{U}$-repairs for Γ are therefore Γ_3, $\{3,4\}$, and $\{1,4\}$. And as can be verified, Γ_3 and $\{1,4\}$ are both $\mathcal{U}$-repairs for Γ, but $\{3,4\}$ is not. As we have noted, it is possible to recognise $\{3,4\}$ as a false $\mathcal{U}$-repair since it is a subset of one of the $\mathcal{U}$-repairs.

3 ROOT JUSTIFICATIONS

We now briefly discuss some preliminary work on an alternative approach to ontology repair. The key difference is to deal with unwanted axioms *simultaneously*, rather than sequentially. The basic notion we need is that of a root justification. Given a Tbox Γ and a set of unwanted axioms $\mathcal{U}$, a set RJ is a *$\mathcal{U}$-root justification* for Γ if and only if it is a U-justification for Γ for some $U \in \mathcal{U}$ (i.e. $RJ \in \mathcal{J}(\mathcal{U})$), and there is no $J \in \mathcal{J}(\mathcal{U})$ such that $J \subset RJ$. We denote the set of $\mathcal{U}$-root justifications for Γ by $\mathcal{RJ}_\Gamma(\mathcal{U})$. As we have seen, for our example the set of all $\mathcal{U}$-justifications is $\mathcal{J}_\Gamma(\mathcal{U}) = \{\{1,3\},\{1,2,4\},\{1,2\}\}$, and therefore the set of $\mathcal{U}$-root justifications for Γ is $RJ_\Gamma(\mathcal{U}) = \{\{1,2\},\{1,3\}\}$.

The significance of root justifications is that they can be used to generate precisely the $\mathcal{U}$-repairs for Γ, in the same way in which U-repairs are generated from justifications for a single unwanted axiom. A subset D of Γ is a *$\mathcal{U}$-diagnosis* for Γ if and only if $D \cap RJ \neq \emptyset$ for every $RJ \in \mathcal{RJ}(\mathcal{U})$. D is a *minimal* $\mathcal{U}$-diagnosis for Γ if and only if there is no $\mathcal{U}$-diagnosis D' (for Γ) such that $D' \subset D$. The set of minimal $\mathcal{U}$-diagnoses for Γ is denoted by $\mathcal{D}_\Gamma(\mathcal{U})$. We then have the following theorem showing that the $\mathcal{U}$-repairs for Γ can be obtained from the $\mathcal{U}$-diagnoses for Γ:

Theorem 2 $\mathcal{R}_\Gamma(\mathcal{U}) = \{\Gamma \setminus D \mid D \in \mathcal{D}_\Gamma(\mathcal{U})\}$.

For our example we have already seen that $RJ_\Gamma(\mathcal{U}) = \{\{1,2\},\{1,3\}\}$. From this it follows that $\mathcal{D}_\Gamma(\mathcal{U}) = \{\{1\},\{2,3\}\}$ and therefore, as indicated by the theorem, that $\mathcal{R}_\Gamma(\mathcal{U}) = \{\{2,3,4\},\{1,4\}\}$.

We have implemented a Protégé 4 plugin[3] for computing root justifications for sets of unwanted axioms (`http://ksg.meraka.org.za/~kmoodley/protege`). We are extending the plugin to compute the $\mathcal{U}$-repairs. The next step will be to compare this approach to ontology repair with both the naïve sequential approach described above, as well as the improved sequential method which uses root unwanted axioms to determine the sequence in which unwanted axioms are eliminated.

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