Modified Euler-Frobenius Polynomials with application to Sampled Data Model

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Outline

- Introduction
- Euler-Frobenius Polynomials
  - Standard
  - Modified
- Preliminary definitions
  - Multiple Integrations
  - Zero Dynamics
- Applications
- Conclusions

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Why?

Euler-Frobenius Polynomials are widely used in math and they play a key role in many engeneering areas.

They appears in in sampled-data models both in linear and non linear systems, polynomial interpolation and splines.

With the evolution of Standard to Modified Euler-Frobenius Polynomials we are trying to find a unified framework to identify the zeroes in the sampled-data models

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Standard Euler-Frobenius polynomials

$$B_r(z) = r! \cdot det M_r$$

$$M_r = \begin{bmatrix} 1 & \frac{1}{2!} & \dots & \frac{1}{(r-1)!} & \frac{1}{r!} \\ 1-z & 1 & \frac{1}{2!} & \dots & \frac{1}{(r-1)!} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ 0 & \dots & 1-z & 1 & \frac{1}{2!} \\ 0 & \dots & 0 & 1-z & 1 \end{bmatrix}$$

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Modified Euler-Frobenius polynomials

$$B'_r(z, f) = r! \cdot det P_r$$

$$P_r = \begin{bmatrix} 1 & \frac{1}{2!} & \dots & \frac{1}{(r-1)!} & \frac{1-f}{r!} \\ 1-z & 1 & \frac{1}{2!} & \dots & \frac{1-f}{(r-1)!} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ 0 & \dots & 1-z & 1 & \frac{1-f}{2!} \\ 0 & \dots & 0 & 1-z & 1-f \end{bmatrix}$$

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Preliminar(1): Multiple integration

Define: $I(r, f, \Delta, g) = \int_{f\Delta}^{\Delta} \int_{f\Delta}^{t_{r-1}} \dots \int_{f\Delta}^{t_1} g(t) dt \dots dt_{r-1}$

And in the special case: $g = 1$

$$I(j, f, \Delta, g) = \frac{\Delta^j (1-f)^j}{j!}$$

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Preliminar(2): Zero-dynamics

Internal Dynamics
when input and initial conditions
are chosen to make output
identically zero

$$
\begin{aligned}
\dot{z}_1(t) &= z_2(t) \\
\dot{z}_2(t) &= z_3(t) \\
\dot{z}_3(t) &= b(z,\eta) + a(z,\eta)u(t) \\
\dot{\eta}(t) &= \eta(z,\eta) \\
y(t) &= z_1(t)
\end{aligned}
$$

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Implementations Outline

- Partial zero-order hold
  - System of integrators
  - Linear system
  - Affine nonlinear system
- Piecewise Constant generalised hold
  - System of integrators
  - Linear system
  - Affine nonlinear system
- Pure time-delay
  - System of integrators
  - Linear system
  - Affine nonlinear system

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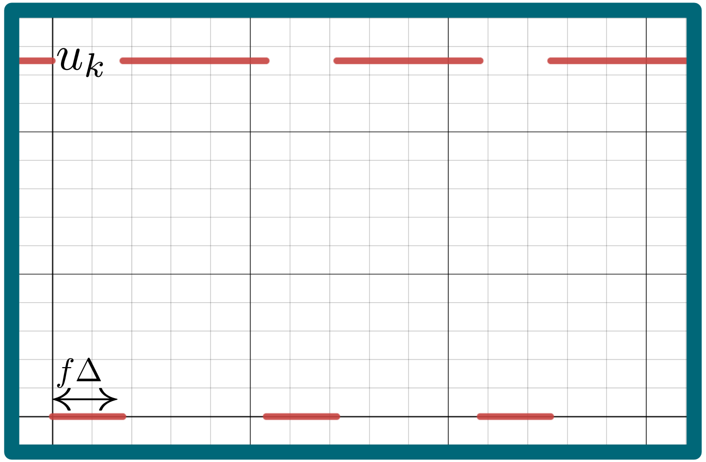
Partial zero-order hold

$$u(t) = \begin{cases} 0 & k\Delta \leq t \leq k\Delta + f\Delta \\ u_k & k\Delta + f\Delta \leq t \leq (k+1)\Delta \end{cases}$$

with $k \in \mathbb{N}$ and $f \in [0,1]$



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Pzoh: System of Integrator $G(s) = 1/s^r$

$$x_{k+1} = \underbrace{\begin{bmatrix} 1 & \Delta & \frac{\Delta^2}{2!} \\ 0 & 1 & \Delta \\ 0 & 0 & 1 \end{bmatrix}}_{A_d} \cdot x_k + \underbrace{\begin{bmatrix} \frac{\Delta^3(1-f)^3}{3!} \\ \frac{\Delta^2(1-f)^2}{2!} \\ \Delta(1-f) \end{bmatrix}}_{B_{PZOH}} \cdot u_k(0)$$

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Pzoh: General Linear Sys

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