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Analysis of a Covid-19 model: Optimal control, stability and simulations

Seda İğret Araz

Siirt University, Department of Mathematics Education, Turkey

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Abstract Mathematical tools called differential and integral operators are used to model real world problems in all fields of science as they are able to replicate some behaviors observed in real world like fading memory, long-range dependency, power law, random walk and many others. Very recently the world has faced a serious challenge since the breakout of corona-virus started in Wuhan, China. The deathly disease has killed about 1720000 and infected more than 2 millions humans around the globe since December 2019 to 21 of April 2020. In this paper, we analyzed a mathematical model for the spread of COVID-19, we first start with stability analysis, present the optimal control for the system. The model was extended to the concept of non-local operators for each case, we presented the positiveness of the system solutions. We presented numerical solutions are presented for different scenarios.

1. Introduction

Since end of December 2019, an outbreak of a deathly disease called COVID-19 has destabilize the world systems, in many sectors including transport (air, marine and roads), economies, education systems, sports, entertainment and many others. Many lives have been lost, many humans have been infected battling for their life in different hospitals around the globe. There are many unknown facts around the genesis, the behavior, the spread patterns and many other biological information's around Covid-19 outbreak. Scientists in many fields of science, technology and engineering, have shifted their attention, time, energies to understand and combat this new enemy of humanity. Many works to find new and adequate vaccine are undergoing in many labs in different countries around the world. Some great results have been obtained as ventilators and many other items have been used to get some recovered patients in many countries. A real proof of progress, however the main aim is to reduce the numbers of infected and deaths, therefore in different countries new measures have been put in place, for instance, closure of airports, lock-down of the countries, social distancing and intensive use of sanitizers. Many investigations have been doing from theoretical to practical point of view with some promising results [1–7]. Mathematicians have suggested some mathematical models with aim to understand the spread and do some simulations for predictions [8–14]. In their suggested models they consider mostly susceptible, recovered, infected and deaths populations, more classes can be added to have more comprehensive and complex systems. Since solutions of such systems can not be obtained ana-

E-mail address: sedaaraz@siirt.edu.tr

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lytically, it is needed numerical methods to solve such complex systems [17,23]. Also, one can find many researches which contain a detailed analysis about differential equations with integer and fractional orders [15–23]. In this paper, we consider the model suggested in [5] the model considered the following classes $S(t), I(t), R(t), U(t)$ describing susceptible, asymptomatic infectious, symptomatic infectious unreported, symptomatic infectious individuals reported by health services. The structure of the paper will be as follow: We start with stability analysis in Section 2, optimal control in Section 3, numerical solution in Section 4.

2. Mathematical model

In this section, we consider a mathematical model that takes into account the population of susceptible, asymptomatic infectious, symptomatic infectious unreported, symptomatic infectious individuals reported by health services. The model was developed in [5] and does not claim to have included all possible scenario about the spread. One few information are used to build the mathematical model. So we handle the following mathematical model

$$\begin{aligned} S'(t) &= -\beta(t)S(t)[I(t)+U(t)] \\ I'(t) &= \beta(t)S(t)[I(t)+U(t)] - wI(t) \end{aligned} \tag{1}$$

$$\begin{aligned} R'(t) &= w_1 I(t) - \mu R(t) \\ U'(t) &= w_2 I(t) - \mu U(t) \end{aligned}$$

where the initial conditions are given as

$$S(t_0) = S_0 > 0, I(t_0) = I_0 > 0, R(t_0) = 0, U(t_0) \geqslant 0. \tag{2}$$

The parameters of the considered model is presented in Table 1.

The total population at time t, denoted by $N(t)$, is defined by

$$N(t) = S(t) + I(t) + R(t) + U(t). \tag{3}$$

The considered model has a unique disease free equilibrium at

$$E_0 = (S_0, I_0, R_0, U_0) = (\beta_0 S_0, 0, 0, 0). \tag{4}$$

We show that all solutions with nonnegative initial data will be nonnegative for all time. Here, we consider the function $\beta(t)$as constant. To show positivity of the solutions, we write the following

$$\begin{aligned} S'(t) = &\ -\beta S(t)[I(t)+U(t)] \\ < &\ -\beta\left(S(t)\left[\max_{t\in I_t} I(t) + \max_{t\in U_t} U(t)\right]\right) \\ &< -\beta\left[S\left(\|I\|_\infty + \|U\|_\infty\right)\right]. \end{aligned} \tag{5}$$

This leads to

$$S(t) < S_0 e^{-\beta\left[\left(\|I\|_\infty + \|U\|_\infty\right)\right]t}. \tag{6}$$

For the second equation

$$\begin{aligned} I'(t) = &\beta S(t)[I(t)+U(t)] - wI(t) \\ \geqslant &- wI(t) \end{aligned} \tag{7}$$

this yields

$$I(t) \geqslant I_0 e^{-wt}. \tag{8}$$

For the third equation

$$\begin{aligned} R'(t) = &w_1 I(t) - \mu R(t) \\ \geqslant &- \mu R(t) \end{aligned} \tag{9}$$

then we get

$$R(t) \geqslant R_0 e^{-\mu t}. \tag{10}$$

For the last equation

$$\begin{aligned} U'(t) = &w_2 I(t) - \mu U(t) \\ \geqslant &- \mu U(t) \end{aligned} \tag{11}$$

then we get

$$U(t) \geqslant U_0 e^{-\mu t}. \tag{12}$$

We have the non-negative set $\Gamma = \{S, I, R, U : S \geqslant 0, I \geqslant 0, R \geqslant 0, U \geqslant 0\}$. If we add these equations of the considered model, we obtain

$$N'(t) = -\mu(R+U) \tag{13}$$

Thus

$$\begin{aligned} N'(t) &\geqslant -\mu N \\ N(t) &\geqslant N_0 . e^{-\mu t}. \end{aligned} \tag{14}$$

2.1. The basic reproduction number

Now we examine the dynamical behavior of the considered system. To do this, we firstly calculate the number R_0 known as the basic reproduction number. If $R_0 < 1$, then the disease will decrease and eventually die out. If $R_0 = 1$, each existing infection causes one new infection. The disease will stay alive and stable, but there will not be an outbreak or an epidemic. If $R_0 > 1$, each existing infection causes more than one new infection. The disease will spread between people, and there may be an outbreak or epidemic.To find reproduction number, we will use the method of next generation matrix [20]. Since the variable of $R(t)$system (1) does not appear in the considered equations, we consider the following system

Table 1 Parameters of the considered model.

Symbol	Interpretation
t_0	Time at which the epidemic started
S_0	Number of susceptible at time t_0
I_0	Number of asymptomatic infectious at time t_0
U_0	Number of unreported symptomatic infectious at time t_0
β	Transmission rate at time t
$1/w$	Average time during which asymptomatic infectious are asymptomatic
f	Fraction of asymptomatic infectious that become reported symptomatic infectious
$w_1 = fw$	Rate at which asymptomatic infectious become reported symptomatic
$w_2 = (1-f)w$	Rate at which asymptomatic infectious become unreported symptomatic
$1/\mu$	Average time symptomatic infectious have symptoms

$$\begin{aligned} I'(t) &= \beta S[I+U] - wI \\ U'(t) &= w_2 I - \mu U \end{aligned} \tag{15}$$

$$S'(t) = -\beta S[I+U].$$

Let $Y = (I, U, S)$, our system can be written as

$$Y' = F - V \tag{16}$$

Here the new infection matrix F and the transition matrix V are defined by

$$F = \begin{bmatrix} \beta SI \\ w_2 I \\ -\beta SI \end{bmatrix}, V = \begin{bmatrix} -\beta SU + wI \\ \mu U \\ \beta SU \end{bmatrix}. \tag{17}$$

The Jacobian of these matrices at disease free equilibrium point are given by

$$JF = \begin{bmatrix} \beta_0 S_0 & 0 & 0 \\ w_2 & 0 & 0 \\ -\beta_0 S_0 & 0 & 0 \end{bmatrix} \text{ where } F = \begin{bmatrix} \beta_0 S_0 & 0 \\ w_2 & 0 \end{bmatrix}, \tag{18}$$

$$JV = \begin{bmatrix} w & -\beta_0 S_0 & 0 \\ 0 & \mu & 0 \\ 0 & \beta_0 S_0 & 0 \end{bmatrix} \text{ where } V = \begin{bmatrix} w_1 + w_2 & -\beta_0 S_0 \\ 0 & \mu \end{bmatrix}$$

and we have the following next generation matrix

$$FV^{-1} = \begin{bmatrix} \beta_0 S_0 & 0 \\ w_2 & 0 \end{bmatrix} \cdot \frac{1}{w\mu} \begin{bmatrix} \mu & \beta_0 S_0 \\ 0 & w \end{bmatrix}. \tag{19}$$

It follows that the spectral radius of the matrix FV^{-1}

$$\rho\left(FV^{-1}\right) = \frac{\beta_0 S_0 (\mu + w_2)}{\mu(w_1 + w_2)}. \tag{20}$$

Thus we get the basic reproduction number

$$R_0 = \frac{\beta_0 S_0 (\mu + w_2)}{\mu(w_1 + w_2)}. \tag{21}$$

We give an example

$$CR(t) = v_1 \exp(v_2 t) - v_3, t \geqslant t_0 \tag{22}$$

with values $v_1 = 0.15, v_2 = 0.38$ and $v_3 = 1$ [5]. It is supposed that the initial values

$$\begin{aligned} &S_0 = 11.000.000, I_0 = \frac{\vartheta_2 \vartheta_3}{f(v_1 + v_2)} = 3.3, \\ &U_0 = \frac{(1-f)(v_1 + v_2)}{\mu + \vartheta_2} I_0 = 0.18. \end{aligned} \tag{23}$$

The time-dependent transmission rate $\beta(t)$is considered as constant and it is calculated as

$$\beta = \left(\frac{v_2 + w_1 + w_2}{S_0}\right)\left(\frac{\mu + v_2}{w_2 + \mu + v_2}\right) = 4.51 \times 10^{-8}. \tag{24}$$

and the initial time is

$$t_0 = \frac{1}{v_2}(\log(v_3) - \log(v_1)) = 5. \tag{25}$$

Thus we have the following basic reproductive number

$$R_0 = \frac{\beta_0 S_0 (\mu + w_2)}{\mu(w_1 + w_2)} = 4.16. \tag{26}$$

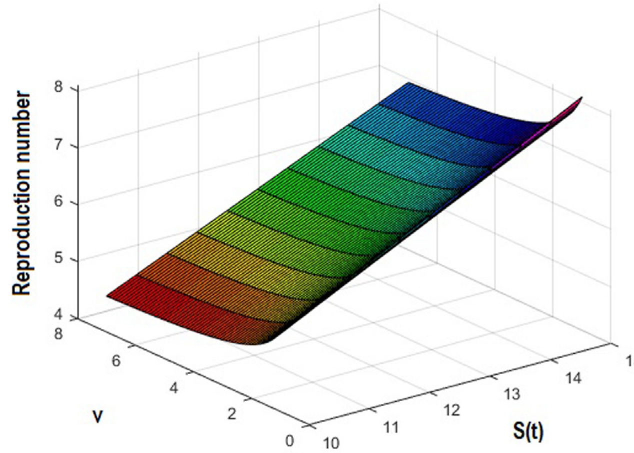

Fig. 1 Reproduction number.

These parameter formulas were calculated in [5]. Using these parameters, we can give for the reproduction number the following simulation. This simulation is done to present reproductive number obtained when susceptible increase from 10 million to 15 million people between 1 and 7 days. (see Fig. 1)

2.2. Stability of the equilibria

We investigate local stability of the disease free equilibrium point $(S_0, 0, 0)$.

Theorem 1. *If $R_0 < 1$, the disease free equilibrium point of system (1) is locally asymtotically stable. If $R_0 = 1$, the disease free equilibrium point of system (1) is stable. $R_0 > 1$, the disease free equilibrium point of system (1) is unstable.*

Proof. We shall recall that our system is given by

$$\begin{aligned} I'(t) &= \beta S(t)[I(t) + U(t)] - wI(t) \\ U'(t) &= w_2 I(t) - \mu U(t) \end{aligned} \tag{27}$$

$$S'(t) = -\beta S(t)[I(t) + U(t)].$$

For this system, we can evaluate Jacobian matrix at disease free equilibrium point

$$J = \begin{bmatrix} -\beta S & -\beta S & 0 \\ \beta S - w & \beta S & 0 \\ -\mu & w_2 & 0 \end{bmatrix} \tag{28}$$

and from here we can obtain the following the characteristic equation

$$\lambda(\beta S v) - \lambda^3 = 0. \quad \square \tag{29}$$

Lemma. *For the obtained reproductive number $R_0 > 1$, a unique equilibrium for endemic case E^* exists and there is no endemic equilibrium if $R_0 = 1$. However the disease is endemic*

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$$U^* < \frac{w_2}{\mu} I^*, \quad R^* < \frac{w_1}{\mu} I^*, \quad \frac{S^*\beta(\mu+w_2)}{\mu(w_1+w_2)} > 1. \tag{30}$$

Therefore a unique endemic exists when $R_0 > 1$. The Jacobian matrix associate to disease-free equilibrium is given as

$$J(E^*) = \begin{bmatrix} -\beta_0 & 0 & 0 & 0 \\ \beta_0 & -w & 0 & 0 \\ 0 & w_1 & -\mu & 0 \\ 0 & w_2 & 0 & -\mu \end{bmatrix}. \tag{31}$$

Thus

$$Tr(J(E^*)) = -(\beta_0 + w + 2\mu) < 0 \tag{32}$$

and

$$Det(J(E^*)) = \beta_0 w \mu^2. \tag{33}$$

3. Global Asymptotic Stability

In this subsection, we present the Lyapunov for different cases when the model is with classical differentiation

$$\dot{L} = \frac{1}{w}\dot{I} + \frac{1}{\mu}\dot{U} \tag{34}$$

and we have

$$\dot{L} = \frac{1}{w}[\beta S(t)[I(t)+U(t)] - wI(t)] + \frac{1}{\mu}[w_2 I(t) - \mu U(t)] \tag{35}$$

$$= \frac{1}{w}\left[\beta S(t)\left[1 + U(t)\frac{1}{I(t)}\right] - w\right] I(t) + \frac{1}{\mu} I(t)\left[w_2 - \mu U(t)\frac{1}{I(t)}\right]$$

From disease, we have

$$\frac{dL}{dt} < \left[\frac{\beta_0 S_0}{w} + \frac{\beta_0 S_0}{w}\frac{U(t)}{I(t)} + \frac{w_2}{\mu} - \frac{U(t)}{I(t)} - 1\right] I(t) \tag{36}$$

$$< \left[\frac{\beta_0 S_0}{w} + \frac{\beta_0 S_0}{w}\frac{w_2}{\mu} + \frac{w_2}{\mu} - \frac{w_2}{\mu} - 1\right] I(t). \tag{37}$$

Thus

$$\frac{dL}{dt} < \left[\frac{\beta_0 S_0}{w} + \frac{\beta_0 S_0}{w}\frac{w_2}{\mu} - 1\right] I(t) < \{R_0 - 1\} I(t) < 0. \tag{38}$$

if $R_0 < 1$.

$$\frac{dL}{dt} = 0 \text{ if } I = 0, \quad \frac{dL}{dt} \geqslant 0 \text{ if } R_0 > 1. \tag{39}$$

Hence, the function L is the Lyapunov function on a largest compact Δinvariant set in $\left\{S, I, R, U \in \Delta : \frac{dL}{dt} \leqslant 0\right\}$ is the point E^*. Thus using Lasalle's invariance principle all solution of the system with initial condition in Δtends E^* when $t \to \infty$ only if $R_0 \leqslant 1$ [25].

4. Local and global stability of the endemic equilibrium

We compute first the Jacobian matrix of the COVID-19 model for endemic equilibrium case

$$JE_* = \begin{bmatrix} \lambda+\beta_0 & 0 & 0 & 0 \\ \beta_0 & \lambda+w & 0 & 0 \\ 0 & w_1 & \lambda+\mu & 0 \\ 0 & w_2 & 0 & \lambda+\mu \end{bmatrix}. \tag{40}$$

We now construct a characteristic equation associate to this model

$$P = \det|I_M\lambda - JE_*| = 0 \tag{41}$$

where I_M is the 4×4 unit matrix. Then we have

$$\det \begin{bmatrix} \lambda+\beta_0 & 0 & 0 & 0 \\ \beta_0 & \lambda+w & 0 & 0 \\ 0 & w_1 & \lambda+\mu & 0 \\ 0 & w_2 & 0 & \lambda+\mu \end{bmatrix}. \tag{42}$$

From the above, we obtain the following polynomial

$$L(\lambda) = \lambda^4 + l_1\lambda^3 + l_2\lambda^2 + l_3\lambda + l_4. \tag{43}$$

The Hurwitz matrix for the characteristic polynomial $L(\lambda)$is written as

$$H = \begin{bmatrix} l_1 & l_3 & 0 & 0 \\ 1 & l_2 & l_4 & 0 \\ 0 & l_1 & l_3 & 0 \\ 0 & 1 & l_2 & l_4 \end{bmatrix}. \tag{44}$$

Then we have

$$H_1 = l_1 > 0 \tag{45}$$

$$H_2 = l_1 l_2 - l_3 > 0$$
$$H_3 = -l_1^2 l_4 + l_1 l_2 l_3 - l_3^2 > 0$$
$$H_4 = -l_1^2 l_4^2 + l_1 l_2 l_3 l_4 + -l_3^2 l_4 > 0$$

5. Optimal control for model

In this section, we discuss the existence of the optimal control. For derivation of the first order necessary conditions for the optimal control, we construct the Hamiltonian of the considered optimal control problem. We shall modify the considered model by adding an optimal control strategies. Modified model is given by

$$\begin{aligned} S'(t) &= -(1-u_1)\beta S(t)[I(t)+U(t)] - u_2 S(t) + u_3 U(t) \\ I'(t) &= (1-u_1)\beta S(t)[I(t)+U(t)] - wI(t) \\ R'(t) &= w_1 I(t) - \mu R(t) + u_2 S(t) \\ U'(t) &= w_2 I(t) - \mu U(t) - u_3 U(t) \end{aligned} \tag{46}$$

where the function u_1, u_2, u_3 states lockdown, quarantine and self isolation strategies respectively. In this section, we aim to find optimal control strategies $\{u_1, u_2, u_3\}$ minimizing lockdown, quarantine, cost of treatment, self isolation at same time while minimizing asymptomatic infectious individuals, symptomatic infectious reported-unreported individuals. So we present the following minimizing functional

$$\min_{(u_1,u_2,u_3)\in Q} J(u_1,u_2,u_3) = \int_0^1 \left[\alpha_1 u_1^2(t) + \alpha_2 u_2^2(t) + \alpha_3 u_3^2(t) + I + U\right] dt \tag{47}$$

on the set of admissible controls

$$Q=\left\{\begin{array}{l}(u_1,u_2,u_3)\in L^{\infty}(0,T)\times L^{\infty}(0,T)\times L^{\infty}(0,T):\\ 0\leqslant u_1(t)\leqslant\widetilde{u}_1, 0\leqslant u_2(t)\leqslant\widetilde{u}_2, 0\leqslant u_3(t)\leqslant\widetilde{u}_3\end{array}\right\}. \tag{48}$$

The parameters α_1,α_2 and α_3 represent the weighted parameters [21].

5.1. Existence of optimal solution

To show the existence of the optimal control for the problem under consideration, we notice that the set of admissible controls Q is, by definition, closed and bounded. It is obvious that there is an admissible pair (u_1,u_2,u_3) for the problem. Hence, the existence of the optimal control comes as a direct result from the Filippove-Cesari theorem [22]. We therefore, have the following result:

We prove the existence of an optimal control pair under the following conditions.

- The set of admissible controls is convex, bounded and closed.
- The right-hand side of the state ODE system is bounded by a linear function in the state and control variables.
- The set of controls and corresponding state variables is nonempty.
- The integrand of the objective functional $J(u_1,u_2,u_3)$is convex on the set Q. The Hessian matrix of this functional is given by;

$$H=\begin{bmatrix}2\alpha_1 & 0 & 0\\ 0 & 2\alpha_2 & 0\\ 0 & 0 & 2\alpha_3\end{bmatrix}. \tag{49}$$

Since the Hessian of of this functional is everywhere positive definite, then the functional $J(u_1,u_2,u_3)$is strictly convex [24].

There exist constants $\alpha=\min\{\alpha_1,\alpha_2,\alpha_3\}>0$ and $\gamma>1$ such that the integrand of the objective functional holds

$$\begin{aligned}\widetilde{J}(u_1,u_2) = &\ \alpha_1u_1^2(t)+\alpha_2u_2^2(t)+\alpha_3u_3^2(t)+I+U\\ \geqslant &\ \alpha_1u_1^2(t)+\alpha_2u_2^2(t)+\alpha_3u_3^2(t)\\ \geqslant &\ c\left(u_1^2(t)+u_2^2(t)+u_3^2(t)\right)\end{aligned} \tag{50}$$

Now we construct the first order necessary conditions for optimal solution for the considered optimal control problem, by using the Hamiltonian H and the Pontryagin's maximum principle. The Hamiltonian is defined by

$$\begin{aligned}H= &\ \alpha_1u_1^2(t)+\alpha_2u_2^2(t)+\alpha_3u_3^2(t)+I+U\\ &+\lambda_1(-(1-u_1)\beta S(t)[I(t)+U(t)]-u_2S(t)+u_3U(t))\\ &+\lambda_2((1-u_1)\beta S(t)[I(t)+U(t)]-wI(t))\\ &+\lambda_3(w_1I(t)-\mu R(t)+u_2S(t))\\ &+\lambda_4(w_2I(t)-\mu U(t)-u_3U(t)).\end{aligned} \tag{51}$$

Then there exists $\lambda\in\mathbb{R}^3$ such that the first order necessary conditions for the existence of optimal control are given by the equations

$$\begin{aligned}\frac{d\lambda_1}{dt} = &\ -\frac{\partial H}{\partial S}=-\begin{pmatrix}-((1-u_1)\beta(I(t)+U(t))-u_2)\lambda_1\\ +((1-u_1)\beta(I(t)+U(t)))\lambda_2+u_2\lambda_3\end{pmatrix}\\ \frac{d\lambda_2}{dt} = &\ -\frac{\partial H}{\partial I}=-\begin{pmatrix}1-(1-u_1)\beta S(t)(\lambda_1-\lambda_2)\\ -w\lambda_2+\lambda_3w_1+\lambda_4w_2\end{pmatrix}\\ \frac{d\lambda_3}{dt} = &\ -\frac{\partial H}{\partial R}=-(-\mu\lambda_3)\\ \frac{d\lambda_4}{dt} = &\ -\frac{\partial H}{\partial U}=-(1-(1-u_1)\beta S(t)(\lambda_1-\lambda_2)+u_3\lambda_1-(\mu+u_3)\lambda_4).\end{aligned} \tag{52}$$

Hence the optimal controls are given as

$$\begin{aligned}u_1 = &\ \frac{S(t)\beta(I(t)+U(t))(\lambda_2-\lambda_1)}{2\alpha_1}\\ u_2 = &\ \frac{S(t)(\lambda_1-\lambda_3)}{2\alpha_2}\\ u_3 = &\ \frac{U(t)(\lambda_4-\lambda_1)}{2\alpha_3}.\end{aligned} \tag{53}$$

and optimality conditions are given by

$$\begin{aligned}u_1^* = &\ \min\left\{\widetilde{u}_1,\max\left\{0,\frac{S(t)\beta(I(t)+U(t))(\lambda_2-\lambda_1)}{2\alpha_1}\right\}\right\}\\ u_2^* = &\ \min\left\{\widetilde{u}_2,\max\left\{0,\frac{S(t)(\lambda_1-\lambda_3)}{2\alpha_2}\right\}\right\}\\ u_3^* = &\ \min\left\{\widetilde{u}_3,\max\left\{0,\frac{U(t)(\lambda_4-\lambda_1)}{2\alpha_3}\right\}\right\}.\end{aligned} \tag{54}$$

6. Numerical application

In this section, we consider the model with non-local operators, in order to include in mathematical formulation some natural law that could be followed by the dynamical processes of the COVID-19 spread. However, it is worth noting that due to the complexities of the system with non-local operators, analytical methods cannot be used in this case. We shall rely on existing numerical scheme to provide approximate solutions for each case [23]. Before doing this, we insure that with nonlocal operators, the solutions are positive in all cases if the initial conditions are positive. The detailed analysis is presented below under some conditions.

6.1. Positive solutions for Caputo-Fabrizio derivative

In this section, we show that under some conditions, the solutions of the model are positives for $\forall t\geqslant 0$, if the initial conditions are positive.

Proof. If $(S_0,I_0,R_0,U_0)\geqslant 0$, then since $\beta(t)$ is positive function and $(S(t),U(t))$ having the same sign, we are sure that

$$\beta S(t)[I(t)+U(t)]>0,\quad \forall t\geqslant 0. \tag{55}$$

We consider the following norm

$$\|f\|_{\infty}=\max_{t\in D_f}|f(t)|. \tag{56}$$

We assume that $\|U\|_{\infty},\|I\|_{\infty}<\infty$, since the number of humans on our planet is finite

$$\begin{aligned}{}_0^{CF}D_t^{\alpha}S(t) = &\ -\beta S(t)[I(t)+U(t)]\\ \geqslant &\ -|\beta|S(t)(|I(t)|+|U(t)|)\end{aligned} \tag{57}$$

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$$\begin{aligned} &\geqslant -\beta\left(\max_{t\in D_I}|I(t)|+\max_{t\in D_U}|U(t)|\right)S(t)\\ &\geqslant -DS(t)\end{aligned}$$

where

$$D=\beta\left(\max_{t\in D_I}|I(t)|+\max_{t\in D_U}|U(t)|\right). \tag{58}$$

Thus

$$\begin{aligned} S(t)&\geqslant S(0)\exp\left(\frac{-\alpha Dt}{M(\alpha)-(1-\alpha)D}\right) \\ &\geqslant S(0)\exp\left(\frac{-\alpha\beta\left(\max_{t\in D_I}|I(t)|+\max_{t\in D_U}|U(t)|\right)t}{M(\alpha)-(1-\alpha)\beta\left(\max_{t\in D_I}|I(t)|+\max_{t\in D_U}|U(t)|\right)}\right).\end{aligned} \tag{59}$$

This shows that $\forall t\geqslant 0, S(t)$is positive for Caputo-Fabrizio case.

With $I(t)$ class, we have using the previous argument that

$$\begin{aligned} {}^{CF}_{0}D^{\alpha}_{t}I(t) &\geqslant -wI(t)\Rightarrow I(t)\\ &\geqslant I(0)\exp\left(\frac{-\alpha wt}{M(\alpha)-(1-\alpha)w}\right).\end{aligned} \tag{60}$$

With $R(t)$ and $U(t)$ classes, we also have

$${}^{CF}_{0}D^{\alpha}_{t}R(t)\geqslant-\mu R(t)\Rightarrow R(t)\geqslant R(0)\exp\left(\frac{-\alpha\mu t}{M(\alpha)-(1-\alpha)\mu}\right) \tag{61}$$

$${}^{CF}_{0}D^{\alpha}_{t}U(t)\geqslant-\mu U(t)\Rightarrow U(t)\geqslant U(0)\exp\left(\frac{-\alpha\mu t}{M(\alpha)-(1-\alpha)\mu}\right).$$

We present positive solutions when the model is with Caputo derivative, the norm and assumptions presented before hold

$$\begin{aligned} {}^{C}_{0}D^{\alpha}_{t}S(t)&=-\beta S(t)[I(t)+U(t)]\\ &\geqslant -DS(t)\end{aligned} \tag{62}$$

then

$$\begin{aligned} S(t)&\geqslant S(0)E_\alpha[-Dt^\alpha],\forall t\geqslant 0\\ I(t)&\geqslant I(0)E_\alpha[-wt^\alpha],\forall t\geqslant 0\\ R(t)&\geqslant R(0)E_\alpha[-\mu t^\alpha],\forall t\geqslant 0\\ U(t)&\geqslant U(0)E_\alpha[-\mu t^\alpha],\forall t\geqslant 0.\end{aligned} \tag{63}$$

Therefore the system has positive solution with Caputo derivative if the initial conditions are positive. We now present the case with ABC derivative

$$\begin{aligned} {}^{ABC}_{0}D^{\alpha}_{t}S(t)&=-\beta S(t)[I(t)+U(t)]\\ &\geqslant -DS(t).\end{aligned} \tag{64}$$

This leads to

$$S(t)\geqslant S(0)E_\alpha\left(\frac{-\alpha Dt^\alpha}{AB(\alpha)-(1-\alpha)D}\right),\forall t\geqslant 0. \tag{65}$$

In the similar way, we show that

$$I(t)\geqslant I(0)E_\alpha\left(\frac{-\alpha wt^\alpha}{AB(\alpha)-(1-\alpha)w}\right),\forall t\geqslant 0. \tag{66}$$

With $R(t)$and $U(t)$classes, we also have

$$\begin{aligned} R(t)&\geqslant R(0)E_\alpha\left(\frac{-\alpha\mu t^\alpha}{AB(\alpha)-(1-\alpha)\mu}\right),\forall t\geqslant 0\\ U(t)&\geqslant U(0)E_\alpha\left(\frac{-\alpha\mu t^\alpha}{AB(\alpha)-(1-\alpha)\mu}\right),\forall t\geqslant 0.\end{aligned} \tag{67}$$

Therefore if $S(0),I(0),R(0),U(0)\geqslant 0$, then all the classes are positive

6.2. Numerical scheme with Caputo-Fabrizio fractional operator

In this section, we consider the model

$$\begin{aligned} {}^{CF}_{0}D^{\alpha}_{t}S(t)&=-\beta S(t)[I(t)+U(t)]\\ {}^{CF}_{0}D^{\alpha}_{t}I(t)&=\beta S(t)[I(t)+U(t)]-wI(t)\\ {}^{CF}_{0}D^{\alpha}_{t}R(t)&=w_1I(t)-\mu R(t)\\ {}^{CF}_{0}D^{\alpha}_{t}U(t)&=w_2I(t)-\mu U(t)\end{aligned} \tag{68}$$

where the differential operator is Caputo-Fabrizio differential operator [18]. For ease, we take as

$$\begin{aligned} S_1(t,S,I,R,U)&=-\beta S(t)[I(t)+U(t)]\\ I_1(t,S,I,R,U)&=\beta S(t)[I(t)+U(t)]-wI(t)\\ R_1(t,S,I,R,U)&=w_1I(t)-\mu R(t)\\ U_1(t,S,I,R,U)&=w_2I(t)-\mu U(t)\end{aligned} \tag{69}$$

then our system becomes

$$\begin{aligned} {}^{CF}_{0}D^{\alpha}_{t}S(t)&=S_1(t,S,I,R,U)\\ {}^{CF}_{0}D^{\alpha}_{t}I(t)&=I_1(t,S,I,R,U)\\ {}^{CF}_{0}D^{\alpha}_{t}R(t)&=R_1(t,S,I,R,U)\\ {}^{CF}_{0}D^{\alpha}_{t}U(t)&=U_1(t,S,I,R,U)\end{aligned} \tag{70}$$

Applying the Caputo-Fabrizio integral operator, at point $t_{\rho+1}$, one can have the following

$$\begin{aligned} S(t_{\rho+1})&=\tfrac{1-\alpha}{M(\alpha)}S_1(t_\rho,S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}S_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\\ I(t_{\rho+1})&=\tfrac{1-\alpha}{M(\alpha)}I_1(t_\rho,S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}I_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\\ R(t_{\rho+1})&=\tfrac{1-\alpha}{M(\alpha)}R_1(t_\rho,S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}R_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\\ U(t_{\rho+1})&=\tfrac{1-\alpha}{M(\alpha)}U_1(t_\rho,S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}U_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\end{aligned} \tag{71}$$

and also at point t_ρ,

$$\begin{aligned} S(t_\rho)&=S(0)+\tfrac{1-\alpha}{M(\alpha)}S_1(t_{\rho-1},S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}S_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\\ I(t_\rho)&=I(0)+\tfrac{1-\alpha}{M(\alpha)}I_1(t_{\rho-1},S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}I_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\\ R(t_\rho)&=R(0)+\tfrac{1-\alpha}{M(\alpha)}R_1(t_{\rho-1},S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}R_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau\\ U(t_\rho)&=U(0)+\tfrac{1-\alpha}{M(\alpha)}U_1(t_{\rho-1},S,I,R,U)\\ &\quad+\tfrac{\alpha}{M(\alpha)}\int_0^{t_{\rho+1}}U_1(\tau,S,I,R,U)(t_{\rho+1}-\tau)^{\alpha-1}d\tau.\end{aligned} \tag{72}$$

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If we substract the above equations, we can have

$$S(t_{\rho+1}) = S(t_\rho) + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} S_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -S_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\int_{t_\rho}^{t_{\rho+1}} S_1(\tau, S, I, R, U)d\tau. \tag{73}$$

$$I(t_{\rho+1}) = I(t_\rho) + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} I_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -I_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\int_{t_\rho}^{t_{\rho+1}} I_1(\tau, S, I, R, U)d\tau.$$

$$R(t_{\rho+1}) = R(t_\rho) + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} R_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -R_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\int_{t_\rho}^{t_{\rho+1}} R_1(\tau, S, I, R, U)d\tau.$$

$$U(t_{\rho+1}) = U(t_\rho) + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} U_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -U_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\int_{t_\rho}^{t_{\rho+1}} U_1(\tau, S, I, R, U)d\tau.$$

If we put Newton polynomial into these equations as the approximations the functions of S, I, R, U and order these equations, we obtain the following scheme for this model

$$S^{\rho+1} = S^\rho + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} S_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -S_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\begin{Bmatrix} \frac{23}{12}S_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho)\Delta t \\ -\frac{4}{3}S_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1})\Delta t \\ +\frac{5}{12}S_1(t_{\rho-2}, S^{\rho-2}, I^{\rho-2}, R^{\rho-2}, U^{\rho-2})\Delta t \end{Bmatrix} \tag{74}$$

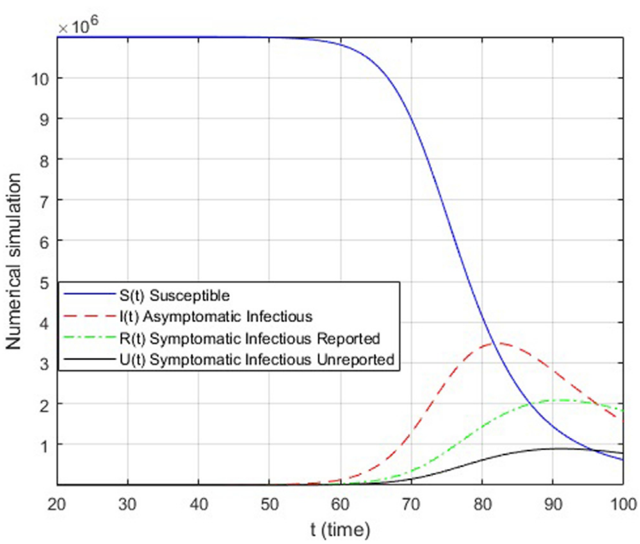

Fig. 2 Numerical simulation for corona model with exponential kernel for $\beta = 4.51 \times 10^{-6}, \alpha = 0.49$.

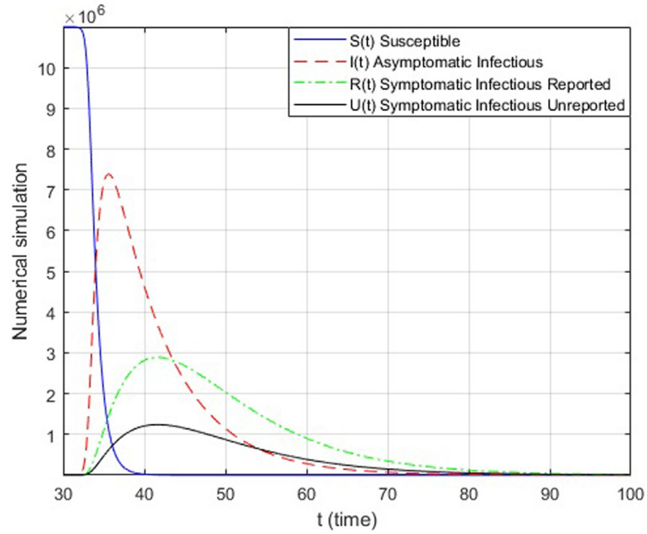

Fig. 3 Numerical simulation for corona model with exponential kernel for $\beta = 6.52 \times 10^{-6}/t, \alpha = 0.74$.

$$I^{\rho+1} = I^\rho + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} I_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -I_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\begin{Bmatrix} \frac{23}{12}I_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho)\Delta t \\ -\frac{4}{3}I_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1})\Delta t \\ +\frac{5}{12}I_1(t_{\rho-2}, S^{\rho-2}, I^{\rho-2}, R^{\rho-2}, U^{\rho-2})\Delta t \end{Bmatrix}$$

$$R^{\rho+1} = R^\rho + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} R_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -R_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\begin{Bmatrix} \frac{23}{12}R_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho)\Delta t \\ -\frac{4}{3}R_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1})\Delta t \\ +\frac{5}{12}R_1(t_{\rho-2}, S^{\rho-2}, I^{\rho-2}, R^{\rho-2}, U^{\rho-2})\Delta t \end{Bmatrix}$$

$$U^{\rho+1} = U^\rho + \frac{1-\alpha}{M(\alpha)}\begin{bmatrix} U_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho) \\ -U_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1}) \end{bmatrix} + \frac{\alpha}{M(\alpha)}\begin{Bmatrix} \frac{23}{12}U_1(t_\rho, S^\rho, I^\rho, R^\rho, U^\rho)\Delta t \\ -\frac{4}{3}U_1(t_{\rho-1}, S^{\rho-1}, I^{\rho-1}, R^{\rho-1}, U^{\rho-1})\Delta t \\ +\frac{5}{12}U_1(t_{\rho-2}, S^{\rho-2}, I^{\rho-2}, R^{\rho-2}, U^{\rho-2})\Delta t \end{Bmatrix}$$

Example 1. We consider the following model

$$\begin{aligned} {}_0^{CF}D_t^\alpha S(t) &= -\beta S(t)[I(t) + U(t)] \\ {}_0^{CF}D_t^\alpha I(t) &= \beta S(t)[I(t) + U(t)] - wI(t) \\ {}_0^{CF}D_t^\alpha R(t) &= w_1 I(t) - \mu R(t) \\ {}_0^{CF}D_t^\alpha U(t) &= w_2 I(t) - \mu U(t) \end{aligned} \tag{75}$$

with initial conditions

$$S(0) = 11 \times 10^6, I(0) = 3.3, R(0) = 1, U(0) = 0.18. \tag{76}$$

With the parameters $w = 1/5, \mu = 0.17$, the numerical simulations are presented in Figs. 2–4.

6.3. Numerical scheme with Atangana-Baleanu fractional operator

In this section, we present numerical scheme for the solution of the following system which model the spread of the 2019-nCoV outbreak emerging in Wuhan. The model having Mittag-Leffler kernel [19] is modified as follows;

$$\begin{aligned} {}_0^{AB}D_t^\alpha S(t) &= -\beta S(t)[I(t) + U(t)] \\ {}_0^{AB}D_t^\alpha I(t) &= \beta S(t)[I(t) + U(t)] - wI(t) \\ {}_0^{AB}D_t^\alpha R(t) &= w_1 I(t) - \mu R(t) \\ {}_0^{AB}D_t^\alpha U(t) &= w_2 I(t) - \mu U(t). \end{aligned} \tag{77}$$

The above system can be revised as follows;

$$\begin{aligned} {}_0^{AB}D_t^\alpha S(t) &= S_1(t, S, I, R, U) \\ {}_0^{AB}D_t^\alpha I(t) &= I_1(t, S, I, R, U) \\ {}_0^{AB}D_t^\alpha R(t) &= R_1(t, S, I, R, U) \\ {}_0^{AB}D_t^\alpha U(t) &= U_1(t, S, I, R, U) \end{aligned} \tag{78}$$

where

$$\begin{aligned} S_1(t, S, I, R, U) &= -\beta S(t)[I(t) + U(t)] \\ I_1(t, S, I, R, U) &= \beta S(t)[I(t) + U(t)] - wI(t) \\ R_1(t, S, I, R, U) &= w_1 I(t) - \mu R(t) \\ U_1(t, S, I, R, U) &= w_2 I(t) - \mu U(t). \end{aligned} \tag{79}$$

If we integrate above system, we write the following

$$\begin{aligned} S(t_{\rho+1}) &= \frac{1-\alpha}{AB(\alpha)} S_1(t_\rho, S, I, R, U) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)} \int_0^{t_{\rho+1}} S_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \\ I(t_{\rho+1}) &= \frac{1-\alpha}{AB(\alpha)} I_1(t_\rho, S, I, R, U) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)} \int_0^{t_{\rho+1}} I_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \end{aligned} \tag{80}$$

$$\begin{aligned} R(t_{\rho+1}) &= \frac{1-\alpha}{AB(\alpha)} R_1(t_\rho, S, I, R, U) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)} \int_0^{t_{\rho+1}} R_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \\ U(t_{\rho+1}) &= \frac{1-\alpha}{AB(\alpha)} U_1(t_\rho, S, I, R, U) \\ &\quad + \frac{\alpha}{AB(\alpha)\Gamma(\alpha)} \int_0^{t_{\rho+1}} U_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \end{aligned}$$

If we do same routine, the above system can be solved numerically as follows

$$\begin{aligned} S^{\rho+1} &= \frac{1-\alpha}{AB(\alpha)} S_1(t_\rho, S, I, R, U) \\ &+ \frac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} S_1(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}) \\ &\times [(\rho - \upsilon + 1)^\alpha - (\rho - \upsilon)^\alpha] \\ &+ \frac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} S_1(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}) \\ -S_1(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}) \end{bmatrix} \\ &\times \begin{bmatrix} (\rho - \upsilon + 1)^\alpha (\rho - \upsilon + 3 + 2\alpha) \\ -(\rho - \upsilon)^\alpha (\rho - \upsilon + 3 + 3\alpha) \end{bmatrix} \\ &+ \frac{\alpha(\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} S_1(t_\upsilon, S^\upsilon, I^\upsilon, R^\upsilon, U^\upsilon) \\ -2S_1(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}) \\ +S_1(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}) \end{bmatrix} \\ &\times \begin{bmatrix} (\rho - \upsilon + 1)^\alpha \begin{bmatrix} 2(\rho - \upsilon)^2 + (3\alpha + 10)(\rho - \upsilon) \\ +2\alpha^2 + 9\alpha + 12 \end{bmatrix} \\ -(\rho - \upsilon)^\alpha \begin{bmatrix} 2(\rho - \upsilon)^2 + (5\alpha + 10)(\rho - \upsilon) \\ +6\alpha^2 + 18\alpha + 12 \end{bmatrix} \end{bmatrix} \end{aligned} \tag{81}$$

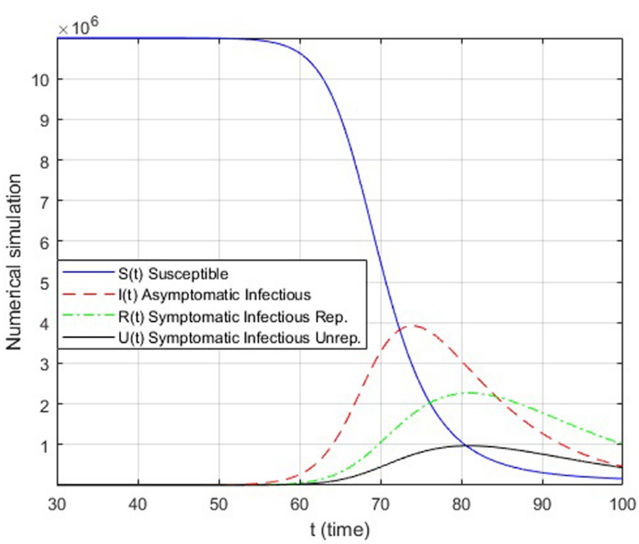

Fig. 4 Numerical simulation for corona model with exponential kernel for $\beta = 5.13 \times 10^{-8}, \alpha = 0.64$.

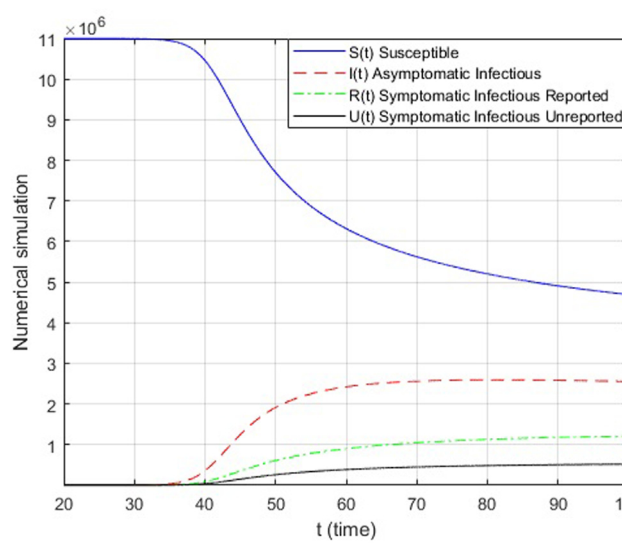

Fig. 5 Numerical simulation for corona model with Mittag-Leffler kernel for $\beta = 4.51 \times 10^{-6}, \alpha = 0.51$.

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$$
\begin{aligned}
I^{\rho+1} = {} & \tfrac{1-\alpha}{AB(\alpha)} I_1\left(t_\rho, S, I, R, U\right) \\
& + \tfrac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} I_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\
& \times\left[(\rho-\upsilon+1)^\alpha-(\rho-\upsilon)^\alpha\right] \\
& + \tfrac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} I_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -I_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{bmatrix} \\
& \times \begin{bmatrix} (\rho-\upsilon+1)^\alpha(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^\alpha(\rho-\upsilon+3+3\alpha) \end{bmatrix} \\
& + \tfrac{\alpha(\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} I_1\left(t_\upsilon, S^\upsilon, I^\upsilon, R^\upsilon, U^\upsilon\right) \\ -2I_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +I_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{bmatrix} \\
& \times \begin{bmatrix} (\rho-\upsilon+1)^\alpha \begin{bmatrix} 2(\rho-\upsilon)^2+(3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2+9\alpha+12 \end{bmatrix} \\ -(\rho-\upsilon)^\alpha \begin{bmatrix} 2(\rho-\upsilon)^2+(5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2+18\alpha+12 \end{bmatrix} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
R^{\rho+1} = {} & \tfrac{1-\alpha}{AB(\alpha)} R_1\left(t_\rho, S, I, R, U\right) \\
& + \tfrac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} R_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\
& \times\left[(\rho-\upsilon+1)^\alpha-(\rho-\upsilon)^\alpha\right] \\
& + \tfrac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} R_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -R_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{bmatrix} \\
& \times \begin{bmatrix} (\rho-\upsilon+1)^\alpha(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^\alpha(\rho-\upsilon+3+3\alpha) \end{bmatrix} \\
& + \tfrac{\alpha(\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} R_1\left(t_\upsilon, S^\upsilon, I^\upsilon, R^\upsilon, U^\upsilon\right) \\ -2R_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +R_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{bmatrix} \\
& \times \begin{bmatrix} (\rho-\upsilon+1)^\alpha \begin{bmatrix} 2(\rho-\upsilon)^2+(3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2+9\alpha+12 \end{bmatrix} \\ -(\rho-\upsilon)^\alpha \begin{bmatrix} 2(\rho-\upsilon)^2+(5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2+18\alpha+12 \end{bmatrix} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
U^{\rho+1} = {} & \tfrac{1-\alpha}{AB(\alpha)} U_1\left(t_\rho, S, I, R, U\right) \\
& + \tfrac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} U_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\
& \times\left[(\rho-\upsilon+1)^\alpha-(\rho-\upsilon)^\alpha\right] \\
& + \tfrac{\alpha(\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} U_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -U_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{bmatrix} \\
& \times \begin{bmatrix} (\rho-\upsilon+1)^\alpha(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^\alpha(\rho-\upsilon+3+3\alpha) \end{bmatrix} \\
& + \tfrac{\alpha(\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \begin{bmatrix} U_1\left(t_\upsilon, S^\upsilon, I^\upsilon, R^\upsilon, U^\upsilon\right) \\ -2U_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +U_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{bmatrix} \\
& \times \begin{bmatrix} (\rho-\upsilon+1)^\alpha \begin{bmatrix} 2(\rho-\upsilon)^2+(3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2+9\alpha+12 \end{bmatrix} \\ -(\rho-\upsilon)^\alpha \begin{bmatrix} 2(\rho-\upsilon)^2+(5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2+18\alpha+12 \end{bmatrix} \end{bmatrix}
\end{aligned}
$$

Example 2. We consider the following model

$$
\begin{aligned}
{}_0^{AB}D_t^\alpha S(t) &= -\beta S(t)[I(t)+U(t)] \\
{}_0^{AB}D_t^\alpha I(t) &= \beta S(t)[I(t)+U(t)]-wI(t) \\
{}_0^{AB}D_t^\alpha R(t) &= w_1 I(t)-\mu R(t) \\
{}_0^{AB}D_t^\alpha U(t) &= w_2 I(t)-\mu U(t)
\end{aligned} \tag{82}
$$

with initial conditions

$$S(0)=11\times10^6, I(0)=3.3, R(0)=1, U(0)=0.18. \tag{83}$$

With the parameters $w=1/5, \mu=0.17$, the numerical simulations are presented in Figs. 5–7.

6.4. Numerical Scheme with Caputo fractional operator

In this section, we now handle the following model

$$
\begin{aligned}
{}_0^{C}D_t^\alpha S(t) &= -\beta S(t)[I(t)+U(t)] \\
{}_0^{C}D_t^\alpha I(t) &= \beta S(t)[I(t)+U(t)]-wI(t) \\
{}_0^{C}D_t^\alpha R(t) &= w_1 I(t)-\mu R(t) \\
{}_0^{C}D_t^\alpha U(t) &= w_2 I(t)-\mu U(t)
\end{aligned} \tag{84}
$$

which has the power-law kernel. This system can be rewritten as follows

$$
\begin{aligned}
{}_0^{C}D_t^\alpha S(t) &= S_1(t,S,I,R,U) \\
{}_0^{C}D_t^\alpha I(t) &= I_1(t,S,I,R,U) \\
{}_0^{C}D_t^\alpha R(t) &= R_1(t,S,I,R,U) \\
{}_0^{C}D_t^\alpha U(t) &= U_1(t,S,I,R,U)
\end{aligned} \tag{85}
$$

where

$$
\begin{aligned}
S_1(t,S,I,R,U) &= -\beta S(t)[I(t)+U(t)] \\
I_1(t,S,I,R,U) &= \beta S(t)[I(t)+U(t)]-wI(t) \\
R_1(t,S,I,R,U) &= w_1 I(t)-\mu R(t) \\
U_1(t,S,I,R,U) &= w_2 I(t)-\mu U(t).
\end{aligned} \tag{86}
$$

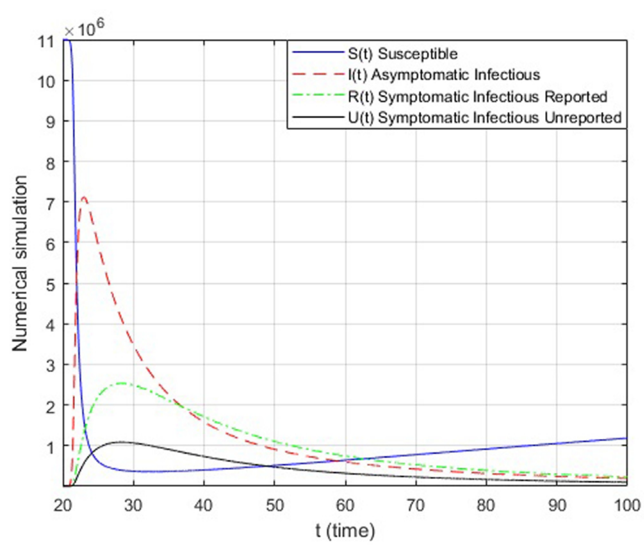

Fig. 6 Numerical simulation for corona model with Mittag-Leffler kernel for $\beta=5.12\times10^{-6}/t, \alpha=0.82$.

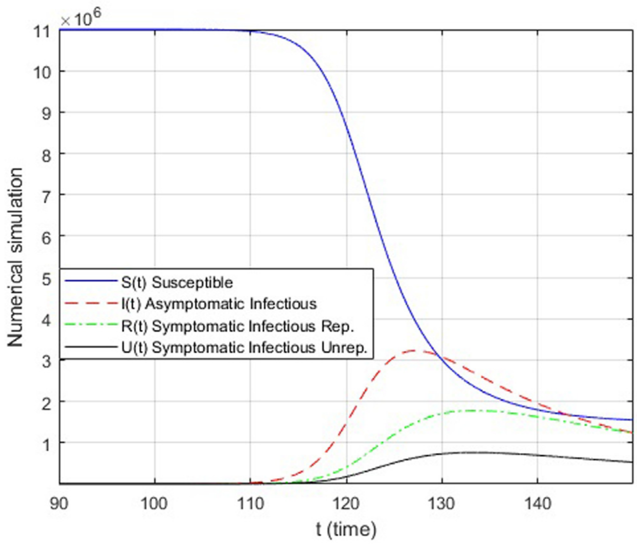

Fig. 7 Numerical simulation for corona model with Mittag-Leffler kernel for $\beta = 4.51 \times 10^{-8}, \alpha = 0.79$.

If we integrate above model, we write the following

$$\begin{aligned} S(t_{\rho+1}) &= \frac{1}{\Gamma(\alpha)} \int_0^{t_{\rho+1}} S_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \\ I(t_{\rho+1}) &= \frac{1}{\Gamma(\alpha)} \int_0^{t_{\rho+1}} I_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \\ R(t_{\rho+1}) &= \frac{1}{\Gamma(\alpha)} \int_0^{t_{\rho+1}} R_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \\ U(t_{\rho+1}) &= \frac{1}{\Gamma(\alpha)} \int_0^{t_{\rho+1}} U_1(\tau, S, I, R, U)(t_{\rho+1} - \tau)^{\alpha-1} d\tau \end{aligned} \tag{87}$$

As we did before, we can have the following numerical approximation

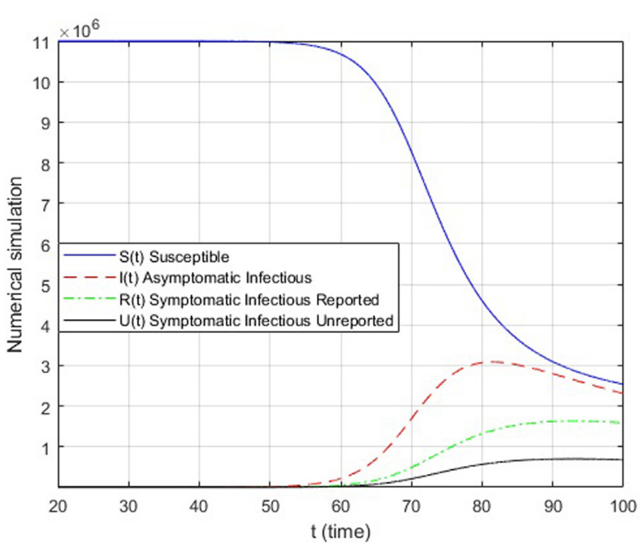

Fig. 8 Numerical simulation for corona model with power-law kernel for $\beta = 4.51 \times 10^{-6}, \alpha = 0.67$.

$$\begin{aligned} S^{\rho+1} = & \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} S_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\ & \times [(\rho-\upsilon+1)^{\alpha} - (\rho-\upsilon)^{\alpha}] \\ & + \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} S_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -S_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array} \right] \\ & \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha}(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^{\alpha}(\rho-\upsilon+3+3\alpha) \end{array} \right] \\ & + \frac{(\Delta t)^{\alpha}}{2\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} S_1(t_{\upsilon}, S^{\upsilon}, I^{\upsilon}, R^{\upsilon}, U^{\upsilon}) \\ -2S_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +S_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array} \right] \\ & \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha} \left[\begin{array}{c} 2(\rho-\upsilon)^2 + (3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2 + 9\alpha + 12 \end{array} \right] \\ -(\rho-\upsilon)^{\alpha} \left[\begin{array}{c} 2(\rho-\upsilon)^2 + (5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2 + 18\alpha + 12 \end{array} \right] \end{array} \right] \end{aligned} \tag{88}$$

$$\begin{aligned} I^{\rho+1} = & \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} I_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\ & \times [(\rho-\upsilon+1)^{\alpha} - (\rho-\upsilon)^{\alpha}] \\ & + \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} I_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -I_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array} \right] \\ & \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha}(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^{\alpha}(\rho-\upsilon+3+3\alpha) \end{array} \right] \\ & + \frac{(\Delta t)^{\alpha}}{2\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} I_1(t_{\upsilon}, S^{\upsilon}, I^{\upsilon}, R^{\upsilon}, U^{\upsilon}) \\ -2I_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +I_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array} \right] \\ & \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha} \left[\begin{array}{c} 2(\rho-\upsilon)^2 + (3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2 + 9\alpha + 12 \end{array} \right] \\ -(\rho-\upsilon)^{\alpha} \left[\begin{array}{c} 2(\rho-\upsilon)^2 + (5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2 + 18\alpha + 12 \end{array} \right] \end{array} \right] \end{aligned}$$

$$\begin{aligned} R^{\rho+1} = & \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} R_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\ & \times [(\rho-\upsilon+1)^{\alpha} - (\rho-\upsilon)^{\alpha}] \\ & + \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} R_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -R_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array} \right] \\ & \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha}(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^{\alpha}(\rho-\upsilon+3+3\alpha) \end{array} \right] \\ & + \frac{(\Delta t)^{\alpha}}{2\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} R_1(t_{\upsilon}, S^{\upsilon}, I^{\upsilon}, R^{\upsilon}, U^{\upsilon}) \\ -2R_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +R_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array} \right] \\ & \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha} \left[\begin{array}{c} 2(\rho-\upsilon)^2 + (3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2 + 9\alpha + 12 \end{array} \right] \\ -(\rho-\upsilon)^{\alpha} \left[\begin{array}{c} 2(\rho-\upsilon)^2 + (5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2 + 18\alpha + 12 \end{array} \right] \end{array} \right] \end{aligned}$$

$$
\begin{aligned}
U^{\rho+1} = \;& \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+1)} \sum_{\upsilon=2}^{\rho} U_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \\
& \times\left[(\rho-\upsilon+1)^{\alpha}-(\rho-\upsilon)^{\alpha}\right] \\
& + \frac{(\Delta t)^{\alpha}}{\Gamma(\alpha+2)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} U_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ -U_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array}\right] \\
& \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha}(\rho-\upsilon+3+2\alpha) \\ -(\rho-\upsilon)^{\alpha}(\rho-\upsilon+3+3\alpha) \end{array}\right] \\
& + \frac{(\Delta t)^{\alpha}}{2\Gamma(\alpha+3)} \sum_{\upsilon=2}^{\rho} \left[\begin{array}{c} U_1\left(t_{\upsilon}, S^{\upsilon}, I^{\upsilon}, R^{\upsilon}, U^{\upsilon}\right) \\ -2U_1\left(t_{\upsilon-1}, S^{\upsilon-1}, I^{\upsilon-1}, R^{\upsilon-1}, U^{\upsilon-1}\right) \\ +U_1\left(t_{\upsilon-2}, S^{\upsilon-2}, I^{\upsilon-2}, R^{\upsilon-2}, U^{\upsilon-2}\right) \end{array}\right] \\
& \times \left[\begin{array}{c} (\rho-\upsilon+1)^{\alpha}\left[\begin{array}{c} 2(\rho-\upsilon)^2+(3\alpha+10)(\rho-\upsilon) \\ +2\alpha^2+9\alpha+12 \end{array}\right] \\ -(\rho-\upsilon)^{\alpha}\left[\begin{array}{c} 2(\rho-\upsilon)^2+(5\alpha+10)(\rho-\upsilon) \\ +6\alpha^2+18\alpha+12 \end{array}\right] \end{array}\right]
\end{aligned}
$$

Example 3. We consider the following model

$$
\begin{aligned}
{}_0^C D_t^{\alpha} S(t) &= -\beta S(t)[I(t)+U(t)] \\
{}_0^C D_t^{\alpha} I(t) &= \beta S(t)[I(t)+U(t)] - wI(t) \\
{}_0^C D_t^{\alpha} R(t) &= w_1 I(t) - \mu R(t) \\
{}_0^C D_t^{\alpha} U(t) &= w_2 I(t) - \mu U(t)
\end{aligned} \tag{89}
$$

with initial conditions

$$S(0) = 11\times 10^6, I(0) = 3.3, R(0) = 1, U(0) = 0.18. \tag{90}$$

With the parameters $w = 1/5, \mu = 0.17$, the numerical simulations are depicted in Figs. 8–10.

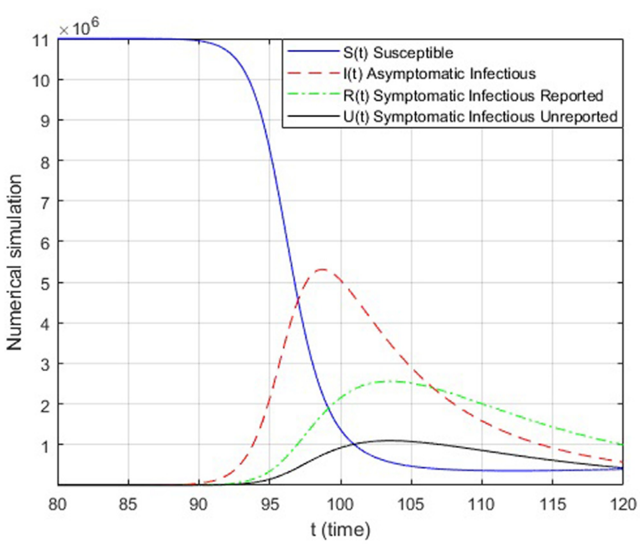

Fig. 9 Numerical simulation for corona model with power-law kernel for $\beta = 7.81\times 10^{-6}/t, \alpha = 0.92$.

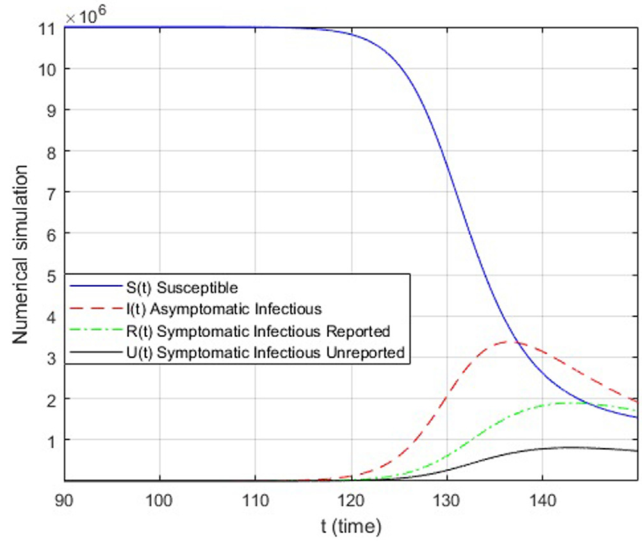

Fig. 10 Numerical simulation for corona model with power-law kernel for $\beta = 4.53\times 10^{-8}, \alpha = 0.82$.

7. Conclusion

In this study, we deal with a mathematical model about Covid-19 spread and present a detailed analysis of the model with classical and nonlocal differential operators by considering 3 cases, power law, fading memory and generalized Mittag-Leffler kernels. We show the positiveness of the solutions of this model with classical and fractional orders. In addition to this, we obtain the reproduction number using the next-generation matrix. We analyze global and local stability for the considered model. We know that control theory has been widely applied in many ordinary and partial differential equations in the last decades with great success. Especially the idea of optimal control could be with great value in epidemiology, to control mathematical models depicting the spread of infectious disease. In this work, optimal control is used to control a mathematical model of COVID-19 using some control strategies. New established numerical scheme based on Newton polynomial is used to solve numerically the extended models with non-local operators. Thus, we present prediction and simulation about a Covid-19 model with these operators.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] S. Al-Sheikh, F. Musali, M. Alsolami, Stability Analysis of an HIV/AIDS Epidemic model with screening, Int. Math. Forum 6 (66) (2011) 3251–3273.
[2] K.M. Owolabi, A. Atangana, Mathematical analysis and computational experiments for an epidemic system with nonlocal and nonsingular derivative, Chaos, Solitons Fract. 126 (2019) 41–49.

[3] T.S. Do, Y.S. Lee, Modeling the Spread of Ebola, Osong Public Health and Research Perspectives 7 (1) (2016) 43–48.
[4] D. Chowell, C.C. Chavez, S. Krishna, X. Qiu, K.S. Anderson, Modelling the effect of early detection of Ebola, Lancet. Infect. Dis. 15 (2) (2015) 148–149.
[5] Z. Liu, P. Magal, O. Seydi, G. Webb, Predicting the cumulative number of cases for the COVID-19 epidemic in China from early data, Math. Biosci. Eng. 17 (4) (2020) 3040–3051.
[6] T.M. Chen, J. Rui, Q.P. Wang, Z.Y. Zhao, J.A. Cui, L. Yin, A mathematical model for simulating the phase-based transmissibility of a novel coronavirus, Infect. Dis. Poverty 9 (2020) 24.
[7] MA. Khan, A. Atangana, Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, Alexandria Eng. J. (2020).
[8] A. Shikongo, K.M. Owolabi, Fractional operator method on a multi-mutation and intrinsic resistance model, Alexandria Eng. J. 59 (4) (2020) 1999–2013.
[9] A. Atangana, S.I. Araz, Mathematical model of COVID-19 spread in Turkey and South Africa: Theory, methods and applications, Adv. Diff. Eqs. (2020).
[10] A. Atangana, Modelling the spread of COVID-19 with new fractal-fractional operators: Can the lockdown save mankind before vaccination?, Chaos, Solitons Fract. 136 (2020) 109860.
[11] P.A. Naik, J. Zu, K.M. Owolabi, Modeling the mechanics of viral kinetics under immune control during primary infection of HIV-1 with treatment in fractional order, Physica A 545 (2020) 123816.
[12] A. Atangana, S. Qureshi, Mathematical modeling of an autonomous nonlinear dynamical system for malaria transmission using Caputo derivative, Fract. Order Anal. Theory Methods Appl. (2020).
[13] A.M. Mishra, S.D. Purohit, K.M. Owolabi, Y.D. Sharma, A nonlinear epidemiological model considering asymptotic and quarantine classes for SARS CoV-2 virus, Chaos, Solitons Fract. 138 (2020) 109953.
[14] M.A. Khan, A. Atangana, Fractional dynamics of HIV-AIDS and cryptosporidiosis with lognormal distribution, Fract. Calculus Med. Health Sci. 167–210.
[15] C. Tunc, M. Gozen, Stability and uniform boundedness in multidelay functional differential equations of third order, Abstract Appl. Anal. (2013) 7. Article ID 248717.
[16] A. Akgül, M. Modanli, Crank-Nicholson difference method and reproducing kernel function for third order fractional differential equations in the sense of Atangana-Baleanu Caputo derivative, Chaos, Solitons Fract. 127, 10–16.
[17] K.M. Owolabi, A. Atangana, Numerical Methods for Fractional Differentiation, Springer Ser. Comput. Math. (2019).
[18] M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel, Prog. Fract. Differ. Appl. 1 (2) (2015) 73–85.
[19] A. Atangana, D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model, Therm. Sci. 20 (2) (2016) 763–769.
[20] P. Driessche, J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Math. Biosci. 180 (1) (2002) 29–48.
[21] N.H. Shah, A.H. Suthar, E.N. Jayswal, Control strategies to curtail transmission of COVID-19, 2020, bioRxiv.
[22] S. Nababan, A filippov-type lemma for functions involving delays and its application to time delayed optimal control problems, Optim. Theory Appl. 27 (3) (1979) 357–376.
[23] A. Atangana, S.I. Araz, New numerical method for ordinary differential equations: Newton polynomial, J. Comput. Appl. Math. 372 (2020) 112622.
[24] M. Barro, A. Guiro, D. Ouedraogo, Optimal control of a SIR epidemic model with general incidence function and a time delays, CUBO A Math. J. 20(2), 53–66.
[25] J.P. LaSalle, The Stability of Dynamical Systems, SIAM Press (1976).

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