Optimal control strategies to prevent the hospital beds collapse during Covid-19 outbreak

**Abstract:** In 2020 the world has faced a serious challenge since the breakout of corona-virus started inWuhan, China. The deathly disease has killed about 1.770.000 and infected more than 80 millions humans around the globe since December 2019 to 27 of December 2020.

The paper presents a new mathematical model for the SARS-CoV-2 virus propagation, designed to include all the possible actions to prevent the spread and to help in the healing of infected people, including the new inoculation to the SARS-CoV-2. The objective of this project is to propose the possibility of optimal controls over the susceptible and the infected subjects considering different cost functions in order to see the effects of different optimised control actions on the evolution of the epidemic spread and in particular how these controls should be tuned in order to avoid the hospital beds collapse. The optimal control analysis was carried out using the Pontryagin’s maximum principle to figure out the optimal strategy necessary to curtail the disease.and the existence of the optimal solution is assessed. Numerical evaluations are developed for a more intuitive and immediate presentation, showing the consequences on the classes of interest.

**Introduction**

Coronavirus disease 2019 (COVID-19) is a disease caused by severe acute respiratory syndrome coronavirus 2 (SARS CoV-2).

Italy has been severely affected[5]. After the first indigenous case on 21 February 2020 in Lodi province, several suspect cases (initially epidemiologically linked) began to emerge in the south and southwest territory of Lombardy[6]. A ‘red zone’, encompassing 11 municipalities where SARS-CoV-2 infection was endemic, was instituted on 22 February 2020, and put on lockdown to contain the emerging threat. A campaign to identify and screen all close contacts with confirmed cases of COVID-19 resulted in taking 691,461 nasal swabs as of 5 April 2020. Of the 128,948 detected cases, 91,246 were currently infected (28,949 hospitalized, 3,977 admitted to intensive care units (ICUs) and 58,320 quarantined at home), 21,815 had been discharged due to recovery and 15,887 had died7. In the early days of the epidemic in Italy, both symptomatic and asymptomatic people underwent screening. A government regulation dated 26 February 2020 limited screening to symptomatic subjects only[8]. On 8 March 2020, to further contain the spread of SARS-CoV-2, the red zone was extended to the entire area of Lombardy and 14 more northern Italian provinces. On 9 March 2020, lockdown was declared for the

entire country[9] and progressively stricter restrictions were adopted. COVID-19 displays peculiar epidemiological traits when compared with previous coronavirus outbreaks of SARS-CoV and MERS-CoV. According to Chinese data[10], a large number of transmissions, both in nosocomial and community settings, occurred through human-to-human contact with individuals showing no or mild symptoms. The estimated basic reproduction number (*R*0) for

SARS-CoV-2 ranges from 2.0 to 3.5[11–13], which seems comparable, or possibly higher, than for SARS-CoV and MERS-CoV. High viral loads of SARS-CoV-2 were found in upper respiratory specimens of patients showing little or no symptoms, with a viral shedding pattern akin to that of influenza viruses[14]. Hence, inapparent transmission may play a major and underestimated role in sustaining the outbreak.

Until the end of December the disease had neither approved medicine nor vaccine and has made governments and scholars search for drastic measures in combating the pandemic. The 26th of December the first 10.000 doses of vaccine have been delivered in Italy but the effectiveness of it is not yet guaranteed and the first side effects have been occured in different countries.

Predictive mathematical models for epidemics [15–18] are fundamental to understand the course of the epidemic and to plan effective control strategies. One commonly used model is the SIR model[19] for human-to-human transmission, which describes the flow of individuals through three mutually exclusive stages of infection: susceptible, infected and recovered. More complex models can accurately portray the dynamic spread of specific epidemics. For the COVID-19 pandemic, several models have been developed for specific classes of infections, to better descrive their propagation and to particularize the specific control actions against its spread.

In this paper a quite rich model is proposed, composed by 8 different classes and the model parameters are identifes on the basis of the available data. To have a more detailed model all the known preventive and active actions that can be put in place are considered, at an organizational and decisional level as well as from a medical point of view, to contain the virus spread. For the aim of our work, the model explains in a better way the compartments of infected people drawing a distinction between different type of infectious and paying attention to those that are in the hospitals.

Regrettably, the spread of the virus and mortality due to COVID-19 has continued to increase daily. Hence, it is imperative to control the spread of the disease particularly using nonpharmacological strategies (and in a second case pharmacological one) such as quarantine, isolation, and public health education.

This work studied the effect of these different control strategies using mathematical modeling and optimal control approach to ascertain their contributions in the dynamic transmission of COVID-19.

In the following paragraph the model is presented and described.

**MATHEMATICAL MODEL**

The mathematical model here adopted is an enrichment of a classical SEQIR one, usually adopted to describe the dynamic of epidemic spreads in presence of a virus incubation phase (E) in which the quarantine compartment is considered. To the standard SEQIR model more classes are added and the possible ways of intervention are modelled in order to make available some numerical evaluations about the possible epidemic diffusion depending on the different strategies. We define our model as *SEIaQI1I2RV,* where each class is defined as follow:

* Susceptible (S): people who are not yet infected but they are potentially plagued by the virus.
* Expose (E): people who have been infected but they still can not spread the virus because of the incubation period.
* Infected undetected (I­a): fraction of population that can infect the susceptible class because they are not yet detected and so they could have contacts with susceptible people.
* Quarantined (Q): fraction of population detected with or without symptoms quarantined and due to this fact they can not have contact with susceptible.
* Hospitalized infected non-ICu (I­1): fraction of population detected, with symptoms and hospitalized not in Intensive Care (IC).
* Hospitalized infected in ICu (I­2): fraction of population detected that due to the heavy symptoms has been hospitalized in Intensive Care (IC).
* Recovered (R): fraction of population healed from the virus and temporarily immune.
* Vaccinated (V): fraction of population vaccinated and immune.

The mathematical model proposed is the following one:

The parameters of the considered model are presented in **Table 1.**

*Discussion on modelling choices:* In the model, we omit the control referring to the swabs considering just the percentage of the positive people. This choice is given by the fact that we are interested in the study of the infected people, so we assume that all the people that are infected not yet detected are positive with a precise percentage given by estimations on real data.

In the model, we have decided to consider a parameter that has the purpose to mitigate the effectiveness of the control in the case in which the control effort is maximum. In a matter of fact, we have supposed that, even if the effort on hospitalised control with respect to non IC units and IC units, is maximum, it is not certain that the outcome of this choice has its maximum effectiveness. About the recoveries we have assumed that it is possible to heal even without drugs only if the infected people are not yet detected and without symptoms (it means in Ia), quarantined (in Q) or hospitalized but not in IC. In the latter case we have considered those infected people that go to the hospital just for a check or would have recovered also without any treatments. On the other side, to be more realistic, the infected people in IC can be healed just through treatments and the usage of ventilator and oxygen, so in this class the only way to be recovered is with a control effort.

**Table 1:** parameters of the considered model

|  |  |
| --- | --- |
| Symbol | Interpretation |
|  | Prior control (social distancing, masks, information campaigns) |
|  | Hospital treatments control over non IC patients (availability of beds, medical staff, use of drugs) |
|  | Hospital treatments control over IC patients (availability of beds in IC units, ventilator, oxygen, medial staff) |
|  | Control over vaccine inoculation and production. |
|  | Number of births. |
|  | Death rate in Italy |
|  | Contact rate |
|  | Incubation period |
|  | Percentage of positive |
|  | Percentage of quarantined people. *(1-p):* percentage of hospitalized patients not in IC |
|  | Percentage of people that from quarantine move to Covid units after complications. |
|  | Percentage of people that from Covid units move to IC units after complications. |
|  | Recovery rate without use of drugs in Ia(*i=1*), Q (*i*=2), I1 (*i*=3) |
|  | Death rate |
|  | Control effectiveness ( *with respect to u1 and with respect to u2)* |
|  | Inverse of the mean time to swab (both referring to the onset of symptoms and the time spent to know about the contact with a positive person) |
|  | Inverse of the mean time to be again susceptible |

**MOTIVATIONS**

In this section we briefly discuss the main reasons about why we exploit an optimal control strategy.

In the past few months Italy has been affected by the second wave of the Covid-19. During this period a lot of infected people are carried to the hospital because of complications. This situation has caused on the whole Italian territory hospitals overcrowding and a collapse of the IC beds’ hospital with very hard consequences in the number of deaths. Due to this the State has taken very heavy decisions at the expense of the economy but most of all of the life of many people.

So, the purpose is to find an optimal control strategy to avoid as much as possible the overcrowding of the hospital minimising the number of infected people and simultaneously minimising also the control effort in such a way that there are mild consequences on the daily life of the Italian people.

Necessario controllo ottimo per minimizzare il carico del controllo usato minimizzando allo stesso tempo il numero di persone infette. Infatti, un problema che tuttora è presente è la difficoltà di gestire i pazienti ospedalizzati (fare un controllo ottimo su infetti in terapia intensiva e non, quindi I1 e I2 e allo stesso tempo minimizzare il controllo sulle cure)

* Chiedere alla professoressa il significato dei pesi imposti sul controllo delle cure (teoricamente il peso aumenta se aumenta la conoscenza della malattia, ossia si sa come trattarla