TABLE 8.3 Nonlinear Regression Models of Test Scores

Dependent variable: average test score in district; 420 observations.

Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Student–teacher ratio (STR)	−1.00 (0.27)	−0.73 (0.26)	−0.97 (0.59)	−0.53 (0.34)	64.33 (24.86)	83.70 (28.50)	65.29 (25.26)
STR^2					−3.42 (1.25)	−4.38 (1.44)	−3.47 (1.27)
STR^3					0.059 (0.021)	0.075 (0.024)	0.060 (0.021)
% English learners	−0.122 (0.033)	−0.176 (0.034)					−0.166 (0.034)
% English learners $\geq$ 10%? (Binary, $HiEL$)			5.64 (19.51)	5.50 (9.80)	−5.47 (1.03)	816.1 (327.7)	
$HiEL \times STR$			−1.28 (0.97)	−0.58 (0.50)		−123.3 (50.2)	
$HiEL \times STR^2$						6.12 (2.54)	
$HiEL \times STR^3$						−0.101 (0.043)	
Included Economic Control Variables							
% eligible for subsidized lunch	Y	Y	N	Y	Y	Y	Y
Average district income (logarithm)	N	Y	N	Y	Y	Y	Y
95% Confidence Intervals for the Effect of Reducing *STR* by 2							
No $HiEL$ interaction	[0.93,3.06]	[0.46,2.48]					
22 to 20					[0.61, 3.25]		[0.54, 3.26]
20 to 18					[1.64, 4.36]		[1.55, 4.30]
$HiEL = 0$			[−0.38, 4.25]	[−0.28, 2.41]			
22 to 20						[0.40, 3.98]	
20 to 18						[1.22, 4.99]	
$HiEL = 1$			[1.48, 7.50]	[0.80, 3.63]			
22 to 20						[−0.98, 2.91]	
20 to 18						[−0.72, 4.01]	
F-Statistics and *p*-Values on Joint Hypotheses							
All STR variables and interactions = 0			5.64 (0.004)	5.92 (0.003)	6.31 (< 0.001)	4.96 (< 0.001)	5.91 (0.001)
STR^2, $STR^3 = 0$					6.17 (< 0.001)	5.81 (0.003)	5.96 (0.003)

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	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$HiEL \times STR$, $HiEL \times STR^2$, $HiEL \times STR^3 = 0$						2.69 (0.046)	
SER	9.08	8.64	15.88	8.63	8.56	8.55	8.57
$\bar{R}^2$	0.773	0.794	0.305	0.795	0.798	0.799	0.798

These regressions were estimated using the data on K–8 school districts in California, described in Appendix 4.1. Regressions include an intercept and the economic control variables indicated by "Y" or exclude them if indicated by "N" (coefficients not shown in the table). Standard errors are given in parentheses under coefficients, and p-values are given in parentheses under F-statistics.

between regressions (1) and (2) to warrant additionally controlling for the logarithm of income in the remaining regressions as a deterrent to omitted variable bias.

Regression (3) in Table 8.3 is the interacted regression in Equation (8.34) with the binary variable for a high or low percentage of English learners but with no economic control variables. When the economic control variables (percentage eligible for subsidized lunch and log income) are added [regression (4) in the table], the class size effect is reduced for both high and low English learner classes; however, the confidence intervals are wide in both cases in both regressions. Based on the evidence in regression (4), the hypothesis that the effect of STR is the same for districts with low and high percentages of English learners cannot be rejected at the 5% level (the t-statistic is $t = -0.58/0.50 = -1.16$).

Regression (5) examines whether the effect of changing the student–teacher ratio depends on the value of the student–teacher ratio by including a cubic specification in STR, controlling for the economic variables in regression (4) [the interaction term, $HiEL \times STR$, is not included in regression (5) because it was not significant in regression (4) at the 10% level]. The estimates in regression (5) are consistent with the student–teacher ratio having a nonlinear effect. The null hypothesis that the relationship is linear is rejected at the 1% significance level against the alternative that it is a polynomial up to degree 3 (the F-statistic testing the hypothesis that the true coefficients on STR^2 and STR^3 are 0 is 6.17, with a p-value of < 0.001). The effect of reducing the class size from 20 to 18 is estimated to be greater than if it is reduced from 22 to 20.

Regression (6) further examines whether the effect of the student–teacher ratio depends not just on the value of the student–teacher ratio but also on the fraction of English learners. By including interactions between $HiEL$ and STR, STR^2, and STR^3, we can check whether the (possibly cubic) population regressions functions relating test scores and STR are different for low and high percentages of English learners. To do so, we test the restriction that the coefficients on the three interaction terms are 0. The resulting F-statistic is 2.69, which has a p-value of 0.046 and thus is significant at the 5% but not at the 1% significance level. This provides tentative evidence that the regression functions are different for districts with high and low percentages of English learners; however, comparing regressions (6) and (4) makes it clear that