6. Statistical Inference: Significance Tests

LPO.8800: Statistical Methods in Education Research

Sean P. Corcoran

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Lecture

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Last time

- Estimators vs. estimates
- Point estimation vs. interval estimation
- \bullet Confidence interval for μ when σ is known
- ullet Confidence interval for π
- \bullet Confidence interval for μ when σ is $\mathit{unknown}$ (using sample standard deviation s, and t-distribution)
- Assumptions behind confidence interval estimation
- Determining minimum sample sizes for desired interval estimate precision

Significance testing

Two approaches to statistical inference about a population mean μ :

- **1** Providing an interval estimate of μ
 - \blacktriangleright We have no a priori idea of what μ is
 - \blacktriangleright The confidence interval represents a range of "likely values" for μ
 - The width of the interval reflects sampling variability in the estimator (its standard error).
- $oldsymbol{0}$ Testing hypotheses about μ
 - \blacktriangleright Begin with a hypothesis about μ
 - \blacktriangleright Ask whether the sample statistic \bar{x} is consistent with this hypothesis
 - Use knowledge of the sampling distribution of x̄ to assess how plausible the hypothesis is
 - This approach is called hypothesis testing or significance testing

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Statistical hypotheses

A statistical **hypothesis** is a statement about a population. It is usually a statement that a parameter takes a particular value or falls in some range of values. Hypotheses come from a variety of sources:

- Past experience or "common wisdom"
- Prior research
- Inductively, from a study of past observations
- Deductively, from theory
- A presumption of "no difference," "no change," "no effect", etc.

A **hypothesis test** or **significance test** asks whether the observed data are *consistent* or *inconsistent* with a hypothesis. That is, how plausible is the hypothesis in light of the observed data?

Statistical hypotheses

- The null hypothesis (H₀): a statement that a population parameter takes a particular value. The null hypothesis is often the *lack* of a hypothesized finding: e.g., "no effect" or "no difference."
- The alternative hypothesis (H₁ or H_a): a statement that the population parameter falls in some alternative range of values (contrary to H₀). The alternative hypothesis is often an "effect" or "difference" that the researcher is testing for.

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Statistical hypotheses

Example: I am interested in testing whether the mean number of study hours per week is higher among Vanderbilt undergraduates than the national average. (This is a *directional* hypothesis since I am stating "higher" and not "different from").

- H₀: the mean study hours per week at Vanderbilt is the same as the national average.
- H₁: the mean study hours per week at Vanderbilt is higher than the national average.

 $H_0: \mu_{vu} = \mu_0$

 $H_1: \mu_{VII} > \mu_0$

 μ_0 represents the population mean for Vanderbilt undergraduates if H_0 is true. Here, μ_0 is the national average.

Statistical hypotheses

Example: I believe home lending in my community is racially biased. (That is, non-white applicants are *less* likely to be approved for home loans. This is again a directional hypothesis.)

- H₀: the proportion of non-white applicants awarded home loans is the same as the proportion of white applicants given home loans.
- H₁: the proportion of non-white applicants awarded home loans is lower than the proportion of white applicants given home loans.

$$H_0: \pi_{nw} = \pi_0$$

$$H_1: \pi_{nw} < \pi_0$$

 π_0 represents the population proportion of non-white applicants awarded home loans if H_0 is true. Here, π_0 is the proportion of white applicants awarded home loans.

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Statistical hypotheses

Note that in both of the above examples H_0 asserts that the population parameter takes a particular value. This value provides an "anchor" for the test. H_1 , on the other hand, asserts that the parameter falls in some *range* of alternative values (e.g., greater than or less than some value).

Also note:

- If H₀ is rejected in favor of H₁ in the first case above (VU study time), then we can also reject H₀ for μ_{vu} < μ₀.
- If H₀ is rejected in favor of H₁ in the second case above (lending), then we can also reject H₀ for π_{nw} > π₀.

The logic of hypothesis testing

 H_0 is presumed to hold unless the evidence strongly suggests otherwise (like a "proof by contradiction").

- A significance test can either reject or not reject the null hypothesis.
- One should not say we "accept" H₀, or that H₀ is "true," only that the evidence is not sufficient to reject it.
- Important: rejection is in a statistical sense—even if rejected there is a chance H₀ is true.



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Hypothesis testing in practice

FDA screening: new drugs must be approved for use by the Food and Drug Administration. According to federal law, drugs must be demonstrated to be both safe and effective.

FDA screening starts from the assumption (H_0) that the drug does not work. It is up to the manufacturer to demonstrate otherwise: that the evidence is sufficient to reject the null hypothesis with high confidence.

The distribution of math SAT scores in the population has a mean of 500 and $\sigma=100.$ You believe that the mean math SAT score in California is higher than the national average (assume σ is the same). To test this hypothesis, you randomly sample n=1,600 California math SAT scores. (Assuming we don't have access to the population of California scores).

- Let μ_c represent the population mean SAT in California.
- H_0 : $\mu_c = 500$
- H₁: μ_c > 500

The null hypothesis is that $\mu_c=\mu_0=500$, i.e., the CA mean is the same as the national average.

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Example 1

We compute the sample mean (\bar{x}) from our random sample of California scores. If the sample mean is "sufficiently higher" than 500 we *reject* H_0 in favor of H_1 . The question is:

- What value of x̄ is "sufficiently higher?"
- What value of \bar{x} would make H_0 seem *implausible*?

Remember, in hypothesis testing, we begin with the assumption H_0 is true. In this example, if H_0 were true, sample means calculated from random samples of size n=1,600 will:

- be normally distributed (per the CLT)
- have a mean of 500.
- have a standard error of $\sigma/\sqrt{n} = 100/\sqrt{1600}$, or 2.5

Note: we are assuming σ is known for this case.

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Example 1

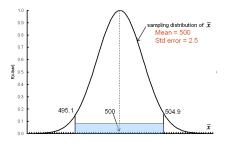


Figure: Distribution of \bar{x} under H_0

If H_0 is true, then 95% of the time \bar{x} calculated from a random sample of n=1,600 will fall between:

$$\mu_0 \pm 1.96(\sigma/\sqrt{n})$$
 $500 \pm 1.96(2.5) = (495.1, 504.9)$

Any realized \bar{x} in this interval wouldn't be that unusual. What about a realized sample mean of $\bar{x}=507$?

Note: the interval above looks like a confidence interval, but it is centered at μ_0 not \bar{x} .

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Example 1

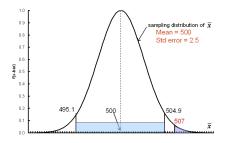


Figure: Distribution of \bar{x} under H_0 , and a realized sample mean of 507

507 is (507 - 500)/2.5 = 2.8 standard errors above μ_0 . The probability of obtaining a sample mean of 507 or higher, assuming H_0 is true, is:

$$Pr(z > 2.8) = 0.0026$$

In other words, *very unlikely*. The value 2.8 is called a **test statistic**. 0.0026 is called the **p-value**.

In Stata: display 1-normal(2.8)

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Example 1

What about a realized sample mean of $\bar{x}=502$? 502 is (502-500)/2.5=0.8 standard errors above μ_0 . The probability of obtaining a sample mean of 502 or higher, assuming H_0 is true, is:

$$Pr(z > 0.8) = 0.2119$$

In other words, not that unlikely. The value 0.8 is our test statistic, and 0.2119 is the p-value.

In Stata: display 1-normal(0.8)

p-values

In general a **p-value** is the probability that a test statistic equals the realized value or a value even more extreme in the direction predicted by H_1 if H_0 is true.

- Above, the probability of obtaining an \bar{x} of 507 or higher was 0.0026.
- Above, the probability of obtaining an \bar{x} of 502 or higher was 0.2119.
- Intuitively, if we obtain an x̄ that would be highly unlikely if H₀ were true (such as 0.0026) then H₀ was probably not true to begin with.
- The less consistent is the test statistic with H₀, the smaller is the p-value.
- A small enough p-value will lead us to reject H₀.

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Significance levels

The researcher decides ahead of time the threshold p-value at which she will conclude the evidence is sufficiently strong against H_0 .

- The threshold value is called the **significance level** of the test, or α .
- Some common significance levels are 0.05, 0.01, 0.10
- ullet When p < 0.05 we say the result is "significant at the 0.05 level."
- When p < 0.01 we say the result is "significant at the 0.01 level," etc.
- The higher is α, the "lower the bar" for rejection of H₀.
- ullet The smaller is lpha, the "higher the bar" for rejection.

Significance levels

- If $p < \alpha$, we reject H_0 in favor of H_1 .
- If $p > \alpha$, we do not reject H_0 .
- Rejection of H₀ is usually referred to as a statistically significant result, effect, or difference.
- Again, even if we cannot reject H₀, this does not mean it is true. In other words, we should not say that we "accept" H₀.
- Rejection only means the test statistic would be unlikely if H₀ were true. It is still possible for H₀ to be rejected when true; in fact, with a 0.05 significance level, this will occur in 5/100 random samples.

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The surprising history(?) of $\alpha = 0.05$

According to Twitter... (and see Goldfarb et al. 2016, p. 175):

An awareness of the history of p-values might help deflate their swollen stature and encourage more judicious use. We were surprised to learn, in the course of writing this article, that the p<0.05 cutoff was established as a competitive response to a disagreement over book royalties between two foundational statisticians. In the early 1920s, [Karl] Pearson, whose income depended on the sale of extensive statistical tables, was unwilling to allow Ronald A. Fisher to use them in his new book. To work around this barrier, Fisher created a method of inference based on only two values: p-values of 0.05 and 0.01 (Hurlbert and Lombardi, 2009).

The surprising history(?) of $\alpha = 0.05$

Fisher himself later admitted that Pearson's more continuous method of inference was better than his binary approach: "no scientific worker has a fixed level of significance at which from year to year, and in all circumstances, he rejects [null] hypotheses; he rather gives his mind to each particular case in the light of his evidence and ideas" (Hurlbert and Lombardi, 2009: 316). A fair interpretation of this history is that we use p-values at least in part because a statistician from the 1920s was afraid that sharing his work would undermine his income (Hurlbert and Lombardi, 2009). Following Fischer, we recommend that authors report p-values and refrain from emphasizing thresholds.

This characterization did (of course) kick off a fierce debate on Twitter. See also Ziliak (2008), "Guinnessometrics: The Economic Foundation's of Student's t" for related history.

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Hypothesis tests about μ

Guide to the next slide:

- Your test begins from the assumption that H_0 is true, in which case \bar{x} has the null sampling distribution (mean of μ_0).
- Suppose H₀ is not true, but some alternative is (H_A), in which case x̄ has the sampling distribution under H_A (mean of μ_A).
- We will not reject H_0 in the yellow region, even if H_0 is false.
- More on this later, in Lecture 7 on power.

Hypothesis tests about μ

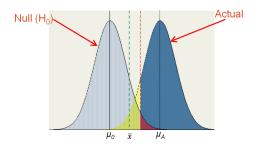


Figure: Distribution of \bar{x} under H_0 and actual distribution

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Example 2

Is average adult body temperature in the population really 98.6 degrees F? Or is it lower? Assume $\sigma=0.6$.

 $H_0: \mu = 98.6$

 $H_1: \mu < 98.6$

We sample 80 adults, and obtain a sample mean body temperature of $\bar{x}=98.4$. Use a significance level of $\alpha=0.05$ to conduct the test.

How likely is it that 80 randomly selected adults would have an $\bar{x}=98.4$ if μ were actually 98.6 (H_0)? If H_0 were true, then \bar{x} is:

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{98.4 - 98.6}{0.6 / \sqrt{80}} = -2.98$$

standard errors below μ (-2.98 is our test statistic). This is pretty unlikely (p=0.001). Because $p < \alpha$, we reject H_0 . Mean body temperature here is lower than 98.6 and the difference is *statistically significant* or *statistically different from 98.6* at the 0.05 (5%) level.

In Stata: display normal(-2.98)

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Example 2

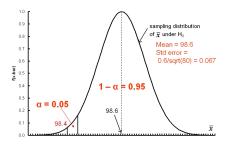


Figure: Distribution of \bar{x} under $H_0 = 98.6$

One- vs. two-sided alternatives

- The alternative hypotheses thus far have been directional, "one-tailed" or "one-sided."
- The tests are constructed so that the rejection region is in one tail of the sampling distribution:
 - ▶ Are mean math SAT scores in California higher than the U.S.?
 - ▶ Is mean adult body temperature lower than 98.6 degrees?
 - ▶ Are fewer non-white applicants awarded home loans than whites?
- "Two-tailed" or "two-sided" alternatives contain values both below and above the value stated in H₀—they are nondirectional.
- Rejecting H₀ means the parameter is significantly different from that stated in H₀, or there is some "effect."

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Example 3

Suppose you would like to test whether mean weekly expenditure on alcoholic beverages by State U undergraduates differs from the national average of \$20. (Nationally, $\sigma=9$, and you assume this is true for State U as well). You decide on a significance level of $\alpha=0.05$.

$$H_0: \mu_{stu} = 20$$

$$H_1: \mu_{stu} \neq 20$$

You take a random sample of n=64 students, and obtain a sample mean of $\bar{x}=\$17$. How likely is it that a random sample of 64 State U students would spend an average amount that differs by \$3 or more from μ , if μ were \$20? (I.e. \$17 or less, \$23 or more).

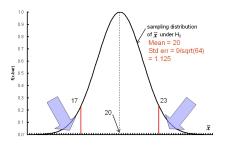


Figure: Two-tailed test example

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Example 3

If H_0 is true, the test statistic for \bar{x} is: $\frac{(17-20)}{9/\sqrt{64}} = -2.67$. Under the standard normal distribution, the probability of obtaining a test statistic this far away from the mean *in either direction* is:

$$2 \times Pr(z < -2.67) = 2 \times 0.0038 = 0.0076$$

Since $p<\alpha$, we reject H_0 in favor of H_1 . It is unlikely a random sample of 64 students will have an \bar{x} this far from the mean if $\mu=20$. Mean weekly alcohol expenditure at State U differs from the national average, and this difference is *statistically significant* at the 0.05 (5%) level.

In Stata: display 2*normal(-2.67)

Check your assumptions

As with interval estimation, we are relying on normality of the sampling distribution to conduct these tests. Need independent random sampling + a normal population distribution *or* a sufficiently large sample size (CLT).

- Example 1: random sample of math SATs. n=1,600 and σ known.
- ullet Example 2: random sample of body temperature. n=80 and σ known
- Example 3: random sample of State U undergrad expenditure on alcohol. n=64 and σ known.

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Logic of *p*-value for two-sided tests

Why do we multiply by 2 when calculating p-values for two-sided tests? In a significance test, we are deciding in advance when we will reject H_0 and how often we will reject H_0 when true. α is the probability of falling in the rejection region when H_0 is true.

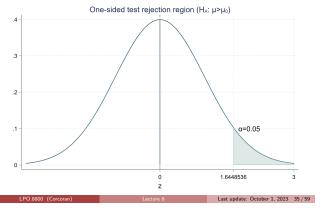
- In a <u>one-sided</u> test: you will only reject when \bar{x} is sufficiently far from μ_0 in the direction of the alternative. (You would not reject if \bar{x} were equally extreme on the other side of the distribution). α is on *one side* of the sampling distribution. We should reject when $p < \alpha$.
- In a two-sided test: you will reject when x̄ is sufficiently far from μ₀ in either direction. α is on both sides of the sampling distribution, changing the set of values that result in rejection. Now we should reject when the one-sided p < α/2, or when the two-sided p < α.

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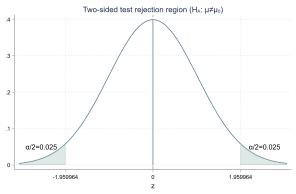
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One-sided test rejection region



Two-sided test rejection region



Logic of *p*-value for two-sided tests

Consider a test statistic of 1.75.

- In a one-sided test, we would reject H_0 if $\alpha = 0.05$. The rejection region includes 1.75 and the one-sided p-value is 0.04 < 0.05
- In a two-sided test, we would <u>not</u> reject H_0 if $\alpha=0.05$. The rejection region does not include 1.75, since it is now split between two tails. The two-sided p-value is 0.08>0.05.

There is some debate over whether one should ever use one-sided tests (two-sided tests are usually the default). One could argue, though, that if theory suggests a directional difference, a one-sided test is appropriate. A two-sided test would produce too many "false negatives." Never make the choice based on the result!

Note: some statistical tests are inherently one-sided (F-test, χ^2)

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Intervals and hypothesis tests

Confidence intervals can be used to test two-sided hypotheses. Whenever p>0.05 in a two-sided test, a 95% confidence interval for μ will contain the H_0 value of μ .

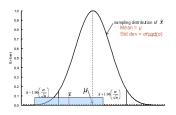


Figure: p>0.05 and confidence interval contains μ_0

Intervals and hypothesis tests

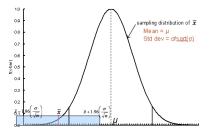


Figure: p < 0.05 and confidence interval does not contain μ_0

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Intervals and hypothesis tests

Suppose we constructed a 95% interval for mean alcohol expenditure μ in Example 3:

$$17 \pm 1.96 \left(\frac{9}{\sqrt{64}}\right) = (14.795, 19.205)$$

Our confidence interval does not support a hypothesized mean of 20—therefore we can reject H_0 . In fact, we can reject $any \mu_0$ outside the interval (14.795, 19.205). We do not reject any μ_0 within this interval.

Womens' height, revisited

Given a sample of n=121 women, $\bar{x}=64$ and $\sigma=3$, our 95% confidence interval for the population mean female height was (63.47, 64.53). Could average female height in the population be 63.5 inches?

$$H_0: \mu = 63.5$$

$$H_1: \mu \neq 63.5$$

63.5 falls within our confidence interval—thus we cannot reject this null hypothesis at the $\alpha=0.05$ level. The evidence is consistent with an H_0 of $\mu=63.5$.

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Womens' height, revisited

Given a sample of n=121 women, $\bar{x}=64$ and $\sigma=3$, our 95% confidence interval for the population mean female height was (63.47, 64.53). Could average female height in the population be **63** inches?

$$H_0: \mu = 63$$

$$H_1: \mu \neq 63$$

63 does **not** fall within our confidence interval—thus we reject H_0 at the $\alpha=0.05$ level. The evidence is **not** consistent with an H_0 of $\mu=63$.

Intervals and hypothesis tests

- A 95% confidence interval allows you to conduct a two-sided hypothesis test at the $\alpha=0.05$ significance level.
- A 99% confidence interval allows you to conduct a two-sided hypothesis test at the $\alpha = 0.01$ significance level.
- A $(1-\alpha)$ % confidence interval allows you to conduct a hypothesis two-sided test at the α significance level.

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Hypothesis test for μ when σ is unknown

All of the examples thus far have assumed that σ is known. This allowed us to assume a standard normal distribution for the test statistic z used to find a p value (if n is large). Of course, σ is rarely known. When s is used in place of σ the test statistic:

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

has a t distribution with n-1 degrees of freedom. The only implication of this change is that the p-values for the test come from the t-distribution.

You are wondering if the average local teen worker is paid above the federal minimum wage of \$7.25/hour. You take a random sample of 20 workers aged 16-19 and calculate a mean hourly wage of $\bar{x}=7.75$ and sample standard deviation of s=1.20. Your hypotheses are:

$$H_0: \mu = \$7.25$$

$$H_1: \mu > \$7.25$$

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Example 4

If H_0 is true, the sample mean of \$7.75 is:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{7.75 - 7.25}{1.20/\sqrt{20}} = 1.866$$

standard errors above the population mean. Using the t-distribution with 19 degrees of freedom, p = P(t > 1.866) = 0.039. We can reject H_0 at the 5% significance level (although not at the 1% level).

In Stata: display ttail(19, 1.866)

Hypothesis test for π

Hypothesis tests for population proportions π are conducted in a manner analagous to the tests above. For example, suppose a state's goal is to exceed last year's proficiency rate of 75% in 4th grade mathematics. The state education department tests a random sample of 100 4th graders and finds that 77.5% are proficient in math ($\hat{\pi}=0.775$). It has decided on a significance level of $\alpha=0.05$. The null and alternative hypotheses are:

 $H_0: \pi = 0.75$

 $H_1: \pi > 0.75$

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Hypothesis test for π

The test statistic is the number of standard errors the realized $\hat{\pi}$ is from the mean under H_0 . Importantly, the standard error in the denominator is calculated using π_0 (the population proportion under H_0), rather than $\hat{\pi}$, since we care about the distribution of the test statistic when H_0 is true:

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1 - \pi_0)}{n}}} = \frac{0.775 - 0.75}{\sqrt{\frac{0.75(1 - 0.75)}{100}}} = 0.577$$

Using the standard normal table, the probability of obtaining a test statistic this far above the mean when H_0 is true is p=P(z>0.577)=0.282. Since this value is not that unlikely (and $p>\alpha$), we cannot reject the null hypothesis that 4th grade proficiency remains at last year's rate of 75%.

In Stata: display 1-normal(0.577)

Check your assumptions

Again we are relying on normality of the sampling distribution to conduct these tests. Need independent random sampling. Our rule of thumb for normality of the sampling distribution of $\hat{\pi}$ is $n\pi \geq 10$ and $n(1-\pi) \geq 10$.

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Statistical tests for μ using Stata

One can use the mean command in Stata to generate a $(1-\alpha)$ % confidence interval for μ , and then use this interval to conduct a hypothesis test with a α significance level. For example:

	. mean achmat(08			
Mean estimation		on	Numbe	er of obs =	500
		Mean	Std. Err.	[95% Conf.	Interval]
	achmat08	56.59102	. 4176799	55.77039	57.41165

Statistical tests for μ using Stata

Alternatively, the ttest command will conduct hypothesis tests for a specified null hypothesis. For example:

. ttest achmat08==57.5

One-sample Variable	Obs	Mean	Ctd Frr	Std. Dev.	195% Conf	Intoruali
variable	ODS	riean	Stu. EII.	sta. Dev.	[33% COHI.	intervar
achmat08	500	56.59102	.4176799	9.339608	55.77039	57.41165
mean = 1	mean (achm	nat08)			t	2.1763
Ho: mean = .	57.5			degrees	of freedom	= 499
Ha: mean	< 57.5	H	la: mean != 5	7.5	Ha: me	an > 57.5
Dr (T < +)			TI S IEIX -			1 - 0 0050

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Statistical tests for π using Stata

One can use the proportion command in Stata to generate a $(1-\alpha)$ % confidence interval for π , and then use this interval to conduct a hypothesis test with a α significance level. For example:

. proportion advmath8, citype(normal)

Proportion estimation Number of obs

491

			Normal		
	Proportion	Std. Err.	[95% Conf.	Interval]	
advmath8					
no	.5397149	.0224934	.4955195	.5839103	
yes	.4602851	.0224934	.4160897	.5044805	

Statistical tests for π using Stata

Alternatively, the prtest command will conduct hypothesis tests for a specified null hypothesis. For example:

```
. prtest advmath == 0.50
One-sample test of proportion
                                               Number of obs
                                                                          491
   Variable
                    Mean
                           Std. Err.
                                                         [95% Conf. Interval]
   advmath8
                 .4602851
                           .0224934
                                                         .4161989
                                                                     .5043714
   p = proportion(advmath8)
                                                                 z = -1.7600
Ho: p = 0.5
    Ha: p < 0.5
                               Ha: p != 0.5
                                                              Ha: p > 0.5
Pr(Z < z) = 0.0392
                          Pr(|Z| > |z|) = 0.0784
                                                         Pr(Z > z) = 0.9608
```

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One-sample t-test calculator in Stata

Stata can calculate test statistics and *p*-values without data, given requisite values:

ttesti #obs #mean #sd #val [, level(#)]

- #obs is the sample size
- #mean is the sample mean
- #sd is the sample standard deviation
- #val is the hypothesized value of μ under H_0
- level(#) is an option for confidence level

Note Stata assumes an unknown σ in ttesti

One-sample t-test calculator in Stata

Try:

• Example 1: $\bar{x} = 507$, $\mu_0 = 500$, $\sigma = 100$, n = 1600

• Example 1: $\bar{x} = 502$, $\mu_0 = 500$, $\sigma = 100$, n = 1600

• Example 2: $\bar{x} = 98.4$, $\mu_0 = 98.6$, $\sigma = 0.6$, n = 80

• Example 3: $\bar{x} = 17$, $\mu_0 = 20$, $\sigma = 9$, n = 64 (two-tailed)

• Example 4: $\bar{x} = 7.75$, $\mu_0 = 7.25$, s = 1.20, n = 20

LPO.8800 (Corcoran)

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One-sample *t*-test calculator in Stata

. ttesti 1600 507 100 500

One-sample t test

	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
×	1,600	507	2.5	100	502.0964	511.9036
mean =	= mean(x) = 500			degrees	t = of freedom =	
	an < 500) = 0.9974		Ha: mean != 5			an > 500 = 0.0026

One-sample proportion test calculator in Stata

Stata can calculate test statistics and p-values without data, given requisite values for a significance test for proportions:

prtesti #obs1 #p1 #p2 [, level(#) count]

- #obs1 is the sample size
- #p1 is the sample proportion
- ullet #p2 is the hypothesized value of π under H_0
- level(#) is an option for confidence level
- · count is an option to use counts rather than proportions

Note: the reported standard error—used for the interval estimate—uses $\hat{\pi}$. The z statistic uses π_0 in the standard error, as expected:

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

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One-sample proportion test calculator in Stata

Try the example: $\hat{\pi} = 0.775$, $\pi_0 = 0.750$, n = 100

. prtesti 100 0.775 0.750

One-sample test of proportion			x: Number of obs = 10	
	Mean Std. Err.		[95% Conf. Interval	
x	. 775	.0417582	.6931554 .856844	
p = proport Ho: p = 0.75	ion(x)		z = 0.577	
Ha: $p < 0.7$ Pr(Z < z) = 0.		Ha: $p != 0.79$ Pr($ Z > z $) = 0		

Next time

- What to report? Statistical significance, p-values, confidence intervals (Romer, 2020; Greenland et al., 2016)
- Practical vs. statistical significance
- Effect size (Kraft, 2020; Hill et al., 2008)
- Type I vs. Type II errors
- Power of the test
- Power analysis