12. Multivariate analyses: introduction

LPO.8800: Statistical Methods in Education Research

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Last time

- Bivariate regression
- Prediction equation, predicted values, residuals (prediction errors)
- Ordinary least squares (OLS)
- Interpreting regression intercept and slope
- Assessing goodness of fit (R^2)
- Conditional mean interpretation of regression
- Inference about the population slope: confidence intervals and hypothesis tests
- Regression diagnostics with residuals

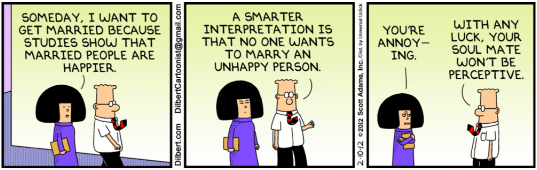
Correlation vs. causality, revisited

Generally speaking, regression slopes (and correlations) *cannot* be interpreted as *causal*. Examples:

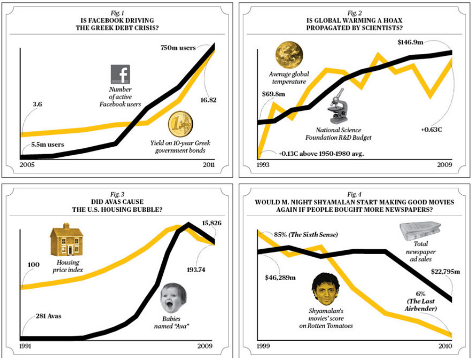
- Russian cholera epidemic: peasants observed that in communities with lots of doctors, there were lots of cholera cases; doctors were murdered.
- SAT prep courses: in 1988 Harvard interviewed its freshmen and found that those who took SAT coaching courses scored 63 points lower than those who did not.
  - A dean concluded that the SAT courses were unhelpful and that "the coaching industry is playing on parental anxiety."

Causal questions imply "all else is held equal."

Correlation vs. causality, revisited

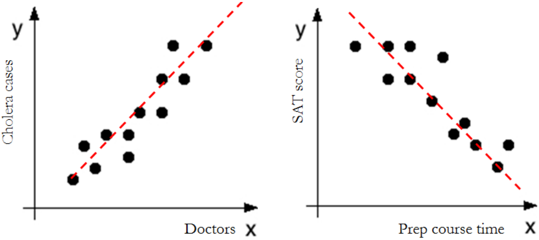


Correlation vs. causality, revisited



Correlation vs. causality, revisited

Imagine collecting data and conducting a simple regression analysis for each case:



In the lefthand figure, each data point is a community. In the righthand figure, each data point is a college applicant.

Correlation vs. causality, revisited

There is clearly an *association* between these pairs of variables, but can we say that variation in X is *causing* variation in Y ($\mathbf{X} \rightarrow \mathbf{Y}$)? There are three possible explanations for a correlation between two variables X and Y: X is causing Y, Y is causing X, or some other factor is causing both.

Criteria for a causal relationship:

- Association between the two variables
- An appropriate time ordering
- Elimination of alternative explanations

Correlation vs. causality, revisited

Considering the above two examples:

- Russian cholera epidemic: it is unlikely doctors (X) caused the cholera cases (Y), since the presence of cholera preceded the arrival of the doctors ($Y \rightarrow X$).

- SAT prep courses: it is *possible* that the prep course worsened SAT performance (the time ordering is appropriate). But it is more likely a third factor explains *both* enrollment in the prep course *and* low SAT scores (e.g., test anxiety, poor prior academic preparation). The association may be **spurious**. One would need to eliminate alternative explanations before making a causal connection.

Correlation vs. causality, revisited

Rigby

Enough with the wind already

Received April 1

Ever since they installed all those big fans up on the hill it's become even windier. Whose bright idea was that?

I've noticed when they're off, we get a nice calm spell. Please turn them off, at least on weekends. (Word count: 40)

JEFF FORBES
Idaho Falls

• **Guest columns, solicited:** 450 words max • **Guest column**

Experimental and observational data

Eliminating alternative explanations can be very difficult to do in social science and education research. The researcher is typically working with *observational* data, and has no control over assignment to "treatment" conditions of interest. Consider again these questions:

- Does smoking cause lung cancer?
- Would a smaller class size improve learning?
- Does education increase labor market productivity and earnings?
- Is parental divorce detrimental to childrens' outcomes?
- Do mask mandates reduce the transmission of infectious diseases?

Experimental and observational data

This is in contrast to the medical researcher who can randomly assign subjects to receive a new drug or a placebo. With this study design, she controls the time ordering, and can confidently attribute any systematic differences in the subjects' outcomes to the drug (and not due to some third factor).

Eliminating alternative explanations

In the absence of random assignment, the elimination of alternative explanations (**confounders**) is difficult to do, and depends on sound research design, data availability, and a good theoretical understanding of factors that affect variation in the outcome Y.

Note: outliers and anecdotal examples of contradictory cases are **not** sufficient for ruling out causal relationships! Causal effects are a description of how X affects Y *on average*, not in a deterministic sense.

- A high-poverty school that is "beating the odds" does not demonstrate that poverty has no effect on academic achievement.
- A smoker that lives to 102 is not proof that smoking does not cause lung cancer.

Controlling for other variables

In practice how does one eliminate alternative explanations for the association between X and Y? **Controlling** involves using statistical techniques to find the correlation between two variables, holding the value of other variables constant.

The variables we wish to remove the effects of—i.e., control for—are called **control variables** or **covariates** (e.g., $X_2, X_3, ..., X_k$).

- We statistically control for a third variable X_2 by examining the relationship between X_1 and Y *conditional on* X_2 (i.e., for fixed values of X_2).

Example 1

Does computer ownership improve 8th grade math achievement?

```
. tabstat achmat08, by(computer) stat(mean n)

Summary for variables: achmat08
     by categories of: computer (computer owned by family in eighth grade?)

computer |      mean         N
---------+--------------------
      no |  54.94897       263
     yes |  58.41321       237
---------+--------------------
   Total |  56.59102       500
------------------------------

. reg achmat08 computer

      Source |       SS       df       MS              Number of obs =     500
-------------+------------------------------           F(  1,   498) =   17.73
       Model |  1496.05776     1  1496.05776           Prob > F      =  0.0000
    Residual |  42030.8486   498  84.3992944           R-squared     =  0.0344
-------------+------------------------------           Adj R-squared =  0.0324
       Total |  43526.9064   499  87.2282693           Root MSE      =  9.1869

------------------------------------------------------------------------------
    achmat08 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    computer |   3.464233   .8228153     4.21   0.000     1.847616    5.080851
       _cons |   54.94897   .5664891    97.00   0.000     53.83597    56.06198
------------------------------------------------------------------------------
```

Example 1

The association between computer ownership and math achievement could be spurious, explained by a third factor correlated with computer *and* math achievement. Let's try "controlling" for SES using 2 groups (low or high):

```
. egen ses2=cut(ses), group(2)

. * the above command creates a new variable 'ses2' that splits 'ses' into two equal-sized groups (low and high)

. table computer ses2, contents(mean achmat08 n achmat08)
```

computer owned by family in eighth grade?	ses2 0	1
no	53.5129 169	57.53085 94
yes	55.46333 78	59.86031 159

The 3.46 point "effect" of computer ownership on math achievement is smaller after conditioning on SES. For high SES students, the "effect" is 2.32 points; for low SES students, 1.95 points.

Example 2

Do AP courses improve high school math achievement?

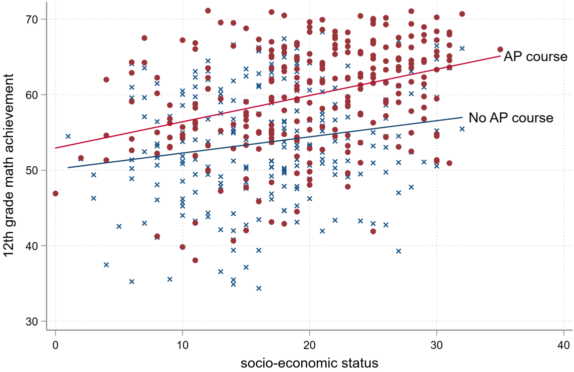
```
. reg achmat12 approg

      Source |       SS           df       MS      Number of obs   =       493
-------------+----------------------------------   F(1, 491)       =     88.17
       Model |  4688.50685         1  4688.50685   Prob > F        =    0.0000
    Residual |   26110.315       491   53.177831   R-squared       =    0.1522
-------------+----------------------------------   Adj R-squared   =    0.1505
       Total |  30798.8219       492  62.5992314   Root MSE        =    7.2923

------------------------------------------------------------------------------
    achmat12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      approg |   6.175654   .6577047     9.39   0.000     4.883391    7.467917
       _cons |   53.69983   .4767134   112.65   0.000     52.76318    54.63648
------------------------------------------------------------------------------
```

Example 2

How does AP course taking vary with SES (quintiles)? How does mean 12th grade math achievement vary with AP *conditional* on SES quintile?

		Mean:			Count:	
SESquint	took AP	no AP	AP	**Diff**	no	yes
1	0.409	52.2	55.7	**3.5**	55	38
2	0.385	52.3	57.1	**4.9**	56	35
3	0.465	55.4	59.4	**4.1**	53	46
4	0.610	54.3	60.4	**6.1**	39	61
5	0.718	55.3	62.9	**7.6**	31	79

Example 2



Controlling for other variables

In each of these cases we would like a single estimate that represents the average difference in 12th grade math achievement *conditional on* SES. This implies some kind of weighted average.

Multiple regression is one way of obtaining such an average.

Multiple regression

Multiple regression allows one to statistically control for other explanatory variables that are ignored in simple regression. With 2 explanatory variables the best fit "line" is:

$$\hat{y} = a + b_1x_1 + b_2x_2$$

With k explanatory variables:

$$\hat{y} = a + b_1x_1 + b_2x_2 + ... + b_kx_k$$

There is now an intercept and k slope coefficients to find.

Multiple regression

As before, the best fit "line" is the one where the intercept a and slope coefficients $b_1, b_2, ..., b_k$ minimize the sum of the squared deviations between the actual data points and the predicted values:

$$\min_{a,\,b} \sum_{i=1}^{n} (y_i - \widehat{y}_i)^2$$

$$\min_{a,\,b} \sum_{i=1}^{n} (y_i - a - b_1 x_1 - b_2 x_2 - ... - b_k x_k)^2$$

Multiple regression: example 1

To implement multiple regression in Stata, continue to use the `regress` command, and include the additional explanatory variables in your variable list:

```
. reg achmat08 computer i.ses2

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(2, 497)       =     21.59
       Model |  3478.87468         2  1739.43734   Prob > F        =    0.0000
    Residual |  40048.0317       497  80.5795407   R-squared       =    0.0799
-------------+----------------------------------   Adj R-squared   =    0.0762
       Total |  43526.9064       499  87.2282693   Root MSE        =    8.9766

------------------------------------------------------------------------------
    achmat08 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    computer |   2.149567   .8465355     2.54   0.011     .4863372    3.812796
      1.ses2 |   4.193894   .8454511     4.96   0.000     2.532795    5.854992
       _cons |   53.45002    .630632    84.76   0.000     52.21098    54.68905
------------------------------------------------------------------------------
```

Multiple regression: example 2

Using SES quintiles as control variables:

```
. reg achmat12 approg i.sesquint

      Source |       SS           df       MS      Number of obs   =       493
-------------+----------------------------------   F(5, 487)       =     26.61
       Model |  6608.74334         5  1321.74867   Prob > F        =    0.0000
    Residual |  24190.0785       487  49.6716191   R-squared       =    0.2146
-------------+----------------------------------   Adj R-squared   =    0.2065
       Total |  30798.8219       492  62.5992314   Root MSE        =    7.0478

------------------------------------------------------------------------------
    achmat12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      approg |   5.218614    .658095     7.93   0.000     3.925558     6.51167
             |
    sesquint |
          2  |   .6226858   1.039324     0.60   0.549    -1.419427    2.664799
          3  |   3.341469   1.018429     3.28   0.001     1.340412    5.342526
          4  |    3.37997   1.023907     3.30   0.001     1.368148    5.391791
          5  |    5.52072   1.013494     5.45   0.000     3.529358    7.512081
             |
       _cons |   51.49928   .7787234    66.13   0.000      49.9692    53.02935
------------------------------------------------------------------------------
```


Multiple regression: example 2

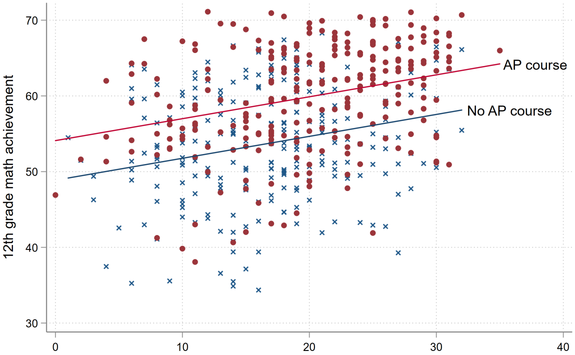
Using continuous SES as control variable:

```
. reg achmat12 approg ses

      Source |       SS           df       MS      Number of obs   =       493
-------------+----------------------------------   F(2, 490)       =     66.29
       Model |  6558.71774         2  3279.35887   Prob > F        =    0.0000
    Residual |  24240.1041       490  49.4696003   R-squared       =    0.2130
-------------+----------------------------------   Adj R-squared   =    0.2097
       Total |  30798.8219       492  62.5992314   Root MSE        =    7.0335

------------------------------------------------------------------------------
    achmat12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      approg |   5.227734   .6528237     8.01   0.000     3.945055    6.510413
         ses |   .2895499    .047092     6.15   0.000     .1970227    .3820771
       _cons |   48.86905   .9103236    53.68   0.000     47.08043    50.65767
------------------------------------------------------------------------------
```

Note: this regression constrains the slope on *ses* to be the same for AP and non-AP students. That is, it finds the best fit regression equation where the slope is the same for these two groups.

Multiple regression: example 2

Using continuous SES as control variable:



Multiple regression: example 3

Regression of monthly wages on experience, controlling for education.

```
. regress wage exper

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  1,   933) =    0.00
       Model |  732.242855     1  732.242855           Prob > F      =  0.9467
    Residual |   152715436   933  163682.139           R-squared     =  0.0000
-------------+------------------------------           Adj R-squared = -0.0011
       Total |   152716168   934  163507.675           Root MSE      =  404.58

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       exper |   .2024031   3.026148     0.07   0.947    -5.736443    6.141249
       _cons |   955.6049    37.4111    25.54   0.000     882.1853    1029.025
------------------------------------------------------------------------------

. regress wage exper educ

      Source |       SS       df       MS              Number of obs =     935
-------------+------------------------------           F(  2,   932) =   73.26
       Model |  20747023.1     2  10373511.5           Prob > F      =  0.0000
    Residual |   131969145   932  141597.795           R-squared     =  0.1359
-------------+------------------------------           Adj R-squared =  0.1340
       Total |   152716168   934  163507.675           Root MSE      =  376.29

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       exper |   17.63777   3.161775     5.58   0.000     11.43275    23.84279
        educ |   76.21639   6.296604    12.10   0.000     63.85922    88.57355
       _cons |  -272.5279   107.2627    -2.54   0.011    -483.0323   -62.02344
------------------------------------------------------------------------------
```

Example: multiple regression

The slope coefficients are now interpreted as **marginal** or **partial** effects: the linear relationship between Y and X_1, *conditional on* (or "holding constant") X_2 and any other included control variables.

- Conditional on years of education (holding constant years of education), we predict that an additional year of work experience is associated with $17.64 additional monthly earnings.
- Conditional on years of work experience (holding constant work experience), we predict that an additional year of education is associated with $76.22 additional monthly earnings.

The *prediction equation* can be used to find the "best prediction" of Y given values of $X_2, ..., X_K$.

Example: multiple regression

For example, let years of experience be $X_1 = 10$ and years of education completed be $X_2 = 14$. Our best prediction of monthly earnings is:

$$\hat{y} = -272.53 + 17.64 * 10 + 76.22 * 14 = 970.95$$

Causality, revisited

Can multiple regression coefficients can be interpreted as causal? In most cases, unfortunately not. While the regression controls for some confounders, there are likely others. This is where careful research design comes in (see later courses!)

When x_1 and x_2 are uncorrelated

When x_1 and x_2 are uncorrelated, the OLS estimators of b_1 and b_2 are:

$$\hat{b}_1 = r_{y1}\frac{s_y}{s_1}$$

$$\hat{b}_2 = r_{y2}\frac{s_y}{s_2}$$

where r_{y1} is the correlation between y and x_1, and r_{y2} is the correlation between y and x_2. (s_1 is the standard deviation of x_1, and s_2 is the standard deviation of x_2). Notice these are equivalent to the formula for $\hat{b}$ in the simple regression case. The r_{y1} and r_{y2} are sometimes called **zero-order correlations**.

When x_1 and x_2 are uncorrelated

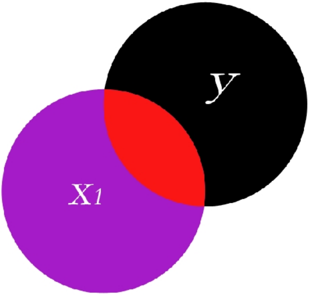
In multiple regression, R^2 can still be used as a measure of fit, interpreted in the same way: the fraction of overall variation in y that is explained by the prediction equation. When x_1 and x_2 are uncorrelated, R^2 is simply:

$$R^2 = r_{y1}^2 + r_{y2}^2$$

(the sum of the two squared zero-order correlations)

- R^2 is the *coefficient of determination*
- R–the square root of R^2–is the *multiple correlation*

Venn diagram with one explanatory variable

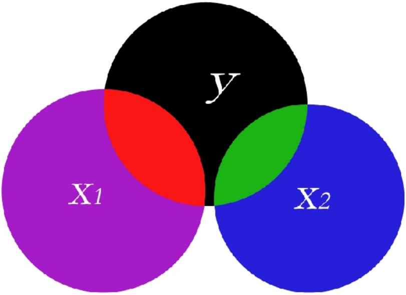


Venn diagram with one explanatory variable

The circle labeled y represents variation in y, and the circle labeled x_1 represents variation in x_1.

- Think of the overlap (red) as variation in y "explained" by variation in x_1
- The red area represents correlation between x_1 and y: information used by the regression to estimate $\hat{b}_1$
- The black area is variation in y *unexplained* by variation in x_1 ("residual" variation)
- The proportion of y covered by x_1 represents the R^2

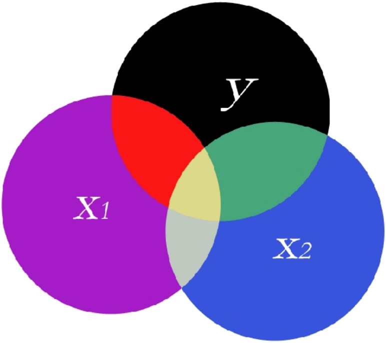
Venn diagram with two explanatory variables - 1



Venn diagram with two explanatory variables - 1

- Think of the overlap between y and x_1 (red) as variation in y "explained" by variation in x_1
- Think of the overlap between y and x_2 (green) as variation in y "explained" by variation in x_2
- x_1 and x_2 do not overlap (they are uncorrelated), so it is easy to attribute variation in y separately to x_1 and x_2
- The red area represents information used by the regression to estimate $\hat{b}_1$
- The green area represents information used by the regression to estimate $\hat{b}_2$
- The black area is variation in y unexplained by x_1 or x_2
- The proportion of y covered by x_1 and x_2 represents R^2

Venn diagram with two explanatory variables - 2



Venn diagram with two explanatory variables - 2

- In this case x_1 and x_2 overlap—they are *correlated* (represented by the yellow area), thus it is not as clear how to attribute variation in y separately to x_1 and x_2
- The red area represents the unique information used by the regression to estimate $\hat{b}_1$
- The green area represents the unique information used by the regression to estimate $\hat{b}_2$
- Both the red and green areas are *smaller* than those in example 1—we have less certainty about how much of y can be attributed to each explanatory variable

When x_1 and x_2 are correlated

When x_1 and x_2 are correlated, the OLS estimators of b_1 and b_2 can be written:

$$\hat{b}_1 = \left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)\left(\frac{s_y}{s_1}\right)$$

$$\hat{b}_2 = \left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)\left(\frac{s_y}{s_2}\right)$$

where r_{12} is the correlation between x_1 and x_2 (and other terms were defined previously). Notice what happens if $r_{12} = 0$ (i.e. if there is no correlation between x_1 and x_2).

Example: private school attendance and math achievement

```
. reg achmat12 private

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =     10.92
       Model |  665.476286         1  665.476286   Prob > F        =    0.0010
    Residual |  30351.3075       498  60.9464005   R-squared       =    0.0215
-------------+----------------------------------   Adj R-squared   =    0.0195
       Total |  31016.7838       499  62.1578833   Root MSE        =    7.8068

------------------------------------------------------------------------------
    achmat12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     private |   2.630139   .7959513     3.30   0.001     1.066303    4.193976
       _cons |   56.22278   .4058571   138.53   0.000     55.42538    57.02019
------------------------------------------------------------------------------

. reg achmat12 private ses

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(2, 497)       =     29.23
       Model |  3264.62388         2  1632.31194   Prob > F        =    0.0000
    Residual |  27752.1599       497  55.8393559   R-squared       =    0.1053
-------------+----------------------------------   Adj R-squared   =    0.1017
       Total |  31016.7838       499  62.1578833   Root MSE        =    7.4726

------------------------------------------------------------------------------
    achmat12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     private |   .5417838    .821064     0.66   0.510    -1.071401    2.154968
         ses |   .3552102   .0520643     6.82   0.000     .2529169    .4575035
       _cons |   50.21781   .9620882    52.20   0.000     48.32755    52.10807
------------------------------------------------------------------------------
```


Example: private school attendance and math achievement

```
. corr achmat12 private ses
(obs=500)

             | achmat12  private      ses
-------------+---------------------------
    achmat12 |   1.0000
     private |   0.1465   1.0000
         ses |   0.3232   0.3728   1.0000

. summ achmat12 private ses

    Variable |        Obs        Mean    Std. Dev.
-------------+------------------------------------
    achmat12 |        500    56.90662    7.884027
     private |        500         .26    .4390735
         ses |        500      18.434    6.924271
```

$$\hat{b}_1 = \left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)\left(\frac{s_y}{s_1}\right)$$

$$\hat{b}_1 = \left(\frac{0.1465 - 0.3232 * 0.3728}{1 - 0.3728^2}\right)\left(\frac{7.884}{0.439}\right) = 0.542$$

Example: private school attendance and math achievement

```
. corr achmat12 private ses
(obs=500)

             | achmat12  private      ses
-------------+---------------------------
    achmat12 |   1.0000
     private |   0.1465   1.0000
         ses |   0.3232   0.3728   1.0000

. summ achmat12 private ses

    Variable |        Obs        Mean    Std. Dev.
-------------+--------------------------------------
    achmat12 |        500    56.90662    7.884027
     private |        500         .26    .4390735
         ses |        500      18.434    6.924271
```

$$\hat{b}_2 = \left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)\left(\frac{s_y}{s_2}\right)$$

$$\hat{b}_2 = \left(\frac{0.3232 - 0.1465 * 0.3728}{1 - 0.3728^2}\right)\left(\frac{7.884}{6.924}\right) = 0.3552$$

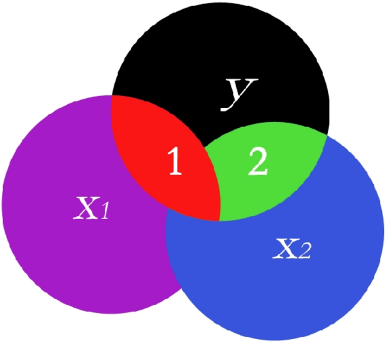
When x_1 and x_2 are correlated

With two explanatory variables the R^2 can be written as:

$$R^2 = r_{y1}^2 + r_{y2|1}^2$$

- First part: the proportion of variation in y explained by x_1
- Second part: the proportion of variation in y explained by x_2 *beyond that explained by* x_1 (a *semi-partial* correlation)

When x_1 and x_2 are correlated



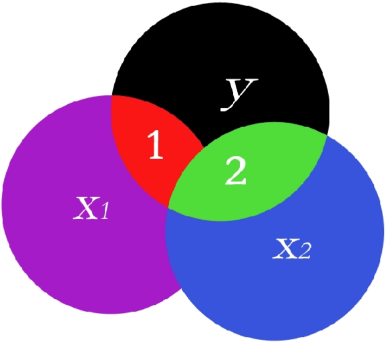
When x_1 and x_2 are correlated

Equivalently, the R^2 can be written as:

$$R^2 = r_{y2}^2 + r_{y1|2}^2$$

- First part: the proportion of variation in y explained by x_2
- Second part: the proportion of variation in y explained by x_1 *beyond that explained by* x_2 (a *semi-partial* correlation)

When x_1 and x_2 are correlated



Semi-partial correlations

The correlations $r^2_{y2|1}$ and $r^2_{y1|2}$ are called *semi-partial* or *part* correlations. They represent the correlation observed between y and that part of x_1 (or x_2) that is uncorrelated with x_2 (or x_1).

Multiple regression and R^2

Some facts about multiple regression and R^2:

- R^2 still ranges between 0 and 1
- R^2 will be high when the x's are highly correlated with y
- R^2 will not fall below the highest R^2 with an individual x
- R^2 cannot *decrease* when additional xs are added to the regression equation
- R^2 will be larger when the explanatory variables are not *redundant*—i.e. their intercorrelation is low
- There is usually diminishing returns to additional explanatory variables (a greater chance of redundancy)

Adjusted R^2

The calculated R^2 tends to overestimate the population R^2 (it is upwardly biased). The smaller is N relative to the number of explanatory variables K, the more R^2 will be inflated. An **adjusted $\mathbf{R^2}$** is often used instead:

$$R^2_{ADJ} = 1 - \left(1 - R^2\right)\frac{(N-1)}{N-K-1}$$

Holding N constant, the adjusted R^2 "penalizes" you for including additional explanatory variables K.

Multicollinearity

Multicollinearity is the condition when explanatory variables in a regression are highly correlated. The consequence of this is that it becomes more difficult to discern how much of the variation in y is "due to" each individual x. This is a bigger problem the smaller the sample size.

Semi-partial (part) correlations

The **semi-partial (or part) correlation** between y and x_1 is the correlation observed between y and that part of x_1 that is uncorrelated with the other x variables.

- The *square* of the semi-partial correlation is the amount by which R^2 decreases when that explanatory variable is excluded.
- It is also the proportion of the variation in y that is explained by x_1 only
- This can be used to assess the relative importance of the explanatory variables (in terms of independent predictive power).
- Could be used to guide model specification.

Can obtain semi-partial correlations in Stata using `pcorr y x1 x2`

Semi-partial (part) correlations

Try this using the math achievement and private school example above.

- Regress *achmat08* on *ses* and *private*, note R^2 (0.1053)
- Regress *achmat08* on *ses* alone, note R^2 (0.1045)
- Regress *achmat08* on *private* alone, note R^2 (0.0215)
- Get squared semi-partial correlations `pcorr achmat08 private ses`

```
. pcorr achmat12 private ses
(obs=500)

Partial and semipartial correlations of achmat12 with

               Partial   Semipartial      Partial   Semipartial   Significance
   Variable    Corr.         Corr.      Corr.^2       Corr.^2          Value
------------+-------------------------------------------------------------------
    private    0.0296        0.0280       0.0009        0.0008         0.5097
        ses    0.2926        0.2895       0.0856        0.0838         0.0000
```