Statistical Methods in Education Research
Vanderbilt University
Prof. Sean P. Corcoran

October 3, 2023
40 total points

Problem Set 5 *Solutions*

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1. (**2 points**) When a recent survey asked whether the government should impose strict laws to make industry do less damage to the environment, a 95% confidence interval for the population proportion responding *yes* was (0.87, 0.90). Would a 99% confidence interval be wider or narrower? Carefully explain why.

   A 99% confidence interval would be **wider**. By definition, a 99% confidence interval contains the true population mean in 99% of random samples, while a 95% confidence interval contains the true population mean in 95% of random samples. For a given sampling distribution of $\hat{\pi}$, greater confidence requires a wider range of possible values.

2. (**4 points**) An education school wants to estimate the mean annual salaries of the school's former students 5 years after graduation. A random sample of 25 such graduates found a sample mean of \$51,288 and a sample standard deviation of \$5,736. Assuming that the population distribution is normal, find a 90% confidence interval for the population mean. Show your work.

   Here n=25, $\bar{x} = 51288$ and $s = 5736$. Although the population distribution is normal, its standard deviation σ is unknown and estimated using the sample standard deviation s. Thus the confidence interval requires a t statistic. With $df = 24$ and $\alpha/2 = 0.05$ (for a 90% confidence interval), the t value used is 1.711 (`display invttail(24,0.05)`), resulting in the confidence interval:

   $$51288 \pm 1.711 \left(\frac{5736}{\sqrt{25}}\right) = (49325, 53251)$$

3. (**6 points**) A graduate school admissions officer has determined that historically, applicants have undergraduate grade point averages that are normally distributed with standard deviation 0.45. From a random sample of 25 applications for the current year, the sample mean grade point average is 3.20.

   (a) Find a 95% confidence interval for the population mean.

Here n=25, $\bar{x} = 3.20$ and $\sigma = 0.45$. Using the z value of 1.96 associated with a 95% confidence interval, the interval estimate is:

$$3.20 \pm 1.96\left(\frac{0.45}{\sqrt{25}}\right) = (3.0236, 3.3764)$$

(b) Based on this sample, a statistician computes a confidence interval for the population mean extending from 3.06 to 3.34. What is the *confidence level* associated with this interval? Show your work or explain how you found your answer.

In this confidence interval the upper bound was calculated as:

$$3.34 = 3.20 + z\left(\frac{0.45}{\sqrt{25}}\right)$$

Solving for z we get 1.556. A z value of 1.556 corresponds to a confidence level of about 88%. $P(z > 1.556) \approx 0.06$ multiplied by 2 is an $\alpha \approx 0.12$.

4. (**3 points**) Find the t value that would be multiplied by the standard error to form a:

(a) 95% confidence interval with a sample size of 5

(b) 95% confidence interval with a sample size of 15

(c) 95% confidence interval with a sample size of 25

(d) 95% confidence interval with degrees of freedom of 25

(e) 99% confidence interval with degrees of freedom of 25

(f) 90% confidence interval with a sample size of 500

t-values can be obtained in Stata using `display invttail(df,p)` where *df* is the degrees of freedom, and p is the right tail probability. (You can also use online lookup tables or the table in the textbook). If the interval has a confidence level $(1 - \alpha)$, then the probability p in each tail is $\alpha/2$. For example, for a 95% confidence interval we find the t-value for which there is an 0.025 probability in the right tail. We can denote this $t_{\alpha/2}$.

(a) for n=5 and df=4, $t_{0.025} =$ **2.777**

(b) for n=15 and df=14, $t_{0.025} =$ **2.145**

(c) for n=25 and df=24, $t_{0.025} =$ **2.064**

(d) for df=25, $t_{0.025} =$ **2.060**

(e) for df=25, $t_{0.005}$ = **2.787**. Notice $\alpha/2 = 0.005$ here.

(f) for n=500 and df=499, $t_{0.05}$ = **1.648**. Notice $\alpha/2 = 0.05$ here and that this t value is quite close to the z value used when p=0.05. As the sample size gets large, the t distribution gets closer to the standard normal (z).

5. (**6 points**) An estimate is needed of the mean travel time from home to school in a large urban school district. The estimate should be correct to within 2 minutes in 95% of random samples. A previous study of school commuting time suggests that 15 minutes is a reasonable approximation for the standard deviation (σ) of commuting time.

   (a) How large of a sample of families is needed to meet this requirement?

   We would like the margin of error for the confidence interval to be 2 minutes:

   $$me = 2 = 1.96 * \frac{15}{\sqrt{n}}$$

   Solving for n we get a minimum sample size of about 216.

   (b) A random sample is selected of the size you reported in part (a). The sample has a standard deviation of 9 minutes, rather than 15. What is the margin of error for a 95% confidence interval for the mean travel time to school?

   Replacing the guessed standard error of 15 with $s = 9$, the margin of error is:

   $$me = 1.96 * \frac{9}{\sqrt{216}} = 1.20$$

   I am using a z of 1.96 instead of t only under the assumption that my n is large enough that $t \approx z$

6. (**6 points**) A question in the General Social Survey asks whether respondents agree or disagree with the following statement: "It is much better for everyone involved if the man is the achiever outside the home and the woman takes care of the home and family." The sample proportion agreeing was 0.36 in 2004 (n=883).

   (a) Show that the estimated standard error for the sample proportion was 0.016.

$$\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}} = \sqrt{\frac{0.36(1-0.36)}{883}} = 0.016$$

.

(b) Show that the margin of error for a 95% confidence interval was 0.03.

$1.96 * 0.016 = 0.030$ (that is, three percentage points)

(c) Construct the 95% confidence interval and interpret it.

$0.36 \pm 1.96 * 0.016 = (\mathbf{0.329}, \mathbf{0.391})$. Over repeated samples, a confidence interval calculated in this way will contain the true population mean 95% of the time. Thus, we have 95% confidence that this interval contains μ.

7. (**8 points**) For this question, use the Stata dataset *Grade4_classrooms.dta* on Github. This file represents a random sample of 4th grade classrooms located in urban school districts in Texas. Each observation is a classroom, and the variables either describe the teacher (e.g., total teaching experience, teacher race/ethnicity), or are an average for the classroom (e.g., reading z-scores, % of students who are economically disadvantaged). All data are from 2006.

```
use https://github.com/spcorcor18/LPO-8800/raw/main/data/Grade4_classrooms.dta, clear
```

(a) Inspect the histograms for the variables *totexp* (the teacher's total years of experience), *maplus* (an indicator of whether the teacher holds a master's degree or higher), and *readz_class* (the mean z-score in reading for the students in the class). How would you describe the shape of these distributions? Note: the mean z-score is based on students' position in the statewide distribution of test-takers. **See attached log**

(b) Provide 90% confidence intervals for each of the variables listed in part (a), and interpret each in words. Note that *maplus* is a binary (dichotomous) variable. **See attached log**

(c) For purposes of constructing confidence intervals, are you concerned about the normality (or lack thereof) in the distributions viewed in part (a)? Explain why or why not. **See attached log**

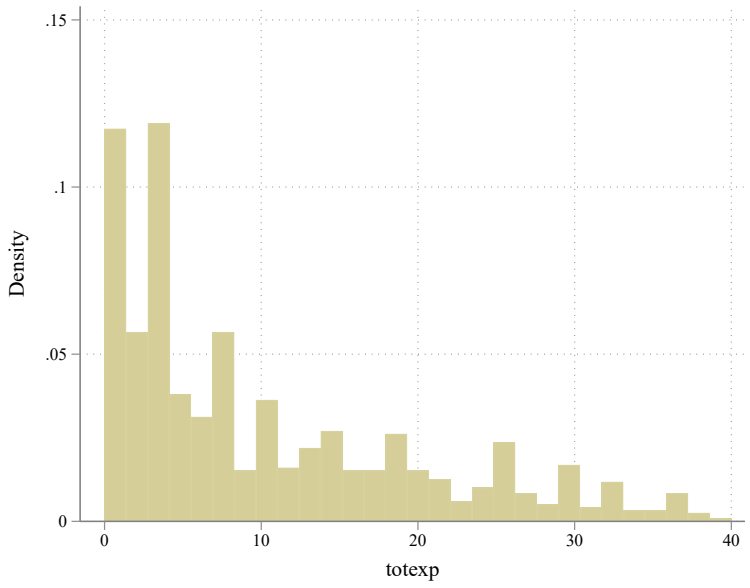
(d) Consider your confidence interval for reading z-scores. Is it consistent with the hypothesis that students in urban school districts perform about the same as the statewide average in reading? Explain your reasoning.

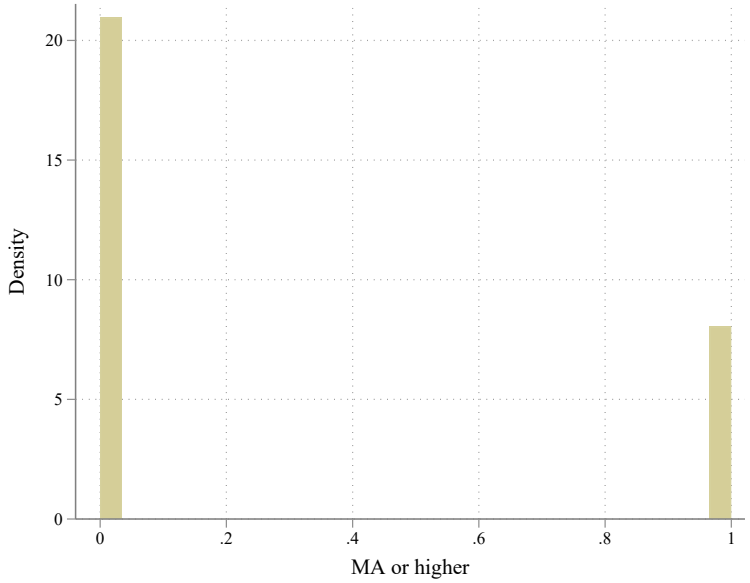
By construction the statewide average reading score is zero. (The z-score was calculated such that the mean is 0). The 95% interval estimate is (-0.06, 0.009). Since the confidence interval is a "range of likely values" and it includes zero, it is consistent with the hypothesis that students in urban districts perform about the same, on average, in reading as the state.

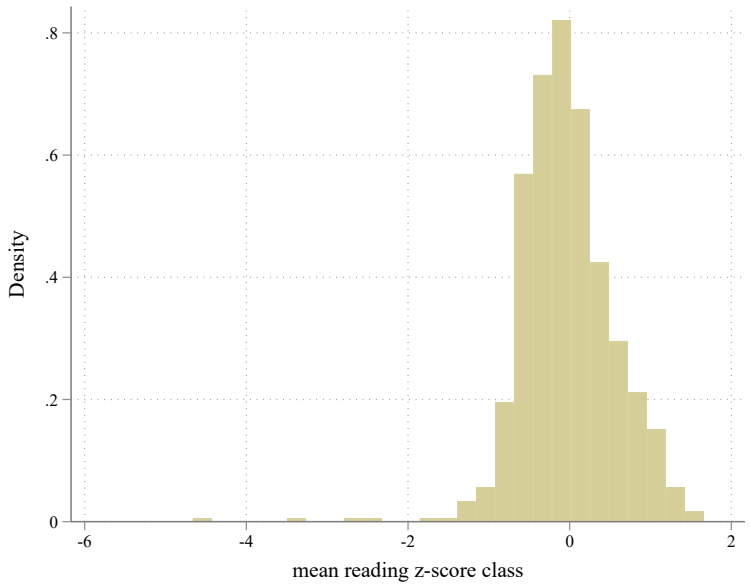
8. (**5 points**) In Lecture 5 you learned how to construct a confidence interval for the population *mean*, but not for the population *median*. There are methods for doing this, but an alternative approach would be to use bootstrap. Continue using the *Grade4_classrooms.dta* dataset from #7. Use the `bootstrap` prefix to draw 1,000 samples with replacement from your data and capture the median (p50) teacher experience (*totexp*) on each draw. The sample size should be the same as your dataset. Save these results to a separate file. Using your saved results, between what two values does the median fall 95% of the time? The `bootstrap` results report a "normal-based" 95% confidence interval. How does this compare to your first answer? Does a "normal-based" confidence interval make sense to use here? Why or why not?

See the attached Stata log for results. The resulting dataset contains the 1,000 medians from 1,000 bootstrapped samples. With 1,000 observations–sorted from smallest values to largest—the 2.5th percentile is obs #25 and the 97.5th percentile is obs #975. The values at these percentiles are 6 and 7. These represent the bootstrapped 95% confidence interval. The "normal-based" confidence interval assumes a normal sampling distribution, which this one clearly is not (it's based on medians, not means—see the histogram for the boostrapped sampling distribution).

```
.
. // ************************************************************************
. // LPO.8800 Problem Set 5 - Solutions to Questions 7-8
. // Last updated: October 1, 2023
. // ************************************************************************
.
. cd "$pset"
C:\Users\corcorsp\Dropbox\_TEACHING\Statistics I - PhD\Problem sets\Problem set 5
.
. // *************
. // Question 7
. // *************
.
. use https://github.com/spcorcor18/LPO-8800/raw/main/data/Grade4_classrooms.dta, clear
.
. // ********
. // Part a
. // ********
.
.         histogram totexp, name(hist1, replace) nodraw
(bin=29, start=0, width=1.3793103)
.         histogram maplus, name(hist2, replace) nodraw
(bin=29, start=0, width=.03448276)
.         histogram readz_class, name(hist3, replace) nodraw
(bin=27, start=-4.6651998, width=.23412534)
.
.         graph combine hist1 hist2 hist3, xsize(3) ysize(7) col(1)
.         graph export hists.pdf, as(pdf) replace
(file hists.pdf written in PDF format)
```


.15
.1
.05
0
Density
0
10
20
30
40
totexp


20
15
10
5
0
Density
0
.2
.4
.6
.8
1
MA or higher

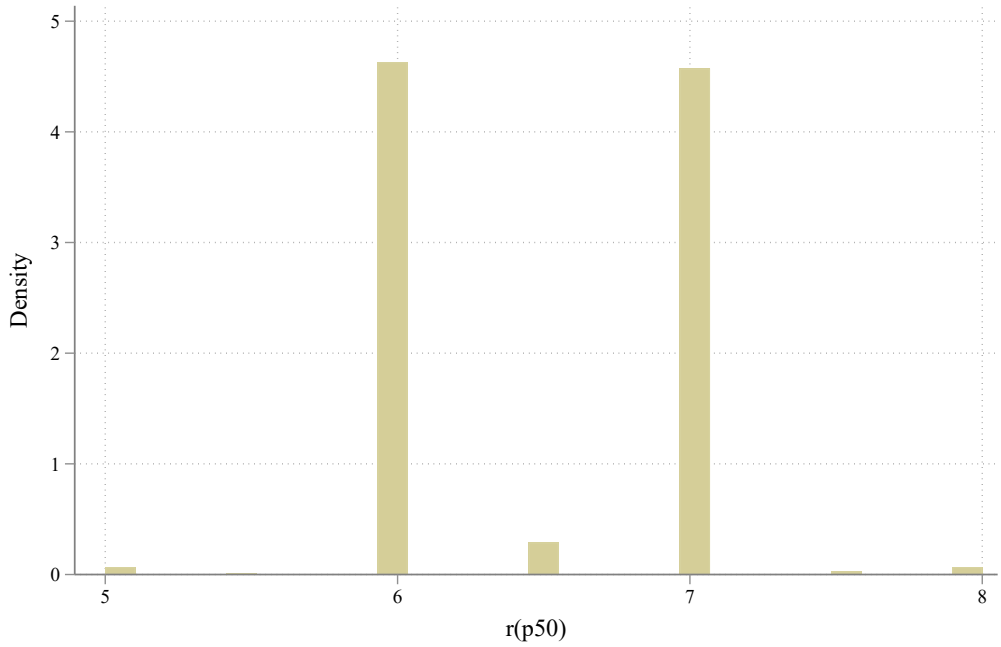

.8
.6
.4
.2
0
Density
-6
-4
-2
0
2
mean reading z-score class

```
.
. // The distribution of teacher experience is right skewed, with a somewhat
. // large mass of inexperienced teachers and long right tail of more experienced
. // teachers. MAplus is a binary variable, so its distribution is concentrated
. // on the values of 0 and 1. Classroom average reading scores are more
. // symmetrical, albeit a bit left skewed.
.
. // **********
. // Parts b-c
. // **********
. // 90% confidence intervals
.
.         mean totexp, level(90)
Mean estimation                   Number of obs   =        858
--------------------------------------------------------------
             |       Mean   Std. Err.     [90% Conf. Interval]
-------------+------------------------------------------------
      totexp |     10.169   .3315576      9.623044    10.71495
--------------------------------------------------------------
.         proportion maplus, level(90) citype(normal)
Proportion estimation             Number of obs   =        858
--------------------------------------------------------------
             |                                      Normal
             | Proportion   Std. Err.     [90% Conf. Interval]
-------------+------------------------------------------------
maplus       |
           0 |   .7226107   .0152846      .6974426    .7477788
           1 |   .2773893   .0152846      .2522212    .3025574
--------------------------------------------------------------
.         mean readz_class, level(90)
Mean estimation                   Number of obs   =        765
--------------------------------------------------------------
             |       Mean   Std. Err.     [90% Conf. Interval]
-------------+------------------------------------------------
 readz_class |  -.0255273   .0208613     -.0598827    .0088282
--------------------------------------------------------------
.
. // The confidence intervals are estimated above. If we can assume the sampling
. // distribution of the sample mean is normal, the 90% confidence interval will
. // contain the population mean in 90% of random samples. Per the CLT, the
. // normality assumption is reasonable if the sample size is large enough. In
. // this case with a sample size of 858 we can be reasonably confident that the
. // sampling distributions are normal, even in cases where the original
. // distribution is very non-normal (like maplus).
.
.
. // *************
. // Question 8
. // *************
.
```

```
.         bootstrap r(p50) , reps(1000) saving(medtotexp, replace): ///               sum
> m totexp, detail
(running summarize on estimation sample)
Warning:  Because summarize is not an estimation command or does not set
          e(sample), bootstrap has no way to determine which observations are
          used in calculating the statistics and so assumes that all
          observations are used.  This means that no observations will be
          excluded from the resampling because of missing values or other
          reasons.
          If the assumption is not true, press Break, save the data, and drop
          the observations that are to be excluded.  Be sure that the dataset
          in memory contains only the relevant data.
Bootstrap replications (1000)
----+--- 1 ---+--- 2 ---+--- 3 ---+--- 4 ---+--- 5
..................................................    50
..................................................   100
..................................................   150
..................................................   200
..................................................   250
..................................................   300
..................................................   350
..................................................   400
..................................................   450
..................................................   500
..................................................   550
..................................................   600
..................................................   650
..................................................   700
..................................................   750
..................................................   800
..................................................   850
..................................................   900
..................................................   950
..................................................  1000
Bootstrap results                                 Number of obs     =        858
                                                  Replications      =      1,000

      command:  summarize totexp, detail
        _bs_1:  r(p50)
------------------------------------------------------------------------------
             |   Observed   Bootstrap                         Normal-based
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _bs_1 |        6.5    .523233    12.42   0.000     5.474482    7.525518
------------------------------------------------------------------------------

.
.         clear
.         use medtotexp
(bootstrap: summarize)
.         rename _bs_1 medtotexp
.         histogram medtotexp
(bin=29, start=5, width=.10344828)
.         graph export medtotexp.pdf, as(pdf) replace
(file medtotexp.pdf written in PDF format)
```




```
.         summ medtotexp, detail
                          r(p50)
-------------------------------------------------------------
      Percentiles      Smallest
 1%            6              5
 5%            6              5
10%            6              5       Obs               1,000
25%            6              5       Sum of Wgt.       1,000
50%          6.5                      Mean              6.499
                        Largest       Std. Dev.       .523233
75%            7              8
90%            7              8       Variance       .2737728
95%            7              8       Skewness       .0144758
99%         7.25              8       Kurtosis       1.796505
.         sort medtotexp
.         list medtotexp if _n==25
      +----------+
      | medtot~p |
      |----------|
  25. |        6 |
      +----------+
.         list medtotexp if _n==(1000-25)
      +----------+
      | medtot~p |
      |----------|
 975. |        7 |
      +----------+
.
. // The resulting dataset contains the 1,000 medians from 1,000 bootstrapped
. // samples. With 1,000 observations--sorted from smallest values to largest---
. // the 2.5th percentile is obs #25 and the 97.5th percentile is obs #975. The
. // values at these percentiles are 6 and 7.
.
. capture log close
```