

### Problem Set 6

**Instructions:** Answer the following questions in their entirety in a separate document. Submit your completed problem set as a PDF document via email to [sean.corcoran@vanderbilt.edu](mailto:sean.corcoran@vanderbilt.edu). Use your last name and problem set number as the filename (e.g., *Obama Problem Set 6.pdf*). Working together is encouraged, but it is expected that all submitted work be that of the individual student.

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1. **(5 points)** A random sample of 172 new college graduates was asked to rate on a scale of 1 (not concerned) to 5 (very concerned) their level of concern about meeting future student loan debt obligations. The sample mean rating was 3.17 and the sample standard deviation was 0.70. Test at the 1% significance level the null hypothesis that the population mean rating is 3.0 against the alternative that it is larger than 3.0. Write down the null and alternative hypotheses, and report your test statistic,  $p$ -value, and conclusion. (Note: this variable is ordinal-measured and arguably not appropriate for such a test. However, we will treat it as an interval measure here).
2. **(5 points)** Children graduating from a particular high school in New York City score an average of 55 on a college-readiness test. A random sample of 36 students from this school is selected to participate in Project Advance, a special program to help high school students prepare for college. At the end of the program, the 36 students are given the college-readiness test. They obtain a sample mean of  $\bar{x} = 58.5$  and a sample standard deviation of  $s = 12$ . Conduct a hypothesis test at significance level  $\alpha = 0.05$  to determine whether students who participate in Project Advance score higher on average on the college readiness test than those in the school as a whole. State your null and alternative hypotheses, your test statistic,  $p$ -value, and conclusion.
3. **(4 points)** Same-sex marriage was legalized across Canada by the Civil Marriage Act enacted in 2005. Was this decision supported by a majority of the Canadian population? A poll conducted in July 2005 of 1,000 Canadians asked whether this bill should stand or be repealed. The responses were: 55% should stand, 39% should repeal, 6% don't know. Let  $\pi$  represent the population proportion of Canadians who believe it should stand. For testing  $H_0 : \pi = 0.50$  against  $H_1 : \pi > 0.50$ :
  - (a) Find the standard error, and interpret in words.
  - (b) Find the test statistic, and interpret.
  - (c) Find the  $p$ -value, and interpret in this context.

(d) What is your conclusion about  $H_0$ ?

Note: consider the “don’t know” group to be part of the “should repeal” group (i.e., those who *didn’t* say the bill should stand.)

4. **(6 points)** According to a union agreement, the mean income for all senior-level assembly-line workers in a large company is \$500 per week. A representative of a women’s group decides to analyze whether the mean income  $\mu$  for female employees matches this norm. From a random sample of nine female employees, the average weekly income is  $\bar{x} = \$410$  and  $s = \$90$ .
- (a) Test whether the mean income of female employees differs from \$500 per week. Report your hypotheses, test statistic,  $p$ -value, and conclusion. Use a significance level of  $\alpha = 0.01$ . **(4 points)**
- (b) Report the  $p$ -value for the one-sided alternative hypothesis  $H_1 : \mu < \$500$ . Would this change your conclusion? Again use  $\alpha = 0.01$ . **(2 points)**
5. **(4 points)** The  $p$ -value from a statistical test for  $\mu$  with  $n=25$  was  $p = 0.05$ .
- (a) Find the  $t$  statistic that has this  $p$ -value for the following alternative hypotheses:  
(i)  $H_1 : \mu \neq 0$ , (ii)  $H_1 : \mu > 0$ , (iii)  $H_1 : \mu < 0$ . **(2 points)**
- (b) Does this  $p$ -value provide stronger, or weaker evidence against the null hypothesis than  $p = 0.01$ ? Explain. **(2 points)**
6. **(5 points)** Below are the results of a hypothesis test about the population mean self concept score in 8th grade (*slfcnc08*), using a random sample of 25 students. Some information from the table has been removed. Given what remains in the table, what is the standard error of the sample mean? Explain how you arrived at your answer.

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. ttest slfcnc08=20
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One-sample t test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
slfcnc08	25	21.96			19.62959 24.29041
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mean = mean(slfcnc08)				t =	
Ho: mean = 20				degrees of freedom = 24	
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Ha: mean < 20		Ha: mean != 20		Ha: mean > 20	
Pr(T < t) = 0.9523		Pr( T  >  t ) = 0.0954		Pr(T > t) = 0.0477	

For questions #7-8, use the Stata dataset *Grade4\_classrooms.dta* on Github. This file represents a random sample of 4th grade classrooms located in urban school districts in Texas. Each observation is a classroom, and the variables either describe the teacher (e.g., total teaching experience, teacher race/ethnicity), or are an average for the classroom (e.g., math  $z$ -scores, % of students who are economically disadvantaged). All data are from 2006.

7. **(14 points)** For this problem you will be conducting two-sided hypothesis tests for the classroom mean math score (*mathz\_class*) and mean reading score (*readz\_class*) in urban Texas school districts.<sup>1</sup>
  - (a) Evaluate the tenability of the normality assumption for these variables or indicate why this assumption is not an issue for this analysis (i.e., that the results of a  $t$ -test would not be compromised by a failure to meet this assumption). **(2 points)**
  - (b) For the state of Texas, the mean classroom math and reading ( $z$ -)scores are both 0. Write down the null and alternative hypotheses for  $t$ -tests to determine whether the mean urban 4th grade classroom average  $z$ -scores for math and reading are zero, or are different from zero. **(2 points)**
  - (c) Using the appropriate Stata command, what are the test statistics and  $p$ -values associated with these tests? **(2 points)**
  - (d) Use the  $p$ -values in each case to determine whether or not  $H_0$  can be rejected in favor of the alternative. Use a significance level of  $\alpha = 0.05$ . **(2 points)**
  - (e) Provide 95% confidence intervals for the mean math and reading  $z$ -scores in urban Texas classrooms. **(2 points)**
  - (f) Use the confidence intervals found in part (e) to conduct the tests in parts (b)-(d). Are the results consistent? Why or why not? **(2 points)**
  - (g) Would you be more or less likely to reject the null hypotheses if the sample size had been 1,000 classrooms, rather than the sample size(s) used above? Explain. **(2 points)**
  
8. **(6 points)** Use the same data from Question #7 to test whether urban Texas classrooms on average exclude more than 14 percent of their students from state testing because of special education accommodations. (The relevant variable is *excl\_spd*). Follow the same procedure as parts (a)-(d) in Question #8 to carry out this test.

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<sup>1</sup>To be clear, these variables are averages of the students' own  $z$ -scores in math and reading in a classroom; they are not the  $z$ -score for the classroom in the distribution of classrooms. This is not important information for the problem, just clarification.