4. Panel data

LPO 8852: Regression II

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Difference-in-differences recap

Difference-in-differences (DD) often relies on *panel data*, with repeat observations of two or more groups (i) over time (t).

$$y_{it} = \beta_0 + \beta_1 treat_i + \beta_2 post_t + \beta_3(treat_i \times post_t) + \gamma X_{it} + u_{it}$$

$treat_i$ is a fixed effect for the treated group, $post_t$ is a time effect equal to one in the post period(s). Regression DD often includes year effects as well.

Difference-in-differences recap

What identification problem does DD solve? Treated observations may differ systematically from untreated observations (β_1)—differencing over time "nets out" these fixed differences and focuses on changes over time. β_2 is intended to capture the change over time that would have occurred for treated observations had they not been treated.

OVB remains if there are omitted variables correlated with *treat* $\times$ *post* and the outcome y. For example: unobserved factors that change differentially for treated observations (implying non-parallel trends).

Difference-in-differences recap

DD was our first attempt to address selection on *unobservables*. Treatment need not be randomly assigned, and treated and untreated units *can* differ systematically prior to treatment (this is captured by β_1, or the group specific coefficients in the generalized DD).

As long as these unobserved differences do not change over time, DD can eliminate the unobserved selection bias.

Panel data

Panel, longitudinal, or *"cross-sectional time series"* data consist of observations on cross-sectional units (e.g., students, schools, hospitals, neighborhoods, counties, states) at multiple points in time.

- N cross-sectional (panel) units and T time periods ($T \geq 2$)
- A *balanced panel* has exactly $N \times T$ observations (T time observations for all N panel units)
- An *unbalanced panel* has T_i observations for panel unit i, where T_i is not the same for all i

Differs from a *pooled cross-section*, although panel methods can be used with this type of data (e.g., Kearney & Levine (2019) example)

Panel data - long

Panel data in *long* format, N students in $T = 4$ years:

studentID	year	readscore	mathscore	incomecat	...
1	1999	75	82	3	
1	2000	78	84	4	
1	2001	80	90	4	
1	2002	78	91	3	
2	1999	91	92	2	
2	2000	94	92	2	
2	2001	80	85	2	
2	2002	87	83	2	
3	1999	62	50	5	
3	2000	70	47	5	
3	2001	75	55	4	
3	2002	73	60	5	

Panel data - wide

Panel data in *wide* format, N students in $T = 4$ years:

studentID	read99	math99	inc99	read00	math00	inc00	read01	...
1	75	82	3	78	84	4	80	
2	91	92	2	94	92	2	80	
3	62	50	5	70	47	5	75	
4	...	...	...					

Panel data - reshape long

Moving between *long* and *wide* format in Stata with `reshape`, beginning with *wide* data

- `i()` contains the time invariant variables (e.g., ID, gender)
- `j()` specifies the time variable to be created (e.g., year)
- The list of time varying variables are "stubs" that end in the *j* suffix

```
reshape long stubnames, i(varlist) j(varname)
```

- If `j()` consists of *string* rather than *numeric* values, use the `string` option
- Example time-varying variable names: *score98, score99, score00* (Stata may have problems with 00 as a `j()` value if `string` option is not used).

Panel data - reshape wide

Moving between *long* and *wide* format in Stata with `reshape`, beginning with *long* data

- `i()` contains the time invariant variables (e.g., ID, gender)
- `j()` specifies the time variable (e.g., year)
- The list of time varying variables are "stubs" that *will* end in the *j* suffix, once converted to wide

```
reshape wide stubnames, i(varlist) j(varname)
```

- After reshaping, Stata allows you to revert back easily without losing information. E.g., after the above command just type `reshape long`
- Most panel regression commands expect the data to be in *long* format.

In-class example 1

Illustration of reshape commands using *Census_states_1970_2000* data:

- Cross sectional unit: state
- Time variable: year (decennial Census years)
- Time-varying variables: median household income, unemployment rate

Stata panel commands

Stata has many useful `xt` commands for working with panel data. Typically these require that you first declare the data to be a panel using `xtset`:

- `xtset` *panelvar timevar*
- The *panelvar* must be numeric. If it is not, you can use encode: **`encode`** *panelvar*, `gen(`*panelvar2*`)`
- It is possible to tell Stata in the `xtset` options what units of time the data represent—e.g., years, quarters, minutes (useful for some purposes)
- `xtset` alone will report back the panel settings

Stata panel commands

Other useful Stata panel data commands for description:

- `xtdescribe`—to see patterns of participation/data availability
- `xtsum`—for descriptive statistics that show between- and within-unit variation
- `xttab`—for one-way tabulations with separate counts within and between units
- `xttrans`—for transition probabilities (movement between categories of a categorical variable)
- `xtline` and `xtline, overlay`—for separate line graphs by panel unit (see in-class example)

xtsum

Decomposition of variation in `xtsum`:

$$s_w^2 = \frac{1}{NT-1}\sum_i \sum_t (x_{it} - \bar{x}_i)^2$$

$$s_b^2 = \frac{1}{N-1}\sum_i (\bar{x}_i - \bar{x})^2$$

$$s_o^2 = \frac{1}{NT-1}\sum_i \sum_t (x_{it} - \bar{x})^2$$

Note $\bar{x}$ is the *grand mean* of x. Can also write:

$$s_w^2 = \frac{1}{NT-1}\sum_i \sum_t (x_{it} - \bar{x}_i + \bar{x})^2$$

because adding a constant ($\bar{x}$) will not affect s_w^2

xtsum

`xtsum` also shows the min and max of:

- x_{it}: overall
- $\bar{x}_i$: between
- $(x_{it} - \bar{x}_i + \bar{x})$: within

Note: on `xtsum`, see also `https://www.stata.com/support/faqs/statistics/decomposed-variances-in-xtsum/`

- Between and within variation do not sum to overall
- Variance estimates are bias-corrected, multiplied by $n/(n-1)$
- With unbalanced panels, s^b is calculated using mean of panel means, not $\bar{x}$ (may not be the same)

In-class example 2

Illustration of `xt` commands using *State_school_finance_panel* data:

- Cross sectional unit: state
- Time variable: year (annual 1990-2010)
- Time-varying variables: various school finance measures

Panel data - advantages

Why use panel data?

- Can help us answer questions not possible with a cross-section or time-series approach
- Can generate *measures* not possible with cross-sectional or time series data (e.g., growth, work spells)
  - If 50% of women are working in year t, does this reflect 50% of women working at any given point, or 50% of women who work all the time?
- Allows us to address selection bias due to unobserved heterogeneity that is fixed over time ("fixed effects")

Selection bias revisited

Lecture 1: interpretation of regression coefficients as causal is often complicated by selection bias. Example:

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

with $E(u_i|x_i) \neq 0$ because we believe potential outcomes are not independent of x. We can attempt to mitigate selection bias through the inclusion of additional covariates or via matching, but this only solves the problem if conditioning on these observables (or the propensity score) eliminates OVB.

In practice we are often more concerned about selection on *unobservables*.

Unobserved heterogeneity

Suppose there are unobserved, fixed differences across units (c_i) that affect the outcome and are (potentially) correlated with the explanatory variable of interest (x_i):

$$y_i = \beta_0 + \beta_1 x_i + c_i + u_i$$

c_i could represent the effects of ability, health, motivation, intelligence, parental resources, managerial quality, organizational culture, state/local policies or regulations, etc.

First difference model

Suppose we have two time periods (T=2) for each cross-sectional unit i, and assume the linear model above applies in both periods:

$$y_{i2} = \beta_0 + \beta_1 x_{i2} + c_i + u_{i2}$$
$$y_{i1} = \beta_0 + \beta_1 x_{i1} + c_i + u_{i1}$$

Now subtract period 1 from period 2 for the "first difference":

$$\Delta y_i = \beta_1 \Delta x_i + \Delta u_i$$
$$y_i^* = \beta_1 x_i^* + u_i^*$$

Because c_i is time-invariant, it differences out of the model. Notice the constant β_0 also differences out.

First difference model

The first difference model can be estimated using OLS, as long as the usual OLS assumptions apply to it:

- The new error term $u_i^* = \Delta u_i$ is uncorrelated with the new explanatory variable, $x_i^* = \Delta x_i$.
- This requires that we have no cross-period correlations between u and x: this is called **strict exogeneity**.
- The x_i must vary over time for at least some i, else they difference out.

In-class example 3

Example using panel of Texas elementary schools:

- `use` *Texas_elementary_panel_2004_2007.dta*
- `xtset` *campus year*
- `xtdescribe`
- `rename` *ca311tar avgpassing*
- `egen` *avgclass* `= rowmean(`*cpctg01a-cpctgmea*`)`
- `reg` *avgpassing avgclass* `if` *year*`==2007` (cross-sectional regression for 2007)

Note: *avgclass* is the mean class size across grades, and *avgpassing* is the school average passing rate across grades and subjects.

In-class example 3

Having declared the dataset as a panel, Stata recognizes the **`d.`** prefix as a "difference operator":

- `reg` *d.avgpassing d.avgclass* `if` *year*`==2007, noconstant`
- This is the first difference regression, using 2007 only (and its lag in the calculation of *d.avgpassing* and *d.avgclass*)
- `d.` can be used after `xtset` or `tsset` (time series set)
- Note suppression of the constant. In theory the constant term differences out. In practice can still estimate with a constant, which allows for a year-to-year time trend.

In-class example 3

A few things to note in example 3:

- Change in coefficient on class size: does it make sense?
- Change in sample size (re: unbalanced panel due to missing values)

A few things to think about:

- Is strict exogeneity likely to hold in this circumstance?
- Where is the identifying variation coming from?
- How much variation is there in the *change* in passing rates (Δy) and class size (Δx)?
- Do outliers dominate the variation in changes?

```
gen davgpassing = d.avgpassing
/* create variable containing FD that can be described */
```


In-class example 3

The first difference model is easily generalizable to multiple years ($T > 2$).

- Each year of data is differenced with the prior year
- 1st period is sacrificed
- Must continue to think about OLS assumptions, e.g. strict exogeneity

```
reg d.avgpassing d.avgclass, noconstant
table year if e(sample)
* note 1st year of data is not used
```

Fixed effects model

Alternatively, in the *(one-way) fixed effects* model, we treat the c_i as parameters to be estimated:

$$y_{it} = \beta_0 + \beta_1 x_{it} + c_i + u_{it}$$

Effectively we are allowing for a *unique intercept* for every cross-sectional unit i. This is feasible to estimate since each i is observed multiple times.

"Least squares dummy variables" approach

Now we are estimating the intercept β_0, slope β_1, and $(N-1)$ intercepts, the "fixed effects." This can be done by including $N-1$ dummy variables in the regression, sometimes called the "least squares dummy variable" (LSDV) model:

- `reg` *avgpassing avgclass* `i.`*campus*
- For this example limit to *year*>=2006 and *houston*==1 so that the number of schools is manageable.
- `areg` is equivalent but suppresses the $(N-1)$ coefficients
- `areg` *avgpassing avgclass*, `absorb(`*campus*`)`

Note omission of first cross sectional unit with `i.campus`. You can control which unit is omitted if desired. See in-class example for interpretation.

“Least squares dummy variables” approach

There are a number of reasons why you might not want to do it this way:

- Could be time-consuming and harder on memory with large datasets (re: you are creating dummy variables for each unique i)
- Soaks up degrees of freedom; may result in the number of regressors exceeding the number of observations
- Often we are not interested in the estimates of the fixed effects themselves, so there is no need to see/report them.
- Exception: recent “school effects” and “teacher effects” studies work explicitly with fixed effects estimates ($\hat{c}_i$)—more on this later

Within transformation

Suppose that panel data are available with multiple observations per i and the model is:

$$y_{it} = \beta_0 + \beta_1 x_{it} + c_i + u_{it} \quad t = 1, ..., T \quad \forall i$$

Within each panel unit i, take the average over t on both sides and subtract the average from each it observation:

$$\bar{y}_i = \beta_0 + \beta_1 \bar{x}_i + c_i + \bar{u}_i$$

$$y_{it} - \bar{y}_i = \beta_1 (x_{it} - \bar{x}_i) + (u_{it} - \bar{u}_i)$$

This is called “de-meaning” or the “within” transformation (sometimes denoted $\ddot{y}_i$). Notice that the intercept β_0 and the c_i “difference out.” c_i differences out only if it is *time invariant*.

Within transformation

Under certain assumptions, an OLS regression of the de-meaned y on the de-meaned x will yield unbiased and consistent estimates of β_1.

$$y_{it} - \bar{y}_i = \beta_1(x_{it} - \bar{x}_i) + (u_{it} - \bar{u}_i)$$

$$y_{it}^* = \beta_1 x_{it}^* + u_{it}^*$$

This is also known as the fixed effects or "within" regression, and extends to more than one explanatory variable $(x_1, ..., x_k)$.

Explanatory variables x_j that are time *invariant* fall out of the model. (They all equal their within-group mean, so the within-transformation equals zero). Examples: gender, race or ethnicity, birthplace ...

Within transformation

The fixed effects or "within" regression model can be estimated using OLS using `xtreg`:

- `xtreg` *`avgpassing avgclass`*`, fe`
- Note `xtset` must have been declared, or specify the cross-sectional unit in the options, e.g. `i(`*`campus`*`)`
- While the fixed effects are not estimated directly, can "back out" a prediction: $\hat{c}_i = \bar{y}_i - \bar{x}_i\hat{\beta}_1$
- `predict` *`schlfe`*`, u`

Note the estimated fixed effects from `xtreg` are not the same as the dummy coefficients from the LSDV model.

Within transformation

The command `xtdata, fe` can be used to transform your data using the within-transformation. However, this is rarely done (in my experience) since it transforms the variables in your dataset. `xtreg` will do the transformation on the fly without affecting your dataset.

Fixed effects model

Compare `xtreg`, `areg` and first difference when $T = 2$

- `xtreg` *`avgpassing avgclass3`*`, fe`
- `areg` *`avgpassing avgclass3`*`, absorb(`*`campus`*`)`
- `reg d.`*`avgpassing`* `d.`*`avgclass3`*`, noconstant`

Fixed effects models

A few notes about `xtreg, fe`

- FE is more efficient (smaller standard errors) than first differencing if the error terms are serially uncorrelated and $T > 2$
- Assumes no correlation in u across units of panel i (some tests for this using user-written `xtscd, xttest3`)
- The estimates of the fixed effects themselves (c_i) are unbiased but inconsistent in large samples. (Why? As the number of panel units grows ($N \to \infty$) the number of parameters to estimate also grows).
- `xtreg` has not historically allowed `svy` specification (for complex sampling designs) but can use `pweights` and `cluster()` option. See also the `mixed` (or `xtmixed`) command for an alternative.

Fixed effects model

Stata actually fits the following model with `xtreg`:

$$(y_{it} - \bar{y}_i + \bar{y}) = \beta_0 + \beta_1(x_{it} - \bar{x}_i + \bar{x}) + (u_{it} - \bar{u}_i + \bar{u})$$

Where the values with a bar but no subscript are the grand means. This includes an intercept which is the average of the fixed effects (c_i).

Fixed effects model

Fixed effects considerations:

- Where is the identification coming from?
- How much variation is there within panel units? When small, one risks imprecise estimates
- For stats on within- and between- school variation can use `xtsum` (described earlier), `xttab` and `xttrans` for categorical variables

`xtsum` *avgpassing avgclass*

Fixed effects model

Other useful output from `xtreg`:

```
.
. xtreg avgpassing avgclass3, fe

Fixed-effects (within) regression               Number of obs      =       350
Group variable: campus                          Number of groups   =       180

R-sq:  within  = 0.0039                         Obs per group: min =         1
       between = 0.0087                                        avg =       1.9
       overall = 0.0032                                        max =         2

                                                F(1,169)           =      0.66
corr(u_i, Xb)  = -0.1079                        Prob > F           =    0.4179

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   avgclass3 |  -.2390704    .294385    -0.81   0.418     -.820216    .3420752
       _cons |   76.80355   6.032673    12.73   0.000     64.89445    88.71265
-------------+----------------------------------------------------------------
     sigma_u |  11.942612
     sigma_e |  7.1632112
         rho |   .7354221   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(179, 169) =     5.27            Prob > F = 0.0000

.     * fixed effects model
```

Fixed effects model

Other useful output from `xtreg`:

- F-test for joint significance of fixed effects (null hypothesis H_0 is that all fixed effects are zero). If rejected, fixed effects model is a reasonable assumption and regular OLS may provide inconsistent estimates. In practice, rarely rejected.
- R^2 *within*: variance "explained" by within-group deviations from group means
- R^2 *between*: variance in group means $\bar{y}_i$ "explained" by the group mean x's: $\bar{x}_i$
- *sigma_u* estimate of the standard deviation in fixed effects (c_i)

Fixed effects model: assumptions for inference

- **FE.1:** linear model $y_{it} = \beta_1 x_{it1} + ... + \beta_k x_{itk} + c_i + u_{it}$
- **FE.2:** cross-sectional units are a random sample
- **FE.3:** x_{it} varies over time for some i, no perfect collinearity
- **FE.4:** $\forall t$, $E(u_{it}|X_i, c_i) = 0$ or the expected value of u given x in *all* time periods is zero (strict exogeneity)
- **FE.5:** $Var(u_{it}|X_i, c_i) = Var(u_{it}) = \sigma_u^2$ - homoskedasticity
- **FE.6:** for $t \neq s$ errors are uncorrelated: $Cov(u_{it}, u_{is}|x_i, c_i) = 0$. No serial correlation.

Under FE.1-FE.4, fixed effects model (and first difference model) is unbiased. Adding FE.5-FE.6, fixed effects model is BLUE. If FE.6 holds, fixed effects is more efficient than the first difference model. Can relax homoskedasticity assumption and calculate `robust` standard errors.

Fixed effects model: assumptions for inference

Note: the econometric theory described here is for "short" panels, with N large relative to T. If the opposite is true in your context, use FE model with caution (see Wooldridge chapter 14, Cameron & Trivedi).

Two-way fixed effects model

The two-way fixed effects model adds another dimension of fixed effects (often time periods). There is no explicit command for two-way models, rather can just include time dummies. Alternatively, `reghdfe`

```
xtreg avgpassing avgclass i.year, fe
* the i.year syntax introduces (T-1) time effects
test _Iyear_2006 _Iyear_2007
* joint test that time effects = 0
reghdfe avgpassing avgclass, absorb(campus year)
```

As with one-way fixed effects model, requires variation across units within time periods t.

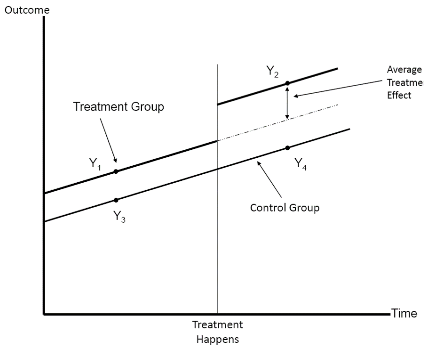
Two-way fixed effects model

The generalized difference-in-differences model is a two-way fixed effects model:

$$y_{it} = \beta_0 + \beta_1(treat_i \times post_t) + \alpha_t + \gamma_i + \delta X_{it} + u_{it}$$

There are cross-sectional unit fixed effects (γ_i) which represent separate intercepts for each unit and time effects (α_t) which represent common variation over time within group.

Two-way fixed effects model



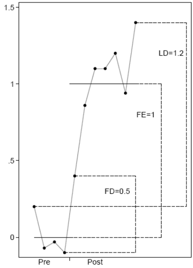
Comparison of models

It is important to be attentive to where the variation in each type of FE model is coming from:

- Fixed effects ("within") model: uses deviations from group (i) means, e.g., mean "pre" vs. mean "post"
- First differences model: uses variation in successive time periods, e.g., just prior to and just after a "treatment" (a change in x)
- **Long differences** is like first differences, but there is a long time span between observations. Here outcomes may be compared well before and well after a "treatment".

To evaluate these in your situation, need some idea of the speed in which x affects y

Comparison of models



Source: Nichols (2007). Figure shows one panel (i)'s contribution to the estimated effect of a treatment that $= 1$ in post period ($t > 4$). Notice the different treatment effects depending on FE, FD, or LD.

Fixed effects models in other applications

Fixed effects models are not exclusively used with panel data in which cross-sectional units i are observed in multiple time periods. They are also used with grouped or clustered data. For example:

- Family fixed effects, where the family is the cross-sectional unit and siblings are the group members (akin to the time dimension)
- School fixed effects with student-level data, where each school has its own intercept

Fixed effects models in other applications

The researcher needs to provide a convincing rationale for why the unobserved variable should be considered fixed across multiple observations (e.g., siblings, years)

- Why did a mother's employment status change between siblings?
- Why did only 1 of 2 siblings participate in Head Start?
- Why did a student switch from a traditional school to a charter school?
- Why did an elementary school receive a new principal?

Fixed effects models: advantages and disadvantages

Advantages:

- Unobserved c_i can be correlated with the explanatory variables
- Slopes estimated using *within-group* (i) variation in x, y

Disadvantages:

- Cannot estimate slope coefficients for <u>time-invariant</u> x
- Fixed effects "remove" a lot of the variation in y
- The "within" model is <u>less efficient</u> (higher standard errors)
- There may be more measurement error (and attenuation bias, see Lecture 5) when relying on within-group *changes* vs. levels
- Group intercepts use up a lot of degrees of freedom

Random effects

An alternative conception of c_i is as a *random* effect, uncorrelated with x_{it}.

$$y_{it} = \beta_0 + \beta_1 x_{it} + \underbrace{c_i + e_{it}}_{v_{it}}$$

Think of v_{it} as a *composite* error consisting of a between-group component (c_i) common to all observations within the group, and a within-group component (e_{it}). It is assumed c_i and e_{it} are independent of one another and:

$$c_i \sim N(0, \sigma_c^2)$$
$$e_{it} \sim N(0, \sigma_e^2)$$

This is the **random effects** or **random intercepts** model.

Random effects

If c_i is uncorrelated with x_{it}, then the composite error term v_{it} is uncorrelated with x_{it}. (We already assume e_{it} is uncorrelated with x_{it}). This means the OLS estimator for β_1 will be unbiased and consistent.

Note: estimation of this model does *not* involve estimating the c_i's as parameters as was done in the LSDV model.

Random effects

The composite error term v_{it} is not, however, i.i.d.:

$$Corr(v_{it}, v_{is}) = \rho = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_e^2} \text{ for } s \neq t$$

The common error for observations in group i (c_i) results in correlation between the composite error in period t (v_{it}) and in period s (v_{is}).

This means OLS is consistent but not efficient, and that traditional standard error formulas assuming i.i.d. errors are incorrect. The ratio above (ρ) is called the **intra-class correlation** (more on this later).

Estimation using GLS (details later): `xtreg, re`.

Success for All example

- Success for All is a whole-school literacy intervention.
- Borman et al. (2005) conducted a randomized evaluation of SFA in 2001-02 and 2002-03 (21 treatment schools and 20 control).
- This was a *cluster-randomized* design with randomization at the school level.
- The data used by Murnane & Willett (*ch7_sfa.dta*) include grade 1 only. The outcome of interest is *wattack*, the student's score on a "Word-Attack" test.

Next slide: an "unconditional" model with no x_{it} estimates variance components σ_c^2 and σ_e^2 and the intra-class correlation ρ. This provides a sense of the degree of between- vs. within-group variance in the outcome.

Random effects with `xtreg`

```
. xtreg wattack, re i(schid)

Random-effects GLS regression                   Number of obs     =      2,334
Group variable: schid                           Number of groups  =         41

R-sq:                                           Obs per group:
     within  = 0.0000                                         min =         10
     between = 0.0000                                         avg =       56.9
     overall = 0.0000                                         max =        134

                                                Wald chi2(0)      =          .
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =          .

------------------------------------------------------------------------------
     wattack |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   477.5356   1.447118   329.99   0.000     474.6994    480.3719
-------------+----------------------------------------------------------------
     sigma_u |  8.8705267
     sigma_e |  17.725757
         rho |  .20027618   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

This example: Success for All impact evaluation (from Murnane & Willett). $\sigma_c^2 = 8.87^2 = \mathbf{78.7}$ and $\sigma_e^2 = 17.73^2 = \mathbf{314.35}$. $\rho = 0.200$.

loneway

loneway (one-way ANOVA) is another handy command for estimating variance components and ICC. (Note the difference in σ_c and ρ from xtreg, re. With unbalanced panels, these will differ slightly).

```
. loneway wattack schid

          One-way Analysis of Variance for wattack: word attack posttest

                                            Number of obs =      2,334
                                               R-squared =      0.2185

    Source                 SS         df      MS            F     Prob > F
-------------------------------------------------------------------------
Between schid          201450.43      40   5036.2607     16.03     0.0000
Within schid           720466.21   2,293   314.20244
-------------------------------------------------------------------------
Total                  921916.63   2,333   395.16358

         Intraclass       Asy.
         correlation      S.E.       [95% Conf. Interval]
         ------------------------------------------------
            0.20993      0.04402      0.12366     0.29621

         Estimated SD of schid effect            9.137203
         Estimated SD within schid               17.72576
         Est. reliability of a schid mean         0.93761
              (evaluated at n=56.56)
```


Random effects with xtreg

```
. xtreg wattack sfa ppvt, re i(schid)

Random-effects GLS regression                   Number of obs      =     2,334
Group variable: schid                           Number of groups   =        41

R-sq:                                           Obs per group:
     within  = 0.1101                                         min =        10
     between = 0.3960                                         avg =      56.9
     overall = 0.1820                                         max =       134

                                                Wald chi2(2)       =    308.21
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
     wattack |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         sfa |   3.440921   2.297268     1.50   0.134    -1.061642    7.943485
        ppvt |   .4851754   .0278075    17.45   0.000     .4306737    .5396771
       _cons |   432.0475   2.972263   145.36   0.000      426.222     437.873
-------------+----------------------------------------------------------------
     sigma_u |  6.9082397
     sigma_e |  16.725172
         rho |  .14574141   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

This regression: includes the treatment indicator (*sfa*) and one covariate (*ppvt*). Note changes in σ_c and σ_e, ρ. The residual variability is reduced with the inclusion of x's.

Random effects

Class size and passing rates in TX class size example:

```
. xtreg avgpassing avgclass, re i(campus)

Random-effects GLS regression                   Number of obs      =     16,062
Group variable: campus                          Number of groups   =      4,326

R-sq:                                           Obs per group:
     within  = 0.0018                                         min =          1
     between = 0.0098                                         avg =        3.7
     overall = 0.0060                                         max =          4

                                                Wald chi2(1)       =       2.74
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =     0.0978

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0442893   .0267548    -1.66   0.098    -.0967277    .0081491
       _cons |   76.21828   .5503649   138.49   0.000     75.13959    77.29698
-------------+----------------------------------------------------------------
     sigma_u |  12.391941
     sigma_e |  6.4870883
         rho |  .78490199   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```


Random effects

Compare to fixed effects: very different slope coefficient estimate.

```
. xtreg avgpassing avgclass, fe i(campus)

Fixed-effects (within) regression               Number of obs      =     16,062
Group variable: campus                          Number of groups   =      4,326

R-sq:                                           Obs per group:
     within  = 0.0018                                         min =          1
     between = 0.0098                                         avg =        3.7
     overall = 0.0060                                         max =          4

                                                F(1,11735)         =      21.30
corr(u_i, Xb)  = -0.1189                        Prob > F           =     0.0000

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.1339024   .0290105    -4.62   0.000    -.1907678   -.0770371
       _cons |   78.09211   .5590819   139.68   0.000     76.99621      79.188
-------------+----------------------------------------------------------------
     sigma_u |  12.997022
     sigma_e |  6.4870883
         rho |  .80056238   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(4325, 11735) = 13.83               Prob > F = 0.0000
```

Random vs. fixed effects

- The RE model is biased and inconsistent if the FE assumptions are more appropriate (correlation between x_{it} and c_i).
- If the RE assumptions hold (no correlation between x_{it} and c_i), both RE and FE are *consistent*. They should give "similar" answers in large samples, but the FE model will be *inefficient* (larger standard errors).
- A sufficiently large difference in point estimates suggests the FE assumption is probably correct and RE is inconsistent.
- The **Hausman test** is a formal test of this.

Hausman test

First use `estimates store` to save your `fe` and `re` estimates. Name them FE and RE, for example.

```
xtreg avgpassing avglcass, fe i(campus)
estimates store FE
xtreg avgpassing avgclass, re i(campus)
estimates store RE
hausman FE RE
```

Hausman test

Null hypothesis: RE assumptions hold, both estimators consistent but RE is efficient. Alternative: RE assumptions do *not* hold and the RE estimator is inconsistent. In the TX example we can reject H_0:

```
. hausman FE RE

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |      FE           RE          Difference          S.E.
-------------+----------------------------------------------------------------
    avgclass |   -.1339024    -.0442893       -.0896131        .0112156
------------------------------------------------------------------------------
                          b = consistent under Ho and Ha; obtained from xtreg
           B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       63.84
                Prob>chi2 =      0.0000
```


Review of GLS

In a linear regression with known heteroskedasticity, we can transform the original data and apply OLS to the transformed data. E.g.:

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

and assume $\text{Var}(u_i) = k_i\sigma_u^2$. The GLS transformation divides the data by $\sqrt{k_i}$. Observations with greater variance get *less* weight. The transformed model satisfies homoskedasticity. Note:

$$\begin{aligned} Var\left(\frac{u_i}{\sqrt{k_i}}\right) &= \frac{1}{k_i} Var(u_i) \\ &= \frac{1}{k_i} k_i \sigma_u^2 \\ &= \sigma_u^2 \end{aligned}$$

GLS estimation of random effects models

The random effects model with one covariate is:

$$y_{it} = \beta_0 + \beta_1 x_{it} + \underbrace{c_i + e_{it}}_{v_{it}}$$

GLS estimation again involves a transformation. Let:

$$\theta = 1 - \sqrt{\frac{\sigma_e^2}{\sigma_e^2 + T\sigma_c^2}}$$

(and note the term under the square root looks like but is different from the ICC). T is the number of observations per group, assuming a balanced panel.

GLS estimation of random effects models

The transformations of y_{it} and x_{it} are:

$$y_{it} - \theta\bar{y}_i$$

$$x_{it} - \theta\bar{x}_i$$

and OLS is estimated on the transformed model:

$$y_{it} - \theta\bar{y}_i = \beta_0(1-\theta) + \beta_1(x_{it} - \theta\bar{x}_i) + (v_{it} - \theta\bar{v}_i)$$

The transformed y_{it} and x_{it} are *quasi-demeaned*. If $\theta = 1$, we have the demeaned (within) model.

GLS estimation of random effects models

θ is not known so it must first be estimated with consistent estimators for σ_e^2 and σ_c^2. Then, $\hat{\theta}$ is used in OLS estimation ("feasible GLS").

$$\hat{\theta} = 1 - \sqrt{\frac{\hat{\sigma}_e^2}{\hat{\sigma}_e^2 + T\hat{\sigma}_c^2}}$$

Consistent estimators for σ_c^2 and σ_e^2 can be obtained using pooled OLS or fixed effects residuals.

GLS estimation of random effects models

One method for estimating σ_c^2 and σ_e^2: note that

$$v_{it} = c_i + e_{it}$$

$$v_{it}v_{is} = (c_i + e_{it})(c_i + e_{is})$$

$$E(v_{it}v_{is}) = \underbrace{E(c_i^2)}_{\sigma_c^2} + \underbrace{E(c_i e_{is})}_{0} + \underbrace{E(c_i e_{it})}_{0} + \underbrace{E(e_{it}e_{is})}_{0}$$

Get the composite residuals $\hat{v}_{it}$ using pooled OLS. The square of the RMSE in this regression estimates σ_v^2. The within-group (i) covariance in $\hat{v}_{it}$ (the sample analog of $E(v_{it}v_{is})$ above) provides a consistent estimate of σ_c^2. Then, $\hat{\sigma}_e^2 = \hat{\sigma}_v^2 - \hat{\sigma}_c^2$.

GLS estimation of random effects models

$$y_{it} - \theta\bar{y}_i = \beta_0(1-\theta) + \beta_1(x_{it} - \theta\bar{x}_i) + (v_{it} - \theta\bar{v}_i)$$

$$\theta = 1 - \sqrt{\frac{\sigma_e^2}{\sigma_e^2 + T\sigma_c^2}}$$

Notice the transformation subtracts a *fraction* of the within-group mean, where the fraction depends on σ_e^2, σ_c^2, and T.

- When $\theta = 0$, the model reduces to pooled OLS
- When $\theta = 1$, the model reduces to fixed effects (within)
- So, the value of θ is indicative of which model RE is closer to

θ gets closer to 1 as between-group variation σ_c^2 grows relative to within-group variation σ_e^2, and as the number of time periods T grows.

GLS estimation of random effects models

Can request $\hat{\theta}$ in `xtreg, re`:

```
. xtreg avgpassing avgclass, re i(campus) theta

Random-effects GLS regression                   Number of obs     =     16,062
Group variable: campus                          Number of groups  =      4,326

R-sq:                                           Obs per group:
     within  = 0.0018                                         min =          1
     between = 0.0098                                         avg =        3.7
     overall = 0.0060                                         max =          4

                                                Wald chi2(1)      =       2.74
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =     0.0978

 ------------------- theta -------------------
  min      5%       median        95%      max
0.5362   0.6529   0.7468      0.7468   0.7468

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0442893   .0267548    -1.66   0.098    -.0967277    .0081491
       _cons |   76.21828   .5503649   138.49   0.000     75.13959    77.29698
-------------+----------------------------------------------------------------
     sigma_u |  12.391941
     sigma_e |  6.4870883
         rho |  .78490199   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

This uses the original unbalanced panel, so $\hat{\theta}$ varies with group size.

GLS estimation of random effects models

Can request $\hat{\theta}$ in `xtreg, re`:

```
. xtreg avgpassing avgclass, re i(campus) theta

Random-effects GLS regression                   Number of obs      =     14,796
Group variable: campus                          Number of groups   =      3,699

R-sq:                                           Obs per group:
     within  = 0.0020                                         min =          4
     between = 0.0138                                         avg =        4.0
     overall = 0.0061                                         max =          4

                                                Wald chi2(1)       =       2.97
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =     0.0848
theta          = .73287384

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0484254   .0280999    -1.72   0.085    -.1035003    .0066494
       _cons |   76.51251   .5742248   133.24   0.000     75.38705    77.63797
-------------+----------------------------------------------------------------
     sigma_u |  11.706021
     sigma_e |  6.4897977
         rho |  .76490175   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

This uses the balanced panel, so $\hat{\theta}$ is constant.

GLS estimation of random effects models

It is useful to consider the error term in the quasi-demeaned model:

$$v_{it} - \theta \bar{v}_i = (1 - \theta) c_i + (e_{it} - \theta \bar{e}_i)$$

Suppose the RE assumption that c_i is uncorrelated with x_{it} does *not* hold. As $\theta \rightarrow 1$, the c_i component of the error term diminishes in importance, the RE estimator tends toward the FE estimator, and any bias associated with RE tends to zero.

MLE estimation of random effects models

Random effects models can also be estimated using **maximum likelihood** in which case all parameters of the model (β's, σ's) are estimated jointly:

```
. xtreg avgpassing avgclass, mle i(campus)

Fitting constant-only model:
Iteration 0:   log likelihood = -53584.523
Iteration 1:   log likelihood = -53584.523

Fitting full model:
Iteration 0:   log likelihood = -53674.187
Iteration 1:   log likelihood = -53583.763
Iteration 2:   log likelihood = -53582.969
Iteration 3:   log likelihood = -53582.969

Random-effects ML regression                    Number of obs     =     14,796
Group variable: campus                          Number of groups  =      3,699

Random effects u_i ~ Gaussian                   Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

                                                LR chi2(1)        =       3.11
Log likelihood  = -53582.969                    Prob > chi2       =     0.0780

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0496391   .0281539    -1.76   0.078    -.1048197    .0055415
       _cons |   76.53576   .5755876   132.97   0.000     75.40763    77.66389
-------------+----------------------------------------------------------------
    /sigma_u |    11.8066   .1481004                      11.51987    12.10047
    /sigma_e |   6.492198   .0436102                      6.407283    6.578237
         rho |   .7678329   .0051631                      .7575916    .7778289
------------------------------------------------------------------------------
LR test of sigma_u=0: chibar2(01) = 1.2e+04            Prob >= chibar2 = 0.000
```


Getting estimates of c_i

As with `xtreg, fe`, one can obtain the $\hat{c}_i$ estimates of the group random effects. Unlike `fe`, these are not coefficient estimates but rather estimated from residuals. The random effects $\hat{c}_i$ can be calculated in two ways:

- Maximum likelihood (following `xtreg, mle`): `predict`
- Empirical Bayes / shrinkage approach: the Best Linear Unbiased Predictors (BLUPs)

Shrinkage approach: multiply $\hat{c}_i$ by a shrinkage factor $\hat{R}_i = \frac{\hat{\sigma}_c^2}{\hat{\sigma}_c^2 + \frac{\hat{\sigma}_e^2}{T_i}}$

where T_i is the number of observations in group i. Examples on next 3 slides.

Getting estimates of c_i: MLE

```
. xtreg avgpassing avgclass, re mle i(campus)

Fitting constant-only model:
Iteration 0:   log likelihood = -53584.523
Iteration 1:   log likelihood = -53584.523

Fitting full model:
Iteration 0:   log likelihood = -53674.187
Iteration 1:   log likelihood = -53583.763
Iteration 2:   log likelihood = -53582.969
Iteration 3:   log likelihood = -53582.969

Random-effects ML regression                    Number of obs     =     14,796
Group variable: campus                          Number of groups  =      3,699

Random effects u_i ~ Gaussian                   Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

                                                LR chi2(1)        =       3.11
Log likelihood  = -53582.969                    Prob > chi2       =     0.0780

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0496391   .0281539    -1.76   0.078    -.1048197    .0055415
       _cons |   76.53576   .5755876   132.97   0.000     75.40763    77.66389
-------------+----------------------------------------------------------------
    /sigma_u |    11.8066   .1481004                      11.51987    12.10047
    /sigma_e |   6.492198   .0436102                      6.407283    6.578237
         rho |   .7678329   .0051631                      .7575916    .7778289
------------------------------------------------------------------------------
LR test of sigma_u=0: chibar2(01) = 1.2e+04            Prob >= chibar2 = 0.000

. predict uhat1, u

. sum uhat1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
       uhat1 |     14,796    8.39e-09    12.24512  -47.43509   23.42125
```


Getting estimates of c_i: BLUP

```
. xtreg avgpassing avgclass, re mle i(campus)

Fitting constant-only model:
Iteration 0:   log likelihood = -53584.523
Iteration 1:   log likelihood = -53584.523

Fitting full model:
Iteration 0:   log likelihood = -53674.187
Iteration 1:   log likelihood = -53583.763
Iteration 2:   log likelihood = -53582.969
Iteration 3:   log likelihood = -53582.969

Random-effects ML regression                    Number of obs     =     14,796
Group variable: campus                          Number of groups  =      3,699

Random effects u_i ~ Gaussian                   Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

                                                LR chi2(1)        =       3.11
Log likelihood  = -53582.969                    Prob > chi2       =     0.0780

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0496391   .0281539    -1.76   0.078    -.1048197    .0055415
       _cons |   76.53576   .5755876   132.97   0.000     75.40763    77.66389
-------------+----------------------------------------------------------------
    /sigma_u |    11.8066   .1481004                      11.51987    12.10047
    /sigma_e |   6.492198   .0436102                      6.407283    6.578237
         rho |   .7678329   .0051631                      .7575916    .7778289
------------------------------------------------------------------------------
LR test of sigma_u=0: chibar2(01) = 1.2e+04            Prob >= chibar2 = 0.000

. gen shrink = _b[/sigma_u]^2 / (_b[/sigma_u]^2 + (_b[/sigma_e]^2)/4)

. gen uhat1s = uhat1*shrink

. summ uhat1s shrink

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
      uhat1s |     14,796    1.16e-08    11.38455  -44.10139   21.77522
      shrink |     14,796    .9297209           0   .9297209   .9297209
```

Getting estimates of c_i: BLUP using xtmixed

```
. xtmixed avgpassing avgclass || campus: , mle

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log likelihood = -53582.969
Iteration 1:   log likelihood = -53582.969

Computing standard errors:

Mixed-effects ML regression                     Number of obs     =     14,796
Group variable: campus                          Number of groups  =      3,699

                                                Obs per group:
                                                              min =          4
                                                              avg =        4.0
                                                              max =          4

                                                Wald chi2(1)      =       3.13
Log likelihood = -53582.969                     Prob > chi2       =     0.0770

------------------------------------------------------------------------------
  avgpassing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    avgclass |  -.0496392   .0280727    -1.77   0.077    -.1046606    .0053823
       _cons |   76.53576   .5741313   133.31   0.000     75.41048    77.66103
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
campus: Identity             |
                   sd(_cons) |    11.8066   .1481006      11.51987    12.10047
-----------------------------+------------------------------------------------
                sd(Residual) |   6.492197   .0436102      6.407283    6.578236
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 11666.05      Prob >= chibar2 = 0.0000

. predict uhat2, reffects

. sum uhat2

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
       uhat2 |     14,796   -6.21e-10    11.38455  -44.10139   21.77523
```


Getting estimates of c_i

The shrinkage factor is smaller for groups with fewer observations (T_i). Their $\hat{c}_i$ is "shrunk" more toward the overall mean group effect of 0.

- RE estimates generally smaller than FE estimates in absolute value
- True for both MLE and EB estimates of the RE, but especially the EB
- The rank order of the $\hat{c}_i$ is usually preserved whether one assumes RE or FE

Random vs. fixed effects

When and where random effects are appropriate:

- As a rule, if the FE assumption holds the RE model is inappropriate. See the Texas class size example, where the Hausman test rejected RE.

- RE is appropriate with grouped or clustered data. See the Success for All example: assignment to treatment was random at the school level, so we need not be concerned about correlation between treatment and the error term. However, the errors are not i.i.d.

See Rabe-Hesketh and Skrondal MLM text for more guidance on RE vs. FE decision.

xttest0

The command `xttest0` (following `xtreg`) provides a formal test for the presence of random effects. H_0 in this case is that the variance across panel units is zero, and thus RE is unnecessary.

```
. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects

        wattack[schid,t] = Xb + u[schid] + e[schid,t]

        Estimated results:
                         |       Var     sd = sqrt(Var)
                ---------+-----------------------------
                 wattack |   395.1636       19.87872
                       e |   279.7314       16.72517
                       u |   47.72378        6.90824

        Test:   Var(u) = 0
                             chibar2(01) =   1266.18
                          Prob > chibar2 =    0.0000
```

Standard errors in panel models

Whether using a FE or RE model, the assumption that errors u_{it} are i.i.d. is not often satisfied in panel data. With repeat observations on the same cross-sectional unit, it is likely that errors are correlated across observations for the same i.

- If y is over-predicted in one period for a given i, it is likely to be over-predicted in the next period.

Standard errors in panel models

The RE model explicitly models the correlation across observations within group. There is an increasing preference, however, for <u>not</u> doing this and adjusting standard errors for within-panel clustering.

- For "short" panels (large N, small T), can use cluster-robust standard errors
- The "cluster" is typically the cross-sectional unit, although when the regressor of interest is aggregated at a higher level (e.g., state), can cluster at that level. Theory requires large N and that higher levels nest the cross-sectional units.
- `vce(robust)` or `robust` in `xtreg` assumes data are clustered
- Cluster-robust standard errors from `areg` are different from those using `xtreg, fe`. It is recommended that you use `xtreg`.