Assignment 2

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Problem 5.31 :
Two cards are drawn simultaneously (or successively without replacement) from a well-shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

Solution :
Let X denote the no.of kings in a draw of 2 cards.

$$\implies \text{Pr(X=0)} = \frac{48_{C_2}}{52_{C_2}} = \frac{188}{221}$$
$$\text{Pr(X=1)} = \frac{4_{C_1} \text{ x } 48_{C_1}}{52_{C_2}} = \frac{32}{221}$$
$$\text{Pr(X=2)} = \frac{4_{C_2}}{52_{C_2}} = \frac{1}{221}$$

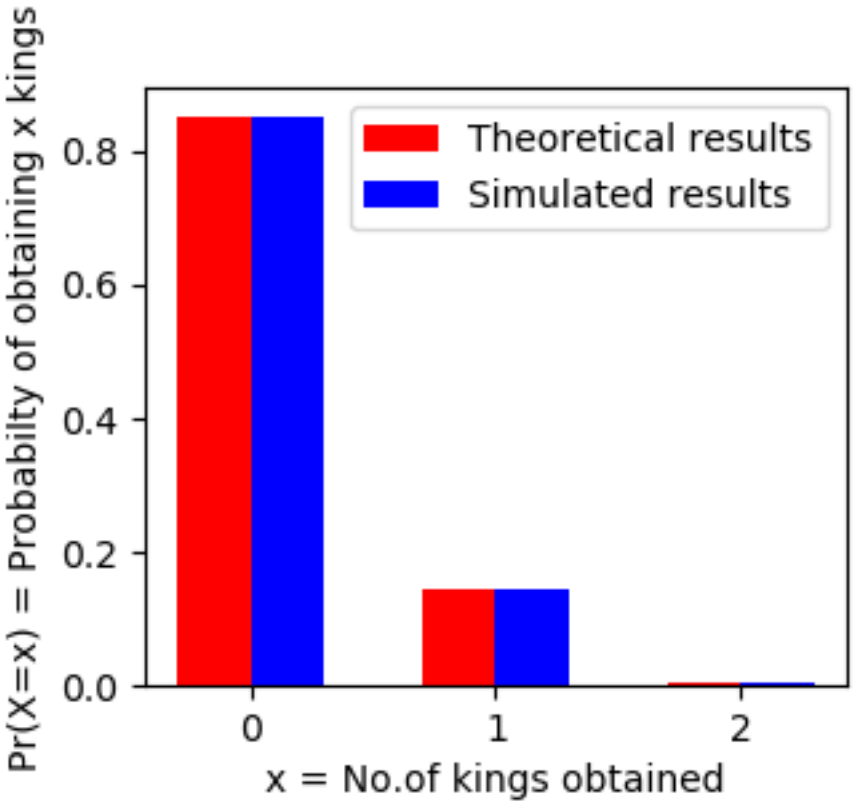
X	0	1	2
Pr(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Above is the probability distribution table of the no.of Kings obtained when two cards are drawn simultaneously from a deck of 52 cards.

Now, Mean of X = E(X)
$$= \sum_{k=0}^{2} k \Pr(X=k)$$
$$= 0 \text{ x } \frac{188}{221} + 1 \text{ x } \frac{32}{221} + 2 \text{ x } \frac{1}{221}$$
$$= \frac{34}{221} = 0.154$$

$$\text{Variance} = \text{E}(\text{X}^2) - [\text{E}(\text{X})]^2$$
$$= \sum_{k=0}^{2} k^2 \Pr(X=k) - \frac{34^2}{221^2}$$
$$= 1^2 \text{ x } \frac{32}{221} + 2^2 \text{ x } \frac{1}{221} - \frac{34^2}{221^2}$$
$$= \frac{6800}{48841} = 0.139$$

$$\text{Standard Deviation } \sigma = \sqrt{\text{Var(X)}}$$
$$= \sqrt{0.139}$$
$$= 0.373$$



Above is the bar graph showing both the simulated and the theoretical results. Both represent almost the same values.