Assignment-3

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Download all latex-tikz codes from

https://github.com/spdanda/AI1103/blob/main/Assignment3/Assignment3.tex

UGC mathA-Dec2017 Q59 :

Let X and Y be independent exponential random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $\Pr(X > 2Y | X > Y)$ is

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{2}{3}$
4. $\frac{3}{4}$

Solution :

Since X and Y are exponential random variables with means'

$$E[X] = 1 \text{ and } E[Y] = \frac{1}{2} \tag{0.0.1}$$

Marginal PDFs of X and Y are given by

$$f_X(x) = e^{-x}, x > 0 \tag{0.0.2}$$

$$f_Y(y) = 2e^{-2y}, y > 0 \tag{0.0.3}$$

CDFs for X and Y are

$$F_X(b) = \int_0^b f_X(x)\, d_x \tag{0.0.4}$$

$$= \int_0^b e^{-x}\, d_x \tag{0.0.5}$$

$$= 1 - e^{-b} \tag{0.0.6}$$

$$F_Y(b) = \int_0^b f_Y(y)\, d_y \tag{0.0.7}$$

$$= \int_0^b 2e^{-2y}\, d_y \tag{0.0.8}$$

$$= \left[-e^{-2y}\right]_0^b \tag{0.0.9}$$

$$= 1 - e^{-2b} \tag{0.0.10}$$

Now,

$$\Pr(X > 2Y | X > Y) = \frac{\Pr(X > 2Y,\, X > Y)}{\Pr(X > Y)} \tag{0.0.11}$$

$$= \frac{\Pr(X > 2Y)}{\Pr(X > Y)} \tag{0.0.12}$$

$$\Pr(X > Y) = \Pr(Y < X) \tag{0.0.13}$$

$$= E[F_Y(X)] \tag{0.0.14}$$

$$= \int_0^\infty F_Y(X)\, f_X(x)\, d_x \tag{0.0.15}$$

$$= \int_0^\infty (1 - e^{-2x})\, e^{-x} d_x \tag{0.0.16}$$

$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right]_0^\infty \tag{0.0.17}$$

$$= (0 + 1) + \frac{1}{3}(0 - 1) \tag{0.0.18}$$

$$= \frac{2}{3} \tag{0.0.19}$$

$$\Pr(X > 2Y) = \Pr\left(Y < \frac{X}{2}\right) \tag{0.0.20}$$

$$= E[F_Y(X/2)] \tag{0.0.21}$$

$$= \int_0^\infty F_Y(X/2)\, f_X(x)\, d_x \tag{0.0.22}$$

$$= \int_0^\infty (1 - e^{-x})\, e^{-x} d_x \tag{0.0.23}$$

$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-2x}}{-2}\right]_0^\infty \tag{0.0.24}$$

$$= (0 + 1) + \frac{1}{2}(0 - 1) \tag{0.0.25}$$

$$= \frac{1}{2} \tag{0.0.26}$$

Putting (0.0.19) and (0.0.26) in (0.0.12)

$$\Pr(X > 2Y \mid X > Y) = \frac{1/2}{2/3} \tag{0.0.27}$$

$$= \frac{3}{4} \tag{0.0.28}$$

$\therefore$ Option 4 is the correct answer.