Assignment-3

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Download all latex-tikz codes from

https://github.com/spdanda/AI1103/blob/main/Assignment2/Assignment2.tex

UGC mathA-Dec2017 Q59 :
Let X and Y be independent random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $\Pr(X > 2Y|X > Y)$ is

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{2}{3}$
4. $\frac{3}{4}$

Solution :
Since X and Y are exponential random variables with means

$$E[X] = 1 \text{ and } E[Y] = \frac{1}{2} \tag{0.0.1}$$

Marginal PDFs of X and Y are given by

$$f_X(x) = e^{-x}, x > 0 \tag{0.0.2}$$
$$f_Y(y) = 2e^{-2y}, y > 0 \tag{0.0.3}$$

Since X and Y are independent

$$f_{XY}(x, y) = f_X(x) \times f_Y(y) \quad x, y > 0 \tag{0.0.4}$$
$$= 2e^{-x}e^{-2y} \tag{0.0.5}$$

Now,

$$\Pr(X > 2Y|X > Y) = \frac{\Pr((X > 2Y) \cap (X > Y))}{\Pr(X > Y)} \tag{0.0.6}$$
$$= \frac{\Pr(X > 2Y)}{\Pr(X > Y)} \tag{0.0.7}$$

$$\Pr(X > 2Y) = \int_0^{\infty}\int_0^{\frac{x}{2}} f_{XY}(x, y) d_y d_x \tag{0.0.8}$$
$$= \int_0^{\infty}\int_0^{\frac{x}{2}} 2e^{-x}e^{-2y} d_y d_x \tag{0.0.9}$$
$$= 2\int_0^{\infty} e^{-x}\left[\frac{e^{-2y}}{-2}\right]_0^{\frac{x}{2}} d_x \tag{0.0.10}$$
$$= \int_0^{\infty} e^{-x}(1 - e^{-x}) d_x \tag{0.0.11}$$
$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-2x}}{-2}\right]_0^{\infty} \tag{0.0.12}$$
$$= (0 + 1) + \frac{1}{2}(0 - 1) \tag{0.0.13}$$
$$= \frac{1}{2} \tag{0.0.14}$$

$$\Pr(X > Y) = \int_0^{\infty}\int_0^{x} f_{XY}(x, y) d_y d_x \tag{0.0.15}$$
$$= \int_0^{\infty}\int_0^{x} 2e^{-x}e^{-2y} d_y d_x \tag{0.0.16}$$
$$= 2\int_0^{\infty} e^{-x}\left[\frac{e^{-2y}}{-2}\right]_0^{x} d_x \tag{0.0.17}$$
$$= \int_0^{\infty} e^{-x}(1 - e^{-2x}) d_x \tag{0.0.18}$$
$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right]_0^{\infty} \tag{0.0.19}$$
$$= \frac{2}{3} \tag{0.0.20}$$

Putting (0.0.14) and (0.0.20) in (0.0.7)

$$\Pr(X > 2Y | X > Y) = \frac{1/2}{2/3} \tag{0.0.21}$$

$$= \frac{3}{4} \tag{0.0.22}$$

$\therefore$ Option4 is the correct answer.