

Result from Challenging Problem 1

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Download all latex-tikz codes from

<https://github.com/spdanda/AI1103/blob/main/Assignment3/Assignment3.tex>

Statement :

A non-negative random variable has zero expectation value if and only if it is zero.

Proof :

Let Y be a non-negative random variable defined on the probability space Ω .

Let A_m be the set of all $\omega \in \Omega$ such that the value of Y is greater than $\frac{1}{m}$ where $m \in N$ i.e.,

$$A_m = \left\{ \omega \in \Omega : Y(\omega) \geq \frac{1}{m} \right\}$$

Given

$$E(Y) = 0 \implies \int_{\Omega} Y dP = 0 \quad (0.0.1)$$

As Y is non-negative,

$$\int_{\Omega} Y dP \geq \int_{A_m} Y dP \quad (0.0.2)$$

Also in the set A_m , $Y \geq \frac{1}{m}$

$$\therefore \int_{A_m} Y dP \geq \int_{A_m} \frac{1}{m} dP \quad (0.0.3)$$

$$= \frac{1}{m} \int_{A_m} dP \quad (0.0.4)$$

$$= \frac{1}{m} \Pr(A_m) \quad (0.0.5)$$

\therefore From (0.0.2) and (0.0.5)

$$\int_{\Omega} Y dP \geq \frac{1}{m} \Pr(A_m) \quad (0.0.6)$$

$$\implies \frac{1}{m} \Pr(A_m) \leq 0 \quad (0.0.7)$$

$$\implies \Pr(A_m) \leq 0 (\because m \in N) \quad (0.0.8)$$

$$\implies \Pr(A_m) = 0 \quad (0.0.9)$$

Also,

$$\Pr(Y \neq 0) = \Pr\left(\bigcup A_m\right)$$

$$(\because \text{In the set } A_m, Y \geq \frac{1}{m} \implies Y \text{ always} > 0) \quad (0.0.10)$$

$$= \Pr(A_1 + A_2 + A_3 + \dots) \quad (0.0.11)$$

$$= 0 (\because \Pr(A_m) = 0 \forall m \in N) \quad (0.0.12)$$

$$\implies \Pr(Y \neq 0) = 0 \quad (0.0.13)$$

$$\implies \Pr(Y = 0) = 1 - \Pr(Y \neq 0) = 1 \quad (0.0.14)$$

Hence, Probability of the random variable equals to 0 is one i.e., the random variable is always equals to 0 in it's domain.

Conversely, If $Y=0$ then

$$\int_{\Omega} Y dP = 0 \quad (0.0.15)$$

$$\implies E(Y) = 0 \quad (0.0.16)$$

Hence proved.