Challenging Problem1

Name: Sai Pravallika Danda, Roll Number: CS20BTECH11013

Download all latex-tikz codes from

https://github.com/spdanda/AI1103/blob/main/Challenging%20Problem1/main.tex

Question :

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then $E(X^{100})$ =?

1. 0
2. 1
3. 2^{100}
4. $2^{100} + 1$

Solution:

Given $E(X) = E(X^2) = 1$, hence variance of X is

$$Var(X) = E(X^2) - (E(X))^2 \quad (0.0.1)$$
$$= 1 - (1)^2 \quad (0.0.2)$$
$$= 0 \quad (0.0.3)$$

Also,

$$Var(X) = E[(X - E(X))^2] \quad (0.0.4)$$

$\implies E[(X - E(X))^2] = 0$
$\implies X - E(X)^2 = 0 \ \ \forall X$
$\implies X = E(X)^2 = 1 \ \ \forall X$
$\therefore$ The random variable X is a constant and equals to 1.
Hence,

$$E(X^{100}) = 1 \quad (0.0.5)$$

Option 2 is the correct answer.