Assignment-3

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Download all latex-tikz codes from

https://github.com/spdanda/AI1103/blob/main/Assignment3/Assignment3.tex

UGC mathA-Dec2017 Q59 :
Let X and Y be independent exponential random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $\Pr(X > 2Y | X > Y)$ is

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{2}{3}$
4. $\frac{3}{4}$

Solution :
Since X and Y are exponential random variables with means'

$$E[X] = 1 \tag{0.0.1}$$
$$E[Y] = \frac{1}{2} \tag{0.0.2}$$

Marginal PDFs of X and Y are given by

$$f_X(x) = e^{-x}, x > 0 \tag{0.0.3}$$
$$f_Y(y) = 2e^{-2y}, y > 0 \tag{0.0.4}$$

Now,

$$\Pr(X > 2Y | X > Y) = \frac{\Pr(X > 2Y\,,\, X > Y)}{\Pr(X > Y)} \tag{0.0.5}$$
$$= \frac{\Pr(X > 2Y)}{\Pr(X > Y)} \tag{0.0.6}$$

$$\Pr(X > Y) = \Pr(Y < X) \tag{0.0.7}$$
$$= E[F_Y(X)] \tag{0.0.8}$$
$$= \int_0^\infty F_Y(X)\, f_X(x) d_x \tag{0.0.9}$$
$$= \int_0^\infty \left(\int_0^x f_Y(y) d_y \right) f_X(x) d_x \tag{0.0.10}$$
$$= \int_0^\infty \left(\int_0^x 2e^{-2y} d_y \right) e^{-x} d_x \tag{0.0.11}$$
$$= \int_0^\infty (1 - e^{-2x}) e^{-x} d_x \tag{0.0.12}$$
$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right]_0^\infty \tag{0.0.13}$$
$$= (0 + 1) + \frac{1}{3}(0 - 1) \tag{0.0.14}$$
$$= \frac{2}{3} \tag{0.0.15}$$

$$\Pr(X > 2Y) = \Pr\left(Y < \frac{X}{2}\right) \tag{0.0.16}$$

$$= E[F_Y(X/2)] \tag{0.0.17}$$

$$= \int_0^{\infty} F_Y(X/2)\, f_X(x) d_x \tag{0.0.18}$$

$$= \int_0^{\infty} \left(\int_0^{\frac{x}{2}} f_Y(y) d_y\right) f_X(x) d_x \tag{0.0.19}$$

$$= \int_0^{\infty} \left(\int_0^{\frac{x}{2}} 2e^{-2y} d_y\right) e^{-x} d_x \tag{0.0.20}$$

$$= \int_0^{\infty} e^{-x}(1 - e^{-x}) d_x \tag{0.0.21}$$

$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-2x}}{-2}\right]_0^{\infty} \tag{0.0.22}$$

$$= (0 + 1) + \frac{1}{2}(0 - 1) \tag{0.0.23}$$

$$= \frac{1}{2} \tag{0.0.24}$$

Putting (0.0.15) and (0.0.24) in (0.0.6)

$$\Pr(X > 2Y | X > Y) = \frac{1/2}{2/3} \tag{0.0.25}$$

$$= \frac{3}{4} \tag{0.0.26}$$

$\therefore$ Option4 is the correct answer.