

Assignment-3

Name: Sai Pravallika Danda, Roll Number: CS20BTECH11013

Download all latex-tikz codes from

<https://github.com/spdanda/AI1103/blob/main/Assignment3/Assignment3.tex>

UGC mathA-Dec2017 Q59 :

Let X and Y be independent random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $\Pr(X > 2Y|X > Y)$ is

- | | |
|------------------|------------------|
| 1. $\frac{1}{2}$ | 3. $\frac{2}{3}$ |
| 2. $\frac{1}{3}$ | 4. $\frac{3}{4}$ |

Solution :

Since X and Y are exponential random variables with means

$$E[X] = 1 \text{ and } E[Y] = \frac{1}{2} \quad (0.0.1)$$

Marginal PDFs of X and Y are given by

$$f_X(x) = e^{-x}, x > 0 \quad (0.0.2)$$

$$f_Y(y) = 2e^{-2y}, y > 0 \quad (0.0.3)$$

Since X and Y are independent

$$f_{XY}(x, y) = f_X(x) \times f_Y(y) \quad x, y > 0 \quad (0.0.4)$$

$$= 2e^{-x}e^{-2y} \quad (0.0.5)$$

Now,

$$\Pr(X > 2Y|X > Y) = \frac{\Pr((X > 2Y) \cap (X > Y))}{\Pr(X > Y)} \quad (0.0.6)$$

$$= \frac{\Pr(X > 2Y)}{\Pr(X > Y)} \quad (0.0.7)$$

$$\Pr(X > 2Y) = \int_0^\infty \int_0^{\frac{x}{2}} f_{XY}(x, y) dy dx \quad (0.0.8)$$

$$= \int_0^\infty \int_0^{\frac{x}{2}} 2e^{-x}e^{-2y} dy dx \quad (0.0.9)$$

$$= 2 \int_0^\infty e^{-x} \left[\frac{e^{-2y}}{-2} \right]_0^{\frac{x}{2}} dx \quad (0.0.10)$$

$$= \int_0^\infty e^{-x} (1 - e^{-x}) dx \quad (0.0.11)$$

$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-2x}}{-2} \right]_0^\infty \quad (0.0.12)$$

$$= (0 + 1) + \frac{1}{2}(0 - 1) \quad (0.0.13)$$

$$= \frac{1}{2} \quad (0.0.14)$$

$$\Pr(X > Y) = \int_0^\infty \int_0^x f_{XY}(x, y) dy dx \quad (0.0.15)$$

$$= \int_0^\infty \int_0^x 2e^{-x}e^{-2y} dy dx \quad (0.0.16)$$

$$= 2 \int_0^\infty e^{-x} \left[\frac{e^{-2y}}{-2} \right]_0^x dx \quad (0.0.17)$$

$$= \int_0^\infty e^{-x} (1 - e^{-2x}) dx \quad (0.0.18)$$

$$= \left[\frac{e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right]_0^\infty \quad (0.0.19)$$

$$= \frac{2}{3} \quad (0.0.20)$$

Putting (0.0.14) and (0.0.20) in (0.0.7)

$$\Pr(X > 2Y|X > Y) = \frac{1/2}{2/3} \quad (0.0.21)$$

$$= \frac{3}{4} \quad (0.0.22)$$

∴ Option4 is the correct answer.