MECA 482-02 Group #4:

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Project: Inertia Pendulum/Reaction Wheel

Fall 2021, Calilfornia State University, Chico

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1. Introduction

This project is one that is commonly used in academia to better understand control systems and has found it's way into the realm of research as well. The premise of the inertia pendulum/reaction wheel is that it is a rotational electromechanical system that uses a mass at the end of a rotational pendulum to act as a balance. This project has two degrees of freedom: one from the pendulum and another from the actuator's rotation.

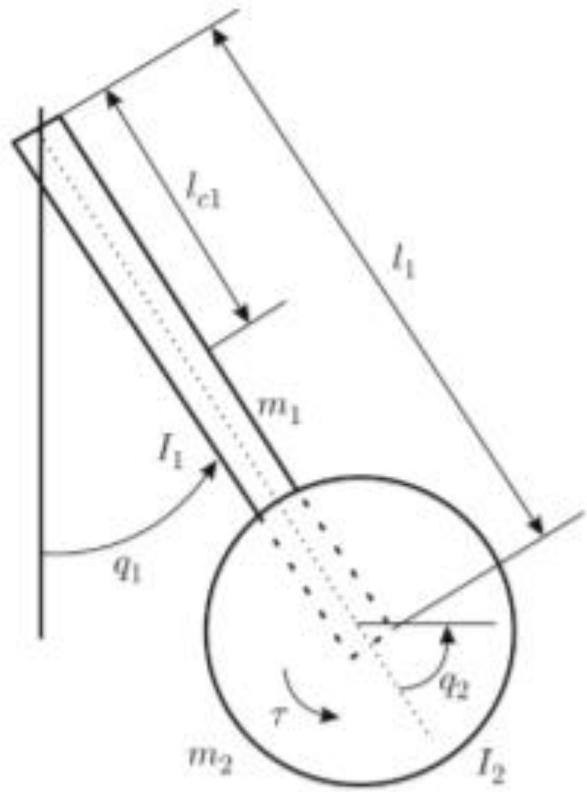


Figure 1. System Schematic

Our goal with this project was to create a desktop toy that, through the use of inertia, would stay upright even after physical contact.

$q_1 = Pendulum\ angular\ position$ $\quad$ $q_2 = Wheel\ angular\ position$

$m_1 = Pendulum\ mass$ $\quad$ $m_2 = Wheel\ mass$

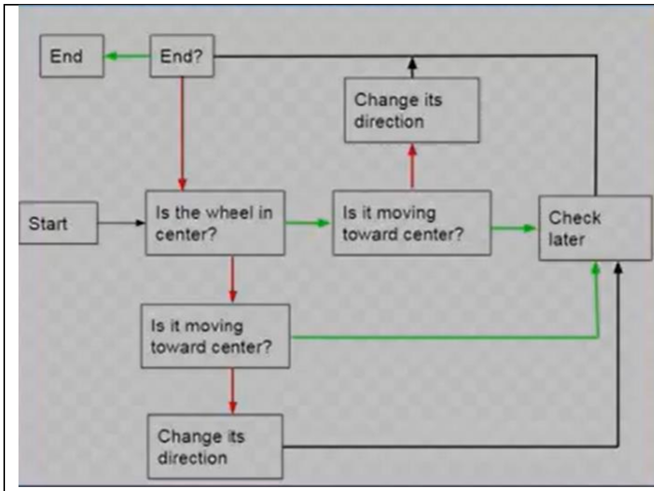
$I_1 = Pendulum\ inertia$ $\quad$ $I_2 = Wheel\ inertia$

$l_1 = Lenth\ of\ pendulum$ $\quad$ $l_{c1} = Center\ of\ mass\ of\ pendulum$

$\tau = Applied\ Torque$

2. Logical Viewpoint/Physical Viewpoint/Capabilities Database

	Figure 2. Logical Viewpoint of our project.



We wanted to take a simple approach that we could easily build upon as necessary. This led to the first question being: Is the wheel in the center.

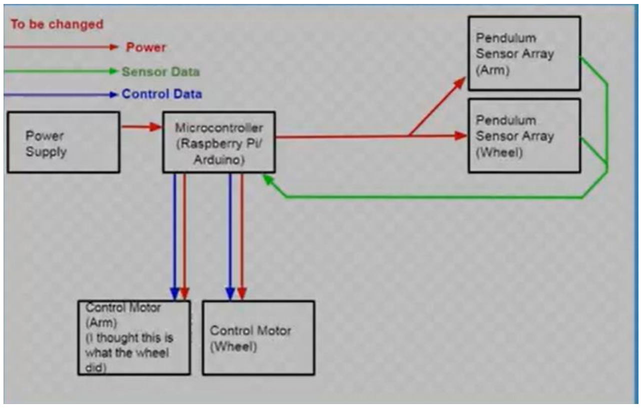


Figure 3. Physical Viewpoint

An attempt at quantifying what components would be used should this project ever be produced.

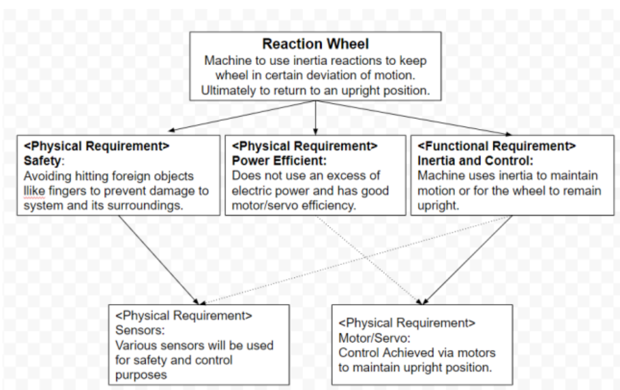


Figure 4. Capabilities Database

A diagram outlining what is required for the project's success as well as factoring in safety conditions as well.

3. <u>CoppeliaSim</u>

The creation of the model was, as expected, the easiest part of this task. What was unexpected, was being able to program proper control via LUA in V-rep's GUI.

Through trial-and-error movement and set boundaries were made using CoppeliaSim's standard ui controls, specifically the use of the dynamic properties of various elements. How fast something would move, to what position, and what range of motion? Ultimately, we were able to make the inertia pendulum aspect stay upright while there was a movement from the reaction wheel, but the issue is this was not done with the models GUI.

Getting a connection to matlab was possible with the help of examples provided, however being able to further add elements of control was the roadblock here. Upon connection to MATLAB, CoppeliaSim would immediately barrage us with error message after error message.

Future work on this project or any involving CoppeliaSim would require a deeper knowledge of UI coding with LUA. One thing that may be considered as well is creating the model in Solidworks then transferring over the more realistic model for simulation, as this would affect masses and dimension on a grander scale.

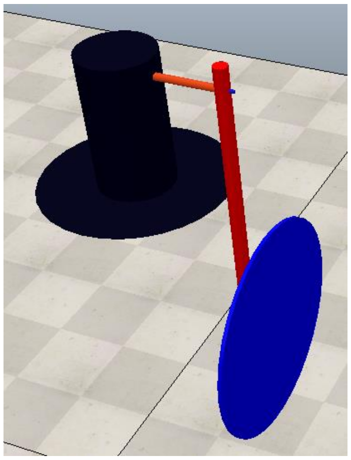


Figure 5. CoppeliaSim model of our Reaction Wheel.

4. Matlab model derivation

The mathematical model is initially obtained by 2 Euler-Lagrange equations below, seeing that the inertia pendulum consists of 2 interacting bodies.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} = 0,$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} = \tau.$$

At this point when the pendulum itself hangs up, there's no torque applied. As for the Lagrangian itself, it's given by the following equations below, where K_1 and K_2 are the respective kinetic energies of the bodies of the pendulum and P_1 and P_2 are their respective potential energies.

$$L = K - P,$$
$$K = K_1 + K_2, \quad P = P_1 + P_2,$$

The equation of the kinetic energy itself can be obtained by adding the kinetic energy of the first mass body (the pendulum) which will translate on a circumference with the radius of the center of mass as well as the kinetic energy of the stick rotating around its center of mass. As far as the wheel goes, obtaining its equation is essentially similar in terms of process. However, the angular velocity has to be considered. As a result, the 2 following equations are formed.

$$K_1 = \frac{1}{2} m_1 l_{c1}^2 \dot{q}_1^2 + \frac{1}{2} I_1 \dot{q}_1^2.$$

$$K_2 = \frac{1}{2} m_2 l_1^2 \dot{q}_1^2 + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2.$$

Assuming $q_1 = 0$, where the arm is basically downwards, the potential energy of both the pendulum and wheel can be determined.

$$P_1 = g m_1 l_{c1} (1 - \cos(q_1)).$$

$$P_2 = g m_2 l_1 (1 - \cos(q_1)).$$

Combining all the previous equations together, the entire Langragian is formed below.

$$L = \frac{1}{2} m_1 l_{c1}^2 \dot{q}_1^2 + \frac{1}{2} I_1 \dot{q}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{q}_1^2 + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2 - g\overline{m}(1 - \cos(q_1)),$$

Following the Langragian calculation, with arrangement in some of the terms, the matrix equivalent is determined.

$$D \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} \overline{m} g \sin(q_1) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix},$$
$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}.$$

It’s always assumed that the determinant ends up being positive, as seen below.

$$\det(D) = d_{11}d_{22} - d_{12}d_{21} > 0.$$

5. References

[1] V.M. Hernandez-Guzman, r. Silva-Ortigoza, Automatic Control with Experiments. Springer International Publishing Ag. 2019

[2] M.W. Spong, P. Corke, R. Lozano. Nonlinear Control of the Reaction Wheel Pendulum. Pergamon. 2001