# ***Problem Description***

The goal of this project is to allocate a group of professors to specific courses at specific times of the week. Three main solution methodologies were considered.

The first solution aimed to model this as a Set Covering problem. This methodology however would require the generation of large and complex sets. After each iteration, the feasibility of the solution needs to be checked. If the solution is infeasible, an additional set will be created and the model would have to be re-run.

The second option was to model the problem as an assignment problem. In this model we had the following decision variable:

The index *j* of this variable represents unique courses. This however was causing issues, as it is hard to model the constraints that shows when the prof and cohort would still be in the course. This is difficult as the number of hours for each course varies and individual continuity constraints need to be added, which makes the problem very complex.

Finally, the model that was used is an assignment problem using the following decision variable:

The index *j* of this variable represents a section of a course as each four hour courses are divided into two sections of two and each five our courses are divided into one section of three hours and another of two hours. Using this new variable, the problem’s difficulty reduced significantly.

# ***Assumptions***

To solve this problem, the assumptions that courses should be taught in sections of two to three hours. This assumption was judged to be realistic as it most courses encountered in university were taught in sections of these lengths. Therefore, *j = 1* represents two hours of MSCI 311 and *j = 2* represents another two hours of MSCI 311. The two sections of the same course are then forced to be scheduled on different days using a constraint.

There’s also an assumption that each day comprises of 8 timeframes. Each timeframe is an hour and a course can only start and finish at the beginning and at the end of each hour, respectively. These timeframes start at 9:00AM every day and ends at 5PM. It is also assumed that the operating days of the school range from Monday to Friday inclusively.

It is assumed that each professor can teach two courses per cohort. When a professor teaches a course that is available for both cohort ***(courses in bold and italic)***, it is considered as two different courses. In other words, if a professor can teach a course such as MSCI 541 which is offered for both cohort of the winter term, then we consider this course as two courses; one course for cohort 1 and the other for cohort 2. However, when scheduling these courses, it is important to make sure that these courses do not overlap with any other course from both cohorts. The following list shows the assumptions made concerning the courses that can be taught by each professor:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Professors | Cohort 1 -A | Cohort 1 - B | Cohort 2 - A | Cohort 2 - B |
| Smucker | 541 | 436 | ***541*** | ***436*** |
| Dimitrov | 541 | 436 | ***541*** | ***436*** |
| Erenay | 431 | 334 | 435 | 433 |
| Bookbinder (RIP) | 334 | 431 | 435 | 433 |
| Vechtomova | 444 | 442 | 423 | ***442*** |
| Hurst | 311 | 454 | 402 | ***454*** |
| Carr | 454 | 442 | ***454*** | ***442*** |
| Khanlyev | 444 | 442 | 435 | ***442*** |
| Gzara | 311 | 454 | 433 | ***454*** |
| Elkadri | 311 | 454 | 423 | ***454*** |

# ***Mathematical Model***

## ***Set/Indices***

## ***Parameters***

The following is a list and description of the parameters used in the model. For more details, please refer to ***Appendix A***.

## ***Variables***

The following is a list of variables and their description:

## ***Constraints***

1. All courses should be scheduled.
2. Each professor can teach at most two courses. These two courses must be for different cohorts.
3. Professors can’t teach two courses during the same timeslot
4. Courses cannot have overlaps if it’s for the same cohort.

*If c = 1:*

*If c = 2:*

*If c = 1:*

*If c = 2:*

1. Courses from the same cohort cannot start at the same time.
2. For courses that are for both cohorts, cohort 2 cannot have classes when these courses are offered. (This constraint is for courses of 2 hours)
3. For courses that are for both cohorts, cohort 2 cannot have classes when these courses are offered. (This constraint is for courses of 3 hours)
4. For courses that are for both cohorts, cohort 2 cannot start another course at the same time.
5. Professors cannot start teaching two courses at the same time.
6. A course can only be assigned to one professor.
7. We need a constraint to relate variable Yik with Xijtdc.
8. We need a constraint to relate variable Zid with Xijtdc.
9. We need a constraint to relate variable Wij with Xijtdc.
10. Professors only teaches courses they can teach.
11. If one part of the course is taught by a course, both part of the course is taught by the prof.
12. A given prof can only start teaching a given course at one given time.
13. Can’t start at last hour of the day if it’s a two hour class.
14. Can’t start at last two out of the day if it’s a 3 hour class.
15. Can’t have two parts of the same class within the same day.
16. A class can only belong to a cohort if it’s intended for the given cohort.

## ***Objective Function***

# ***Solution Approach***

## ***Decomposition Structure***

## ***Lagrangian Bound***

## ***Branch and Price***

## ***Heuristics***

# ***Solution***

## ***Initial Solution***

Initially, the following solution was obtained:

*Cohort 1:*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t d | 1 | 2 | 3 | 4 | 5 |
| 1 | I=3, j=8 | I=6, j=2 |  | I=2, j=14 | I=3, j=7 |
| 2 | I=3, j=8 | I=6, j=2 | I=5, j=17 | I=2, j=14 | I=3, j=7 |
| 3 | I=2, j=13 |  | I=5, j=17 | I=9, j=22 | I=1, j=19 |
| 4 | I=2, j=13 | I=7, j=16 | I=7, j=15 | I=9, j=22 | I=1, j=19 |
| 5 | I=4, j=3 | I=7, j=16 | I=7, j=15 | I=9, j=22 |  |
| 6 | I=4, j=3 | I=5, j=18 | I=6, j=1 | I=1, j=20 | I=4, j=4 |
| 7 | I=4, j=3 | I=5, j=18 | I=6, j=1 | I=1, j=20 | I=4, j=4 |
| 8 |  |  |  |  |  |

*Cohort 2:*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t d | 1 | 2 | 3 | 4 | 5 |
| 1 | I=4, j=12 |  |  | I=2, j=14 | I=4, j=11 |
| 2 | I=4, j=12 | I=5, j=5 |  | I=2, j=14 | I=4, j=11 |
| 3 | I=2, j=13 | I=5, j=5 |  | I=9, j=22 | I=1, j=19 |
| 4 | I=2, j=13 |  |  | I=9, j=22 | I=1, j=19 |
| 5 |  | I=6, j=21 | I=5, j=6 | I=9, j=22 | I=3, j=9 |
| 6 | I=3, j=10 | I=6, j=21 | I=5, j=6 | I=1, j=20 | I=3, j=9 |
| 7 | I=3, j=10 | I=6, j=21 |  | I=1, j=20 |  |
| 8 |  |  |  |  |  |

As it can be observed, many courses are scheduled during lunchtime. This is due to the fact that in the objective function, the weights to avoid morning courses, lunchtime courses and evening courses are equal. However, due to personal experiences, time off during lunchtime was considered most important. For this reason, the weight for classes during lunch time have been increased. Since it is a minimization problem, courses scheduled at that time would be avoided.

## ***Final Solution***

A re-run of the model was conducted after increasing the weight in the objective function for courses scheduled during lunchtime. This resulted in schedules with lunch breaks, which is a better schedule. The following is the final solution:

*Cohort 1:*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t d | 1 | 2 | 3 | 4 | 5 |
| 1 |  |  | I=5, j=18 | I=9, j=22 | I=3, j=3 |
| 2 | I=3, j=4 | I=5, j=17 | I=5, j=18 | I=9, j=22 | I=3, j=3 |
| 3 | I=3, j=4 | I=5, j=17 |  | I=9, j=22 | I=3, j=3 |
| 4 |  |  |  |  |  |
| 5 | I=7, j=15 | I=7, j=16 | I=2, j=14 | I=1, j=20 | I=4, j=7 |
| 6 | I=7, j=15 | I=7, j=16 | I=2, j=14 | I=1, j=20 | I=4, j=7 |
| 7 | I=4, j=8 | I=6, j=1 | I=1, j=19 | I=2, j=13 | I=6, j=2 |
| 8 | I=4, j=8 | I=6, j=1 | I=1, j=19 | I=2, j=13 | I=6, j=2 |

*Cohort 2:*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t d | 1 | 2 | 3 | 4 | 5 |
| 1 | I=4, j=12 | I=6, j=21 |  | I=9, j=22 |  |
| 2 | I=4, j=12 | I=6, j=21 | I=10, j=5 | I=9, j=22 | I=4, j=11 |
| 3 |  | I=6, j=21 | I=10, j=5 | I=9, j=22 | I=4, j=11 |
| 4 |  |  |  |  |  |
| 5 |  |  | I=2, j=14 | I=1, j=20 | I=3, j=9 |
| 6 | I=3, j=10 | I=10, j=6 | I=2, j=14 | I=1, j=20 | I=3, j=9 |
| 7 | I=3, j=10 | I=10, j=6 | I=1, j=19 | I=2, j=13 |  |
| 8 |  |  | I=1, j=19 | I=2, j=13 |  |

# ***Discussion and Conclusion***

***Appendix A***

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Course Code | 311-a | 311-b | 334-a | 334-b | 423-a | 423-b | 431-a | 431-b | 433-a | 433-b | 435-a | 435-b | 436-a | 436-b | 442-a | 442-b | 444-a | 444-b | 541-a | 541-b | 402-a | 454-a |
| index j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| H(j) | 2 | 2 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 |

|  |  |  |
| --- | --- | --- |
| Course index j | Cohort 1 | Cohort 2 |
| 1 | 1 | 0 |
| 2 | 1 | 0 |
| 3 | 1 | 0 |
| 4 | 1 | 0 |
| 5 | 0 | 1 |
| 6 | 0 | 1 |
| 7 | 1 | 0 |
| 8 | 1 | 0 |
| 9 | 0 | 1 |
| 10 | 0 | 1 |
| 11 | 0 | 1 |
| 12 | 0 | 1 |
| 13 | 1 | 0 |
| 14 | 1 | 0 |
| 15 | 1 | 0 |
| 16 | 1 | 0 |
| 17 | 1 | 0 |
| 18 | 1 | 0 |
| 19 | 1 | 0 |
| 20 | 1 | 0 |
| 21 | 0 | 1 |
| 22 | 1 | 0 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Prof. Name | j i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| Smucker | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| Dimitrov | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| Erenay | 3 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Bookbinder | 4 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Vechtomova | 5 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| Hurst | 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| Carr | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| Khanlyev | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| Gzara | 9 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Elkadri | 10 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |