



An Assessment of Requested Seed Core Support for Expansion Based on an Analysis of Giving Prepared 21 Aug 2013

Executive Summary

Weekly giving totals from the Seed congregation from January 2010 through 11 August 2013 were analyzed with several mathematical models. Statistical analysis was used to attempt to detect if giving, following Seed's Facility Planning team's late May 2013 request for increased giving, actually increased. One model, attempting to characterize week-to-week variation, was unsatisfactory. A second model, which assumes no change has occurred since Jan 2010 in Seed giving rates, fit the data surprisingly well. However, subsequent analysis discredits this model. The third model accounts for month-to-month variation in giving and explains enough of the total variation to identify statistically significant correlations between giving and both month and year. A final minor modification of this model provided the best overall analysis and good correlation between giving, month, and year.

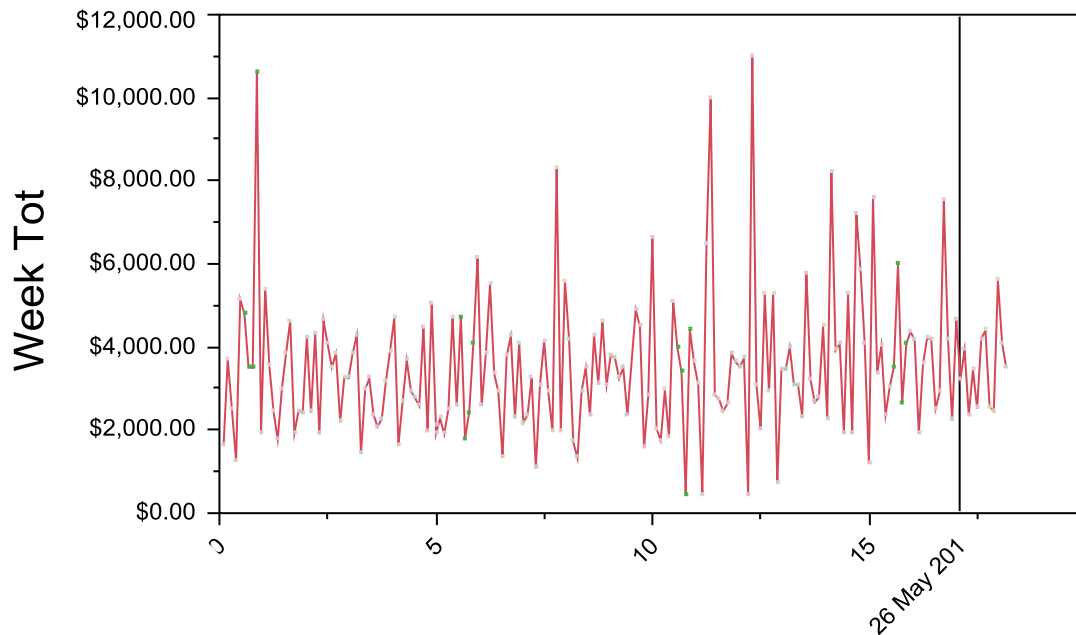
Each model is used to evaluate the question of whether post-request giving has increased enough to be statistically discernable with reasonable certainty. None of the models are able to detect that giving increased. The strongest models clearly detect a statistically significant decrease in giving since the request (e.g. see [Figure 12](#)). Analysis also affirms that if the Seed congregation had changed giving as requested, the performed analyses would have been capable of detecting it.

Finally, the monthly giving total models are used to predict the 2013 total giving ([Table 1](#)) assuming no giving changes (contrary to the findings). Finally, average additions and reductions in giving for each month of the year are quantified in [Figure 13](#).

Source Data

Weekly totals for Seed congregation giving both via PayPal and the offering, including mailed in payments, from the beginning of 2010 till 11 August 2013 were tallied and provided by John Earling and are shown in [Figure 1](#).

Figure 1. Seed Weekly Giving Totals



Cumulative Week Number (1 = 3Jan2010)

Events

- 30 April 2013 Seed Deacon Minigroup announcement of the upcoming Deacons meeting including minimal details of expansion plans
- 5 May 2013 Seed Deacons presentation
- 16 May 2013 Seed Core Minigroup announcement explaining that the roof was failing and Seed would be replacing it. Replacement occurred that week.
- 26 May 2013 Seed Core presentation
- 30 May 2013 Seed Core Minigroup announcement relaying Facilities Planning Team short term plan including the requested Core financial support.

Requested Core Financial Support for Seed Short-term Expansion Plans

Quoting the 30 May Minigroup announcement, the financial request made of the Core was:

“This is what ‘aggressive pay-off’ means if completely funded by Core giving:

\$38,000 between 38 regularly giving families:

- 24Mo payoff: \$10 per week per family
- 36Mo Payoff: \$7 per week per family
- 48Mo Payoff: \$5 per week per family”

This means one would like to detect something between a $38 * \$5 = \190 per week increase and $38 * \$10 = \380 per week giving increase. June 2013 had 5 Sundays so the requested monthly

total increase would be between \$950 and \$1,900. July 2013 had 4 Sundays so the requested monthly total increase would be between \$760 and \$1,520.

Analysis of Giving Data

Weekly Giving Model

As [Figure 1](#) shows, giving varies from week to week. We desire to detect an increase in giving beginning 30 May. But what would an increase be evaluated in comparison to?

Using 2010, 2011, 2012, and pre-19May 2013 data, the following model and SAS Inc. JMP software version 10.0.2, an average giving model was regressed:

$$\text{Week Giving Total} = \text{Giving}_{Avg} + f(\text{year}) + f(\text{week of the year})$$

Specifically,

$$\text{Week Giving Total} = a_1 + \text{Match}(\text{Year}) \begin{bmatrix} 2010 \rightarrow b_1 \\ 2011 \rightarrow b_2 \\ 2012 \rightarrow b_3 \\ 2013 \rightarrow b_4 \end{bmatrix} + \text{Match}(\text{Week}) \begin{bmatrix} 1 \rightarrow c_1 \\ 2 \rightarrow c_2 \\ 3 \rightarrow c_3 \\ \vdots \\ 52 \rightarrow c_{52} \end{bmatrix}$$

This model allows for year by year changes in giving as well as estimates of giving predicted for each week of the year as established by available giving data for that week for each year.

Model parameters shown above were regressed using data from Jan 2010 thru 12 May 2013. The initial regression result (not shown) shows 4 weeks where actual giving is more than \$4,000 different from that predicted. These weeks are the 4 high peak giving weeks observable in [Figure 1](#). Assuming these weeks are unusual and unrepresentative of “typical giving”, these weeks were excluded and the model again regressed against the remaining data. This model result is shown in [Figure 2](#).¹

¹ Prior to the Sunday giving count being completed by John Earling, occasionally one week’s giving would be counted in the subsequent week’s giving. This would not only lead to unusually high giving totals for those weeks, but also inaccurately low giving totals for the uncounted weeks. In the results shown here, the low weeks were not excluded from the analysis. Subsequently, this entire analysis was repeated excluding both the high and the low values. The results are not shown but led to the same overall conclusions with slightly different model parameters and p-values.

Figure 2. Average Giving Total Model

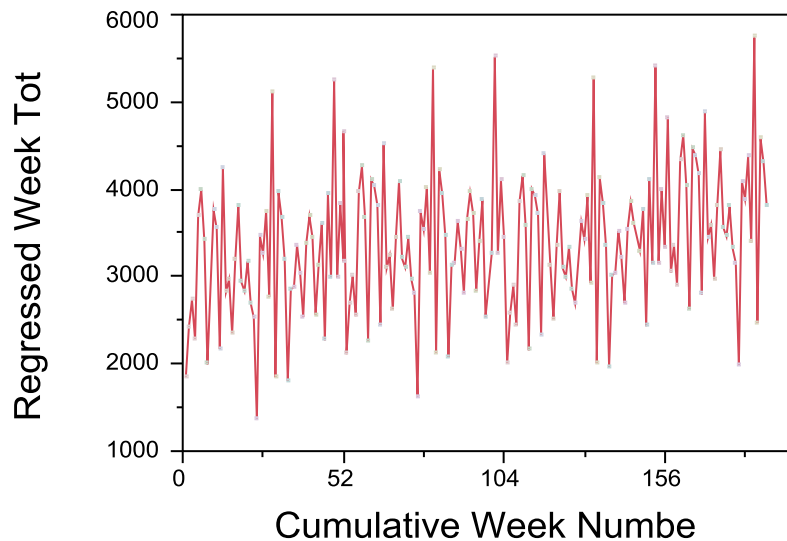
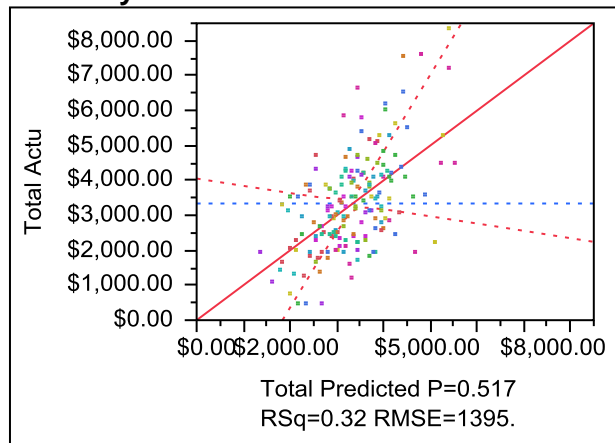


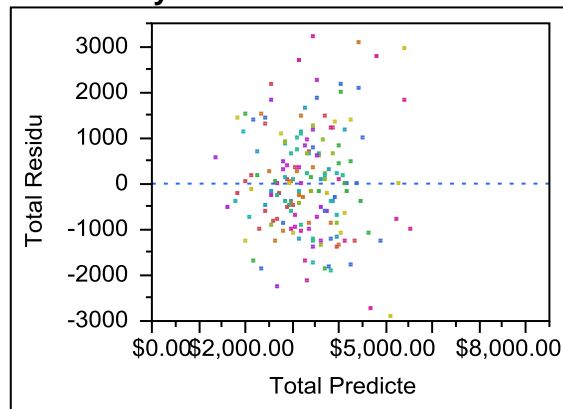
Figure 3 shows an evaluation of the regression result above. The 172 data values used are fitted by a model with 57 parameters: the overall mean (a_1), year with 4 levels (b_1 , b_2 , b_3 , and b_4), and week with 52 levels (c_1 , c_2 , ... c_{52}). That is a lot of parameters. In fact, neither of the two factors, year nor week, evaluated to be statistically justifiable as the p-values 0.46 and 0.50 (respectively) show. This is another way of acknowledging that the total giving shown in Figure 1 is a relatively flat, though noisy, trend. One might argue that the year-to-year variation of the week giving total is obviously real (justified) because the entire year giving amount is an exactly knowable number, therefore the weekly mean value is also exactly knowable and is certainly not the same each year. However, this argument doesn't actually address the question about whether the underlying intended giving rate by the collection all families that give at Seed changed from year to year. The model attempts to characterize that difficult to know collective trend. Also, grouping the 52 individual weeks of data doesn't show enough week-to-week differences to statistically justify the "week" model term. In the end, the model only "explains" 32% of the variation in the data ($R^2 = 0.318$). Finally, the residuals have no apparent structure. This is generally a good model result. However, they also wouldn't if the original giving data variation is random (no special year-to-year nor week-to-week variation).

Figure 3. Weekly Giving Total Regression Evaluation

Actual by Predicted Plot



Residual by Predicted Plot



Summary of Fit

RSquare	0.318066
RSquare Adj	-0.00526
Root Mean Square Error	1395.117
Mean of Response	3340.531
Observations (or Sum Wgts)	172

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Year	3	3	5095911	0.8727	0.4575
Week #	52	52	100202667	0.9900	0.5049

Model of Weekly Giving Total

$$Week\ Total = \$3415 + Match(Year) \begin{bmatrix} 2010 \rightarrow -\$264 \\ 2011 \rightarrow \$6 \\ 2012 \rightarrow -\$103 \\ 2013 \rightarrow \$360 \end{bmatrix} + Match(Week) \begin{bmatrix} 1 \rightarrow -\$1310 \\ 2 \rightarrow -\$730 \\ 3 \rightarrow -\$875 \\ \vdots \\ 52 \rightarrow \$1511 \end{bmatrix}$$

Obviously, not all Week parameter level values are shown above.

Despite the evidence of an unjustifiably complicated model, I use it for the sake of the analysis with the reasoning that we expect there could be year-to-year and week-to-week factors that should help explain giving totals. For example, attendance in the summer weeks is known to drop off and attendance has probably increased over the analysis period.^{2,3} So, even if there is not enough power to statistically justify week-to-week and year-to-year variation estimates, we can use the available data to estimate it in case it really is there.

So, the model regressed on the 3 Jan 2010 to 19 May 2013 data is used (extrapolated) to predict the estimated weekly giving for the weeks from 19 May 2013 to 11 Aug 2013. Then the entire collection of weekly giving totals is used to address the initial question. Specifically, to evaluate whether giving increased after 26 May 2013, we calculate a t-test on the residuals (Actual giving – Model Avg) for all the weeks prior to 26 May 2013 compared to the residuals

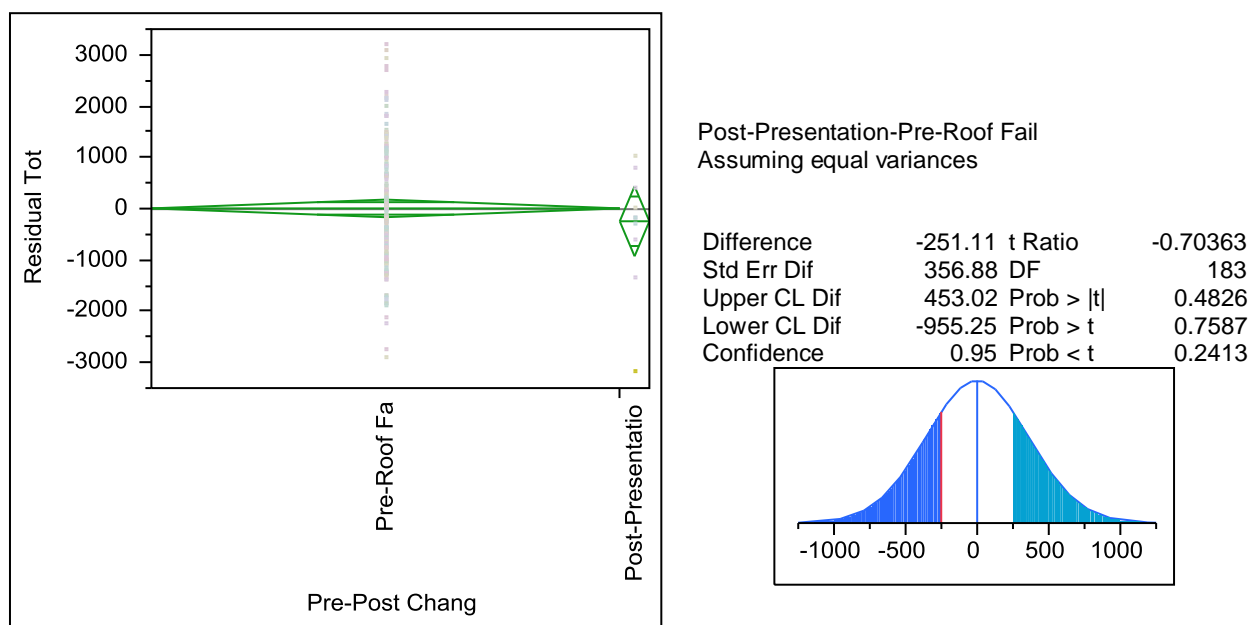
² Analysis is shown elsewhere by the Seed Facilities Planning team, et al.

³ Of course increased attendance does not necessarily result in increased giving.

for the giving totals for weeks after 26 May 2013. We use the residuals because they characterize the difference between the actually giving weekly total and the model's regressed weekly total. The residuals for totals of weeks prior to 26 May 2013 average to nearly zero because we used those data to regress the model.⁴ If giving increased from the model average after 26 May 2013 then the residuals for those months will be positive. The t-test allows one to evaluate if the specific estimate of the difference is large enough to say it is unlikely to have occurred by chance.

Figure 4 shows the t-test analysis. The analysis shows that the mean of the post-26 May 2013 giving residuals is lower than the pre-19 May 2013 residuals by \$251. No t-test is even needed to answer if that is a \$190-\$380 increase. However, it is also worth mentioning that the p-value (Prob < t = 0.24) evaluating if there is a statistically significant decrease does not support that conclusion either.

Figure 4. Post-26 May 2013 vs. Pre 19 May 2013 Giving



Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Pre-Roof Fail	174	4.43	87.02	-167.3	176.13
Post-Presentation	11	-246.68	346.11	-929.6	436.20

Std Error uses a pooled estimate of error variance

There is one week, 14 July, that is nearly \$3,200 below the average (The point in the bottom, right corner of the residual plot). If that single data point is excluded from the analysis the difference goes from -\$251 to \$41. However, that estimated increase is not enough for the

⁴ It is not exactly zero because the model was regressed with data through May 19 since Deacons knew by this time but did the t Test comparing before and after May 26.

whole collection of points to be statistically significantly different from zero ($p\text{-value}=0.38$). In other words, even excluding the low value, an increase is not large enough to detect.

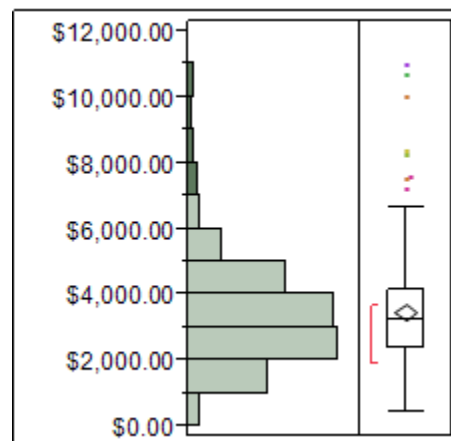
For a point to be \$3,200 below the model predicted average seems odd considering the overall average weekly giving is only \$3,238. The reason that residual is so far below the model is that the model predicts that particular week to be a high giving week, \$5,739. This is perhaps because in past years people may have missed church on the July 4 weekend and gave their offering the following week. This oddity again raises questions about the complex model used to estimate the mean.

Single Mean Model

Instead of using a complicated model, the simplest model can be used: the single mean model supposes that giving has been roughly constant, though variable, from Jan 2010. This model estimates all weekly totals to be part of a single population best estimated by the mean.

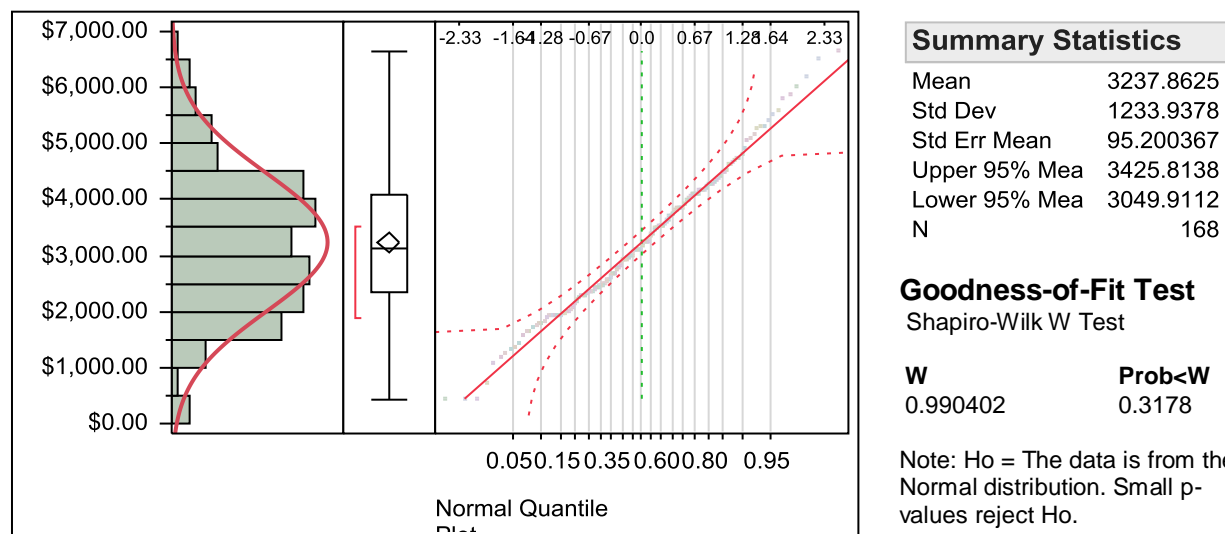
A histogram, shown in [Figure 5](#), shows all pre-19 May 2013 values as a single population.

Figure 5. Histogram of All weekly Giving Totals



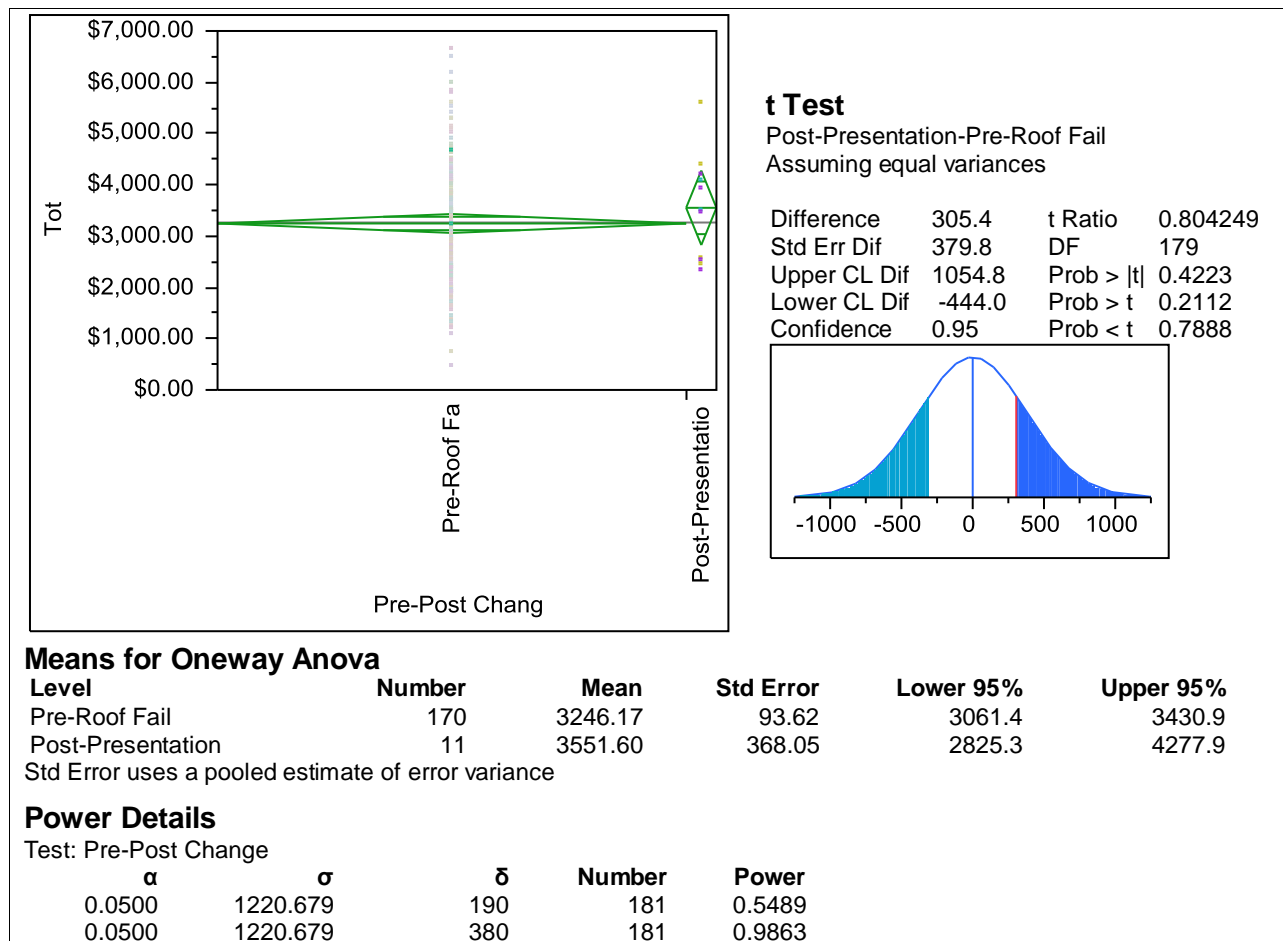
Note the sub-population of high-giving weeks above the limit of the whisker plot. Those weeks are “unusually large” and were excluded so as to not distort the estimation of the mean of the “typical” giving. This turns out to be only 8 weeks out of the total 189 weekly giving amounts. The subsequent population analysis following exclusion of these values is shown in [Figure 6](#).

Figure 6. Mean of "Typical Weekly Giving Totals"



This analysis is pretty well behaved. Both the normal quantile plot and the goodness of fit test favorably affirm this sub-population is fairly “normal”. The normal distribution is characterized by a mean of \$3,237.86 per week with a standard deviation of \$1,233.94. Using the single mean model the t-Test analysis can be repeated and is shown in [Figure 7](#).

Figure 7. Comparison of Giving Before and After Increase Request



The above analysis directly compares the 170 weekly totals up to and including 26 May 2013 against the 11 weeks of data after 26 May 2013. The post-request giving total increase estimate is \$305. However, this amount is not large enough (p-value for Prob > t = 0.21) to be able to say with sufficient statistical certainty that the difference isn't just from chance. Specifically, there is a 21% chance, given weekly giving totals that exactly average at a fixed \$3,237.86 but vary "normally" with a standard deviation of \$1,220, of getting the 11 latest giving values being \$305 higher than the average. It is customary to expect the probability of obtaining the result by chance to be less than 5% before declaring it to be statistically "certain" ($p < 0.05$).⁵

The power analysis shows that the probability of detecting a \$190 change in the last 11 values with any 181 weekly totals with a standard deviation of \$1,220 is only 55%. However, if the difference really was \$380 there is a 98% chance we would detect it with that amount of data. So, Seed does not seem to have increased giving to achieve the 24 month payoff but it is not possible to say, with the described analyses, that Seed has indeed increased its giving at all.

⁵ Most scientists don't conclude $p > 0.2$ to be statistically significant. Some consider $p \sim 0.1$ to be "marginally" statistically significant.

In fact, the 95% confidence interval for the estimate of Seed's post-May 26 average giving amount per week is \$2,825 to \$4,278. This range includes the well behaved mean of \$3,237.86 that seems to have been characteristic of Seed weekly giving since Jan 2010. The 95% confidence interval for that long-term giving average is \$3,061 to \$3,430.

Monthly Mean Model

The "Weekly Giving" analysis above shows that a weekly model of giving data is not statistically justified. It seems reasonable to ask if a monthly model might do better:

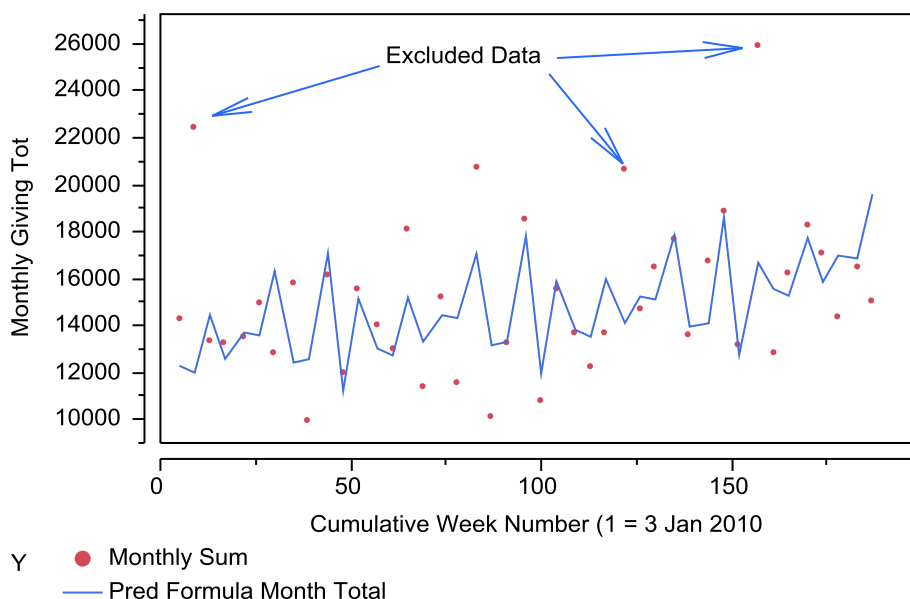
$$\text{Monthly Giving Total} = \text{Giving}_{Avg} + f(\text{year}) + f(\text{Month of the year})$$

Specifically,

$$\text{Monthly Giving Total} = a_1 + \text{Match}(\text{Year}) \begin{bmatrix} 2010 \rightarrow b_1 \\ 2011 \rightarrow b_2 \\ 2012 \rightarrow b_3 \\ 2013 \rightarrow b_4 \end{bmatrix} + \text{Match}(\text{Month}) \begin{bmatrix} 1 \rightarrow c_1 \\ 2 \rightarrow c_2 \\ 3 \rightarrow c_3 \\ \vdots \\ 12 \rightarrow c_{52} \end{bmatrix}$$

The model tested includes the year term again in case the month term removes enough of the variability to bring out the statistical significance of the year-to-year variation. A plot of the regression result is shown in [Figure 8](#) with analysis of the result shown in [Figure 9](#). Again, regression was performed using only the pre-26 Mar 2013 data.

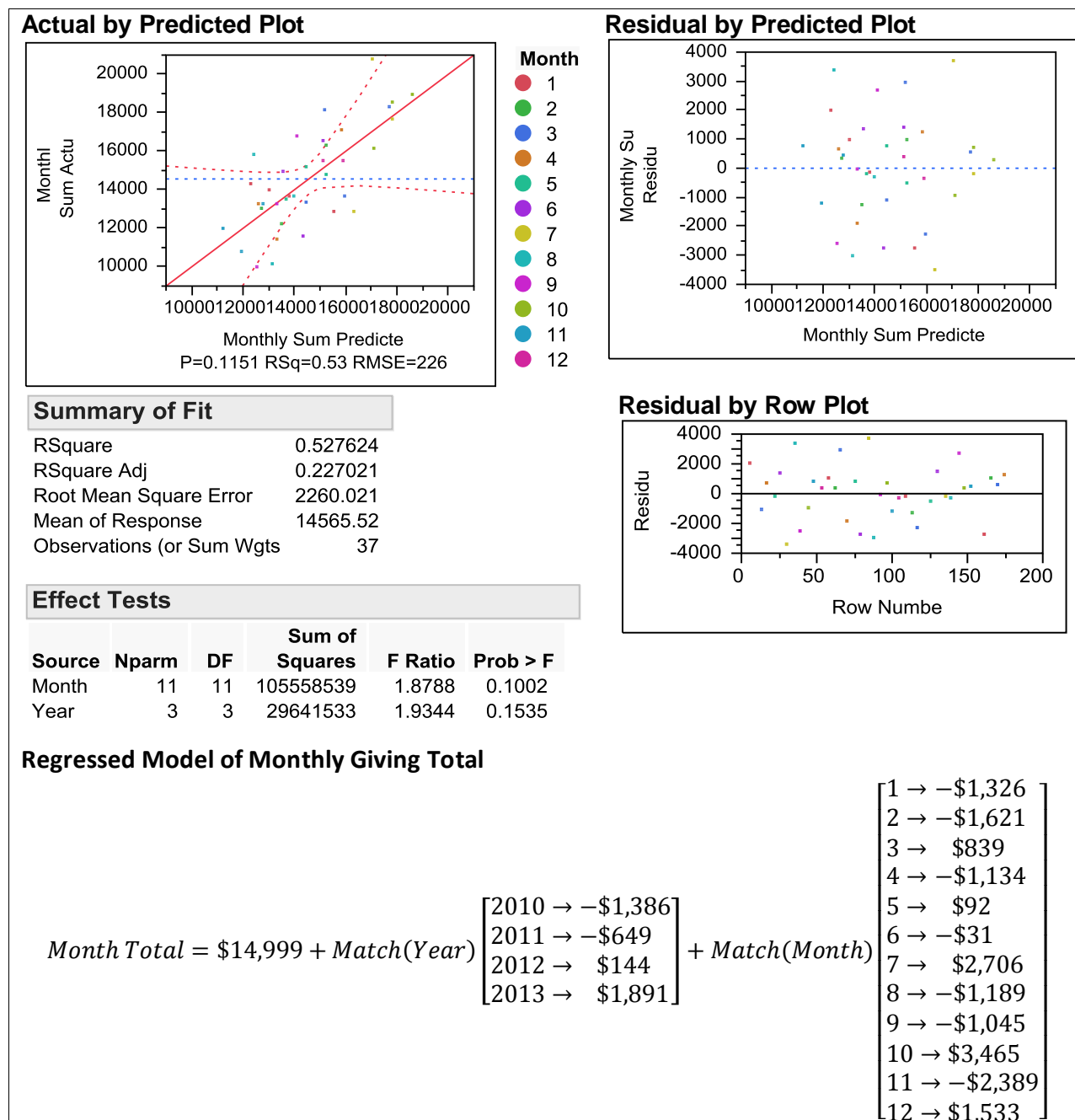
Figure 8. Monthly Mean Model Result



The excluded values again correlate to the months containing the highest weekly giving spikes. However, the values excluded were identified based on residuals from the initial monthly

model regression results with no points excluded. “Averaging out” of high giving weeks with lower ones within a month left fewer months apparently being outliers.

Figure 9. Analysis of Monthly Mean Model Regression

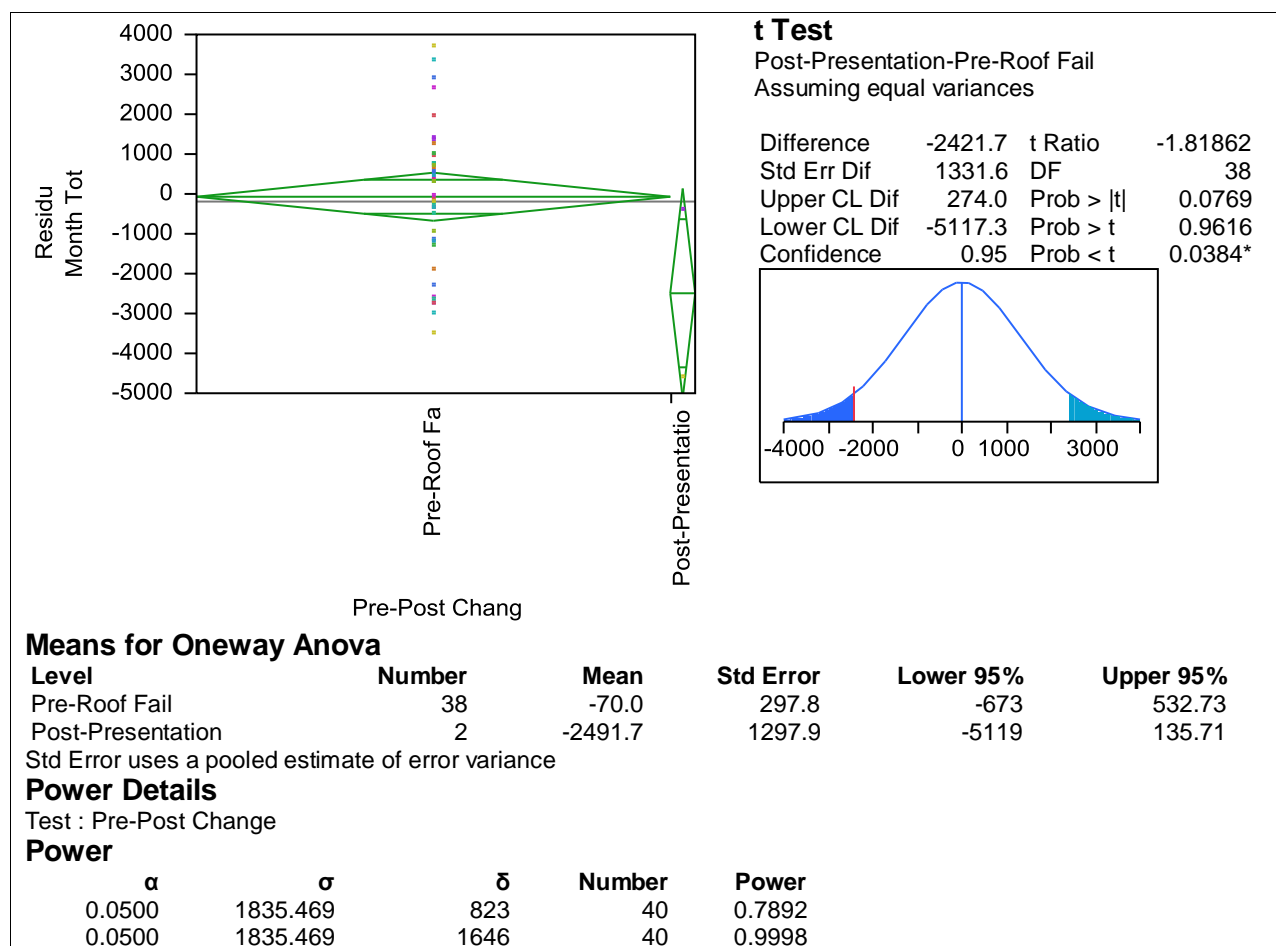


The monthly model fits better than the weekly model. While Month and Year parameter p-values, 0.10 and 0.15 respectively, are still not highly statistically significant, they are more so. The model appears to “explain” 53% of the total variation (see R^2). The resultant monthly total uncertainty, characterized by the Root Mean Square Error (RSME) is \$2,260. In comparison, the

weekly model RSME was \$1,395, which when compiled to a monthly total uncertainty would be between \$2,790 and \$3,119 depending on if the month is a 4 or 5 Sunday month.⁶ Finally, the model residuals don't appear to show any structure, a good model fit characteristic. In this case, both the residual by predicted and residual by row plots are shown.

Analysis of residuals (the difference between actually monthly giving and model predictions) to answer the question about if giving changed following the facilities planning expansion request is shown in Figure 10.

Figure 10. Comparison of Giving Before and After Increase Request



The difference estimated by this model in post-request giving is -\$2,422 and is a statistically distinguishable decrease from the pre-request giving (Prob < t = 0.038 < 0.05). As a reminder, the requested monthly giving increase for a 24-month loan pay-off equates to between \$1,520 and \$1,900 per month depending on if the month has 4 or 5 Sundays. The model does not distinguish between 4 and 5-Sunday months. So \$1,646, the difference for the average 4.333

⁶ Variability in each week was "summed" using the standard linear "variances add" approach: $Var(x) = \sigma^2$, $Var(\sum_{i=1}^N x_i) = \sum_{i=1}^N Var(x_i)$. This approach assumes covariance between months is zero. Strictly, this is not true since low counts one month might naturally lead to high counts the next month.

Sunday month, is used in the power analysis. The power analysis indicates the available data (38 months before and 2 months after the request) was virtually certain to detect the difference if it actually existed and was at least \$1,646. Even if the monthly difference is only \$823 there is a 79% chance of detecting it with the available data. This “power” is considered “good” in most cases.

Though the following analysis is not shown here, the upper (one-sided) $1-\alpha=0.80$ prediction interval for a single month’s giving total with the 38 pre-request residuals is \$1,620. That means, that in order to have a 80% probability of actually identifying a future single month’s increase as being outside the distribution described by the 38 pre-request residuals, the increase over the predicted month’s giving would have to be \$1,620 or larger. June 2013’s giving was \$400 under the model’s prediction of \$16,860. So, that “decrease” was not large enough to conclude that the core had indeed decreased giving.

More than just low, the July residual from the model prediction, however, was -\$4,584 which is perhaps enough to flag it as an outlier. (It is barely visible in the lower triangle of the post-presentation diamond in the plot.) Note that residual is well outside the range of residuals for the “pre-Roof fail” group. This large residual resulted in the 2-month average of -\$2,492. It should be noted that July 2013 was a 4-Sunday month. July 2010 was also a 4-Sunday month, but 2011 and 2012 were 5-Sunday month years. This is likely to have affected the July month model parameter which makes it the second largest giving month. However, June 2013 was a 5-Sunday month and was a 4-Sunday month in 2010, 2011, and 2012. So it being down \$400, with consideration to the number of giving Sundays each year, is particularly low.

Modified Monthly Mean Model

In the Monthly Mean model described above, year was included as a nominal variable. That is, a separate model parameter is regressed for each year:

$$f(\text{year}) = \text{Match}(\text{Year}) \begin{bmatrix} 2010 \rightarrow b_1 \\ 2011 \rightarrow b_2 \\ 2012 \rightarrow b_3 \\ 2013 \rightarrow b_4 \end{bmatrix}$$

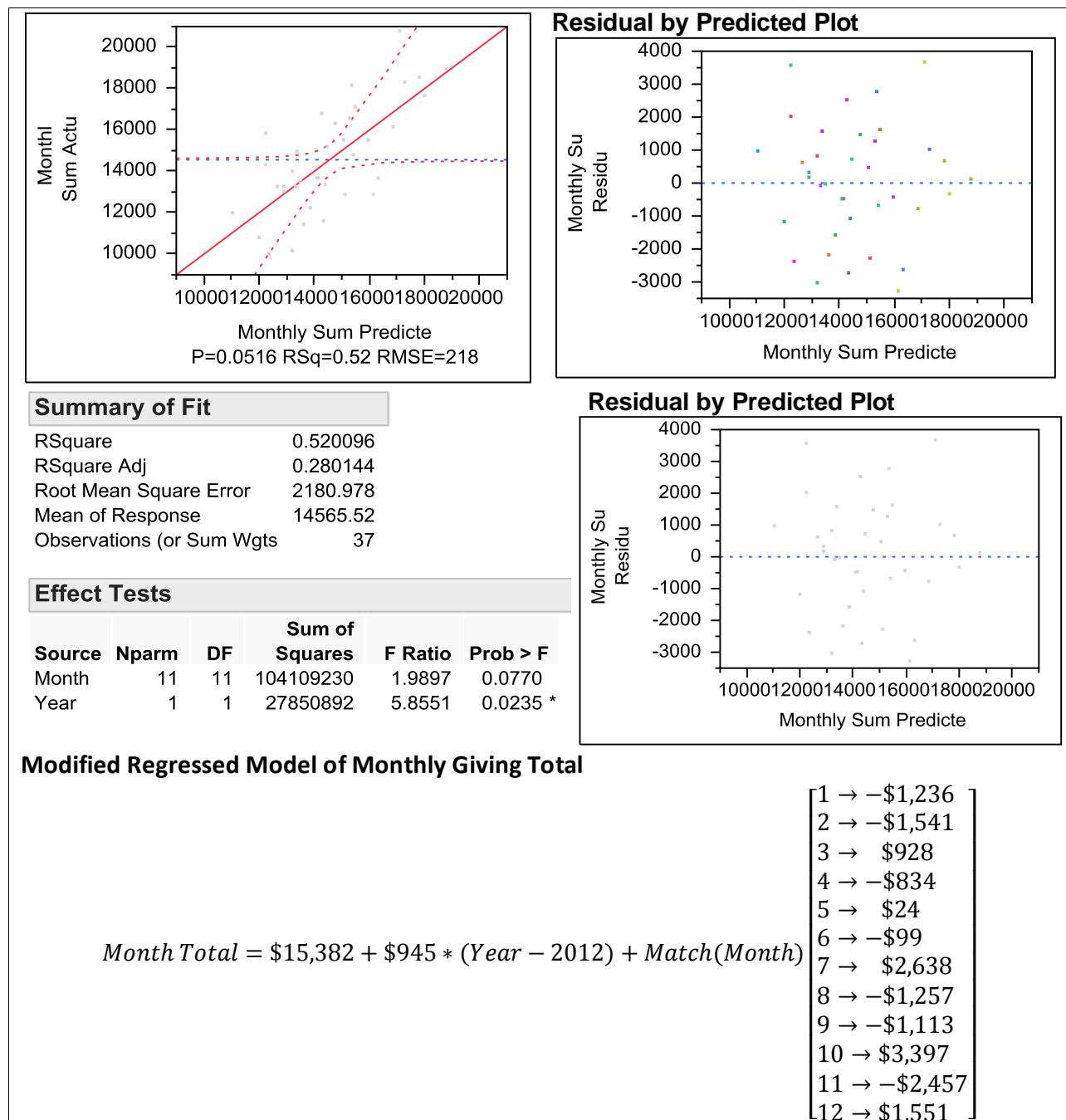
An alternative year function that consumes fewer degrees of freedom uses year as a continuous variable:

$$f(\text{year}) = b_1 * \text{Year}$$

The nominal variable function assesses whether a difference can be detected between each of the individual years. The continuous variable model assesses whether there is a trend (slope) with time. Either model is a potentially appropriate and superior model. Which is better depends on the data being fit. [Figure 11](#) shows the evaluation of this alternate monthly giving total model:

$$\text{Monthly Giving Total} = a_1 + b_1 * \text{Year} + \text{Match}(\text{Month}) \quad \begin{bmatrix} 1 \rightarrow c_1 \\ 2 \rightarrow c_2 \\ 3 \rightarrow c_3 \\ \vdots \\ 12 \rightarrow c_{52} \end{bmatrix}$$

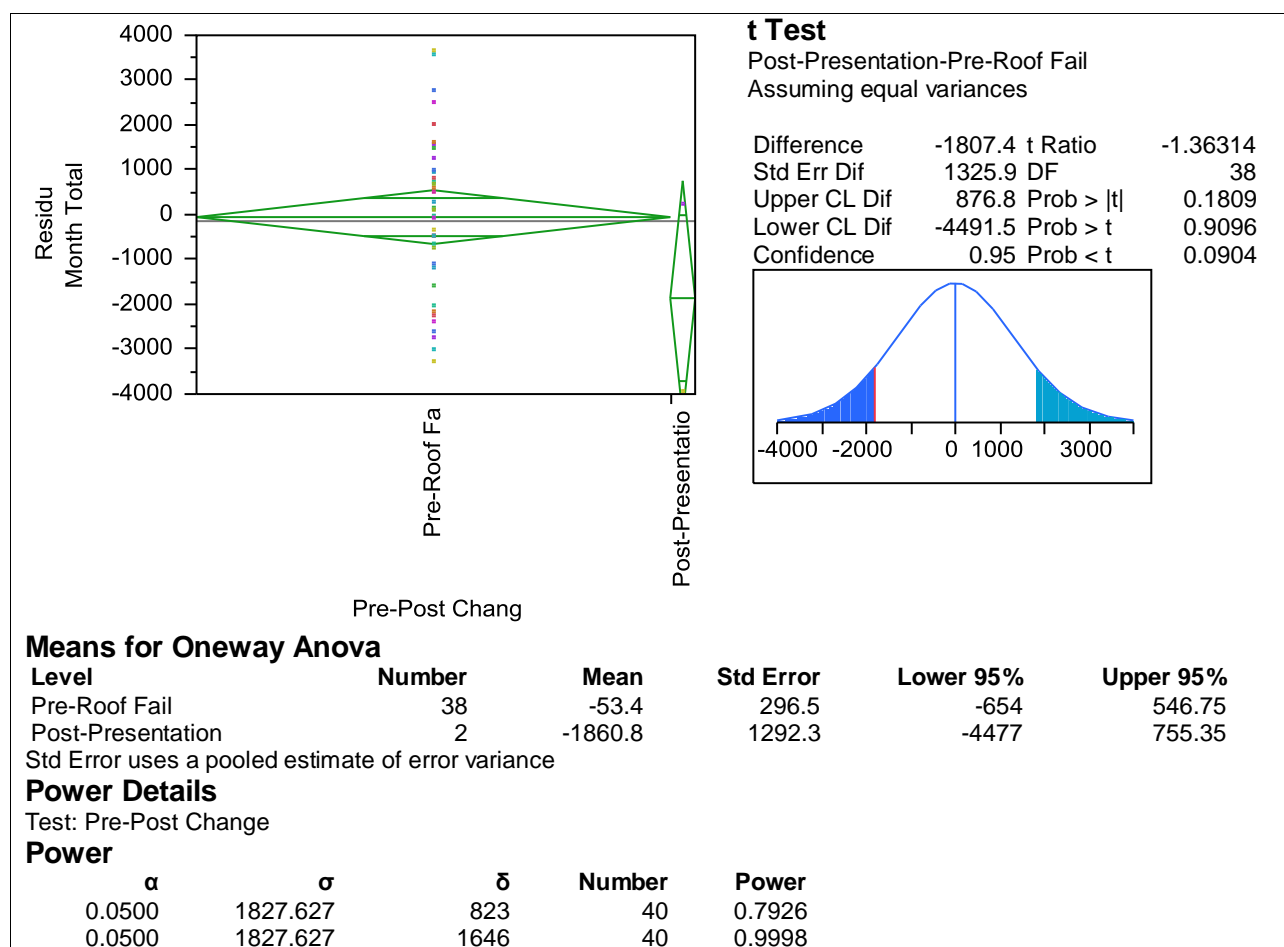
Figure 11. Analysis with the Alternate Monthly Model



This model confirms that monthly giving totals are a function of year ($p = 0.02 < 0.05$). Giving has increased an average of \$945 per month per year since 2010.⁷ Also, the month term becomes very legitimately marginally statistically significant ($p = 0.077$). While R^2 isn't improved, the model still explains about 52% of all variation in monthly giving totals, using fewer degrees of freedom improves the R^2_{Adj} . The RSME is even slightly reduced to \$2,181 from \$2,260 in the previous model. The residuals are well behaved and the model's overall model p-value = 0.05.

Analysis of residuals to answer the question about if giving changed following the facilities planning expansion request is shown in Figure 12.

Figure 12. Comparison of Giving Before and After Increase Request



The estimated difference between monthly giving totals before and after the Seed Facilities Planning proposal is -\$1,807 ($p = 0.09$). The July 2013 estimate, in this case, is not as much of an outlier. It is not clear if that is more or less concerning.

⁷ This trend may seem odd since 2010 and 2011 giving were very similar and 2012 and 2013 are quite similar. However, the average increase is the regression result and is consistent with the b_i 's shown in the previous (Monthly Mean) model. These two results firmly refute the single mean model.

2013 Annual Giving Prediction

One final interesting prediction that can be made using the regression models is the extrapolation of the model to predict the total 2013 giving. This is shown in [Table 1](#).

Table 1. Actual and Predicted Annual Giving

Year	Actual Giving	Predicted Giving	
		Monthly Model	Modified Model
2010	173,804.56	163,360 ^a	161,897 ^a
2011	172,200.50	172,201	173,240
2012	197,492.07	181,723 ^b	184,584 ^b
2013	117,808.86 ^c	202,686	195,928

^a – 2010 had 1 unusually high giving week value excluded

^b – 2013 had 2 unusually high giving week values excluded

^c – This total only includes weeks through Aug 11th.

The modified monthly giving total model predicts that annual giving for 2013 will be about \$196K. This prediction assumes there is NO post-request change. The predictions simply use the available 2013 data to estimate the 2013 year giving model parameter taking into consideration all the month-to-month variation regressed from all the data. This, of course, assumes no unusual circumstances occur (including the reduction detected above) and that future giving is predicted by past performance which is obviously not assured.

Finally, Figure 13 shows graphically the variation in monthly giving totals. These “adders” are the average result experienced over the 2010 to Aug 2013 period.

Figure 13. Month-to-Month Giving “Adder”

