2018 Model Extension

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Overview

This notebook extends the modeling done in 2013 to include data through much of 2018.

Load Data

Loading cleaned and transformed giving data:

```
df <- readRDS("../data/Cleaned and Transformed Giving Data.rds")
head(df)
```

```
##   week.ending    month year    total monthly.giving.families
## 1  2010-01-31  January 2010 14241.15                      45
## 2  2010-02-28 February 2010 22437.50                      41
## 3  2010-03-28    March 2010 13317.00                      32
## 4  2010-04-25    April 2010 13232.50                      36
## 5  2010-05-30      May 2010 13478.15                      40
## 6  2010-06-27     June 2010 14930.50                      43
##   SundaysInMonth MonthsGivingPerWeek
## 1              5            2848.230
## 2              4            5609.375
## 3              4            3329.250
## 4              4            3308.125
## 5              5            2695.630
## 6              4            3732.625
```

Columns to Accomodate Analysis

I'll add a few columns to enable asking analysis "what if's" and adjust for anomolies:

```
library("dplyr", quietly=TRUE, warn.conflicts = FALSE)
df <- mutate(df,
             excluded = FALSE,
             removed.amount = 0,
             original.total = total)
```

Accommodating Unusual Gift

12/26/2016 had an unusual single gift skewing analyses. Other large gifts have been recieved over this period but this gift was at least twice all others. I'm removing the amount of the unusual gift from the month's total. Previously I excluded that month all-together, but I've commented that line out so no values are excluded.

```
df$removed.amount[as.Date(df$week.ending) == "2016-12-25"] <- 20000
df$total <- df$original.total - df$removed.amount
#df$excluded[as.Date(df$week.ending) == "2016-12-25"] <- TRUE
```

Note: we're not excluding the 3 months consider "outliers" in the 2013 analysis. Now with 9 years of history, those months don't appear as much to be outliers.

Modeling

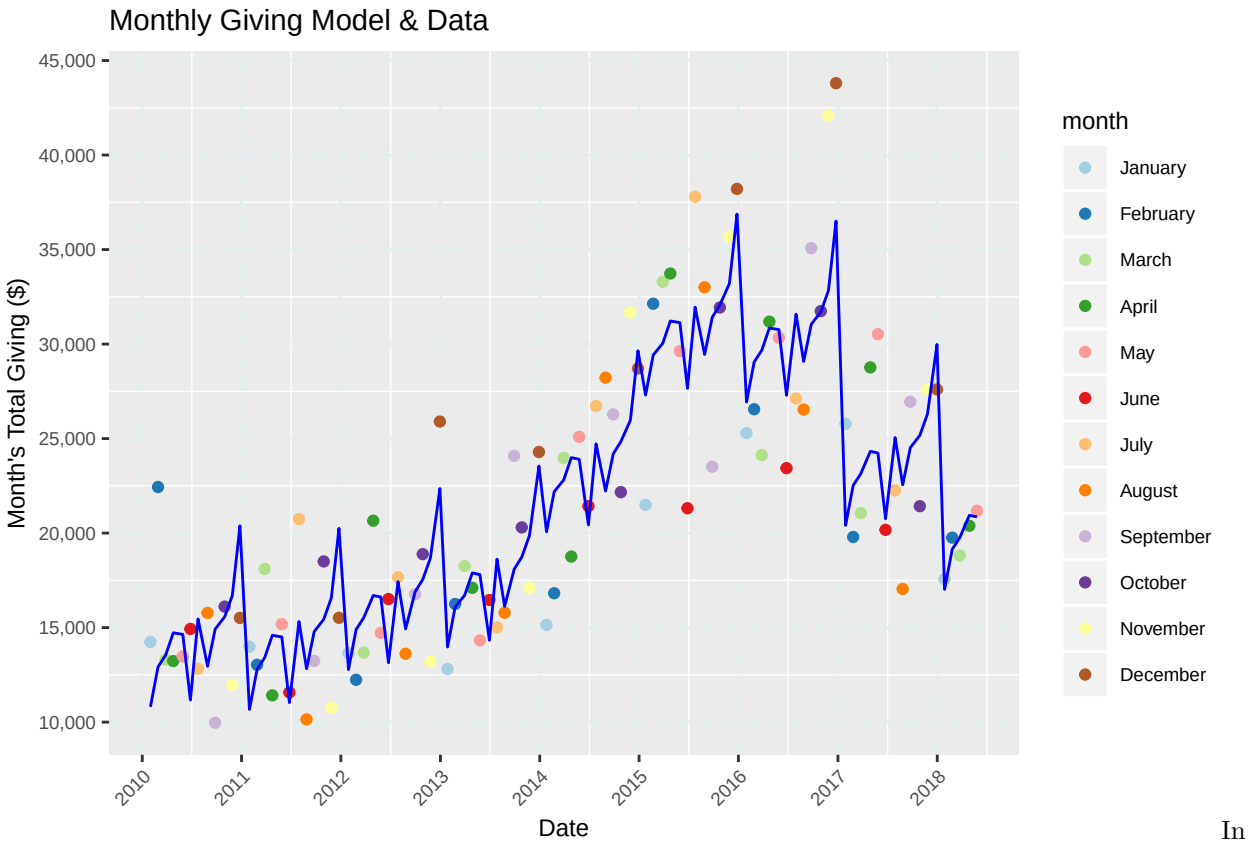
Regressing the R version of the 2013 model:

$$Monthly\ Giving = a + b_{year} + c_{month}$$

```
mod <- lm(
  formula = total ~
    year + month,
  data = df[df$excluded!=TRUE,]
)
```

. . . gives the following

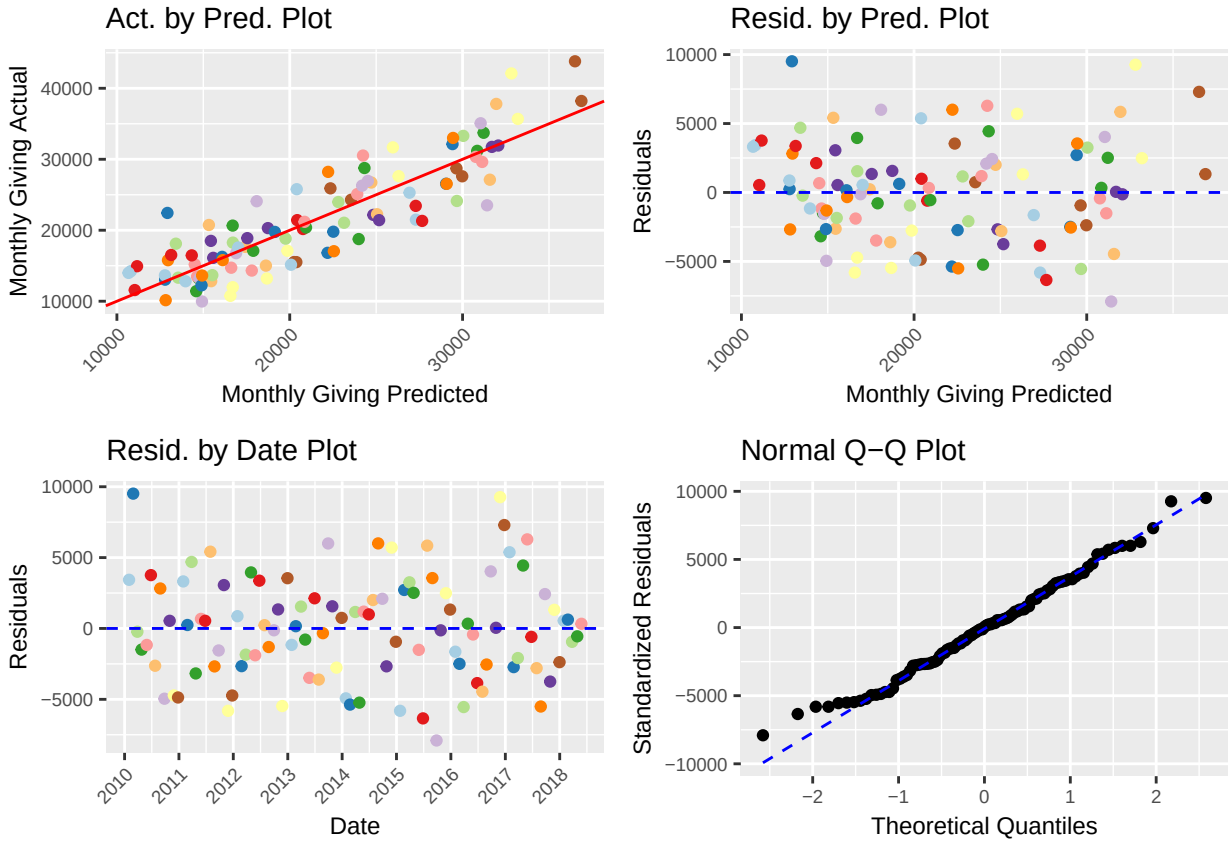
Model Predictions



In this case it is far more clear that giving was increasing through 2016 which would be expected to improve the statistical significance of the *year* model term over what was seen in the 2013 analysis.

Model Fit Assessment

```
## Analysis of Variance Table
##
## Response: total
##           Df     Sum Sq   Mean Sq F value    Pr(>F)
## year       8 3920266253 490033282  30.198 < 2.2e-16 ***
## month     11  588525930  53502357   3.297  0.000896 ***
## Residuals 81 1314433124  16227569
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Both model terms are highly statistically significant. The residuals seem nicely randomly scattered. I don't see any particularly concerning patterns in the plots including model biases by month (dot color). The quantile plot shows very normally distributed residuals.

Regressed Model Details

```
##
## Call:
## lm(formula = total ~ year + month, data = df[df$excluded != TRUE,
##     ])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7904.4 -2659.0    46.1  2486.6  9513.2
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)     10807.3     1741.1   6.207 2.18e-08 ***
## year2011         -133.7     1644.6  -0.081  0.93542
## year2012         1974.0     1644.6   1.200  0.23352
## year2013         3165.1     1644.6   1.925  0.05779 .
## year2014         9265.5     1644.6   5.634 2.47e-07 ***
## year2015        16494.5     1644.6  10.030 7.42e-16 ***
## year2016        16124.2     1644.6   9.805 2.05e-15 ***
## year2017         9597.9     1644.6   5.836 1.06e-07 ***
## year2018         6215.8     2198.7   2.827  0.00592 **
## monthFebruary    2117.0     1899.0   1.115  0.26824
## monthMarch       2739.3     1899.0   1.443  0.15301
## monthApril       3918.4     1899.0   2.063  0.04227 *
## monthMay         3833.6     1899.0   2.019  0.04682 *
## monthJune         359.5     1968.9   0.183  0.85559
## monthJuly        4648.5     1968.9   2.361  0.02063 *
## monthAugust      2149.1     1968.9   1.092  0.27828
## monthSeptember   4115.6     1968.9   2.090  0.03973 *
## monthOctober     4765.5     1968.9   2.420  0.01774 *
## monthNovember    5895.7     1968.9   2.994  0.00365 **
## monthDecember    9574.9     1968.9   4.863 5.59e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4028 on 81 degrees of freedom
## Multiple R-squared:  0.7743, Adjusted R-squared:  0.7213
## F-statistic: 14.62 on 19 and 81 DF,  p-value: < 2.2e-16
```

Model Generalization

Due to R's conventions, the model coefficients, as regressed by R:

$$\textit{Monthly Giving} = a + b_{year} + c_{month}$$

are regressed relative to $month_1$=January ($c_{January} = 0$) and $year_1$=2010 ($b_{2010} = 0$). This is why there is no year2010 nor monthJanuary coeffiencients. A more useful set of references for forecasting would be "an average month" and "an average year". Thus we can modify the model as such:

$$\begin{aligned}\textit{Monthly Giving} &= a + (\overline{b_{year}} + b_{year} - \overline{b_{year}}) + (\overline{c_{month}} + c_{month} - \overline{c_{month}}) \\ &= \left[a + \overline{b_{year}} + \overline{c_{month}}\right] + \left(b_{year} - \overline{b_{year}}\right) + \left(c_{month} - \overline{c_{month}}\right)\end{aligned}$$

where $\overline{b_{year}}$ is the mean of b_{year} coefficients, including $b_{2010} = 0$, and $\overline{c_{month}}$ is the mean of the c_{year} coefficients, including $c_{January} = 0$. The term $\left[a + \overline{b_{year}} + \overline{c_{month}}\right]$ is then the regressed *Monthly Giving* for an average

month in an average year over the period covered by the data. Of course no actual month or year is the average, and to reproduce a given month and year prediction the shifted coefficients $(b_{year} - \overline{b_{year}})$ and $(c_{month} - \overline{c_{month}})$ must be employed. However, this facilitates using the model for monthly variance predictions in the upcoming year since:

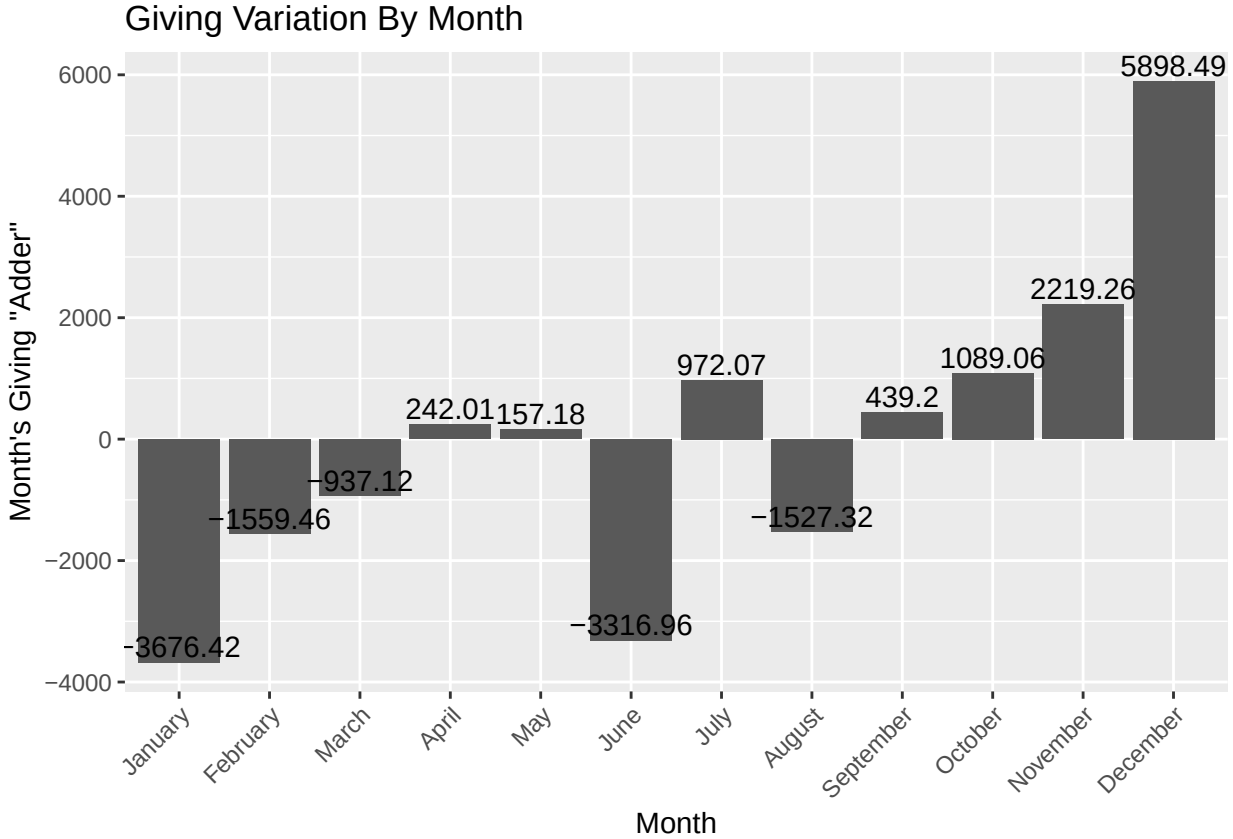
$$\begin{aligned} \textit{Average Monthly Giving}_{\textit{next year}} &= \left[a + \overline{b_{year}} + \overline{c_{month}}\right] + \left(b_{\textit{next year}} - \overline{b_{year}}\right) \\ &= \frac{\textit{Projected Income}_{\textit{next year}}}{12} \end{aligned}$$

So even though we don't know $b_{next\ year}$ we can make an estimation for next year's total *Projected Income* and get monthly estimates based on the annual estimate:

$$\textit{Monthly Giving}_{\textit{next year}} = \frac{\textit{Projected Income}_{\textit{next year}}}{12} + \left(c_{month} - \overline{c_{month}}\right)$$

Monthly Variation

This plot shows the resulting *month* regression coeffiecients shifted by $\overline{c_{month}}$ capturing month-to-month variation. $((c_{month} - \overline{c_{month}}))$ The shape of the yearly repeated pattern in the prediction are the result of these terms.



This plot is generally similar to Figure 13 in the 2013 Analysis with the exception of November. Since it is based on 9 years of Seed history, I'll judge that it is more representative of an average year.