# INTRODUCTION TO THE COURSE

A computational introduction to
stochastic differential equations

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# Content


1. Prerequisites
2. Arrangements
3. Syllabus (6 credits)
4. Reading materials

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# PREREQUISITES

The course is PhD level, but Master students are welcome to attend as well.

You should know

- Linear algebra.
- Real analysis (not essential but would be very useful).
- Probability theory.
- Ordinary differential equations.
- Numpy, even better, JAX.

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# Arrangements

The course has 6 credits.

You can upgrade to 9 credits by additionally doing a project work. We detail these later.

The course grade is pass or fail.

# Arrangements

The course consists of lectures, exercises(, and project work).

To get 6 credits, you need to:

- Actively participate all the lectures.
- Pass the four exercise assignments. Each exercise assignment has a passing rule.

# Arrangements

To get 9 credits, you need to in addition do a project work.

The aim of the project is to give you a chance to apply what you have learnt in this course for your own research, and possibly to aim for a publication. For example, developing a generative diffusion model for materials generation.

If you intend, you should discuss with the course responsible well in advance to select a topic to work on. The deadline to hand-in the project report is 19.12.2025.

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# Syllabus

## Lecture 1

Stochastic differential/integral equations.

This lecture explains what a stochastic differential equation (SDE) is, in particular, the meaning of $\mathrm{d}W(t)$ in

$$\mathrm{d}X(t) = a\big(X(t)\big)\,\mathrm{d}t + b\big(X(t)\big)\,\mathrm{d}W(t).$$

We also explain the celebrated Itô's formula and the Markov property which makes SDEs data-scalable models.

# Syllabus

## Lecture 2

Numerical solution to stochastic differential equation.

This lecture shows computational methods to simulate SDEs. For example, Euler–Maruyama and Milstein's method.

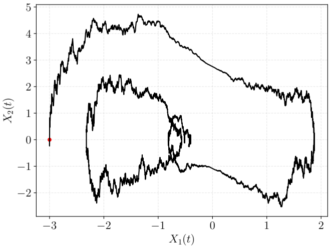


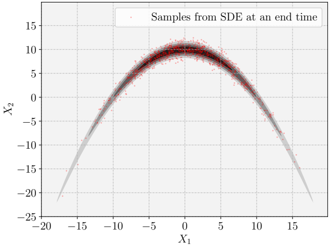

Figure: A simulation of a 2D Duffing–van der Pol SDE.

# Syllabus

## Lecture 3

Distribution and statistics of SDE solutions.

- Compute $\mathbb{E}[\phi(X(t)) \mid X(s)]$ for any function $\phi$ of interests.
- Probability density function of the solution (i.e., Kolmogorov forward equation/Fokker–Planck, see the animation in the github repository).
- Stationary solution to the Kolmogorov forward equation (e.g., Langevin dynamics).



# Syllabus

## Lectures 4

Linear SDEs and stochastic filtering.

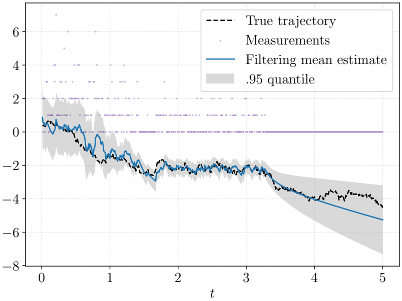
Linear SDEs are an important class of SDEs, especially for generative diffusion models. This lectures explains the basic properties of linear SDEs.

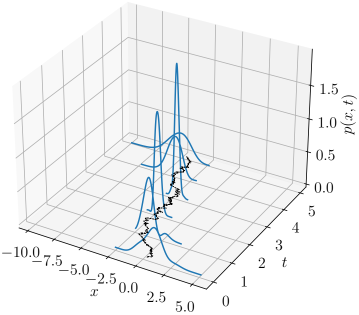
Imagine that you have a latent process described by an SDE, and some observation/data of the process. To estimate the latent process based on the data is a filtering problem.

For example, Kalman filtering.

# Syllabus

## Lectures 4



# Syllabus

## Lectures 4

The filtering has a great number of applications.

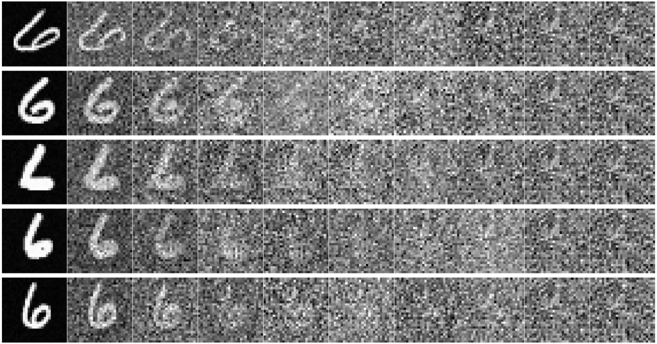
- The Apollo program.
- Object tracking, navigation, and control.
- Time series analysis.
- Regression, interpolation, and extrapolation.
- Posterior generative diffusion models.
- ...

# Syllabus

## Lectures 5 - 6

Generative diffusion models.

Everything essential about generative diffusion models.



# Project

Ideally you should come up with your own project topic.

## Project topic examples

- (Computational biology) Use stochastic SIR models to describe infectious diseases (e.g., COVID-19)
- (Quantitative finance) Analyse interest rates with financial SDE models (e.g., Cox–Ingersoll–Ross and Chan–Karolyi–Longstaff–Sanders)
- (Scientific computing) Solve PDEs with Feynman–Kac formula
- (Machine learning) Solve Gaussian process regression with SDEs
- (Machine learning) Diffusion-based generative models
- (Computer vision) Diffusion-based vision-language models
- (Statistics) Exact SDE sampling (see, e.g., Beskos, 2005)
- (Statistics) Metropolis adjusted Langevin algorithm

# Syllabus

If we count from 06.11.2025, then approximately you have 1 month + 14 days to prepare the project work.

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# Reading materials

## Basic

- Hui-Hsiung Kuo. Introduction to stochastic integration. Springer, 2006. (very recommended)
- Simo Särkkä and Arno Solin. Applied stochastic differential equations. Cambridge University Press, 2019.
- Gabriel Lord, Catherine Powell, and Tony Shardlow. An introduction to computational stochastic PDEs. Cambridge University Press, 2014.

# Reading materials

## More theoretical exposition

- Ioannis Karatzas and Steve E. Shreve. Brownian motion and stochastic calculus. Springer, 2nd edition, 1991.
- Jean-François Le Gall. Brownian motion, martingales, and stochastic calculus. Springer, 2013.
- René Schilling and Lothar Partzsch. Brownian motion. De Gruyter, 2012.

# Reading materials

## Generative diffusion models

- Conditional sampling within generative diffusion models.
- Taming diffusion models for image restoration: a review.

Tack!