Please submit your answer as an email to nathanael.bosch@uni-tuebingen.de and zheng.zhao@it.uu.se.

Exercise on "Probabilistic Numerics for Ordinary Differential Equations"

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1 Simulate the Lotka–Volterra system

Initial Value Problem. The Lotka–Volterra system is a system of two ordinary differential equations (ODEs) that describes the dynamics of a predator-prey system. It is given by the ODEs

$$\dot{x} = \alpha x - \beta xy,$$
$$\dot{y} = \gamma xy - \delta y,$$

where x and y are the number of prey and predators, respectively. For this exercise, consider an initial value $x(0) = 1$, $y(0) = 1$, parameters $(\alpha, \beta, \gamma, \delta) = (1.5, 1.0, 3.0, 1.0)$, and a time span $t \in [0, 20]$.

Prior. As in the lecture, use a 2-times integrated Wiener process as your prior — only that this time the ODE problem is 2-dimensional. So, we use an independent 2-times integrated Wiener process prior for each dimension. This amounts to having transition matrices of the form

$$A_2(h) = A(h) \otimes I_2,$$
$$Q_2(h) = Q(h) \otimes I_2,$$

where $A(h), Q(h)$ are exactly the matries from the lecture, and I_2 is the 2×2 identity matrix. Adjust the projection matrices E_0, E_1 accordingly.

Likelihood and data. As in the lecture slides.

Task. *Simulate the Lotka–Volterra system with a probabilistic numerical ODE solver.* Don't forget to implement the calibration to get non-arbitrary uncertainty estimates. You are free to select an appropriate step size. Filtering posteriors are sufficient for this exercise (but do keep in mind that smoothing posteriors provide strictly more meaningful trajectory estimates!).

2 Please provide feedback on the lecture!

The URL to the google form is `https://forms.gle/7cabFm35uC3zjWew9`.