national center for ontological research
ncor

University at Buffalo
The State University of New York

# *Zebra Logic*

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Co-Director, National Center for Ontological Research
Affiliate Faculty, *Institute of Artificial Intelligence and Data Science*

# *Outline*

- Basic Logic Refresh
- Zebra Puzzle

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# *First-Order Logic*

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  - Predicates
  - Variables
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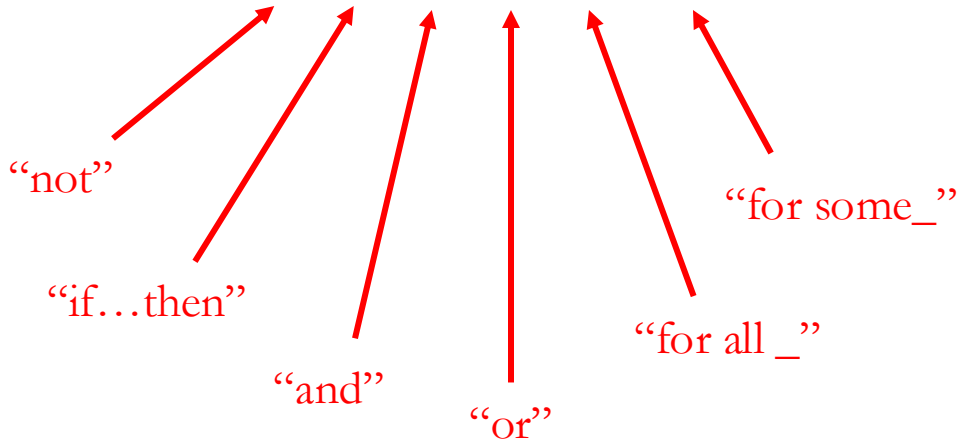

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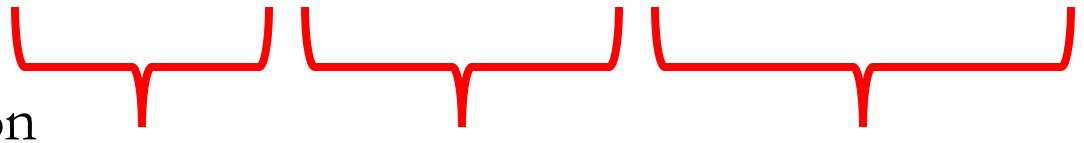

"_ is P"
"_ is R"

"_ is P to _"
"_ is R to _"

"_ is P to _ and _"
"_ is R to _ and _"

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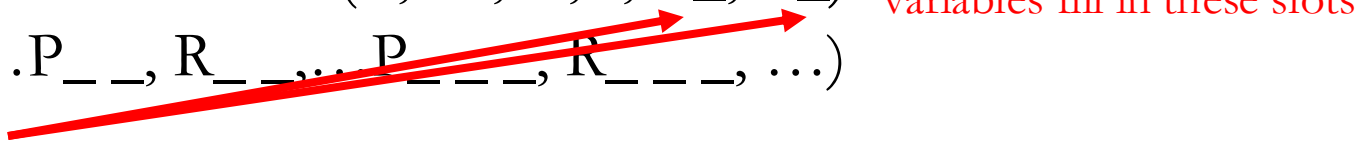

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variables fill in slots for predicates too

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The variable associated with “V” binds variables associated with predicates within its *scope*

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In this case all the variables are within the scope of “V”

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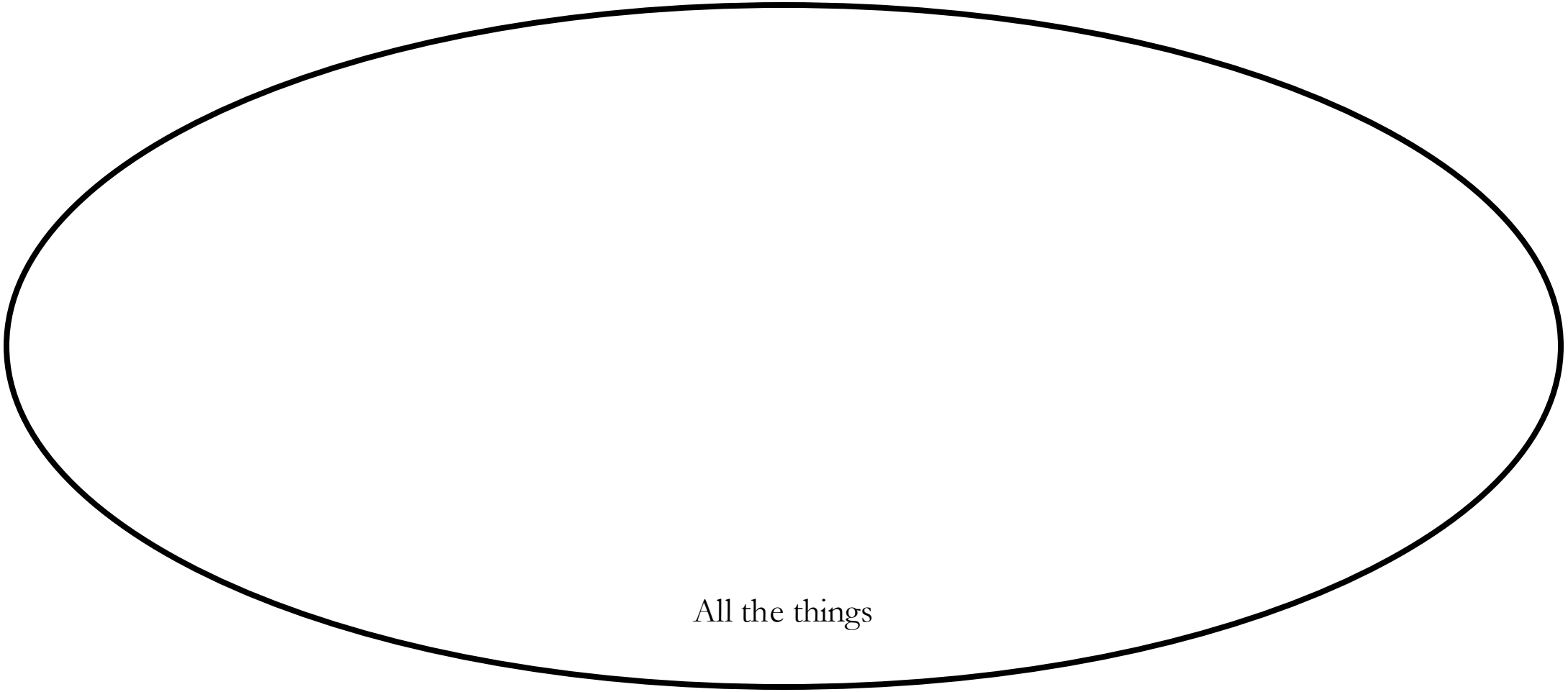

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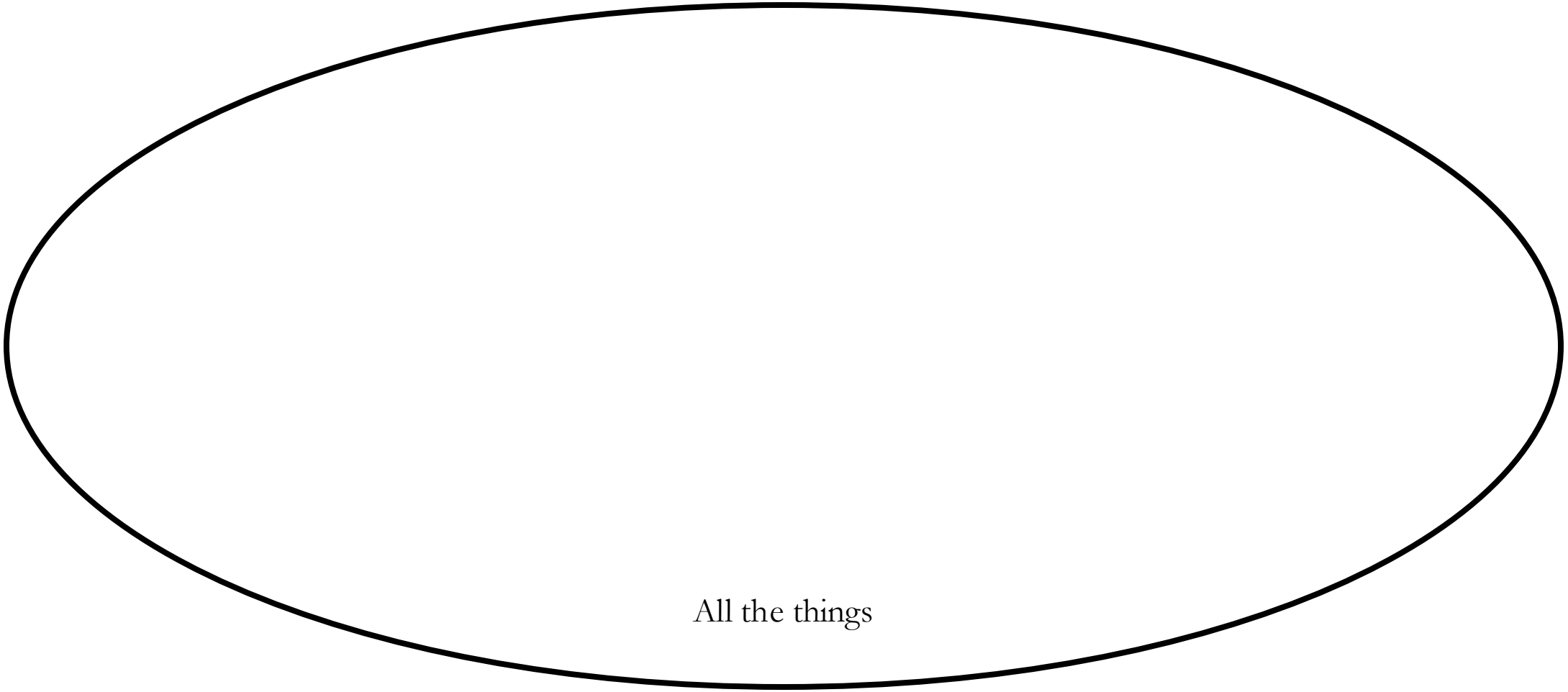

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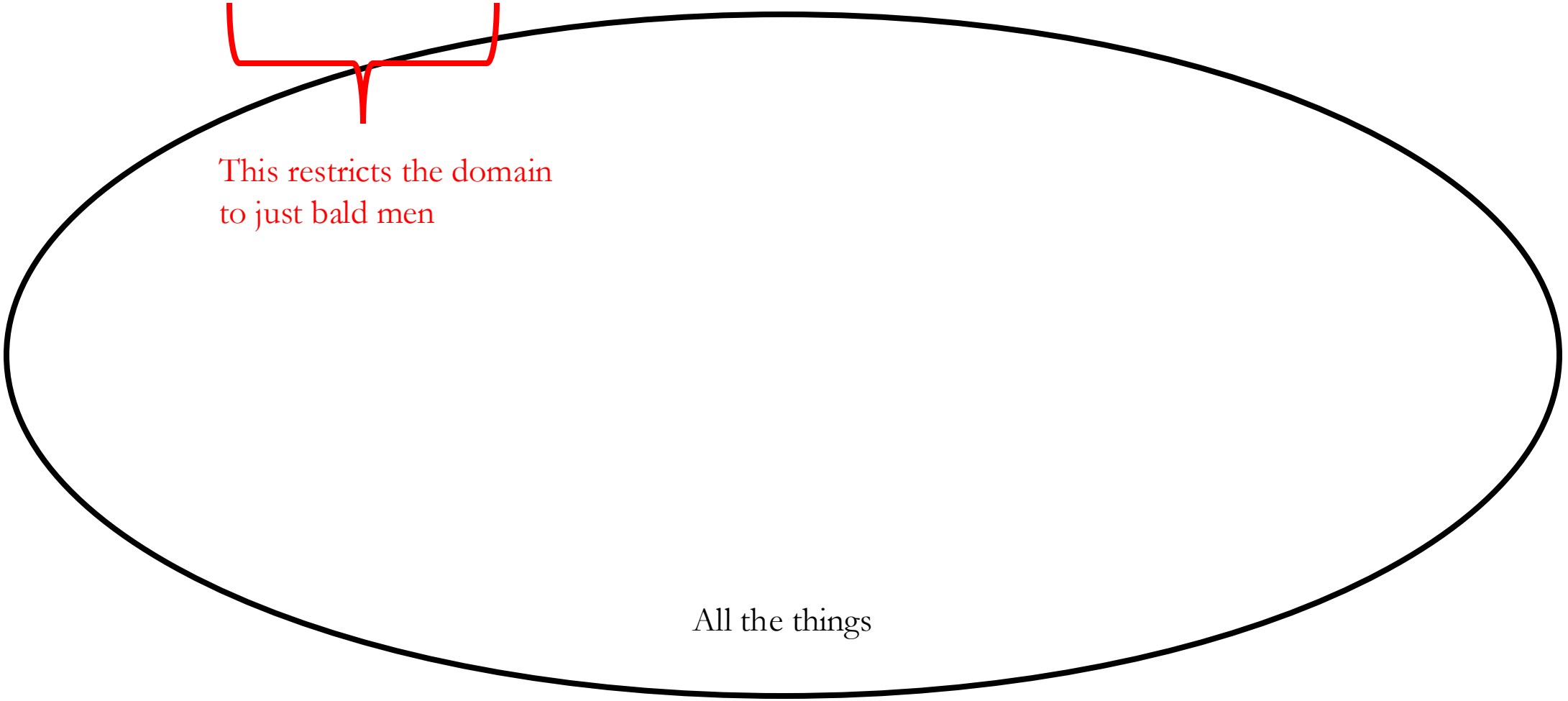

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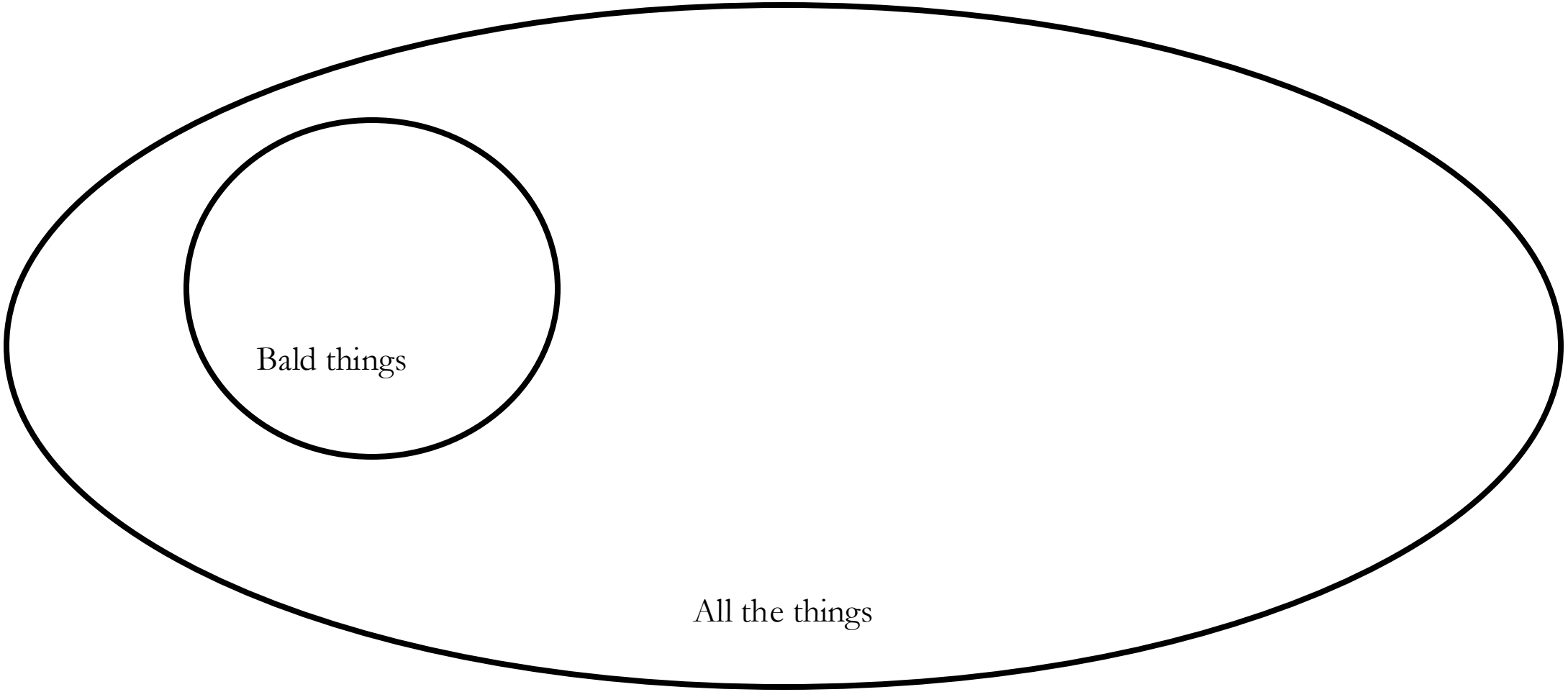

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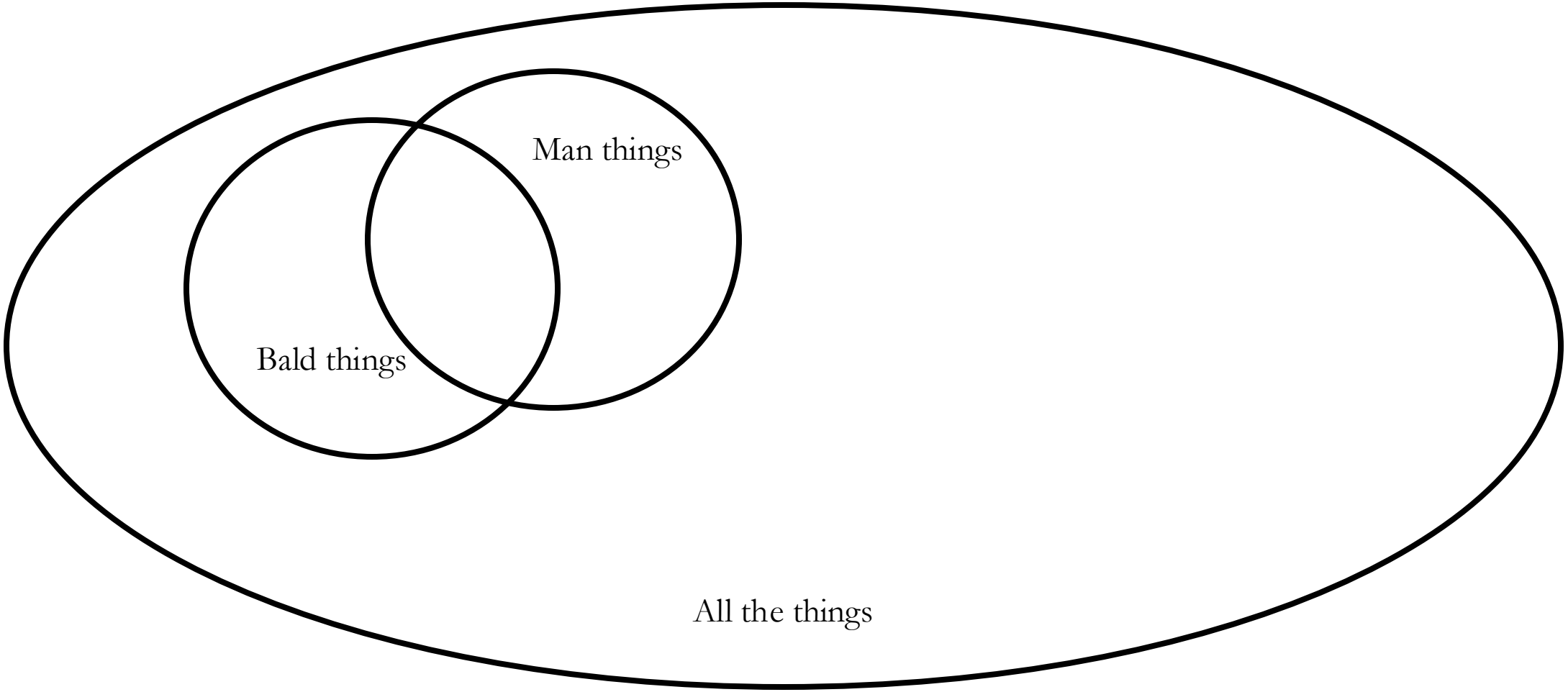

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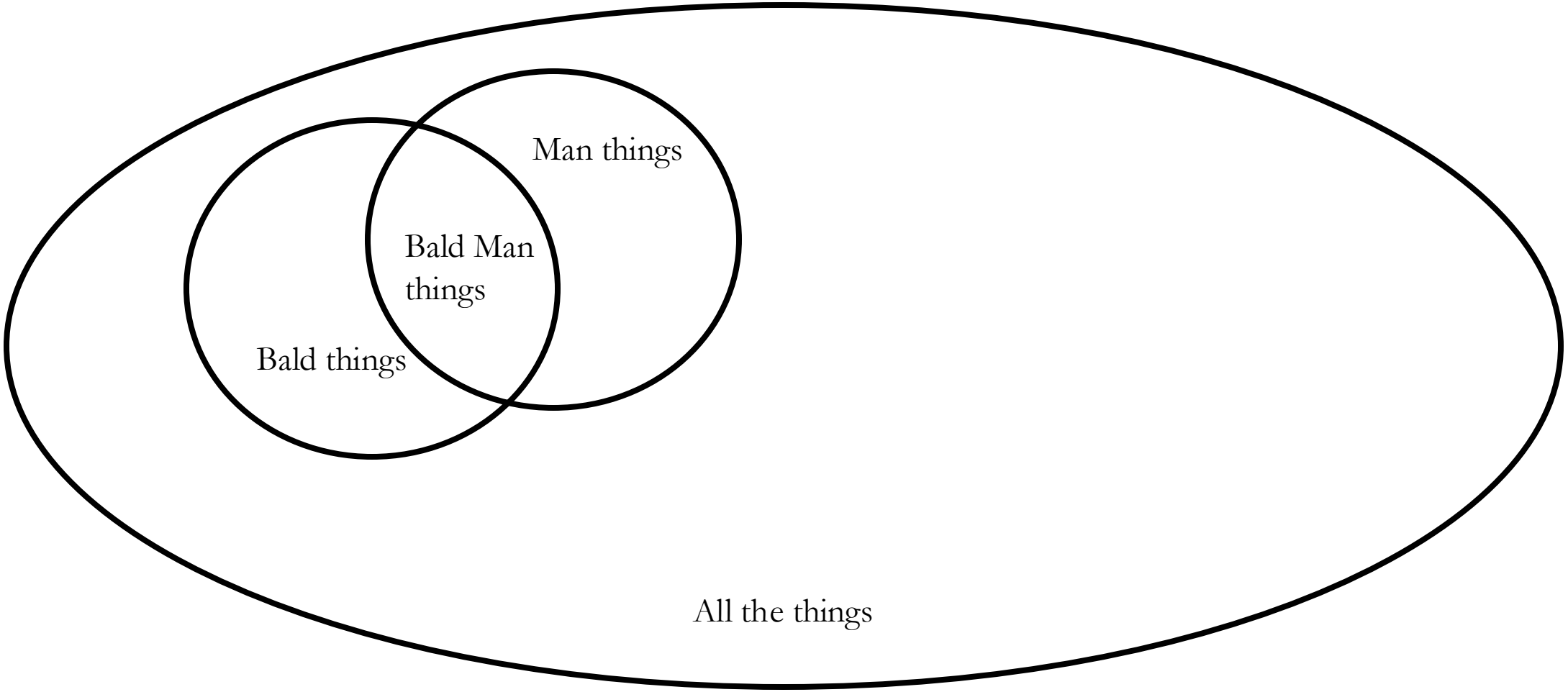

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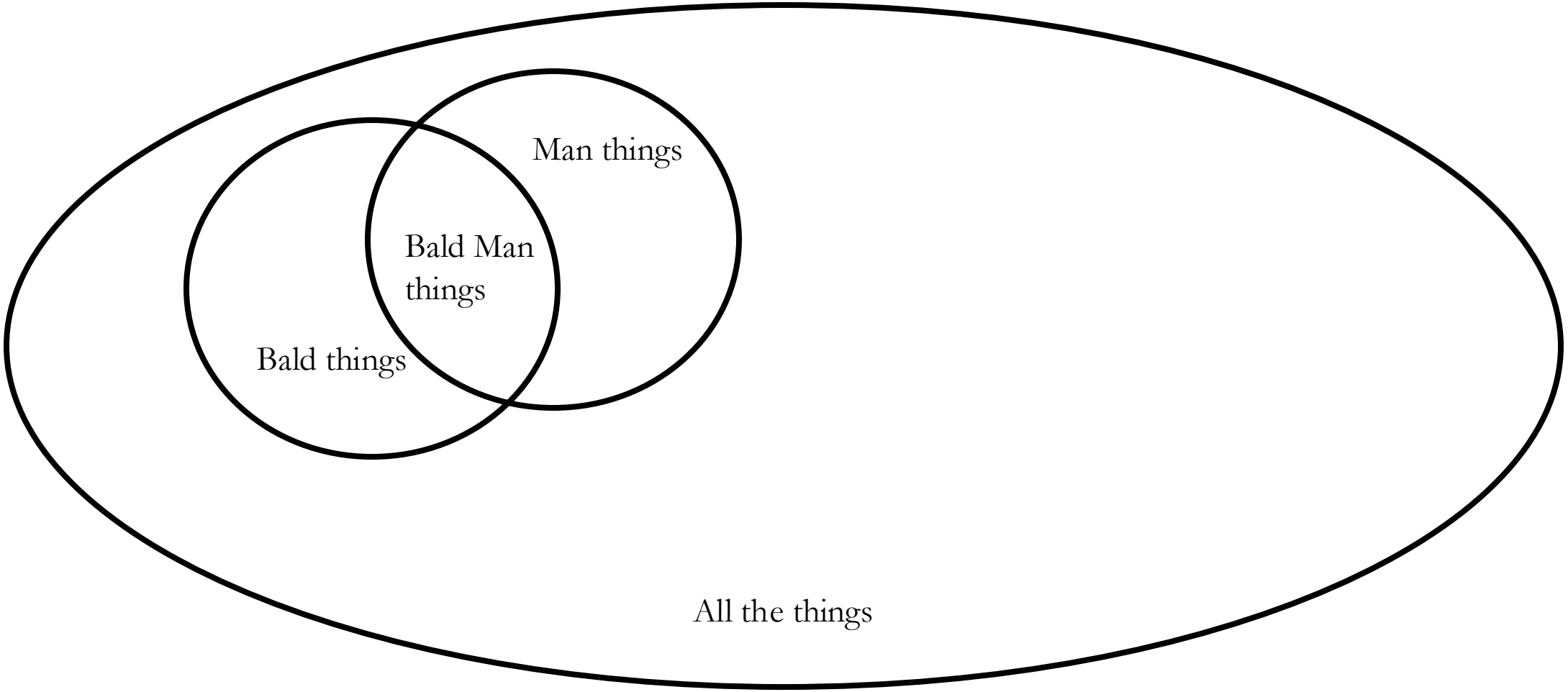

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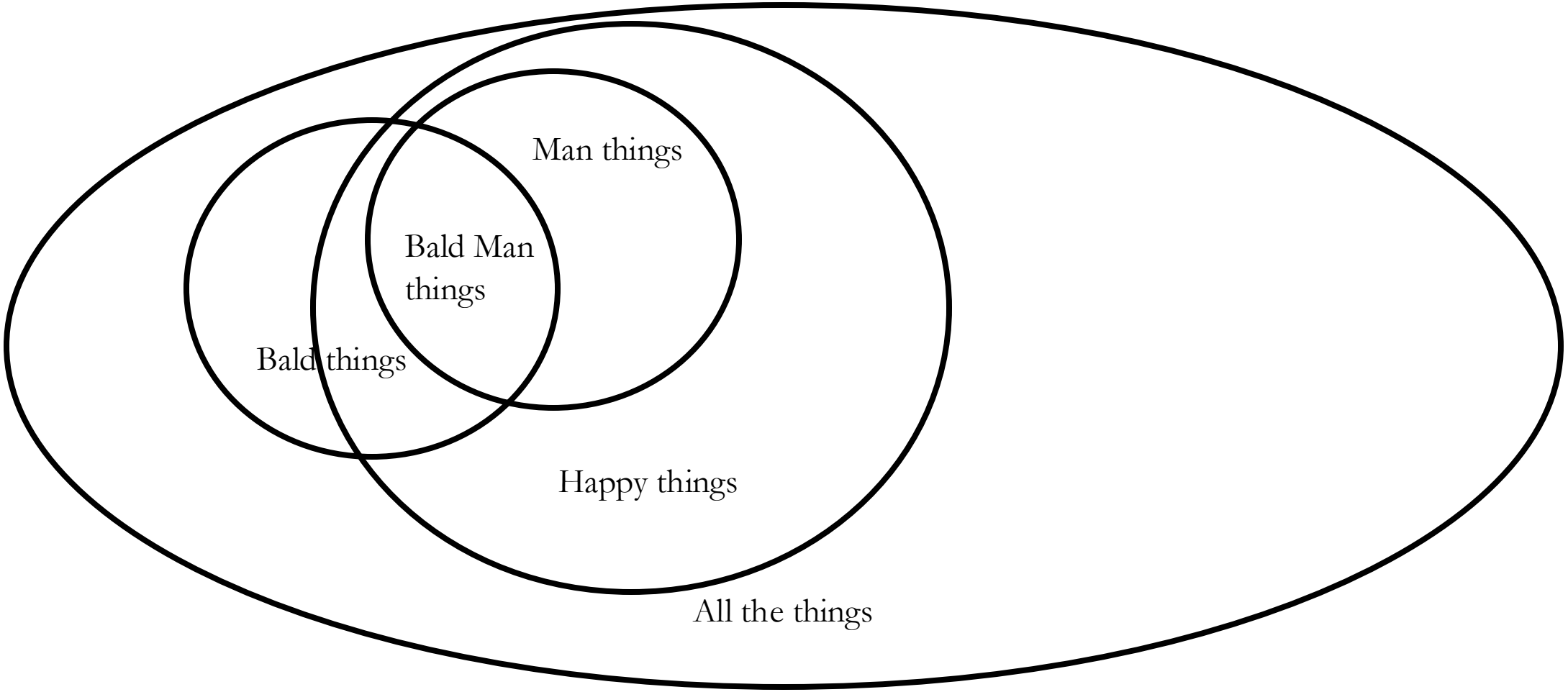

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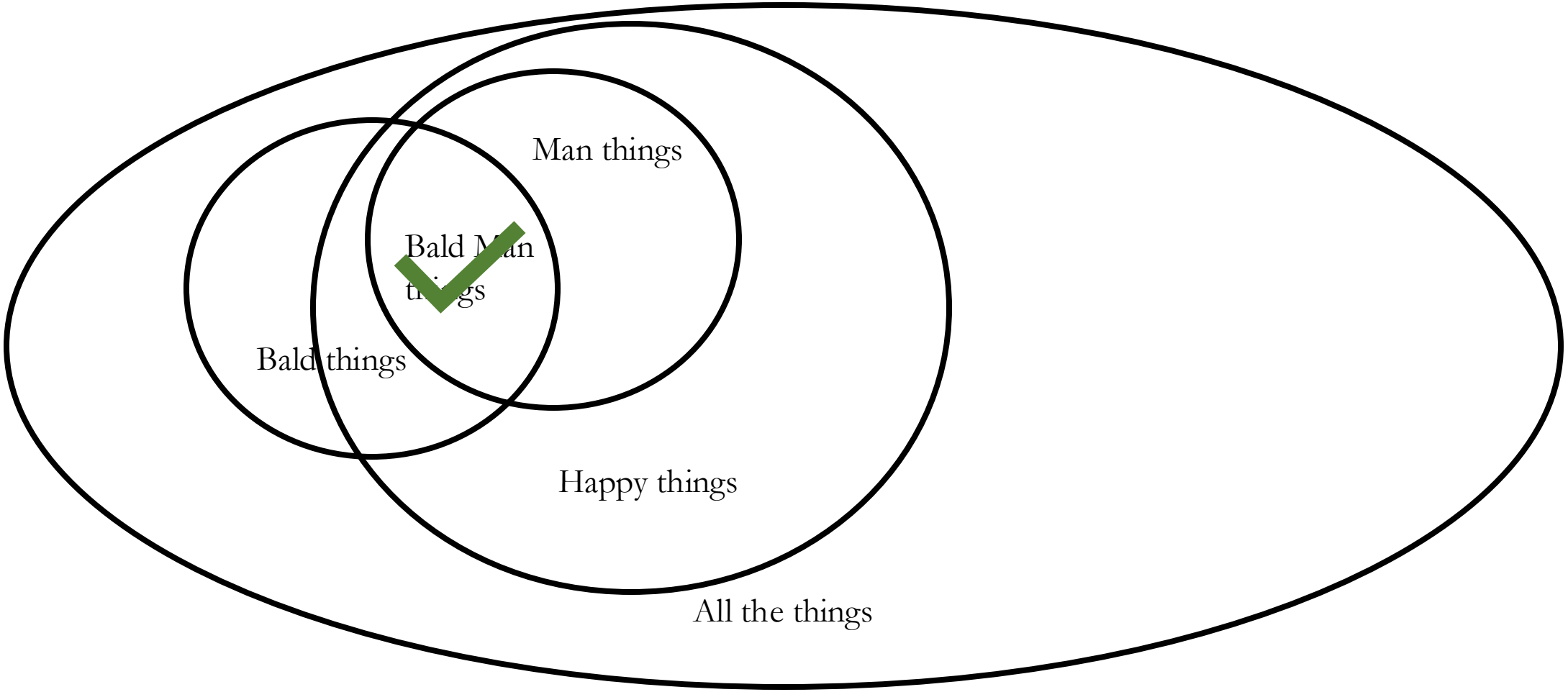

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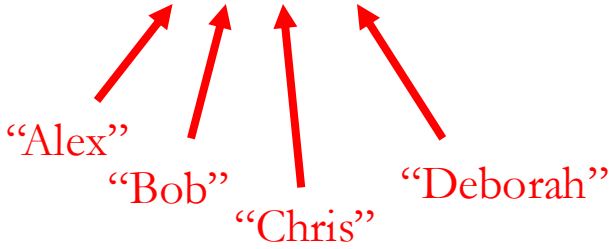


# *Supplemented FOL*

- Compare:

- Someone is bald and happy
  - Ex(Bx & Hx)

- John is bald and happy
  - (Bj & Hj)

# ***Binary Relations***

- FOL includes predicates, e.g. *is red*, *is bald*, and relations, e.g. *is part of*, *is between*, *is next to*, etc.

- For example (give John's arm the name 'a'):
  - John has an arm and it is part of John
  - part of(a, j)
  - John has a sister, Kellye
  - is related to(j, k)
  - John is between Sam and Deborah
  - is between (j, s, d)

# *FOL is Too Powerful!*

- Once we add ternary relations, the formal language becomes *undecidable*
- What this means – roughly – is that we can't determine in FOL for every expression whether that expression is false or we just haven't found a counterexample to it
- In other words, it's impossible in FOL hard to prove every negative

# *Restricting FOL*

- Because we don't want our computers to loop forever when we ask them to check our expressions, researchers have turned to more restrictive versions of FOL

- These are known as **description logics** (or 'guarded fragments' of FOL)

- The logical language you see in Protégé is a description logic

# *Description Logic & Protégé*

- Basically, description logics start with FOL, and add the following constraints:

- Only binary relations are allowed
- Names for particulars are in the language
- The same object may have multiple aliases

# *Importance*

- Why care about the logic? Because that's what computers will read
- When we introduce terms into an ontology, there is an implicit hierarchy logic involved
- But it's very simple; the real strength of ontologies is in linking parts of hierarchies to other parts of hierarchies
- That allows for many inferences to be drawn automatically

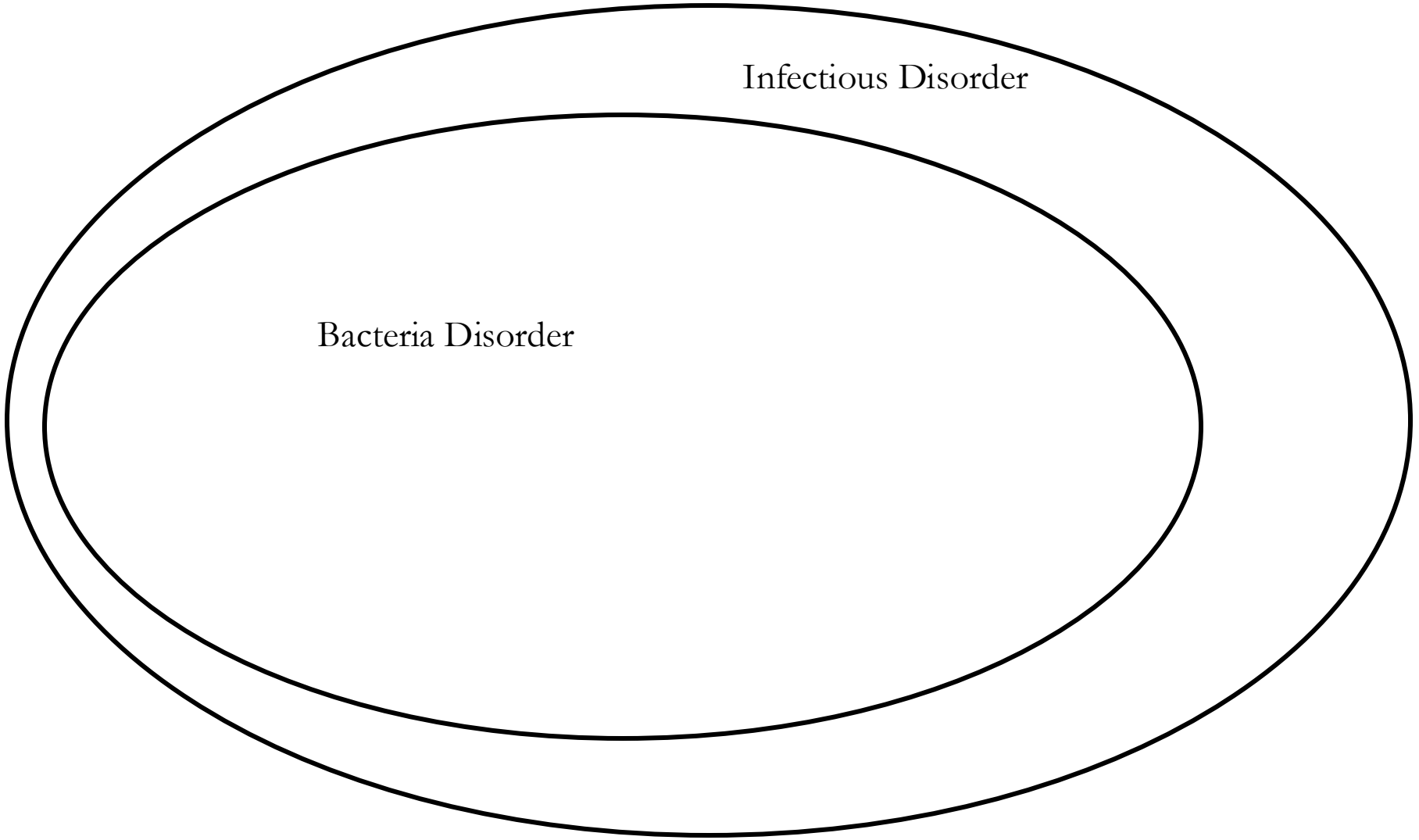
# *Example: Bacteria Disorder*

- Suppose we just add the term ‘bacteria disorder’ as a subclass of ‘infectious disorder’ to protégé but don’t add anything else

- The only thing a computer would know then is that anything that’s a bacteria disorder is an infectious disorder

- It wouldn’t even know bacteria disorders must have bacteria involved in them!

# *Possibilities*



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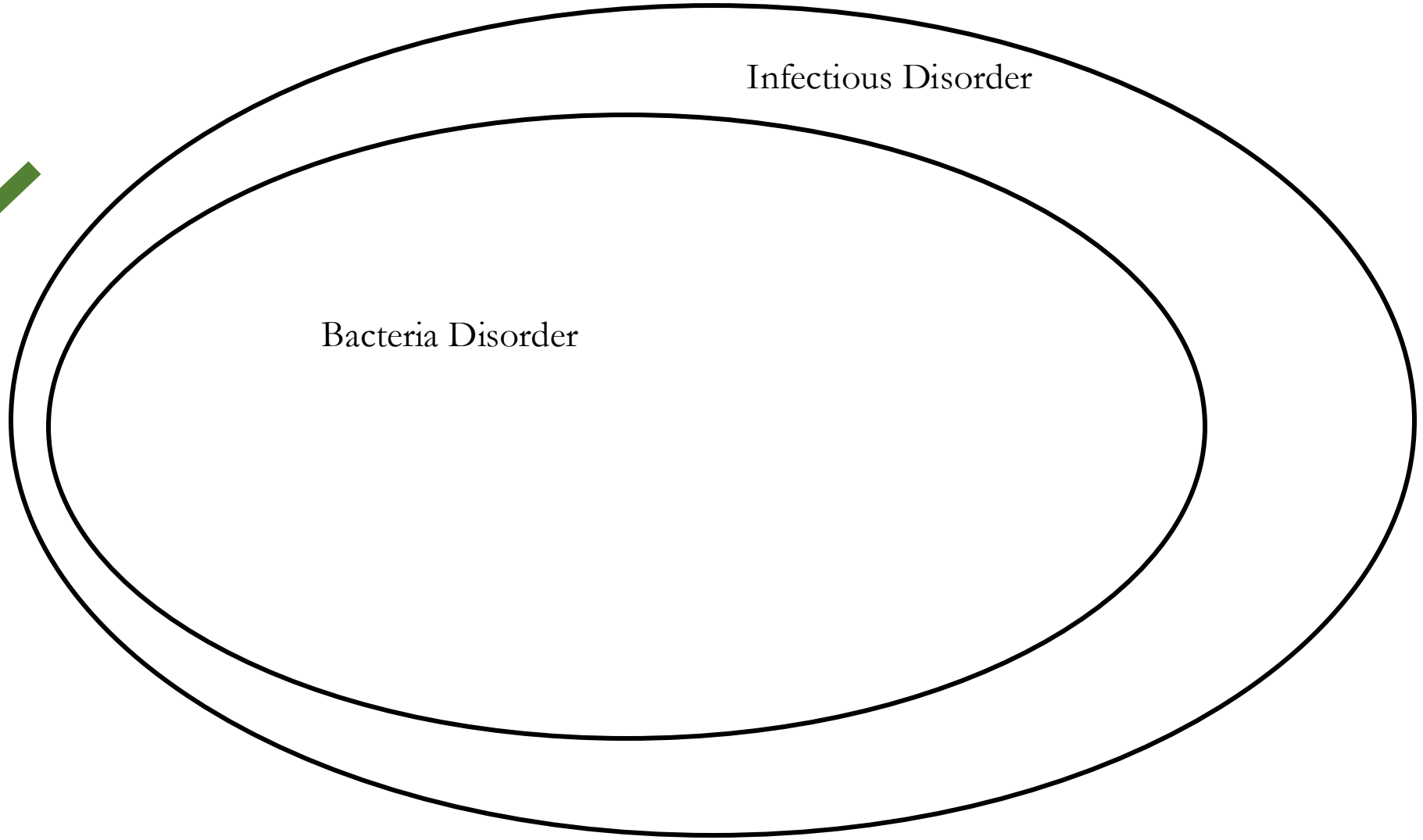

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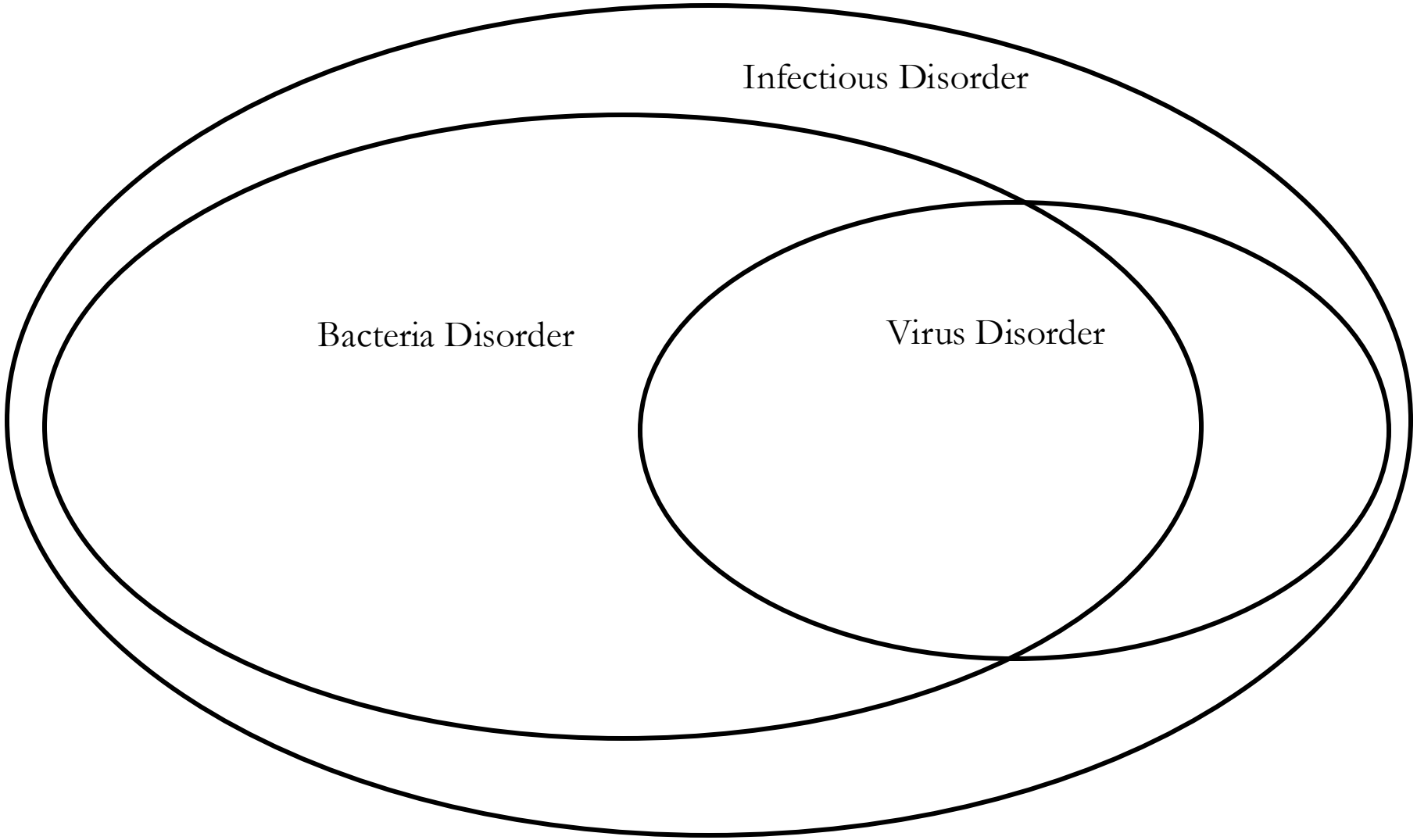
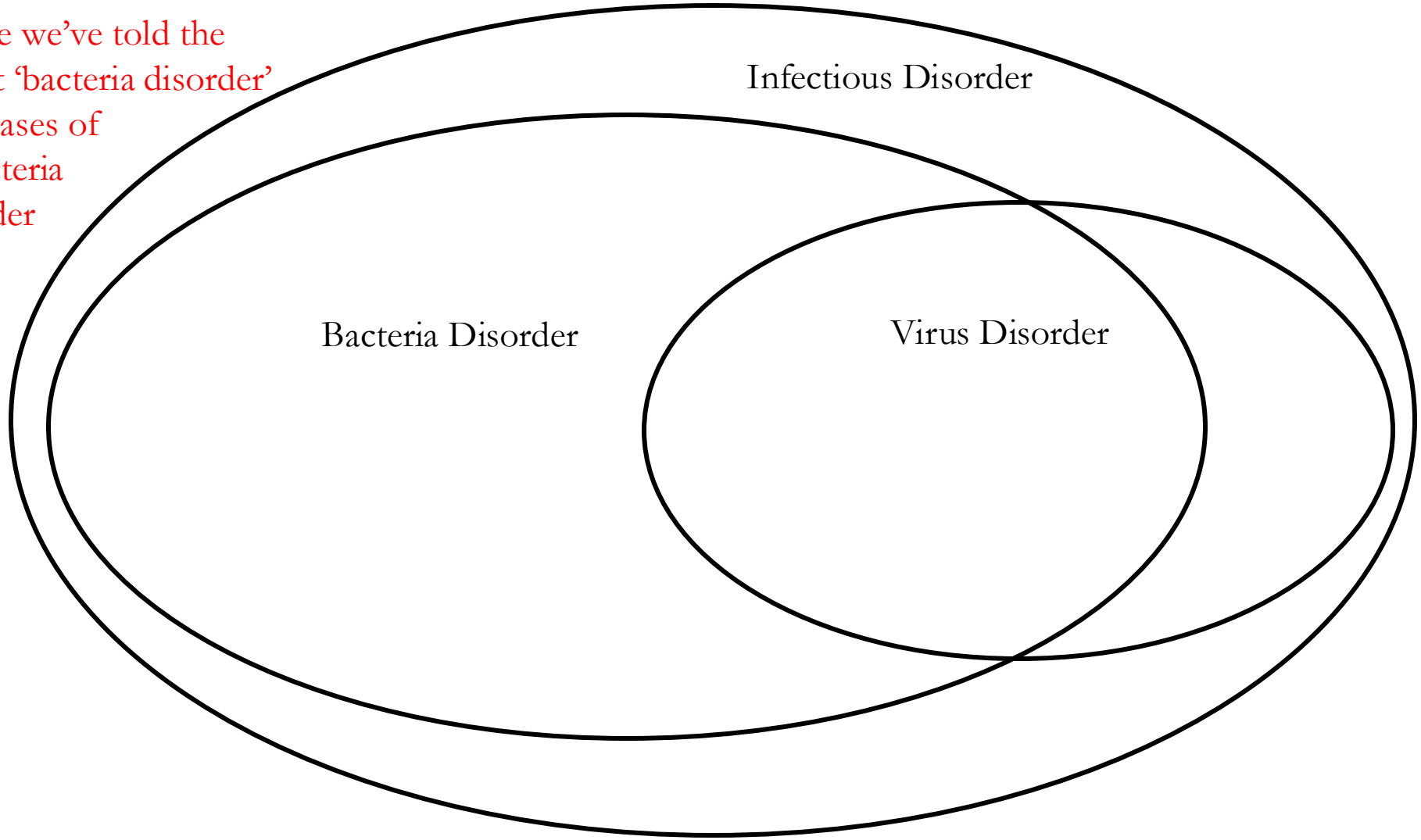


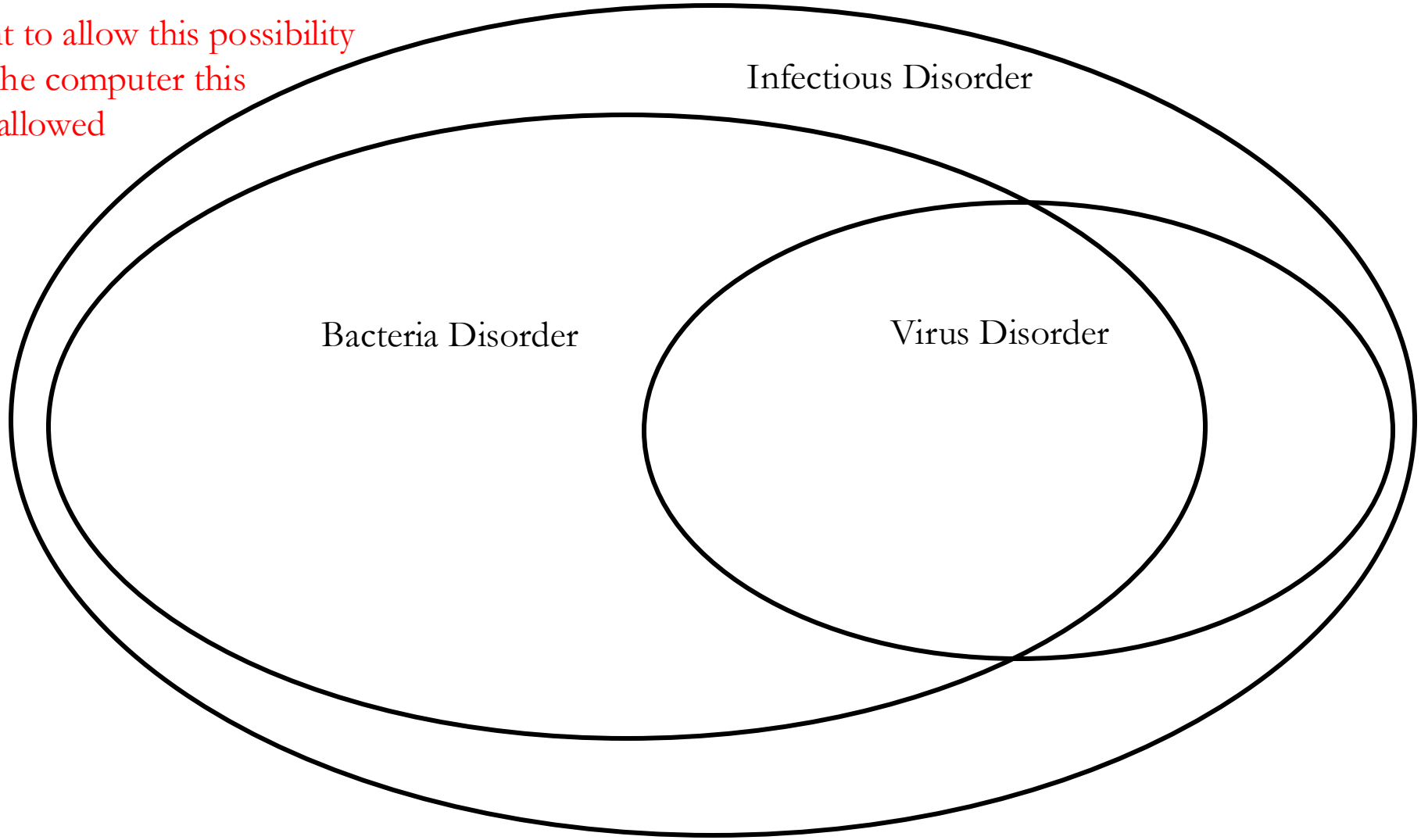
# *Possibilities*

Given what little we've told the computer about 'bacteria disorder' it would allow cases of overlapping bacteria and virus disorder



# *Possibilities*

If we don't want to allow this possibility we have to tell the computer this situation is not allowed



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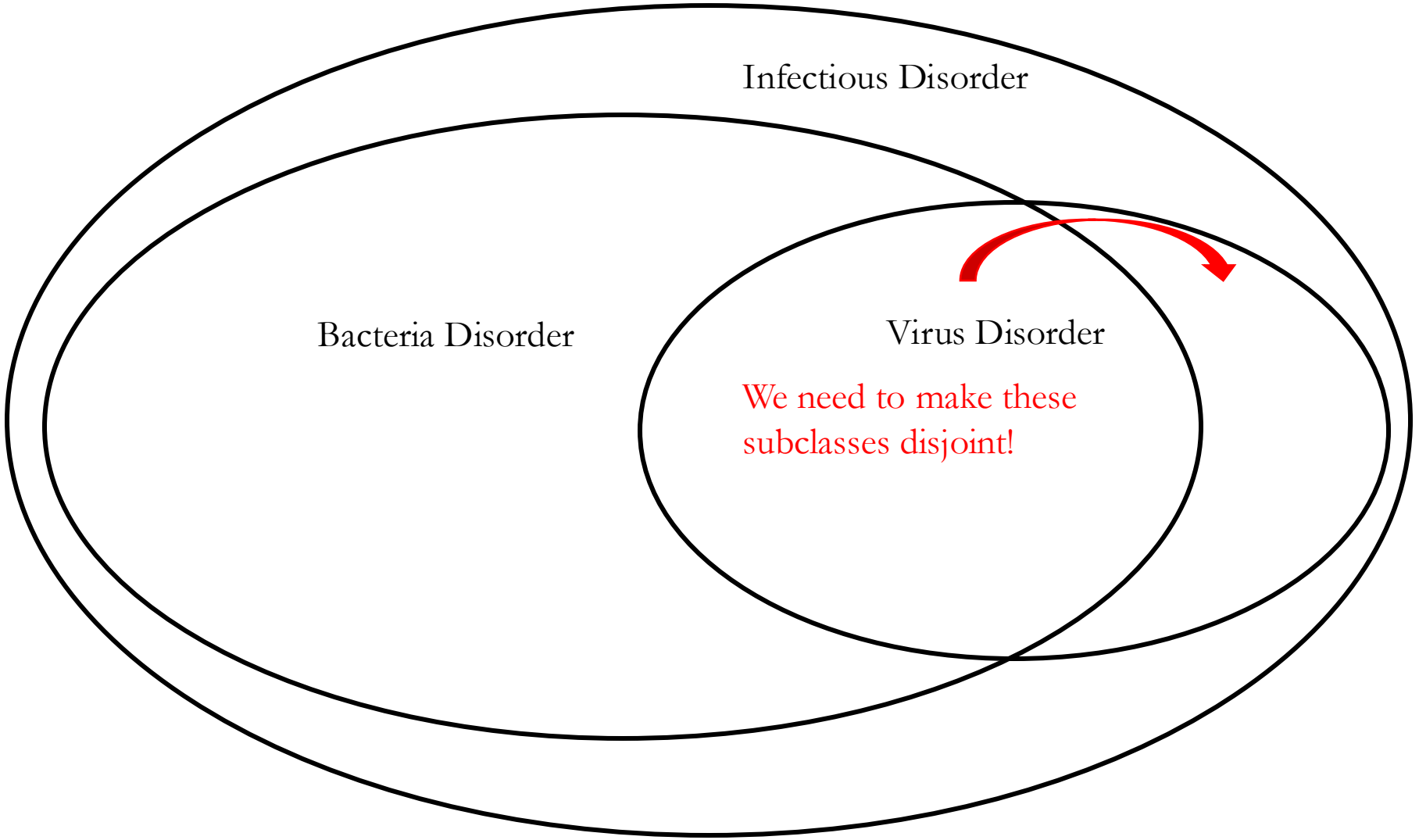
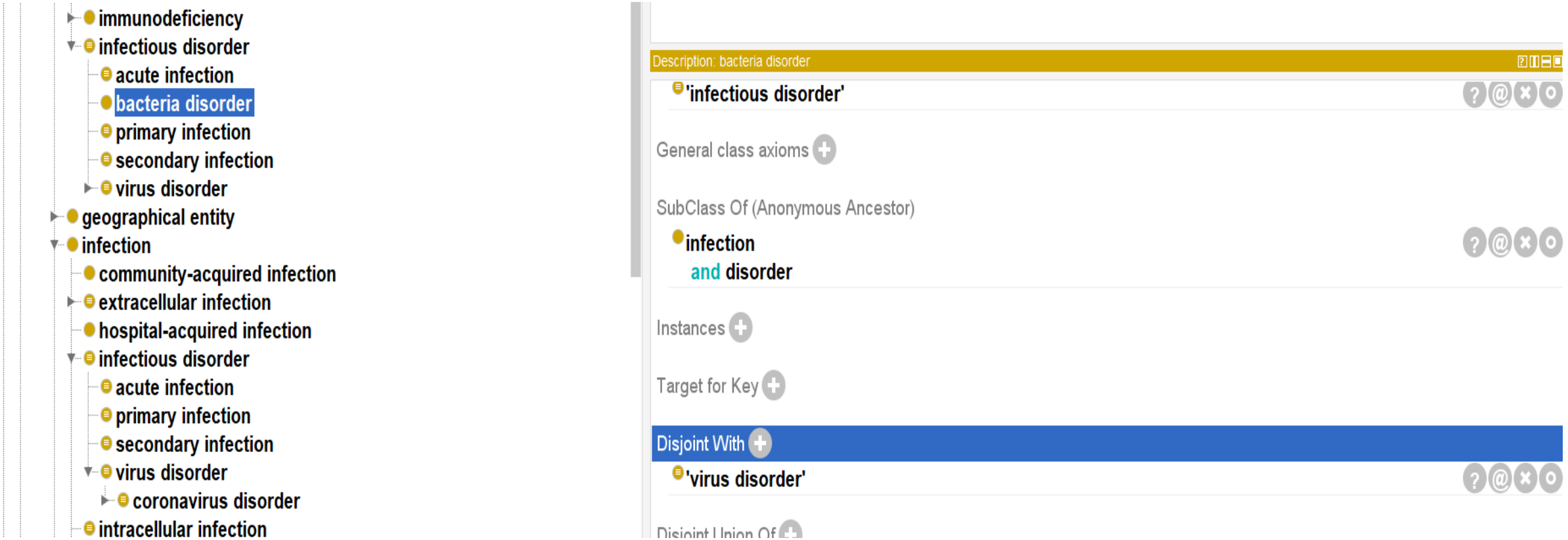


# *Protégé Disjoint*



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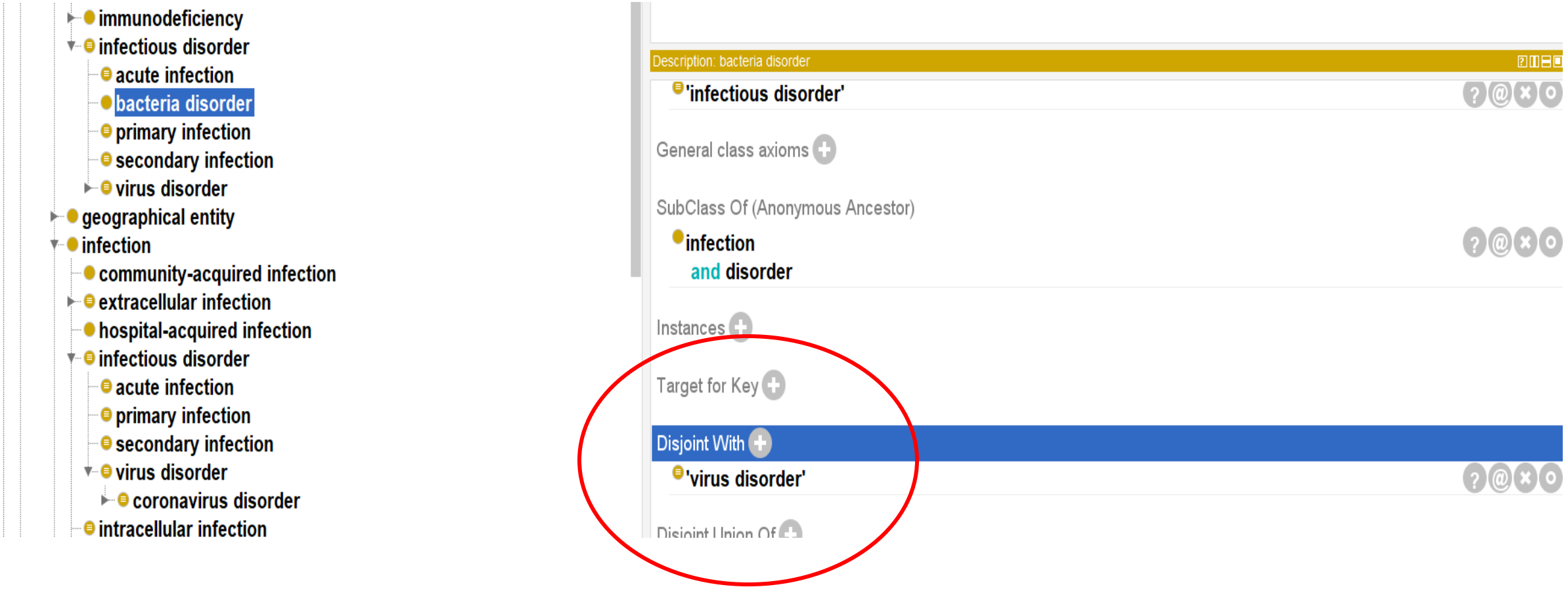

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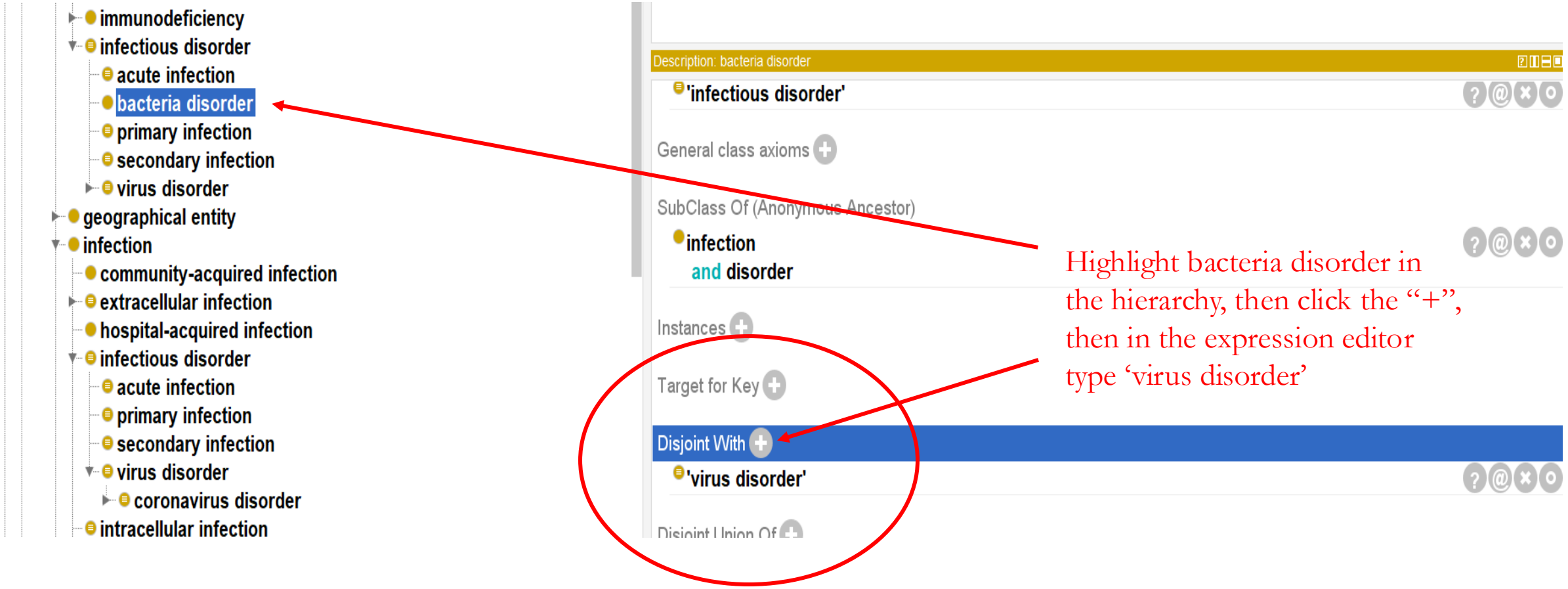

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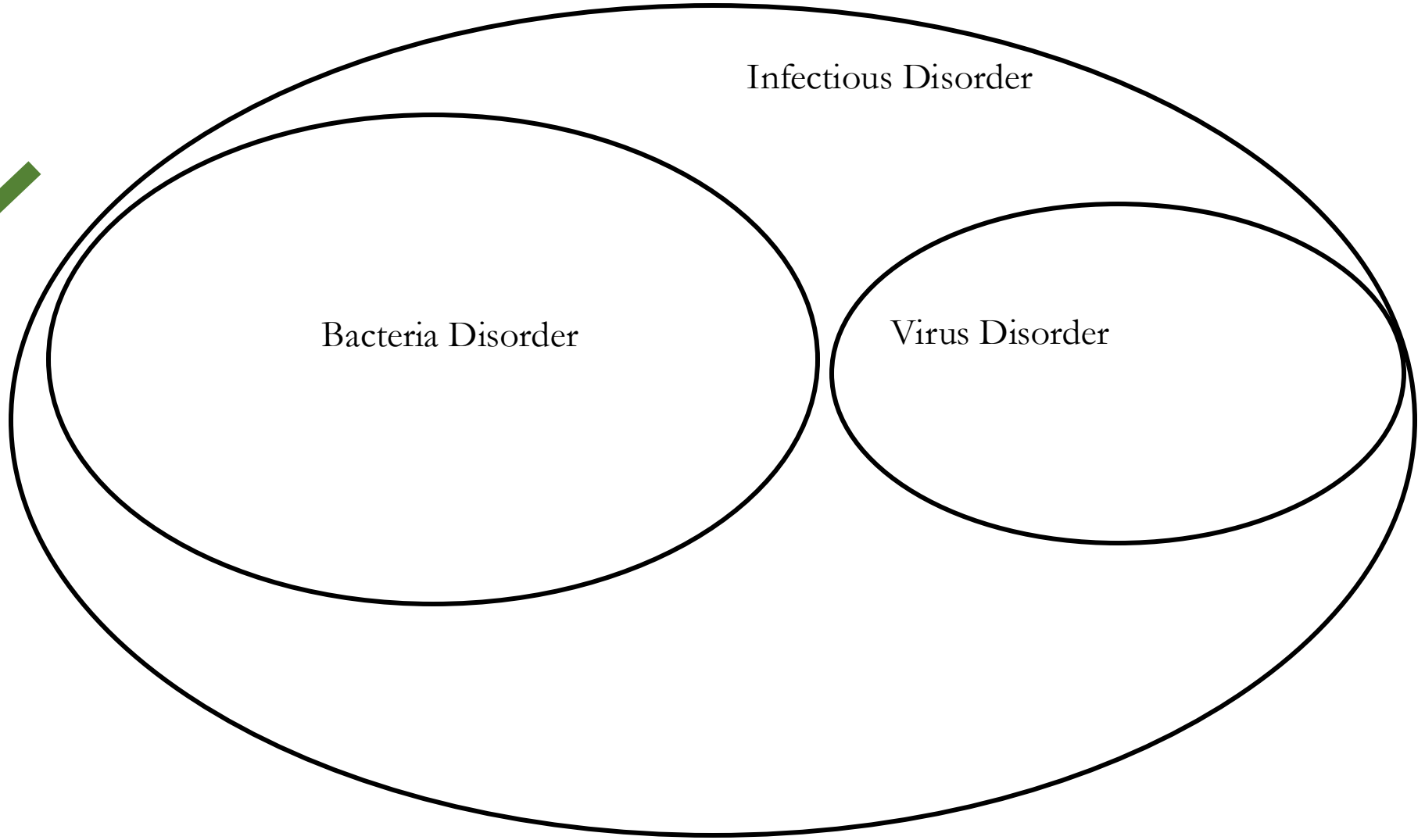
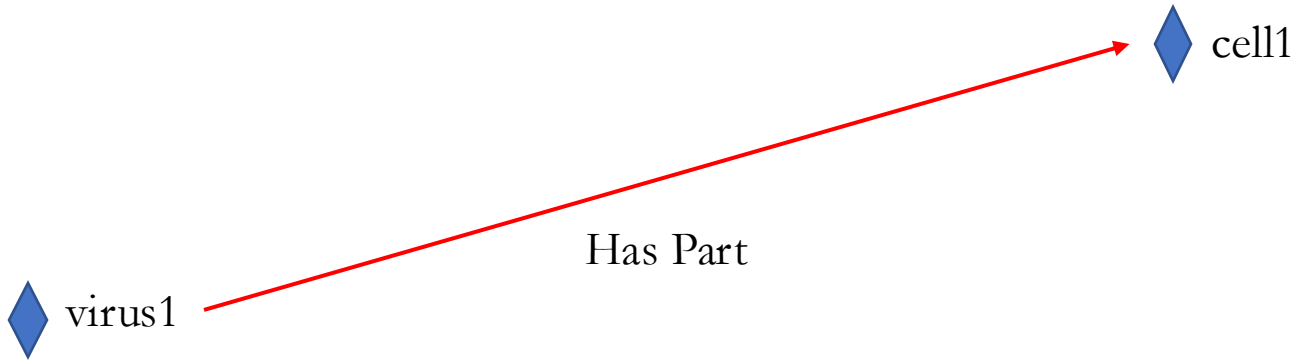
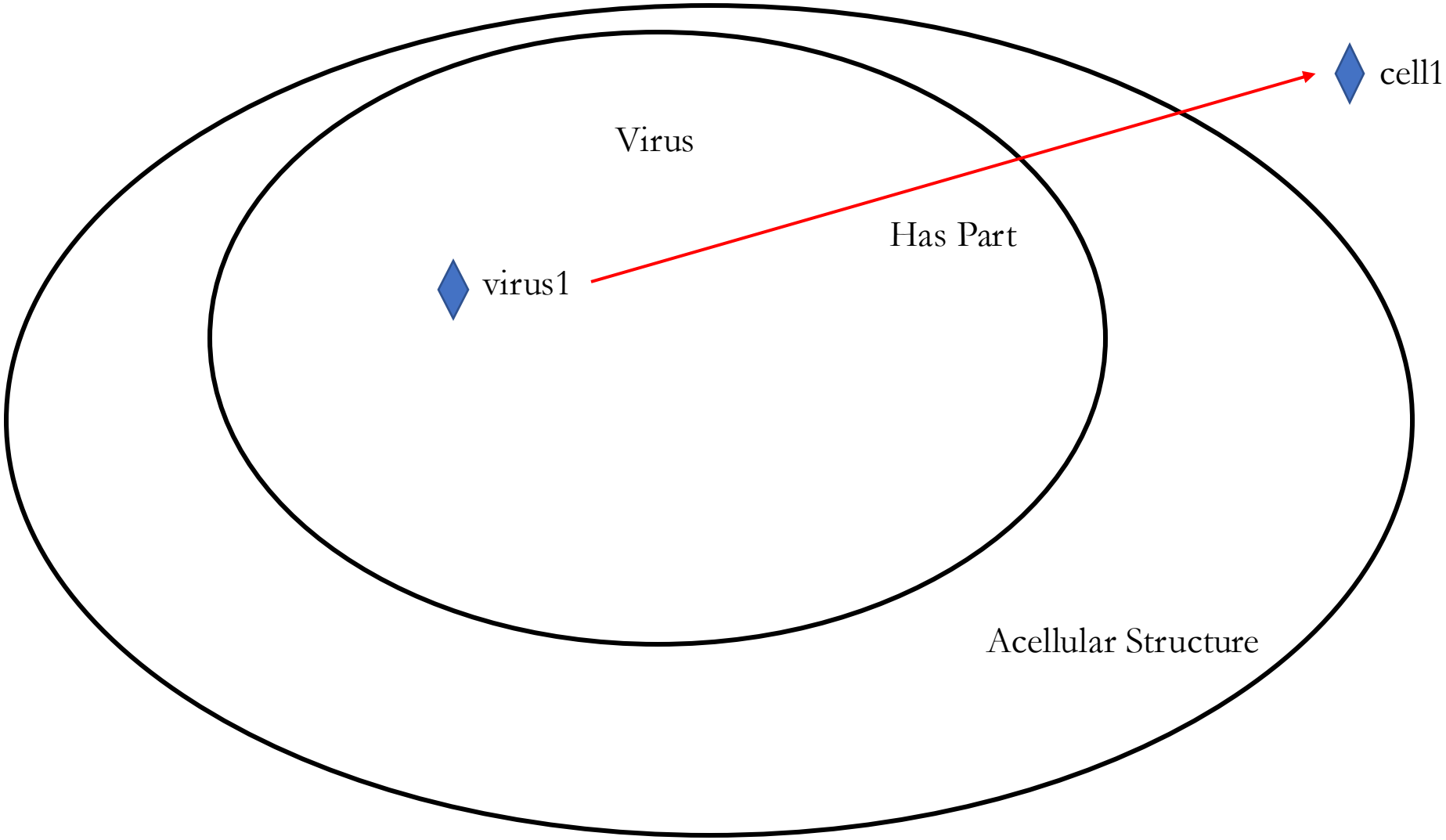
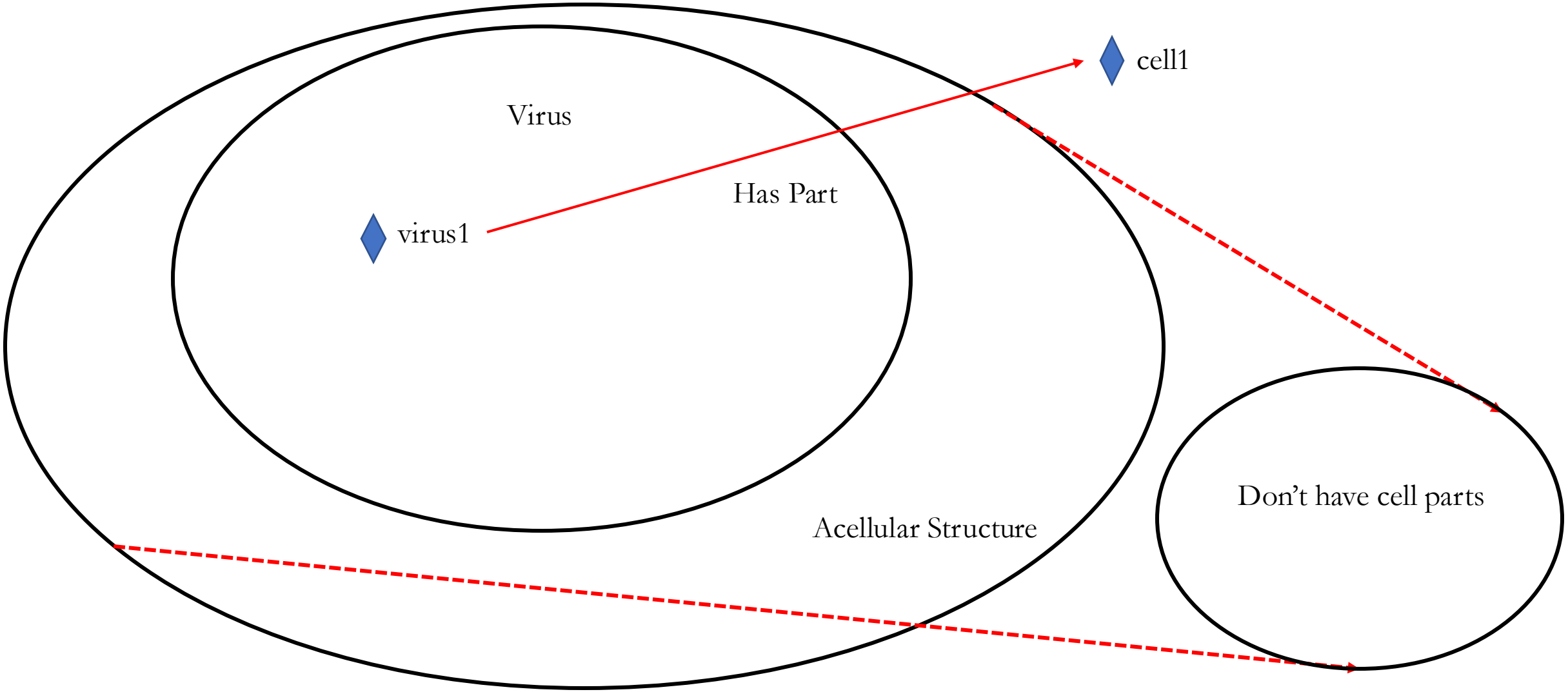


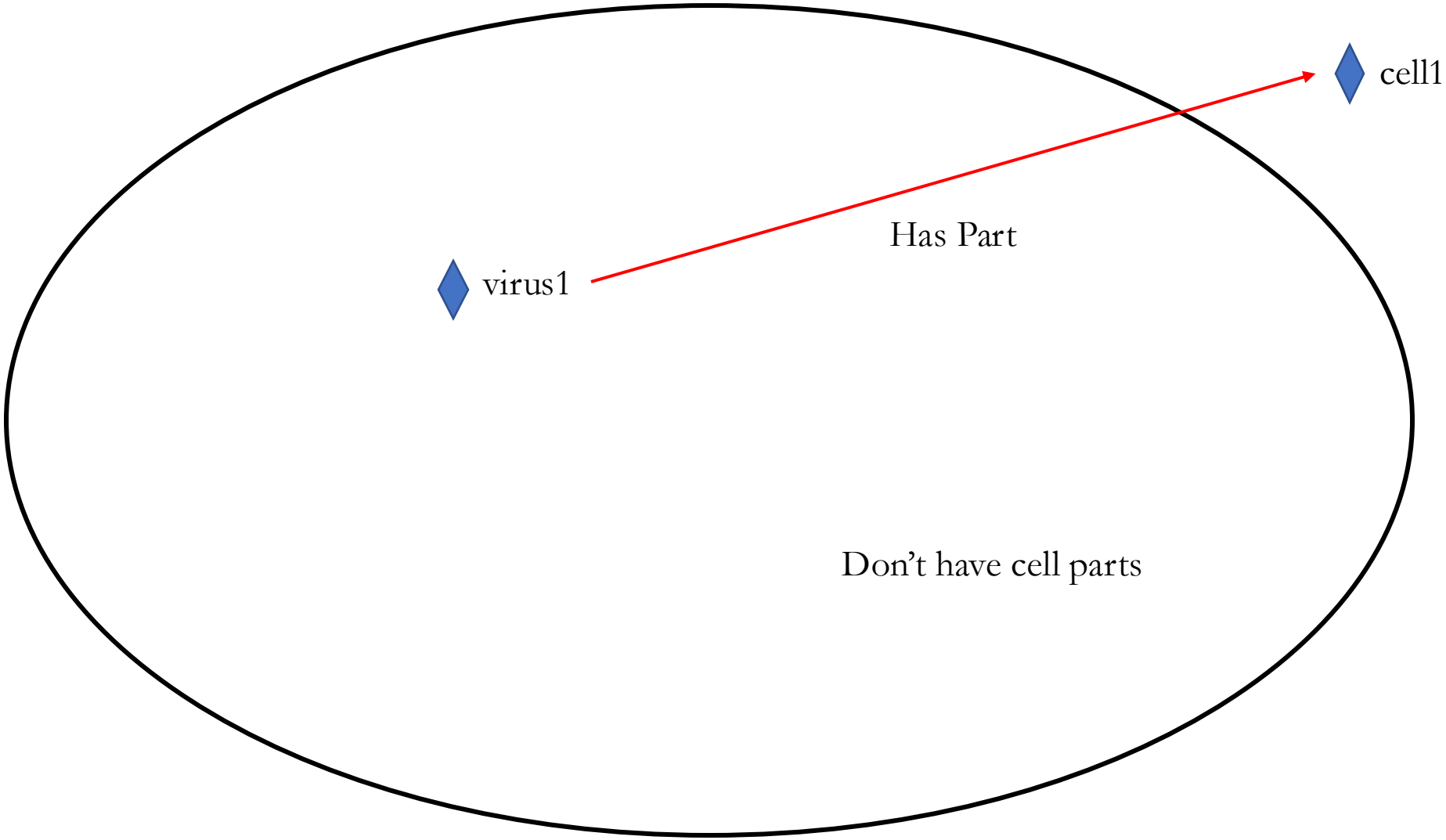
# *Individuals*

- I'm going to introduce an individual 'virus1' that is a virus
- And an individual 'cell1' that is a cell
- I do this in the interest of showing you how protégé will check your work for you, and protect you from introducing inconsistent items to the ontology
- Specifically, I'll assert (falsely) that virus1 has cell1 as a part…


cell1
Has Part
virus1


cell1
Virus
Has Part
virus1
Acellular Structure


cell1
Virus
Has Part
virus1
Acellular Structure
Don’t have cell parts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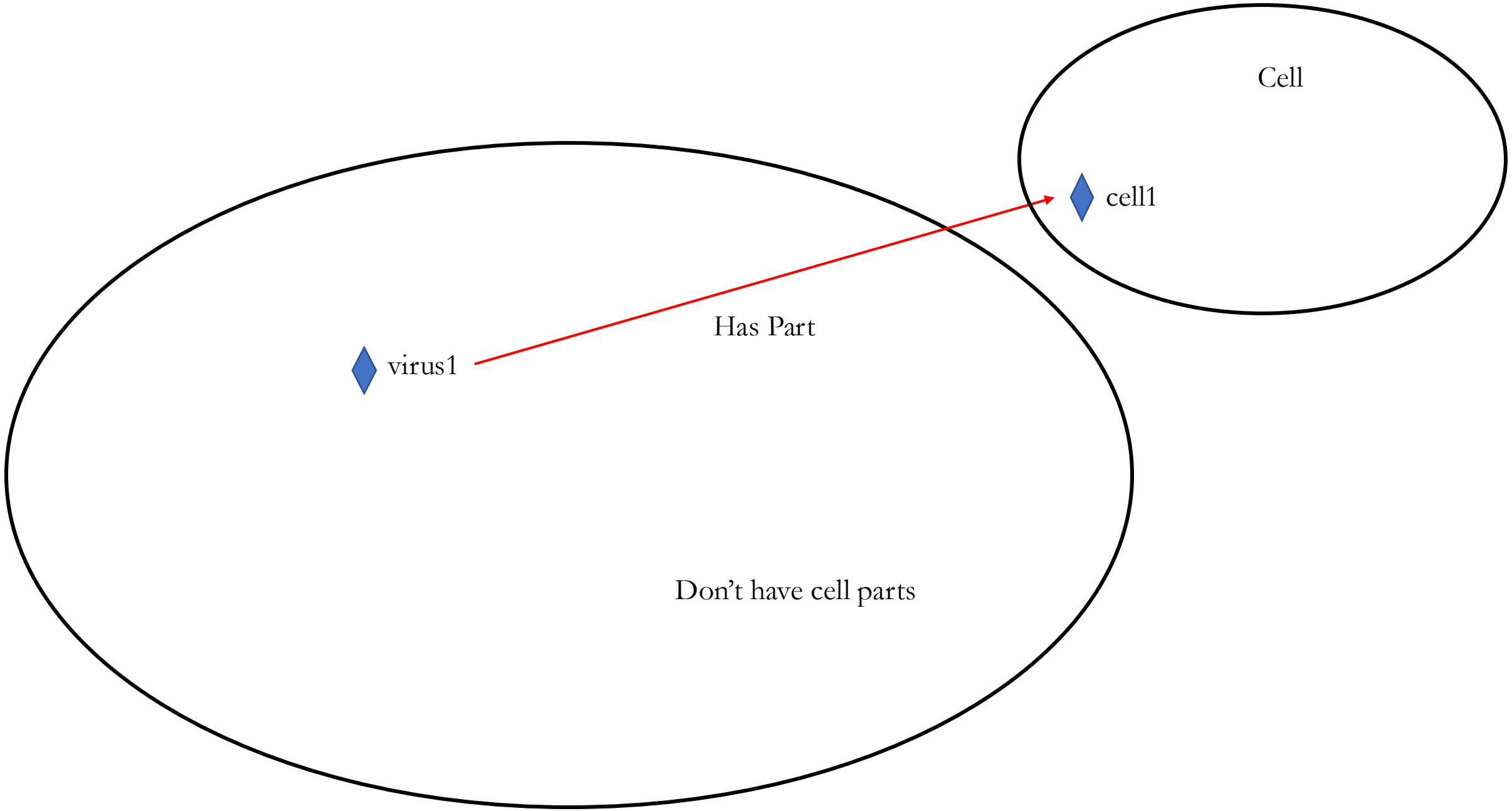
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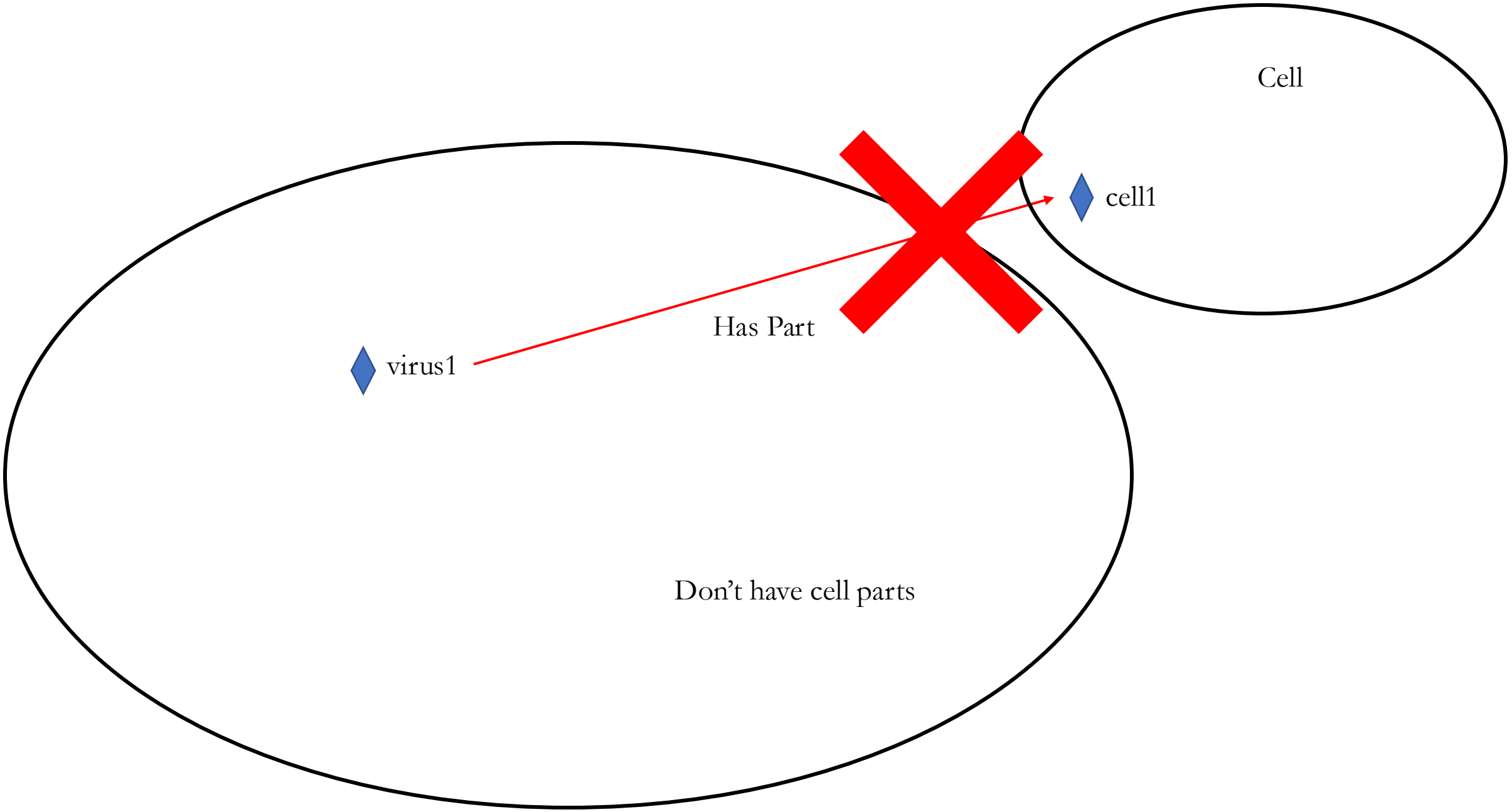
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# *Zebra Puzzle*

1. There are five houses.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house.
5. The Ukrainian drinks tea.
6. The green house is immediately to the right of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in a house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese man smokes Parliaments.
15. The Norwegian lives next to the blue house.

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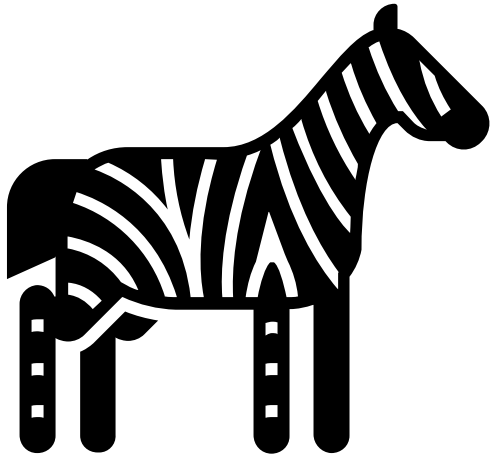

**WHO OWNS THE ZEBRA?**

rdfs:comment [language: en]

Note 1: Each house is painted exactly one color; each house is painted a different color.
Note 2: Each house has exactly one human occupant of distinct nationality and exactly one distinct pet is owned by that human.
Note 3: Each human occupant drinks exactly one distinct beverage and smokes exactly one distinct brand of cigarettes.

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**Classes allow you to represent uncertainty; you know that there are five men, you are trying to uncover instance-level facts about them**

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15. The Norwegian lives next to the blue house.

- **Beverage**
- **Cigarette**
- **Color**
- **House**
- **Man**
- **Pet**

# *Zebra Puzzle*

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**We do this by representing logical relationships between instances of classes using object properties**

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- drinks
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- lives_in
- owned_by
- owns
- right_of
- smoked_by
- smokes

# *Zebra Puzzle*


**If R is the inverse of S, then for any <x,y> in R, <y,x> is in S**

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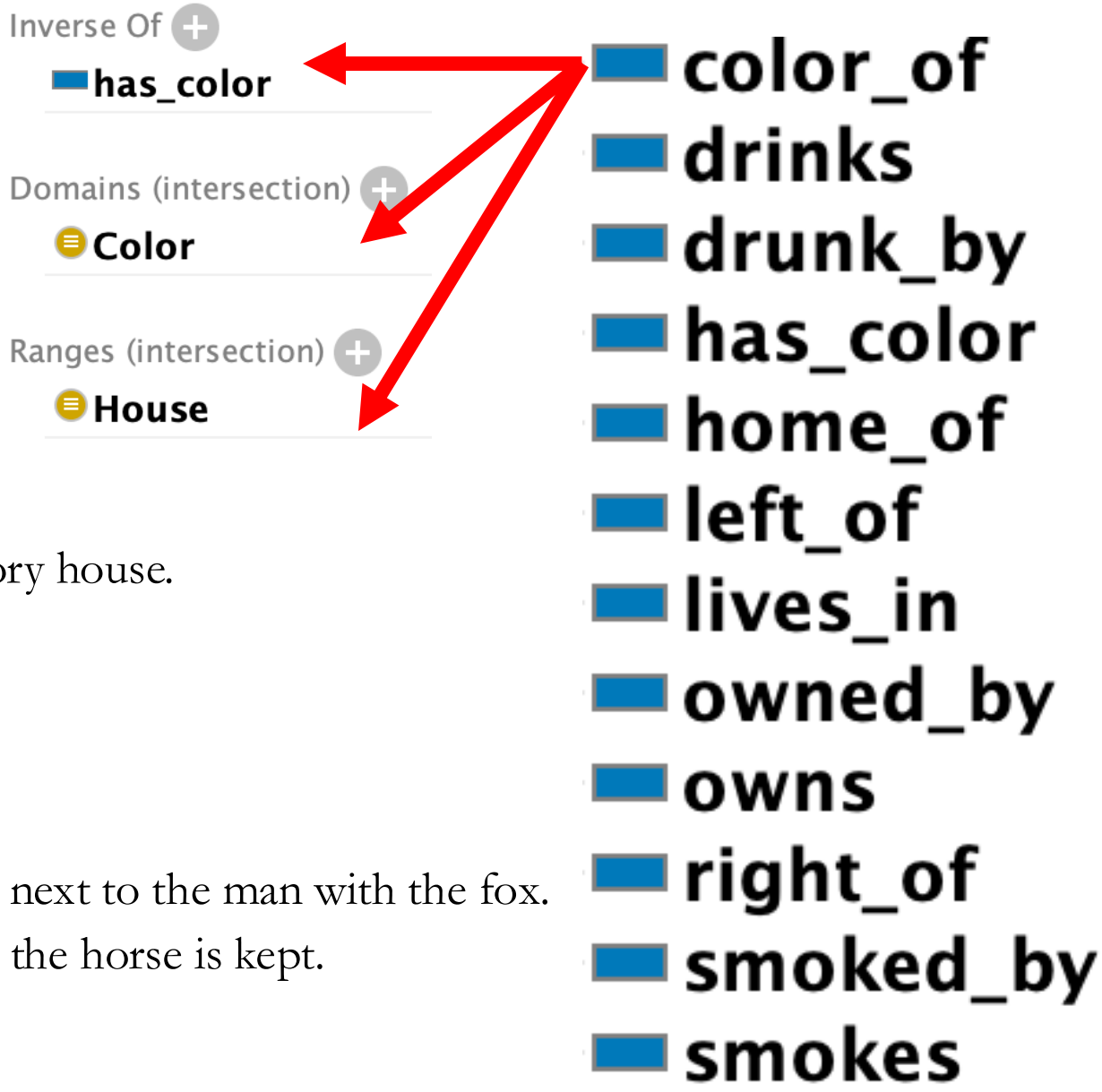


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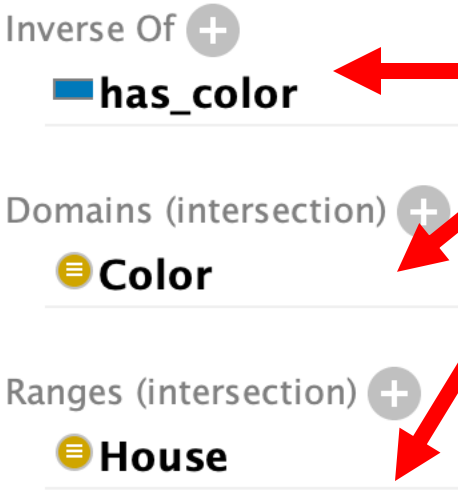



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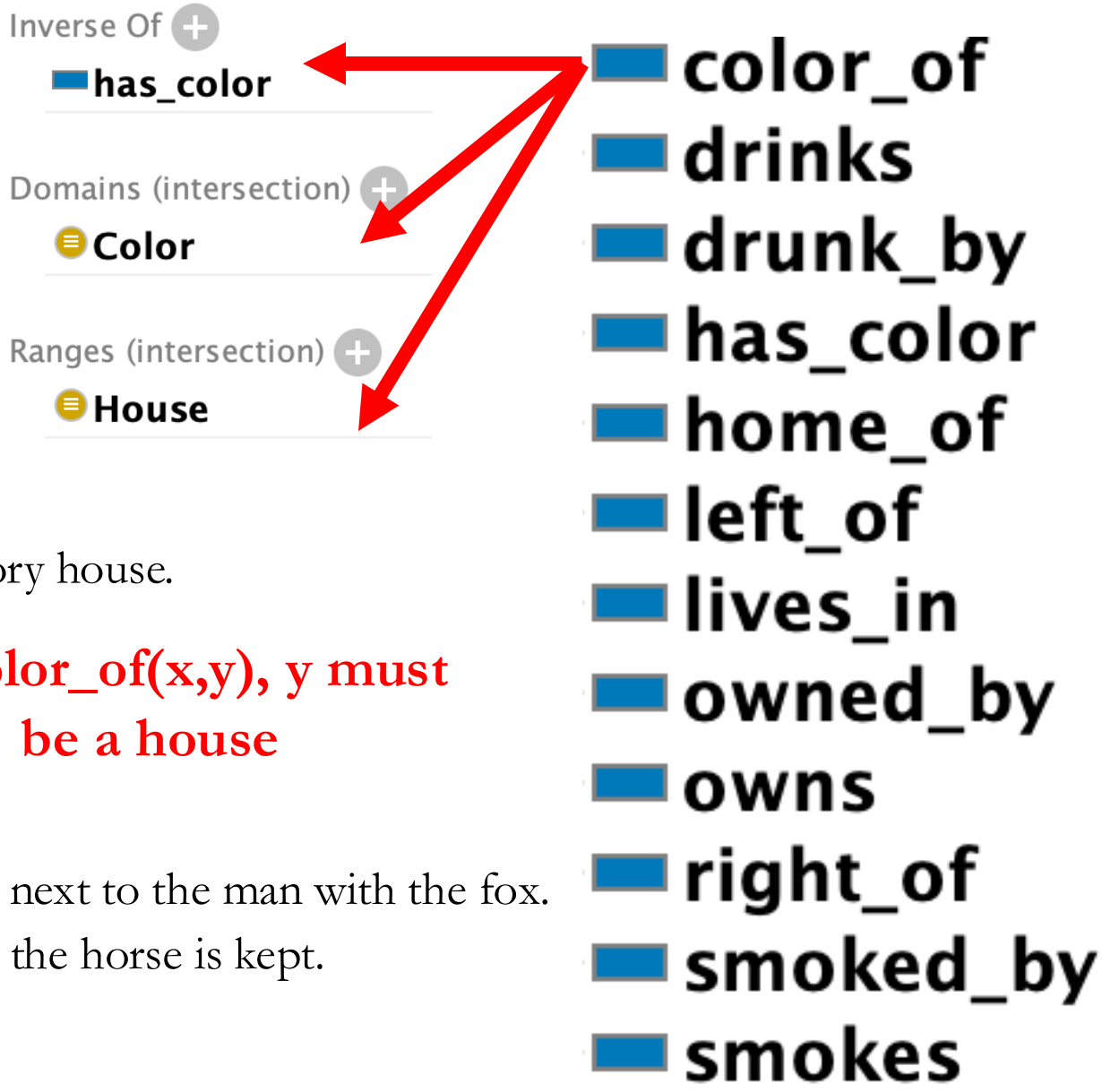
**For color_of(x,y), x must be a color**

# *Zebra Puzzle*

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**For color_of(x,y), y must be a house**

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**For color_of(x,y), there is a 1-1 relationship between color and house**

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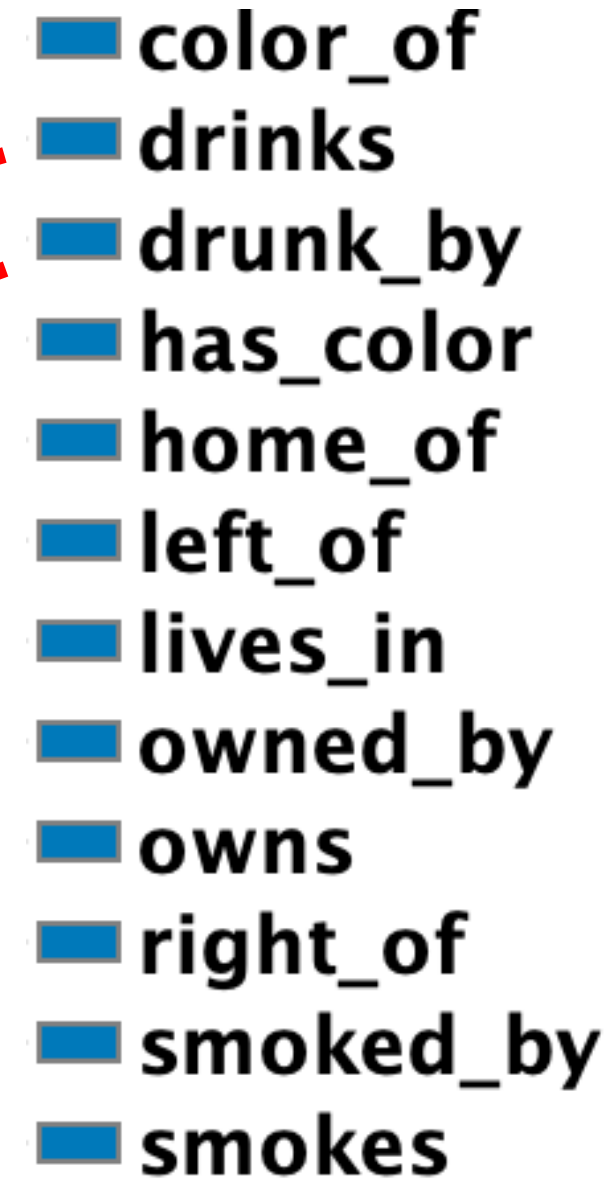

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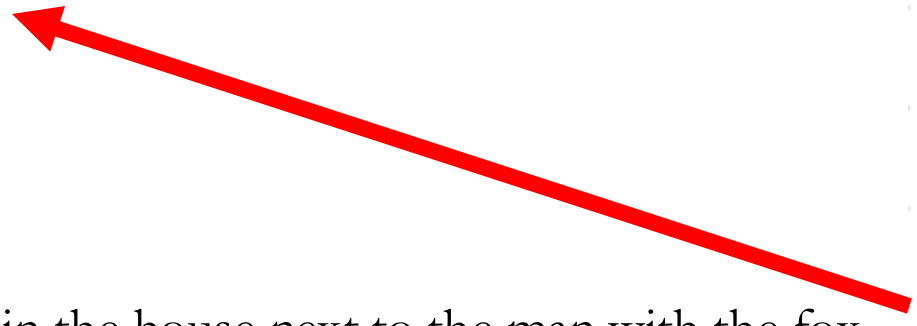

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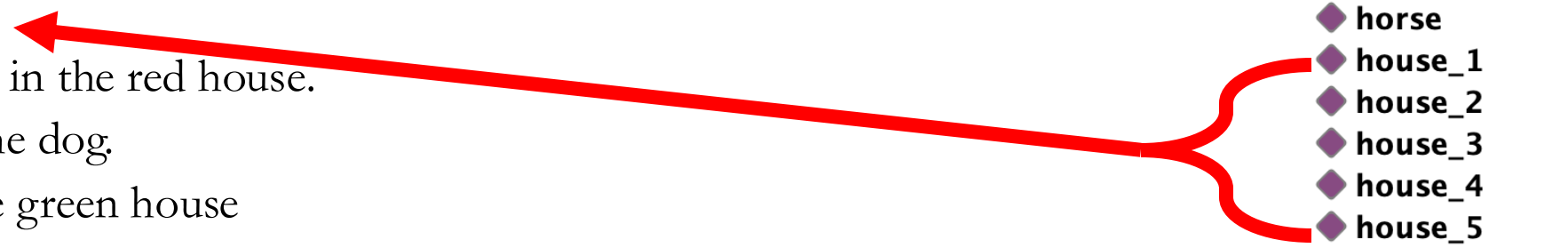

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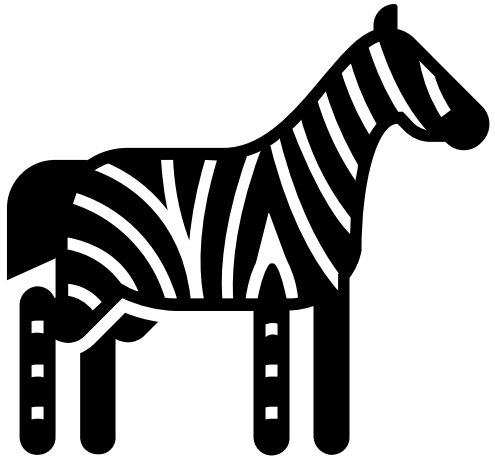

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Color

color_of some (left_of some (has_color value ivory))

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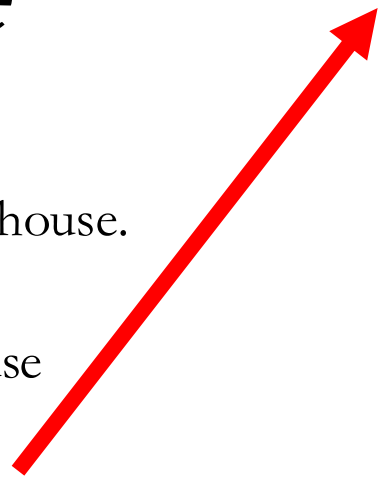

green is the color of some x

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# *Zebra Puzzle*

color_of some (right_of some (has_color value ivory))

...and since the domain of color_of is colors and the range is houses, it follows that x is a house

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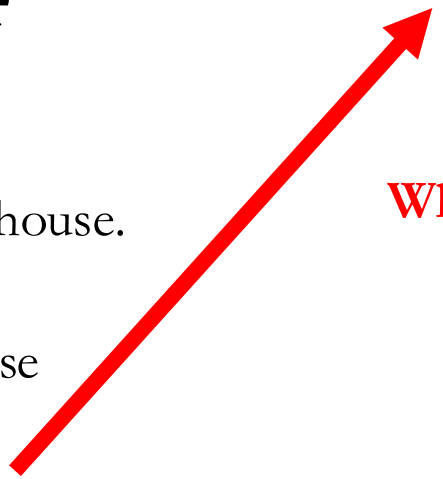

Where that house is to the right of some x

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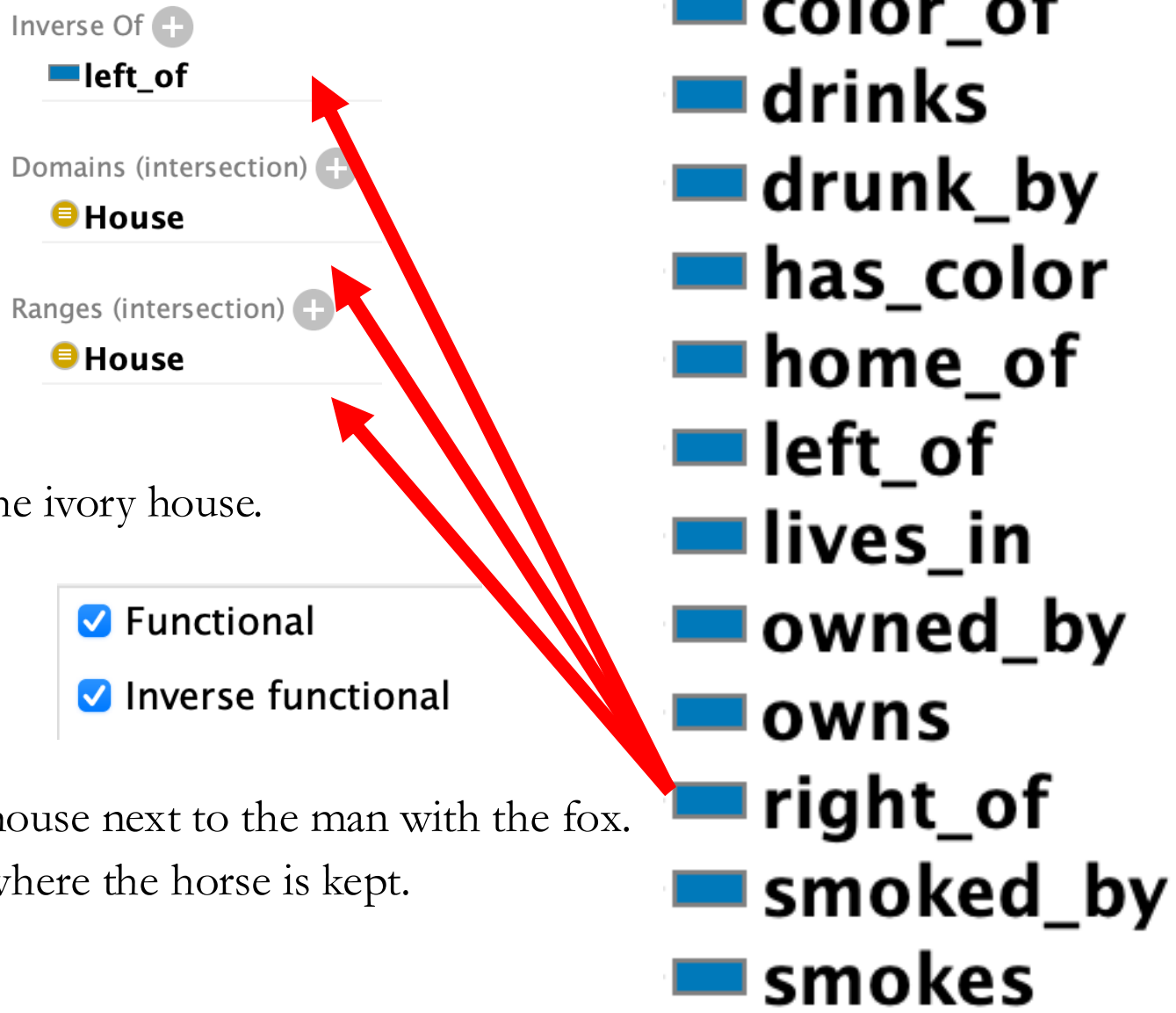

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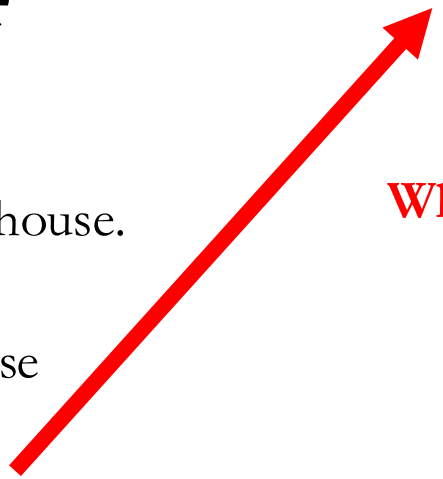

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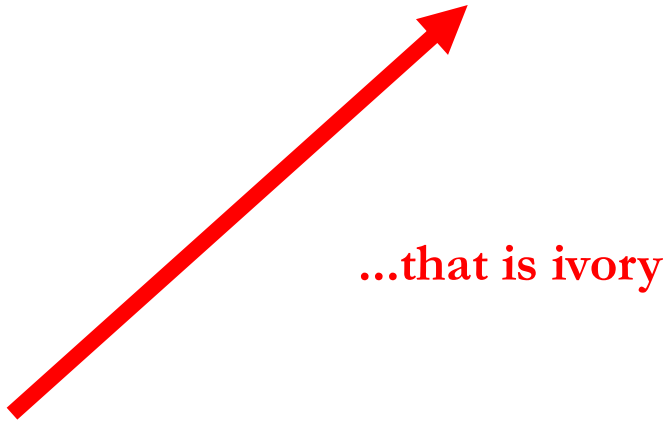
- spaniard
- tea
- ukrainian
- water
- yellow
- zebra

# *Zebra Puzzle*

color_of some (right_of some (has_color value ivory))




1. There are five houses.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house
5. The Ukrainian drinks tea.
6. The green house is immediately to the right of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in a house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese man smokes Parliaments.
15. The Norwegian lives next to the blue house.

- blue
- chesterfields
- coffee
- dog
- englishman
- fox
- green
- horse
- house_1
- house_2
- house_3
- house_4
- house_5
- ivory
- japanese
- kools
- lucky_strikes
- milk
- norwegian
- old_gold
- orange_juice
- parliaments
- red
- snail
- spaniard
- tea
- ukrainian
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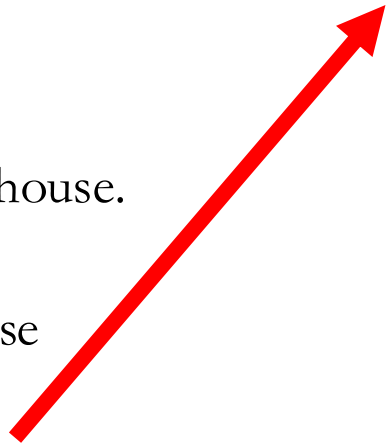
- zebra

# *Zebra Puzzle*

color_of some (right_of some (has_color value ivory))



**Altogether, green is the color of some house x that is to the right of some house y that has color ivory**

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2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house
5. The Ukrainian drinks tea.
6. The green house is immediately to the right of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
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