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Long Short-Term Memory in Recurrent Neural Network

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Outline

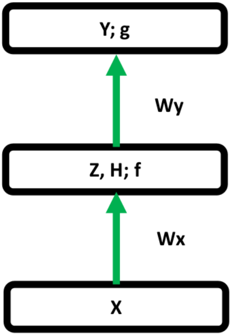
Machine Learning Review

Discrimitive Model learns $P(Y|X)$
Generative Model learns $P(X, Y)$, or $P(X)$
Supervised Learning uses pairs of input X and output Y, and aims to learn $P(Y|X)$ or $f : X \longrightarrow Y$
Unsupervised Learning uses input X only, and aims to learn $P(X)$ or $f : X \longrightarrow H$, where H is "better" representation of X
Models can be **stochastic** or **deterministic**

In this presentation, we focus on **deterministic, supervised, discrimitive model** based on **Recurrent Neural Network (RNN)/Long Short-Term Memory (LSTM)**

Applications: Speech Recognition, Machine Translation, Online Handwriting Recognition, Language Modeling, Music Composition, Reinforcement Learning, etc.

input: $X \in \mathbb{R}^Q$
target output: $Y \in \mathbb{R}^D$
predicted output: $\hat{Y} \in \mathbb{R}^D$
hidden: $H, Z \in \mathbb{R}^P$
weights: $W_X \in \mathbb{R}^{P \times Q}, W_Y \in \mathbb{R}^{D \times P}$
activations: $f : \mathbb{R}^P \longrightarrow \mathbb{R}^P, g : \mathbb{R}^D \longrightarrow \mathbb{R}^D$
loss function: $L : \mathbb{R}^D \times \mathbb{R}^D \longrightarrow \mathbb{R}$
loss: $E \in \mathbb{R}$
objective: minimize total loss E for $(X^{(i)}, Y^{(i)})_{i=1\dots N}$ training examples



Forward Propagation:
$Z = W_X \cdot X$
$H = f(Z)$
$\hat{Y} = g(W_Y \cdot H)$
$E = L(Y, \hat{Y})$

Activation functions

Common activation functions $f : X \longrightarrow Y$ on $X, Y \in \mathbb{R}^D$:

Linear: $f(X) = X$

Sigmoid/Logistic: $f(X) = \frac{1}{1+e^{-X}}$

Rectified Linear (ReLU): $f(X) = max(0, X)$

Tanh: $f(X) = tanh(X) = \frac{e^X - e^{-X}}{e^X + e^{-X}}$

Softmax: $f : Y_i = \frac{e^{X_i}}{\sum_{j=1}^{j=D} e^{X_i}}$

Most activations are **element-wise** and **non-linear**
Derivatives are easy to compute

NN: Backpropagation (1960s-1981)

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Backpropagation (i.e. chain rule):

assume $g(X) = X$,

$L(Y, \hat{Y}) = \frac{1}{2}\sum_{i=1}^{D}(Y_i - \hat{Y}_i)^2$:

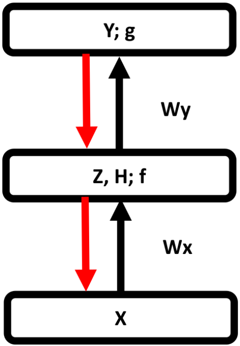
$\frac{\partial E}{\partial \hat{Y}} = \hat{Y} - Y$, $\frac{\partial \hat{Y}}{\partial H} = W_Y$, $\frac{\partial H}{\partial Z} = f'(Z)$

$$\frac{\partial E}{\partial Z} = \frac{\partial E}{\partial \hat{Y}}\frac{\partial \hat{Y}}{\partial H}\frac{\partial H}{\partial Z} = (W_Y^T \cdot (\hat{Y} - Y)) \odot f'(Z)$$

$$\frac{\partial E}{\partial W_Y} = \frac{\partial E}{\partial \hat{Y}}\frac{\partial \hat{Y}}{\partial W_Y} = (\hat{Y} - Y) \otimes H$$

$$\frac{\partial E}{\partial W_X} = \frac{\partial E}{\partial Z}\frac{\partial Z}{\partial W_X} = \frac{\partial E}{\partial Z} \otimes X$$

*abusing matrix calculus notations

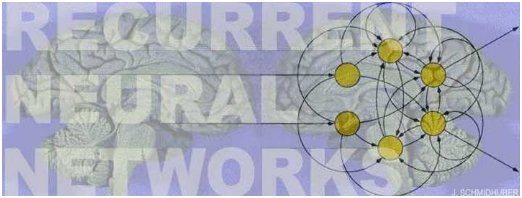


Stochastic Gradient Descent

learning rate: $\epsilon > 0$

$$W = W - \epsilon\frac{\partial E}{\partial W}, \forall W \in \{W_Y, W_X\}$$

RNN (1980s-)



Why Recurrent?

- Human brain is a recurrent neural network, i.e. has feedback connections
- A lot of data is sequential and dynamic (can grow or shrink)
- RNNs are Turing-Complete [Siegelmann and Sontag, 1995, Graves et al., 2014]

Unrolling RNNs

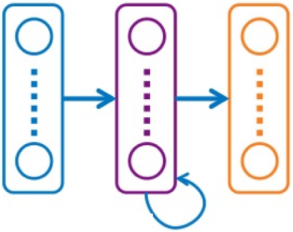


Figure: RNN

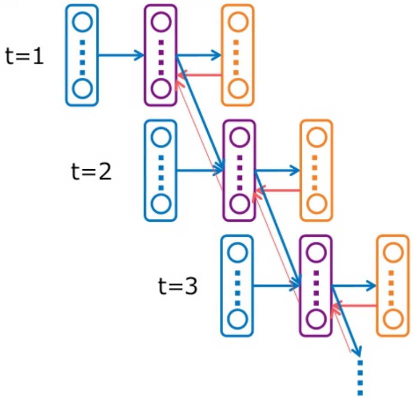


Figure: Unrolled RNN

RNN (1980s-)

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Forward Propagation:

Given H_0

$Z_t = W_X \cdot X_t + W_H \cdot H_{t-1}$

$H_t = f(Z_t)$

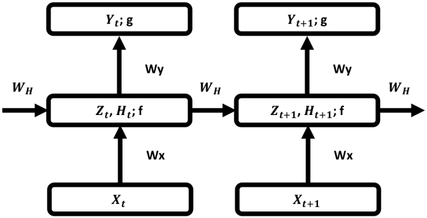
$\hat{Y}_t = g(W_Y \cdot H_t)$

$E_t = L(Y_t, \hat{Y}_t)$

$E = \sum_{t=1}^{T} E_t$

RNN=Very Deep NN w/ tied weights

Sequence learning: unknown input len $\longrightarrow$ unknown output len



Vanishing/exploding gradients (1991)

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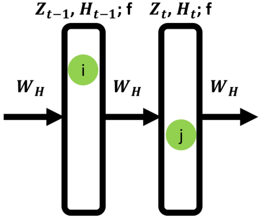
Fundamental Problem in Deep Learning [Hochreiter, 1991]:



Let $v_{j,t} = \frac{\partial E}{\partial Z_{j,t}}$

$v_{i,t-1} = f'(Z_{i,t-1}) \cdot \sum_{j=1}^{P} w_{ji} \cdot v_{j,t}$

$$\frac{\partial v_{i,t-q}}{\partial v_{j,t}} = \sum_{l_1=1}^{P} \cdots \sum_{l_{q-1}=1}^{P} \prod_{m=1}^{q} f'_{l_m}(Z_{l_m,t-m}) \cdot w_{l_m,l_{m-1}}$$

$|f'_{l_m}(Z_{l_m,t-m}) \cdot w_{l_m,l_{m-1}}| > 1.0 \forall m \longrightarrow$ explodes

$|f'_{l_m}(Z_{l_m,t-m}) \cdot w_{l_m,l_{m-1}}| < 1.0 \forall m \longrightarrow$ vanishes

Gradient vanishes/increases exponentially in terms of time steps T

e.g. “bufferfly effect”

Early work on RNN (1980s-90s)

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Exact gradient (TOO HARD!(back then)):

- **Backpropagation Through Time** (BPTT) [Werbos, 1990]

Approximate gradient:

- **Real Time Recurrent Learning** (RTRL) [Robinson and Fallside, 1987, Williams and Zipser, 1989]
- **Truncated BPTT** [Williams and Peng, 1990]

Non-gradient methods:

- **Random Guessing** [Hochreiter and Schmidhuber, 1996]

Unsupervised "pretraining":

- **History Compressor** [Schmidhuber, 1992]

Others:

- **Time-delay Neural Network** [Lang et al., 1990], **Hierarchical RNNs** [El Hihi and Bengio, 1995], **NARX** [Lin et al., 1996] and more...

sources: [Graves, 2006, Sutskever, 2013, Schmidhuber, 2014]

Pretraining RNN (1992)

History compressor (HC) [Schmidhuber, 1992] = unsupervised, greedy layer-wise pretraining of RNN
(Recall in standard DNN: **unsupervised pretraining** (AE,RBM) [Hinton and Salakhutdinov, 2006]$\longrightarrow$ **supervised** [Krizhevsky et al., 2012])

Forward Propagation:

Given $H_0, X_0, Y_t = X_{t+1}$

$Z_t = W_{XH} \cdot X_{t-1} + W_H \cdot H_{t-1}$

$H_t = f(Z_t)$

$\hat{Y}_t = g(W_Y \cdot H_t + W_{XY} \cdot X_t)$

$E_t = L(Y_t, \hat{Y}_t)$

$E = \sum_{t=0}^{T-1} E_t$

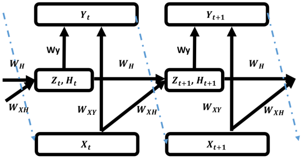


Figure: HC

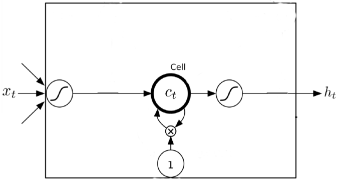
$X' \longleftarrow \{X_0, H_0, (t, X_t) \; \forall \; t \ni X_t \neq \hat{Y}_{t-1}\}$
$\longrightarrow$ train new HC using X'

Greedily pretrain RNN, using "surprises" from previous step

Basic "LSTM" (1991-95)

Idea: Have separate linear units that simply deliver errors [Hochreiter and Schmidhuber, 1997]

Forward Propagation*:
Let $C_t \in \mathbb{R}^P$ = "cell state",
Given H_0, C_0
$C_t = C_{t-1} + f_1(W_X \cdot X_t + W_H \cdot H_{t-1})$
$H_t = f_2(C_t)$
BPTT: Truncate gradients outside the cell
*approximately



Original LSTM (1995-97)

Idea: Have additional controls for R/W access (input/output gates) [Hochreiter and Schmidhuber, 1997]

Forward Propagation*:

Given H_0, C_0

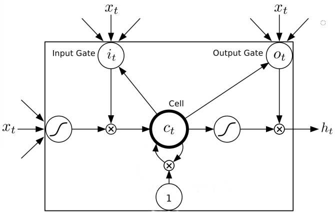
$C_t = C_{t-1} + i_t \odot f_1(W_X \cdot X_t + W_H \cdot H_{t-1})$

$H_t = o_t \odot f_2(C_t)$

$i_t = \sigma(W_{i,X} \cdot X_t + W_{i,C} \cdot C_{t-1} + W_{i,H} \cdot H_{t-1})$

$o_t = \sigma(W_{o,X} \cdot X_t + W_{o,C} \cdot C_t + W_{o,H} \cdot H_{t-1})$

BPTT: Truncate gradients outside the cell



*approximately

Modern LSTM (full version) (2000s-)

Idea: Have control for forgeting cell state (forget gate) [Gers, 2001, Graves, 2006]

Forward Propagation:

Given H_0, C_0

$C_t = f_t \odot C_{t-1} + i_t \odot f_1(W_X \cdot X_t + W_H \cdot H_{t-1})$

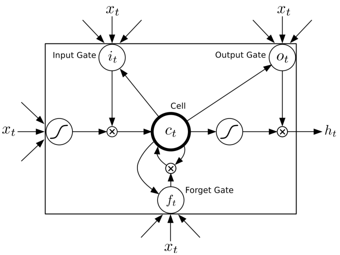
$H_t = o_t \odot f_2(C_t)$

$i_t = \sigma(W_{i,X} \cdot X_t + W_{i,C} \cdot C_{t-1} + W_{i,H} \cdot H_{t-1})$

$o_t = \sigma(W_{o,X} \cdot X_t + W_{o,C} \cdot C_t + W_{o,H} \cdot H_{t-1})$

$f_t = \sigma(W_{f,X} \cdot X_t + W_{f,C} \cdot C_{t-1} + W_{f,H} \cdot H_{t-1})$

BPTT: Use exact gradients



Recent Progress in RNN

Echo-State Network (ESN): [Jaeger and Haas, 2004]

- only train hidden-output weights
- other weights drawn from carefully-chosen distribution and fixed

Hessian-Free (HF) optimization: [Martens and Sutskever, 2011]

- approximate second-order method
- outperformed LSTM on small-scale tasks

Momentum methods: [Sutskever, 2013]

- Could we get **some** good results without HF?
- Yes! With “good” intialization & aggressive momemtum scheduling
- Nesterov’s accelerated gradient?

DEMO!

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