Calculate the mean, median, mode and standard deviation for the problem statements 1& 2.

Problem Statement 1:

The marks awarded for an assignment set for a Year 8 class of 20 students were as follows:

6 7 5 7 7 8 7 6 9 7 4 10 6 8 8 9 5 6 4 8

Solution:

Mean = (6 + 7+ 5+ 7+ 7+ 8+ 7+ 6 +9 +7 +4 +10+6+ 8+ 8+ 9+ 5+ 6+ 4+ 8)/20 = **6.85**

Median = [4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10] = (7+7)/2 = **7**

Mode = [4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10] = **7**

Problem Statement 2:

The number of calls from motorists per day for roadside service was recorded for a particular month:

28, 122, 217, 130, 120, 86, 80, 90, 140, 120, 70, 40, 145, 113, 90, 68, 174, 194, 170, 100, 75, 104, 97, 75, 123, 100, 75, 104, 97, 75, 123, 100, 89, 120, 109

Solution:

Mean = (sum of all numbers)/35 = **107.51**

Median = [28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90, 97, 97, 100, 100, 100, 104, 104, 109, 113, 120, 120, 120, 122, 123, 123, 130, 140, 145, 170, 174, 194, 217] = **100**

Mode = [28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90, 97, 97, 100, 100, 100, 104, 104, 109, 113, 120, 120, 120, 122, 123, 123, 130, 140, 145, 170, 174, 194, 217]= **75**

Problem Statement 3:

The number of times I go to the gym in weekdays, are given below along with its associated probability:

x = 0, 1, 2, 3, 4, 5

f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01

Calculate the mean no. of workouts in a week. Also evaluate the variance involved in it.

Solution:

Here N = 6

X	F(x)	X*f(x)	$(x-\mu)^2$
0	0.09	0	$(0-0.3583)^2$ = 0.1283
1	0.15	0.15	$(0.15-0.3583)^2$ = 0.0433
2	0.40	0.80	$(0.80-0.3583)^2$ = 0.1950
3	0.25	0.75	$(0.75-0.3583)^2$ = 0.1534
4	0.10	0.40	$(0.40-0.3583)^2$ = 0.0017
5	0.01	0.05	$(0.05-0.3583)^2$ = 0.0950

Mean = (0+0.15+0.80+0.75+0.40+0.05)/6 = **0.3583**

Variance = $sum((xi-\mu)^2)/N$ = (0.1283+0.0433+0.1950+0.1534+0.0017+0.0950)/6 = **0.1027**

Problem Statement 5:

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample?

Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution:

Here N = 6,

the probability of success= p = 30% = 0.3,

the probability of failure q = 1 – p = 1 – 0.3 = 0.7

Probability of having 2 faulty LED, here x = 2

According to Binomial distribution

$P(x,N,p) = C_x^N\, p^x\, q^{N-x}$

C_x^N = N!/x!(N-X)! = 6!/2!(6-2)! = 15

$P(x, N, p) = 15 * 0.3^2\ 0.7^{6-2}$

$P(x, N, p) = 15 * 0.9 * 0.2401$

$\boldsymbol{P(x = 2) = 3.2413}$

The mean of the distribution (µx) is equal to N * P.

µx = 6* 0.3

µx = 1.8

The standard deviation (σx) is sqrt[N * P * (1 - P)].

(σx) = sqrt[6 * 0.3 * 0.7]

(σx) = 1.12

Problem Statement 6:

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Solution:

For Gaurav, p = 0.75, q = 0.25 , N = 8

For Barakha, p = 0.45, q = 0.55 , N = 12

a) each of them will solve 5 questions correctly

According to Binomial distribution
$P(x, N, p) = C_x^N\ p^x\ q^{N-x}$

For Gaurav with N = 8 and x = 5 C_x^N = N!/x!(N-X)! = 8!/5!(8-5)! = 56 $P(x, N, p) = 56 * 0.75^5 * 0.25^{8-5}$ $\boldsymbol{P(5) = 0.2076} = \boldsymbol{20.76}\%$ **Gaurav has 20.76% probability of solving 5 questions correctly**	For Barakha with N = 12 and x = 5 C_x^N = N! /x! (N-X)! = 12!/5!(12-5)! = 792 $P(x, N, p) = 792 * 0.45^5 * 0.55^{12-5}$ $\boldsymbol{P(5) = 0.2224} = \boldsymbol{22.24}\ \%$ **Barakha has 22.24% probability of solving 5 questions correctly**

b) in cases of 4 and 6 correct solutions

For Gaurav x = 4 C_x^N = N!/x!(N-X)! = 8!/4!(8-4)! = 56 $P(x, N, p) = 70 * 0.75^4 * 0.25^{8-4}$ $\boldsymbol{P(4) = 0.0865 = 8.65\%}$	For Barakha x = 4 C_x^N = N!/x!(N-X)! = 12!/4!(12-4)! = 56 $P(x, N, p) = 495 * 0.45^4 * 0.55^{12-4}$ $\boldsymbol{P(4) = 0.1699 = 16.99\%}$
For Gaurav x = 6 C_x^N = N!/x!(N-X)! = 8!/6!(8-6)! = 56 $P(x, N, p) = 28 * 0.75^6 * 0.25^{8-6}$ $\boldsymbol{P(6) = 0.3114 = 31.14\%}$	For Barakha x = 6 C_x^N = N!/x!(N-X)! = 12!/6!(12-6)! = 56 $P(x, N, p) = 924 * 0.45^6 * 0.55^{12-6}$ $\boldsymbol{P(6) = 0.2123 = 21.23\%}$

Problem Statement 7:

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers. Give a pictorial representation of the same to validate your answer.

Solution:

The Poisson distribution with mean μ = 72 per hour = 1.2 $per\ min$ = 4.8 $per\ 4\ min$.

a) 5 customers

P(5) = $e^{-\mu} \frac{\mu^k}{k!}$ = = $e^{-4.8} \frac{4.8^5}{5!}$

P(k = 5) = 0.1747 = 17.47%

b) not more than 3 customers

P(k <= 3) = P(0) + P(1) + P(2) + P(3)

= $e^{-4.8} \frac{4.8^0}{0!} + e^{-4.8} \frac{4.8^1}{1!} + e^{-4.8} \frac{4.8^2}{2!} + e^{-4.8} \frac{4.8^3}{3!}$

= 0.0082 + 0.0395 + 0.0948 + 0.1516

P(k <= 3) = 0.2941 = 29.41%

c) more than 3 customers

P(k > 3) = 1 – P(k < 3)

= 1 - P(0) + P(1) + P(2)

$= 1 - e^{-4.8} \ \frac{4.8^0}{0!} + e^{-4.8} \ \frac{4.8^1}{1!} + \ e^{-4.8} \ \frac{4.8^2}{2!}$

= 1 - 0.0082 + 0.0395 + 0.0948

P(k > 3) = 0.8575 = 85.75%

Problem Statement 9:

Let the continuous random variable D denote the diameter of the hole drilled in an aluminum sheet. The target diameter to be achieved is 12.5mm. Random disturbances in the process often result in inaccuracy. Historical data shows that the distribution of D can be modelled by the PDF, f(d) = 20e−20(d−12.5), d ≥ 12.5. If a part with diameter > 12.6 mm needs to be scrapped, what is the proportion of those parts? What is the CDF when the diameter is of 11 mm?

What is the conclusion of this experiment?

Solutions:

Here PDF is f(x) = $\int_{12.5}^{\infty} 20e^{-20(d-12.5)}$

P(d > 12.6) = $\int_{12.6}^{\infty} 20e^{-20(d-12.5)}$

= $20 \int_{12.6}^{\infty} e^{-20(d-12.5)}$

= $20 \int_{12.6}^{\infty} \frac{e^{-20(d-12.5)}}{-20}$

= $- \int_{12.6}^{\infty} e^{-20(d-12.5)}$

P(d > 12.6) = 0.135

Proportion of scrap in between 12.5 to 12.6

P(12.5 < d < 12.6) = $\int_{12.5}^{12.6} 20e^{-20(d-12.5)}$

= $20 \int_{12.5}^{12.6} \frac{e^{-20(d-12.5)}}{-20}$

= $- \int_{12.5}^{12.6} e^{-20(d-12.5)}$

= 1 – 0.135

P(12.5 < d < 12.6) = 0.865

To make the scrap minimum, we need to adjust the diameter close to 12.5.

Problem Statement 10:

Please compute the following:

a) P(Z > 1.26), . P(Z < −0.86), P(Z > −1.37), P(−1.25 < Z < 0.37), . P(Z ≤ −4.6)

P(Z > 1.26) = 1 - P(Z < 1.26) = 1 - 0.8962 = **0.1038**

P(Z < −0.86) = 0.1949

P(Z > −1.37) = 0.9147

P(−1.25 < Z < 0.37) = P(Z < 0.37) – P(Z < - 1.25) = 0.6443 - 0.1056 = **0.5387**

P(Z ≤ −4.6) = 0

b) Find the value z such that ($Z > z$) = 0.05

Z = -1.64

c) Find the value of z such that ($-z < Z < z$) = 0.99

In Z table 2.5 + 0.08 = 0.9951

Finding P(Z<-2.58) = 0.0049

($-z < Z < z$) = P(Z<2.58) - P(Z<-2.58) = 0.9951 – 0.0049 = 0.9902

So **Z = 2.58**

Problem Statement 11:

The current flow in a copper wire follow a normal distribution with a mean of 10 mA and a variance of 4 (mA)2. What is the probability that a current measurement will exceed 13 mA? What is the probability that a current measurement is between 9 and 11mA? Determine the current measurement which has a probability of 0.98.

Solution:

Here, μ = 10 & σ = 4

Considering it as normal distribution,

Z = (x – μ) / σ

1. Probability that a current measurement will exceed 13 mA

   x = 13

   calculating Z

   Z > (13 - 10)/4 = 3/4

   Z > 0.75

   So from Z table we get,

   $P(Z > 0.75) = 1 - P(Z < 0.75) = 1 - 0.7734 = 0.2266$

   [**Probability = 22.66%**]

2. What is the probability that a current measurement is between 9 and 11mA

   $9 < Z < 11$

   So we calculate Z values for both values of X = 9 & X = 11

   $(9 – 10) / 4 < Z < (11 – 10) / 4$

   $-1/4 < Z < 1/4$

   $-0.25 < Z < 0.25$

   $P(-0.25 < Z < 0.25) = P(Z < 0.25) – P(Z < -0.25) = 0.5987 – 0.4013$

   $P(-0.25 < Z < 0.25) = 0.1974$

   [**Probability = 19.74%**]

3. Determine the current measurement which has a probability of 0.98

   Here P-value is given 0.98.
   Using Z calculator we get Z= 2.05 for P = 0.98
   Now calculate X with the help of Z score

   $Z = (x – \mu) / \sigma$

   $2.05 = (x - 10) / 4$

   $8.2 = x – 10$

   x = 18.2 mA

Problem Statement 12:

The shaft in a piston has its diameter normally distributed with a mean of 0.2508 inch and a standard deviation of 0.0005 inch. The specifications of the shaft are 0.2500 ∓ 0.0015 inch. What proportion of shafts are in sync with the specifications? If the process is centered so that the mean is equal to the target value of 0.2500, what proportion of shafts conform to the new specifications? What is your conclusion from this experiment?

Solution:

Mean = 0.2508 & Variance = 0.0005

Specification = 0.2500 ∓ 0.0015

1. What proportion of shafts are in sync with the specifications

   $0.2500 - 0.0015 < Z < 0.2500 + 0.0015$

   $(0.2485 - 0.2508) / 0.0005 < Z < (0.2515 - 0.2508)/ 0.0005$

   $P(-4.6 < Z < 1.4) = P(Z < 1.4) - P(Z < -4.6)$

   $= 0.9192 - 0$

   $= 0.92$

   So nearly 92% proportion of shafts are in sync with the specifications 0.2500 ∓ 0.0015

2. With mean = 0.2500

   $(0.2485 - 0.2500) / 0.0005 < Z < (0.2515 - 0.2500)/ 0.0005$

   $P(-3 < Z < 3) = P(Z < 3) - P(Z < -3)$

   $= 0.9987 - 0.0013$

   $= 0.9974$

   So nearly 99% proportion of shafts are in sync with the specifications 0.2500 ∓ 0.0015