HW 12 - Graph Theory

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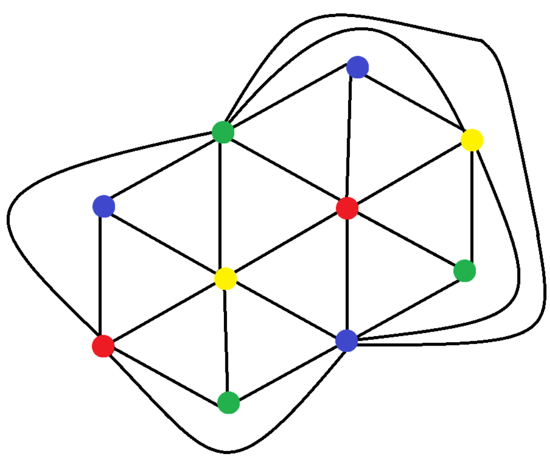
1.



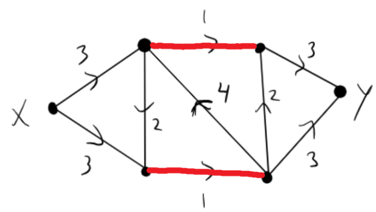
3.

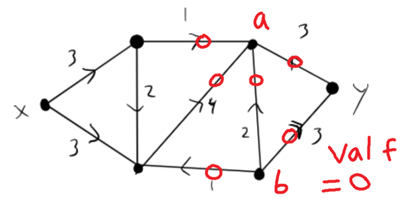
Base Case: k=1. $A^1 = A$, and the (i, j)th entry of A is the number of directed edges from v_i to v_j. This holds.

Inductive Step: Assume that the (i, j)th entry of A^k is the number of directed walks of length k from v_i to v_j, for some $k > 1$. We now show that the (i, j)th entry of A^{k+1} is the number of directed walks of length $k+1$ from v_i to v_j. We use the definition of matrix multiplication. Specifically, the (i, j)th entry of A^{k+1} is equal to the dot product of the $i-th$ row of A^k and the $j-th$ column of A. Since the (i, j)th entry of A^k is the number of directed walks of length k from v_i to v_j, and the $j-th$ column of A is the vector of the number of directed edges from each vertex to v_j, it follows that the (i, j)th entry of A^{k+1} is the number of directed walks of length $k+1$ from v_i to v_j.

$\square$

4.





The red edges indicated in our first network clearly satisfy the conditions to be a cut and are minimal since no other combination of edges to produce a cut in N have a smaller capacity. By the Maximum Flow - Minimum Cut Theorem, the first digraph has a maximum flow of 2.

In the second network, vertex a has all inbound edges. To have a flow, rate in must equal rate out for all intermediate vertices, so these edges are 0 in any flow. Similarly, vertex b has all outbound edges, so these edges are 0 in any flow. Therefore $val f = 0$ for any flow and the maximum flow is 0.

□