Polarimetric modeling and calibration of the Aerosol-UA space mission instruments

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Abstract

We have developed the polarimetric models for the scanning polarimeter ScanPol and the multi-spectral imager polarimeter MSIP, which are main instruments of the Aerosol-UA space mission. The polarimeters developed to measure of degree of linear polarization (DoLP) and angle of linear polarization (AoLP) of sun light scattered by clouds and aerosols with high precision and high spatial resolution. The models of polarimeters include main sources of systematic polarimetric errors such as finite extinction ratios and offsets of polarizers, and birefringence of telescopes which are all described by corresponding well known Mueller matrices. Signal's zero level bias, difference in gain of polarimeters' channels and some instrumental depolarization factor are modeled too as an additive value and scalar multipliers correspondingly. Entirely the models are the products of specified Mueller matrices and scalar values that are tend to describe precisely a transformation of incoming light's polarization and intensity during its propagation through the ScanPol and MSIP channels. The developed models allowed proposing some promising calibration procedures that provide precisely assess and hereby effectively compensating the all mentioned instrumental polarization imperfections of polarimeters. As usual the calibration process in cases of both polarimeters implies the using of high quality reference sources of linear polarized and depolarized light and radiometric reference sample. It is assumed the ground-based stage calibration the both ScanPol and MSIP instruments, in-orbit permanent calibration of ScanPol instruments by the using on a board reference units, and in-orbit intercalibration of ScanPol and MSIP, since their fields of view are overlapped. To assess the effectiveness of proposed calibration procedures the number of numerical experiments was carried out. They demonstrated an excellent compensation of ScanPol and MSIP polarization imperfections by ground-based calibration procedures. Thought the potentially reached imperfections' compensation here is restricted only by quality of reference sources and signal-to-noise ratio. In-orbit calibration procedure demonstrates an acceptable result providing to measure the DoLP with required error < 0.15% and AoLP with error < 0.2 degrees (for DoLP>0.2) for the ScanPol.

Keywords: atmosphere, aerosol, climate, polarimeter, degree of linear polarization, calibration

Introduction

The ScanPol polarimeter and the multi-spectral imager polarimeter MSIP are main instruments of the space mission Aerosol-UA (Milinevsky et al., 2015; 2016). The ScanPol is the multi-channel scanning polarimeter with six solar reflectance spectral bands in the near ultraviolet (NUV), visible (VIS) and near infrared (NIR) spectral channels centered in the wavelengths 370, 410, 555, 865, 1378, 1610 nm, that measure the first three Stokes parameters *I*, *Q* and *U* of the reflected by atmospheric aerosols and Earth surface radiation at about 200 viewing directions in between scanning angles +50° and –60° degrees from nadir. The polarimeter is designed to acquire spatial, temporal, and spectral-polarimetric measurements simultaneously to minimize instrumental "parasitic" effects and effects of "false" polarizations due to scene movement. Simultaneity is provided by separation of initial spatial field by pair conjugated telescopes and pair Wollaston prisms which polarization axis are oriented at 45 degrees. One telescope in pair provides simultaneous measurements intensities of the linear polarization components in orthogonal planes at 0° and 90° to the meridional plane of the instrument, while the other telescope simultaneously measures equivalent intensities in orthogonal planes at 45° and 135°. Polarization–intensity scanning of the ScanPol is achieved by the use of a two-mirror system with the aluminum mirrors oriented in the way when any polarization introduced at the first reflection is compensated for by the second reflection. In Figure 1 the optical layout of ScanPol polarimeter is shown. Since the design of the ScanPol polarimeter is partially based on RSP and APS/Glory scanning polarimeters (Cairns et al., 1999; Peralta et al., 2007), the some peculiarities of the polarimetric model and calibration procedure are similar to all three instruments. We use the minimum set of the retrieval requirements for the ScanPol polarimeter that has been formulated and discussed in Mishchenko and Travis (1997), and Mishchenko et al. (2004, 2007a).

The MSIP instrument is the multispectral wide-angle imager–polarimeter, which as a part of the Aerosol-UA space mission will provide collection of images on the state of the atmosphere and surface in the area, where the ScanPol polarimeter will measure. The concept and details of the MSIP imager-polarimeter optical layout design has been proposed by (Sinyavskii et al., 2013).

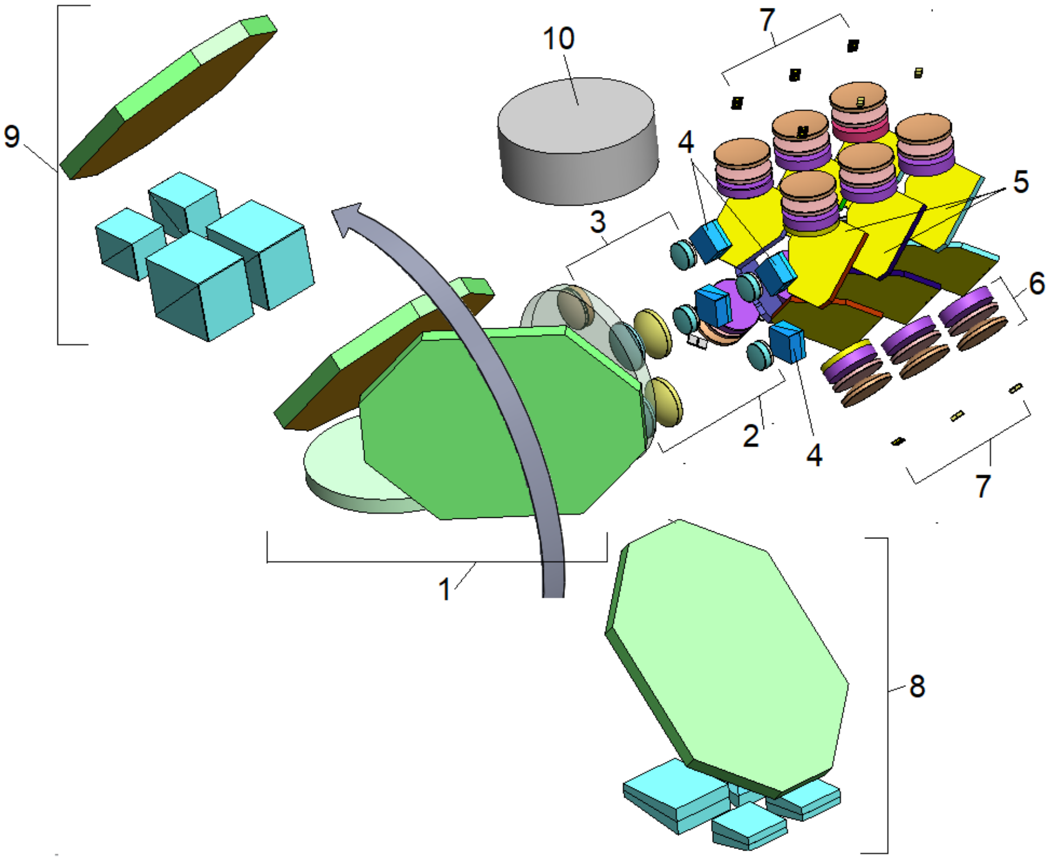

Figure1. ScanPol polarimeter optical layout: 1 – mirror scanning system; 2 – input lens and collimator of the NUV–VIS spectral channel, 3 – input lens and collimator of the NIR spectral channel, 4 – Wollaston prisms; 5 – dichroic mirrors; 6 – camera lens and interference filters for spectral channels; 7 – the NUV–VIS and NIR sensors; 8 –quartz wedge depolarizers; 9 – Glan prisms polarizer; 10 – black body.

The MSIP polarimeter consists of five optical channels with the wide-angle field-of-view (60x60 degrees) in nadir across flying trajectory. Three optical channels are polarizing and two are photometric. Polarizing channels measure Stokes parameters *I*, *Q* and *U* with central wavelength 410, 555, and 865 nm with the spectral full width at half maximum (FWHM) 20 nm. Two photometric channels of the MSIP will serve to obtain image in eight spectral wavebands to retrieve the aerosol optical depth. The first photometric channel has central wavelength 410, 443, 470, and 490 nm, with spectral FWHM 20 nm, and the second photometric channel has central wavelength 555, 670, 865, and 910 nm with spectral FWHM 20...40 nm. The design of MSIP allows measuring the first three Stokes vector components simultaneously in wide field-of-view without restrictions by f-number of the MSIP optical system.

The image-separation system of the MSIP provides separation of the initial input image on four equal images that are polarized by four sheet polarizers with azimuths 0, 90, 45 and 135 degrees. Polarized images are projected by camera lens on four different square areas of the single CCD image sensor. The MSIP spatial resolution is 6 km in the projection on the Earth surface, which corresponds to the instantaneous field-of-view of the ScanPol polarimeter. The number of phase angles for measuring the single observation area is at least 15. Optical layout of MSIP is shown in Figure 2.

To describe polarization systems of the ScanPol and the MSIP polarimeters we use self-consistent way by using the Stokes-Mueller formalism. Targeted Stokes parameters I, Q, U are elements of Stokes vector **S**:

$$\mathbf{S} = (I, Q, U, V) = (I_{0^\circ} + I_{90^\circ} = I_{45^\circ} + I_{135^\circ}, I_{0^\circ} – I_{90^\circ}, I_{45^\circ} – I_{135^\circ}, I_R – I_L)$$

where values I_i are intensities passed through linear/circular polarizers as it is described in general.

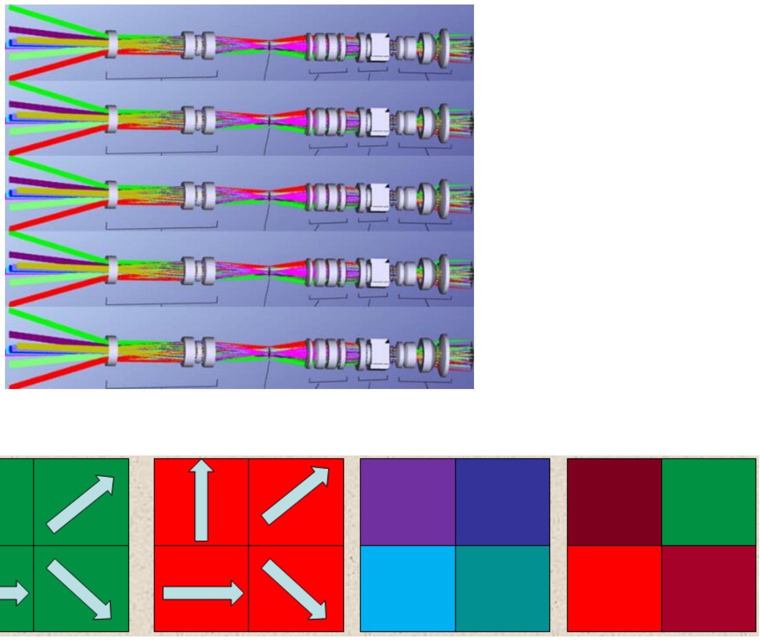

Figure 2. Optical layout concept for the MSIP imager-polarimeter: the light path in five optical channels (top) and field-of-view of the MSIP polarimeter channels – three polarimetric and two intensity channels.

The ScanPol and the MSIP polarimeters designed do not provided the measurements of the incoming light circular polarization (V Stokes parameter), because the circular polarization component of atmospheric scattering is typically at least two orders of magnitude smaller than the linear polarization and has no effect on aerosol parameters retrieving from polarimetric data. Commonly used polarization parameters are p – degree of linear polarization (DoLP), and θ – azimuth of the linear polarization (AoLP):

$$\mathrm{DoLP} = p = \sqrt{q^2 + u^2}\,,\ \mathrm{AoLP} = \theta = \frac{1}{2}\arctan(u/q)\,,$$

where $q=Q/I$ and $u=U/I$ are normalized Stokes parameters of incoming light.

Another useful form of equation defines Stokes parameters I, Q, U of partially linear polarized light via p and θ as below

$$\mathbf{S} = (I, Ip\cos(2\theta), Ip\sin(2\theta), 0) \qquad (1)$$

The ScanPol polarimeter uses a pair of crossed mirrors to scan the field-of-view. The light enters then into refractive telescope before being analyzed by a Wollaston prism. The MSIP imager-polarimeter doesn't include mirror scan system so the light enters straight into the telescopes and then analyzed by the polarizer film set.

In order to determine how the measurements in the ScanPol and MSIP channels relate to the input Stokes vector it is necessary to model the primary sources of imperfection in a polarimeters of the ScanPol and MSIP types.

In modeling the ScanPol system we are concerned by the potential introducing of instrumental polarization by the scanning mirrors set. The expected mismatch of the correct position between the ScanPol mirrors is the primary source of the instrumental polarization. The telescope lenses can also introduce retardance caused by stress. We expect that the Wollaston prism performance will be perfect and can be excluded as a significant factor in degrading system performance. However the effects of depolarization and cross-talk we considered as the factor of the polarizer performance degradation for simplicity (Cairns and Geogdzhayev, 2010). In addition, we take into account that the polarizer films are generally have less extinction ratios than prism polarizers. That means less polarization contrast between parallel and cross polarization. The following Mueller matrices are therefore needed for the ScanPol instruments description. The Mueller matrix M_{TMS} (*Two Mirror System*) for a pair of the crossed mirrors in an arbitrary orientation α_M is:

$$\mathbf{M}_{TMS} = \begin{pmatrix} A & \cos(2\alpha_M)B & \sin(2\alpha_M)B & 0 \\ -\cos(2\alpha_M)B & -\cos^2(2\alpha_M)A-\sin^2(2\alpha_M)\cos(\delta_{\text{ps2}}-\delta_{\text{ps2}}) & \cos(2\alpha_M)\sin(2\alpha_M)\left(\cos(\delta_{\text{ps2}}-\delta_{\text{ps2}})\text{-}A\right) & \sin(2\alpha_M)\sin(\delta_{\text{ps2}}-\delta_{\text{ps2}}) \\ -\sin(2\alpha_M)B & \cos(2\alpha_M)\sin(2\alpha_M)\left(\cos(\delta_{\text{ps2}}-\delta_{\text{ps2}})\text{-}A\right) & -\sin^2(2\alpha_M)A-\cos^2(2\alpha_M)\cos(\delta_{\text{ps2}}-\delta_{\text{ps2}}) & -\cos(2\alpha_M)\sin(\delta_{\text{ps2}}-\delta_{\text{ps2}}) \\ 0 & \sin(2\alpha_M)\sin(\delta_{\text{ps2}}-\delta_{\text{ps2}}) & -\cos(2\alpha_M)\sin(\delta_{\text{ps2}}-\delta_{\text{ps2}}) & \cos(\delta_{\text{ps2}}-\delta_{\text{ps2}}) \end{pmatrix} \qquad (2)$$

where $A = \frac{1}{2}\left(\frac{\mathrm{R}_{\mathrm{ps1}}}{\mathrm{R}_{\mathrm{ps2}}} + \frac{\mathrm{R}_{\mathrm{ps2}}}{\mathrm{R}_{\mathrm{ps1}}}\right)$, $B = \frac{1}{2}\left(\frac{\mathrm{R}_{\mathrm{ps1}}}{\mathrm{R}_{\mathrm{ps2}}} - \frac{\mathrm{R}_{\mathrm{ps2}}}{\mathrm{R}_{\mathrm{ps1}}}\right)$, $C = |R_{p1}||R_{p2}||R_{s1}||R_{s2}|$, $R_{ps} = \frac{|R_p|}{|R_s|}$,

$\delta_{ps} = \delta_p - \delta_s$.

$R_{p,s} = |R_{p,s}| e^{i\delta_{p,s}}$ – complex reflection indexes of mirror.

The Mueller matrix for a retarder with the phase shift between two linear polarization components δ in an arbitrary orientation α_r is:

$$\mathbf{M}_T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\alpha_T) + \sin^2(2\alpha_T)\cos(\delta_T) & \cos(2\alpha_T)\sin(2\alpha_T)\left(1-\cos(\delta_T)\right) & \sin(2\alpha_T)\sin(\delta_T) \\ 0 & \cos(2\alpha_T)\sin(2\alpha_T)\left(1-\cos(\delta_T)\right) & \sin^2(2\alpha_T) + \cos^2(2\alpha_T)\cos(\delta_T) & -\cos(2\alpha_T)\sin(\delta_T) \\ 0 & -\sin(2\alpha_T)\sin(\delta_T) & \cos(2\alpha_T)\sin(\delta_T) & \cos(\delta_T) \end{pmatrix} \quad (3)$$

and for a polarizer with extinction ratio 1/*e* in an arbitrary orientation α_r the expression is

$$\mathbf{M}_A = \frac{1+e}{2}\begin{pmatrix} 1 & \frac{1-e}{1+e}\cos(2\alpha_A) & \frac{1-e}{1+e}\sin(2\alpha_A) & 0 \\ \frac{1-e}{1+e}\cos(2\alpha_A) & \cos^2(2\alpha_A) + 2\frac{\sqrt{e}}{1+e}\sin^2(2\alpha_A) & \frac{\left(1-\sqrt{e}\right)^2}{1+e}\cos(\alpha_A)\sin(\alpha_A) & 0 \\ \frac{1-e}{1+e}\sin(2\alpha_A) & \frac{\left(1-\sqrt{e}\right)^2}{1+e}\cos(2\alpha_W)\sin(2\alpha_A) & 2\frac{\sqrt{e}}{1+e}\cos^2(\alpha_A) + \sin^2(\alpha_A) & 0 \\ 0 & 0 & 0 & 2\frac{\sqrt{e}}{1+e} \end{pmatrix} \quad (4)$$

Here $e = \frac{t_\perp}{t_\parallel}$ – ratio of polarizer's transmissions for input light with linear polarization crossed and parallel to polarizer's axis. *e* – is the parameter that is inversely proportional to extinction ratio.

1 ScanPol polarimeter and calibration models

1.1 Modelling

In Figure 3 an equivalent polarization scheme of the single spectral channel of the ScanPol polarimeter is shown.

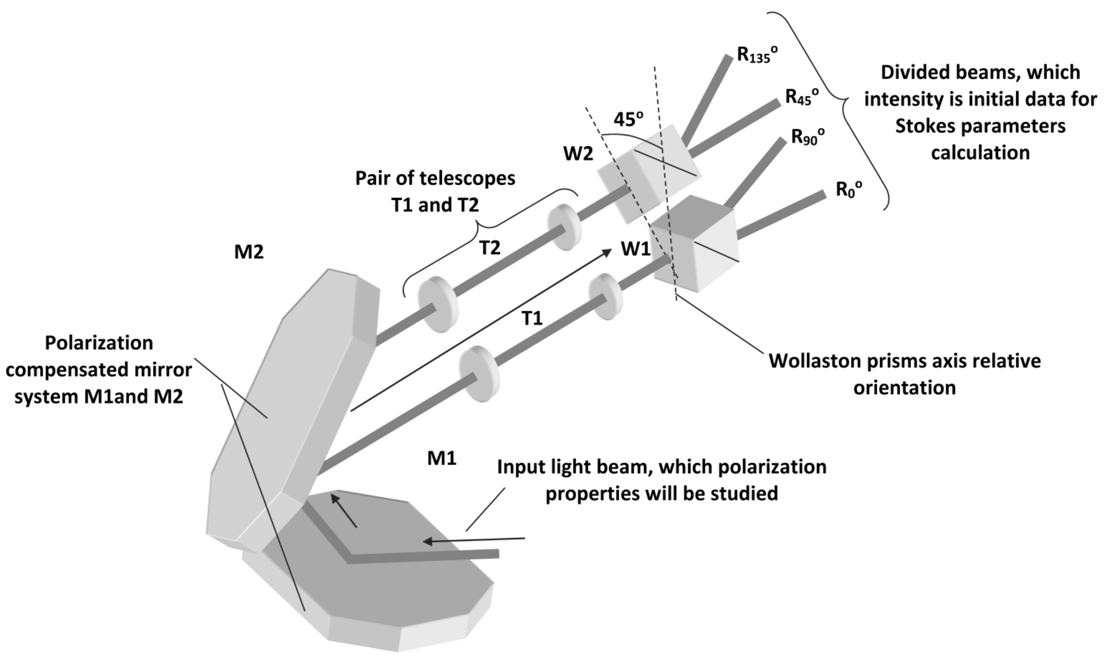

Figure 3. An equivalent polarization scheme of the ScanPol polarimeter single spectral channel.

We include possible offset in orientations of polarization axes of prisms, finite extinction ratios of prisms, polarization imperfections of telescopes and inequality in gain of the spectral ScanPol channels for the model development. From Figure 3 it follows that the each intensity channel can be described by equation:

$$R = K\left[\mathbf{M}_A(e, \alpha_A + \varepsilon)\cdot\mathbf{M}_T(\delta_T, \alpha_T)\cdot\mathbf{M}_{TMS}(R_{ps1}R_{ps2}, \delta_{ps1}, \delta_{ps2}, \alpha_M)\cdot\mathbf{S}_{scene}\right]_0 + D, \qquad (5)$$

where D – is zero bias of the channel; $\mathbf{S}_{scene}$ – Stokes vector for polarization of input scene, $\mathbf{M}$ – Mueller matrices of corresponding optical elements; K – scalar value that describes isotropic gain of channel; ε – the small value of the some offset of the Wollaston prism orientation from the meridional plane. We expect that the orthogonality of the beam polarization within prism will be almost perfect since it is determined by the crystalline structure of the Wollaston elements. A zero subscript in the equation (5) indicates that we operate with the incoming light intensity *I* (the first Stokes parameter). All other parameters in the equation (5) and their physical meaning are explained clearly in the equations (2)–(4).

We use an approach described in details by Cairns and Geogdzhayev (2010). According to consideration in (Cairns and Geogdzhayev, 2010) we can derive an approximated equations that relates to the measured raw digital values $R_{0°,\,90°,\,45°,\,135°}$ (see Figure 3) from the ScanPol analog-to-digital converter unit (ADC) by the normalized targeted Stokes parameters q and u of the incoming light at the input window of scanning system:

$$\begin{aligned}\frac{RD_{0^o}-K1\cdot RD_{90^o}}{RD_{0^o}+K1\cdot RD_{90^o}}&=\frac{a_q^{-1}\left[(-q+q_{inst})\cos(2\varepsilon_1)+(-u+u_{inst})\sin(2\varepsilon_1)\right]}{1+q_{inst}q+u_{inst}u}\\ \frac{RD_{45^o}-K2\cdot RD_{135^o}}{RD_{45^o}+K2\cdot RD_{135^o}}&=\frac{a_u^{-1}\left[-(-q+q_{inst})\sin(2\varepsilon_2)+(-u+u_{inst})\cos(2\varepsilon_2)\right]}{1+q_{inst}q+u_{inst}u}\end{aligned}\qquad(6)$$

where subscript indices 0º, 90º, 45º and 135º mark corresponding four intensity channels; $RD_{0^o,90^o,45^o,135^o}=R_{0^o,90^o,45^o,135^o}-D_{0^o,90^o,45^o,135^o}$ – zero corrected ADC data; $D_{0^o,90^o,45^o,135^o}$ – define the zero level biases. For the ScanPol polarimeter the values $D_{0^o,90^o,45^o,135^o}$ are the dark measurements; *K1*, *K2* are intra-telescope calibration factors to equalize non-polarized input responses:

$$K1=\frac{\left(RD_{0^o}\right)}{\left(RD_{90^o}\right)},\qquad K2=\frac{\left(RD_{45^o}\right)}{\left(RD_{135^o}\right)};\qquad(7)$$

ε_1 and ε_2 – offsets of the Wollaston prism transmission axes from the exact corresponded angle position ($0^o+\varepsilon_1$, $90^o+\varepsilon_1$, $45^o+\varepsilon_2$, $135^o+\varepsilon_2$,); q_{inst}, u_{inst} – stray parameters, which are additional values to the targeted Stokes parameters q and u, appeared due to parasitic instrumental polarization of the ScanPol instrument mirrors and the telescopes imperfection; a_q, a_u – are polarization scaling factors that include the stray depolarization of light in the ScanPol channel due to polarizer imperfection and the inhomogeneity of the telescopes and mirrors anisotropy in the areas of the incoming light intersection:

$$a_q = \frac{1+e_1}{1-e_1},\; a_u = \frac{1+e_2}{1-e_2} \tag{8}$$

where $1/e_1$, $1/e_2$ are extinction ratios of two Wollaston prisms.

From equations (6) we can calculate the targeted Stokes parameters q and u by iterations as described in (Cairns and Geogdzhayev, 2010) or using the accurate approximation:

$$\begin{aligned} q &= \frac{q_{inst} s_2 \left(s_1 + u_{inst}\Delta I_{0^o,90^o}\right) - \left(1+u_{inst}{}^2\right)\left(c_2 \Delta I_{0^o,90^o} - s_1 \Delta I_{45^o,135^o}\right) + c_1 q_{inst}\left(c_2 + u_{inst}\Delta I_{45^o,135^o}\right)}{s_1 s_2 + c_2 q_{inst}\Delta I_{0^o,90^o} + s_2 u_{inst}\Delta I_{0^o,90^o} - q_{inst} s_1 \Delta I_{45^o,135^o} + c_1\left(c_2 + u_{inst}\Delta I_{45^o,135^o}\right)} \\ u &= \frac{c_1 c_2 u_{inst} - \left(s_2 + q_{inst}{}^2 s_2 - c_2 q_{inst} u_{inst}\right)\Delta I_{0^o,90^o} - c_1\left(1+q_{inst}{}^2\right)\Delta I_{45^o,135^o} + s_1 u_{inst}\left(s_2 - q_{inst}\Delta I_{45^o,135^o}\right)}{s_1 s_2 + c_2 q_{inst}\Delta I_{0^o,90^o} + s_2 u_{inst}\Delta I_{0^o,90^o} - q_{inst} s_1 \Delta I_{45^o,135^o} + c_1\left(c_2 + u_{inst}\Delta I_{45^o,135^o}\right)} \end{aligned} \tag{9}$$

where:

$$c_{1,2} = \cos(\varepsilon_{1,2}),\; s_{1,2} = \sin(\varepsilon_{1,2}),\; \Delta I_{0^o,90^o} = \frac{RD_{0^o} - K1 \cdot RD_{90^o}}{RD_{0^o} + K1 \cdot RD_{90^o}} a_q,\; \Delta I_{45^o,135^o} = \frac{RD_{45^o} - K2 \cdot RD_{135^o}}{RD_{45^o} + K2 \cdot RD_{135^o}} a_u .$$

Therefore, we can retrieve the targeted polarization Stokes parameters for input light from equation (9) after determination of the calibration coefficients for the ScanPol instrument measurement channels.

1.2 Ground-based calibration approach

We developed the ground-based polarimetric calibration, which consists of two stages. At the first stage, we calibrate the static part of the ScanPol polarimeter with the scanning mirror system is removed. The linear polarized light with highest DoLP and periodically variable AoLP is used as input signal for calibration. This configuration is described by the equation (5) when matrix of the mirror system is excluded. In that case the general equation for the zero compensated signal is:

$$
\begin{aligned}
RD(t) &= K \cdot \left[\mathbf{M}_A(e, \alpha_A + \varepsilon) \cdot \mathbf{M}_T(\delta, \alpha) \cdot \begin{bmatrix} I & Q & U & 0 \end{bmatrix}^T \right]_0 = \\
&= IK\ (p/a) \Big[q \Big[\cos\big(2(\alpha_A + \varepsilon)\big) \Big[\big(\cos(\delta) - 1\big) \sin^2(2\alpha_T) + 1 \Big] - \tfrac{1}{2} \sin\big(2(\alpha_A + \varepsilon)\big) \sin(4\alpha_T) \big(\cos(\delta) - 1\big) \Big] + \\
&+ u \Big[\sin\big(2(\alpha_A + \varepsilon)\big) \Big[\big(\cos(\delta) - 1\big) \cos^2(2\alpha_T) + 1 \Big] - \tfrac{1}{2} \cos\big(2(\alpha_A + \varepsilon)\big) \sin(4\alpha_T) \big(\cos(\delta) - 1\big) \Big] \Big] + IK
\end{aligned}
$$

(10)

where p is the real DoLP value of the input polarization and q, u – are the real normalized Stokes parameters of linear input polarization, when the variable AoLP values ($\theta = \Omega\ t$) are equal $q = \cos(2\Omega t)$ and $u = \sin(2\Omega t)$. The value I is an intensity of input signal, which is supposed to be constant for all values of the angle θ.

We can transform the equation (10) in few ways:

$$
\begin{aligned}
RD(t) &= IK + IK\,(p/a)\big[\cos(2\Omega t)B + \sin(2\Omega t)C\big] = \\
&= IK + IK\,(p/a)\,p_{AT}\big(\cos(2\Omega t)\cos(2\alpha_{AT}) + \sin(2\Omega t)\sin(2\alpha_{AT})\big)
\end{aligned} \quad (11)
$$

where

$$
\begin{aligned}
&B = \cos\big(2(\alpha_A + \varepsilon)\big) \Big[\big(\cos(\delta) - 1\big) \sin^2(2\alpha_T) + 1 \Big] - \tfrac{1}{2} \sin\big(2(\alpha_A + \varepsilon)\big) \sin(4\alpha_T) \big(\cos(\delta) - 1\big) \\
&C = \sin\big(2(\alpha_A + \varepsilon)\big) \Big[\big(\cos(\delta) - 1\big) \cos^2(2\alpha_T) + 1 \Big] - \tfrac{1}{2} \cos\big(2(\alpha_A + \varepsilon)\big) \sin(4\alpha_T) \big(\cos(\delta) - 1\big) \quad (12) \\
&p_{AT} = \sqrt{B^2 + C^2}; \qquad \cos(2\alpha_{AT}) = B/\sqrt{B^2 + C^2}; \quad \sin(2\alpha_{AT}) = C/\sqrt{B^2 + C^2};
\end{aligned}
$$

in equations (12) the parameter p_{AT} only has the physical meaning. It is an additional depolarization introduced by the birefringent telescope and due to polarizers offset.

As it is seen from the equation (11) there is no sense to separate parameters $pa^{-1}p_{AT}$, because there are no possibilities to proceed with that. Therefore we introduce the effective depolarization parameter a' for the prism polarizer as value $a' = a / pp_{AT}$. Finally, we rewrite α_{AT} as $\alpha_{AT} = \alpha_A + \varepsilon'$, where ε' is the effective polarizer's offset. Then the equation (10) is transformed to

$$RD(t) = IK + IKa'^{-1}[\cos(2\Omega t)\cos(2(\alpha_A + \varepsilon')) + \sin(2\Omega t)\sin(2(\alpha_A + \varepsilon'))]. \qquad (13)$$

According to the equation (13), we replace the sequence of the imperfect telescope and polarizer in polarimetric channel by the single equivalent polarizer with effective depolarization parameter a' and effective offset ε'.

Note that the equation (13) is written in form of the Fourier series where the value of zero harmonic is $\mathrm{a}_0 = IK$, and the values of second harmonics by cosine and sine are $\mathrm{a}_2 = IK\, a'^{-1}\cos(2(\alpha_A + \varepsilon'))$ and $b_2 = IK\, a'^{-1}\sin(2(\alpha_A + \varepsilon'))$, correspondingly. To determine the mentioned calibration parameters for all intensity channels we need to obtain appropriate harmonics' values, therefore:

$$\alpha_A + \varepsilon' = \frac{1}{2}\arctan(\mathrm{b}_2 / \mathrm{a}_2); \;\; a'^{-1} = \sqrt{\mathrm{a}_2{}^2 + \mathrm{b}_2{}^2} / \mathrm{a}_0; \qquad (14)$$

We determine K values as the relations of zero harmonics in different intensity channels:

$$K1 = \mathrm{a}_0\big|_{\alpha_A = 0^o} / \mathrm{a}_0\big|_{\alpha_A = 90^o}; \; K2 = \mathrm{a}_0\big|_{\alpha_A = 45^o} / \mathrm{a}_0\big|_{\alpha_A = 135^o}; \; C12 = \mathrm{a}_0\big|_{\alpha_A = 0^o} / \mathrm{a}_0\big|_{\alpha_A = 45^o}. \qquad (15)$$

Here the parameter C12 is inter-telescopes gain differences. Another way to determine values K is to use the non-polarized light source. Since the Stokes parameters q and u are zero for this case, the values $K1$, $K2$ and $C12$ are determined as relation of the corresponding intensities at output of the Wollaston prisms:

$$K1 = I_{0^o} / I_{90^o}\ ;\ K2 = I_{45^o} / I_{135^o}\ ;\ C12 = I_{0^o} / I_{45^o}\ . \qquad (16)$$

In that way we have got the calibrated static part of the ScanPol instrument and ready to attach epy mirror system in the model.

We apply the model for the Stokes vector parameters that arrives at the telescopes in the form described in (Cairns and Geogdzhayev, 2010):

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} I + q_{inst}Q + u_{inst}U \\ I\,q_{inst} - Q \\ I\,u_{inst} - U \\ V' \end{bmatrix} \qquad (17)$$

The parameters, which have to be determined, are q_{inst} and u_{inst} that are approximately equal to elements $m_{12} = \cos(2\alpha_M)B$ and $m_{13} = \sin(2\alpha_M)B$ of normalized Mueller matrix of mirror system (see the equation (2) for $\mathbf{M}_{TMS}$), respectively. From the equation (17) the instrumental parameters q_{inst} and u_{inst} are the Stokes parameters at output of the mirrors system for non-polarized incoming light. Therefore, they can be measured accurately by the calibrated part of the polarimeter. They are obtained from the equation (9) as left part when at right we set q_{inst}=0, u_{ins}=0 and use corrected values $\varepsilon_{1,2}{}'$ and $a_{1,2}{}'$. To determine the final normalized Stokes parameters q and u of the input scene we should use the equations in form (9) with corrected values $\varepsilon_{1,2}{}'$ and $a_{1,2}{}'$.

We can show that the arbitrary polarization instrumental imperfection of the ScanPol polarimeter can be corrected precisely by applying approach, which we used above to compensate the telescopes imperfection. For example, the arbitrary polarization due to the instrumental imperfections of the ScanPol polarimeter can be reduced using the new effective parameters $a_{1-4}{}'', \varepsilon_{1-4}{}''$ and the new effective gain value $K1,2'$. There is no need to input any instrumental polarizations, which is the significant advantage of that approach. However, all calibration parameters will be function of the prism, telescope and mirror imperfections and will be different for all intensity channels, which is disadvantage of the approach. Therefore the orbital calibration of the ScanPol polarimeter becomes more complicated, since the number of the reference units at the orbit is restricted to one depolarizer (non-polarized reference assemble, one per telescope) and one prism polarizer with fixed orientation (polarized reference assemble, one per telescope).

1.3 On orbit calibration approach

Orbital calibration procedure of the ScanPol polarimeter is based on the APS/Glory approach described in (Cairns and Geogdzhayev, 2010). The orbital calibration is provided using the reference units that have installed inside the ScanPol instrument, which shown in Figure 4. The depolarizers unit is the quartz wedge depolarizers, which guarantees the non-polarized with zero DoLP value reference light at the input window of the ScanPol. The dark unit is the dark chamber with the light adsorbing coating to set the zero level bias for the registering electronics of the ScanPol. The solar unit is the scattering diffuse plate with well-known the bidirectional reflectance distribution function (BRDF) is used for the radiometric calibration of the ScanPol intensity channels. The polarizers unit is a set of four Glan prism polarizers with the fixed well known orientation. The polarizers unit produces the light with the linear polarization of high DoLP value and well known AoLP value. The system of the crossed mirrors is rotated and guides the light scattered from the nadir direction which transmitted through and scattered from the reference samples to the ScanPol polarimeter measurement channels with axes that perpendicular to the plane of Figure 4 and parallel to the axis of the mirror system rotation.

The sequence of the algorithm for orbital calibration of the ScanPol instrument is following. First, we determine the mean value of "zero" raw data in the polarization channels:

$$D_{0^o,90^o,45^o,135^o} = \frac{\sum_{k=0}^{i} D_{0^o,90^o,45^o,135^o,\beta_k}}{(i-1)}, \quad (18)$$

where $D_{0^o,90^o,45^o,135^o,\beta_k}$ values are raw data of the ADC signal when the mirror system (MS) looks at the dark unit at the angle β_k (β_k is changed in the $\beta \div \beta_i$ range when MS looks at extended dark unit, see the shaded sector in Figure 4). Then all ADC raw data obtained at the other MS direction angles are corrected by that "zero" value averaged between β_1–β_i angles:

$$RD_{0^o,90^o,45^o,135^o} = R_{0^o,90^o,45^o,135^o} - D_{0^o,90^o,45^o,135^o}.$$

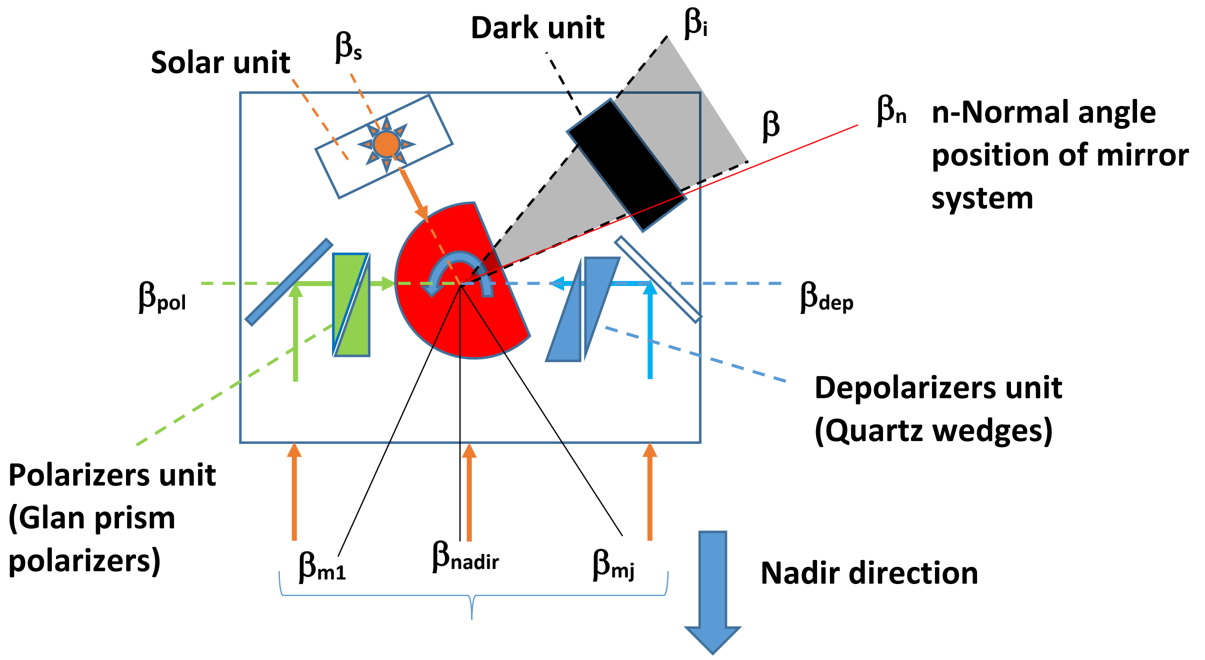

Figure 4. The scanning system layout of the ScanPol instrument.

Second, we calculate the radiometric coefficient A. We obtain the ADC intensity data from calibrated source at the direction MS to the angle β_s for calculation of the general radiometric calibration coefficient A for on-orbit intensity calibration. The radiometric calibration coefficient A allows to obtain the "zero"-compensated raw data from ADC to transfer raw data to the intensity data according the National Institute of Standards and Technology (NIST, https://www.nist.gov/) methods.

The calibration values $K1'$ and $K2'$ include the previously determined values q_{inst}, u_{inst} and a_q', a_u' that are supposed to be stable. When the MS looks to nadir through depolarizers unit (MS position –β_{dep}, see Figure 4) the values $q=0$ and $u=0$, therefore the equations (6) is simplified to:

$$\begin{aligned} \frac{RD_{0^o} - K1' \cdot RD_{90^o}}{RD_{0^o} + K1' \cdot RD_{90^o}} &= a_q'^{-1}\left[q_{inst}\cos(2\varepsilon_1') + u_{inst}\sin(2\varepsilon_1')\right] \\ \frac{RD_{45^o} - K2' \cdot RD_{135^o}}{RD_{45^o} + K2' \cdot RD_{135^o}} &= a_u'^{-1}\left[-q_{inst}\sin(2\varepsilon_2') + u_{inst}\cos(2\varepsilon_2')\right] \end{aligned} \tag{19}$$

From (19) we obtain refined calibration coefficients $K1'$ and $K2'$:

$$K1' = \left(\frac{1 - a_q' q_{inst}'}{1 + a_q' q_{inst}'}\right)\frac{RD_{0^0}}{RD_{90^0}};\ K2' = \left(\frac{1 - a_u' u_{inst}'}{1 + a_u' u_{inst}'}\right)\frac{RD_{45^0}}{RD_{135^0}}; \tag{20}$$

where

$$\begin{aligned} q_{inst}' &= q_{inst}\cos(2\varepsilon_1') + u_{inst}\sin(2\varepsilon_1') \\ u_{inst}' &= -q_{inst}\sin(2\varepsilon_2') + u_{inst}\cos(2\varepsilon_2') \end{aligned} \tag{21}$$

For the calibration of the stray depolarization coefficients a_q' and a_u', we use the stable calibration coefficients $K1'$ and $K2'$. When the MS looks to nadir through polarizers unit (the MS position – β_{pol}, see Figure 4) the incoming beam light polarization is linear with well-defined azimuth, which values are set during the laboratory pre-flight calibration procedure. Therefore the Stokes parameters of the incoming light are known exactly and they are used as calibration coefficients q_{cal} and u_{cal}. Substituting values q_{cal} and u_{cal} into (6), we can refine depolarization calibration parameters a_q' and a_u', knowing other calibration values:

$$a_q' = \frac{q'_{cal} + q'_{inst}}{\dfrac{RD_{0^0} - K1' \cdot RD_{90^0}}{RD_{0^0} + K1' \cdot RD_{90^0}}(1 + q_{cal}q_{inst} + u_{cal}u_{inst})}; \; a_u' = \frac{u'_{cal} + u'_{inst}}{\dfrac{RD_{45^0} - K2' \cdot RD_{135^0}}{RD_{45^0} + K2' \cdot RD_{135^0}}(1 + q_{cal}q_{inst} + u_{cal}u_{inst})}; (22)$$

where

$$\begin{aligned} q'_{cal} &= -q_{cal}\cos(2\varepsilon_1') - u_{cal}\sin(2\varepsilon_1') \\ u'_{cal} &= q_{cal}\sin(2\varepsilon_2') - u_{cal}\cos(2\varepsilon_2') \end{aligned} \qquad (23)$$

To minimize and equalize the relative errors in $a_q{}'$ and $a_u{}'$ refinements it is supposed to be reasonable to use the reference polarization with comparable values of q_{cal} and u_{cal}. This condition is fulfilled for the AoLP value equal of 22.5º, which is the set angle for the reference Glan polarizers. Using calibrated parameters determined above, we can retrieve the Stokes parameters of scene from equation (9) and then the DoLP value is:

$$\mathrm{DoLP} = p = \sqrt{q^2 + u^2} \qquad (24)$$

Azimuth of linear polarization (AOLP):

$$\mathrm{AoLP} = \theta_{real} = \frac{1}{2}arctg\left(\frac{u}{q}\right) - \theta_{TMS}, \qquad (25)$$

where $\theta_{TMS} = 90^o - \beta_{nadir}$ because the azimuth angle of polarization is rotated synchronously with the scanning MS rotation.

2 Multispectral imager-polarimeter MSIP calibration model

2.1 Multispectral imager modelling

The concept of the imager-polarimeter optical layout design has been proposed in the paper (Sinyavskii et al., 2013). The design allows measuring the Stokes vector components simultaneously in wide field-of-view without limitations by the f-number of an optical system. The MSIP optical system (see Figure 5) consists of collimator (1), composited polarizing element (2), system of separation image to four images (4), camera lens (6) and image sensor (7).

In the MSIP channel the four-sector film polarizers are set with polarization angles 0º, 45º, 90º, 135º (Figure 5b). The system of deflection prisms (achromatic in 420–850 nm spectral band) serves for separation of the input image to four images (see separation system in Figure 6). Camera lens creates the image of scene simultaneously in four channels in the 420–850 nm spectral bands.

In Figure 6 the main elements of one MSIP channel which influences on quality of measurements of polarization parameters p and θ in the scene image are shown. The lens of the telescope T (collimator, see Figure 6) could have areas with mechanical tensions that produce additional birefringence. We describe this distortions by the Mueller matrix (3) at each pixel (i, j) with $\delta_{T(i,j)}$ – birefringence value and $\alpha_{T(i,j)}$ – birefringence axes position. In the further consideration we omit (i,j) indices meaning that all calculations are related to the separate pixel of the image sensor.

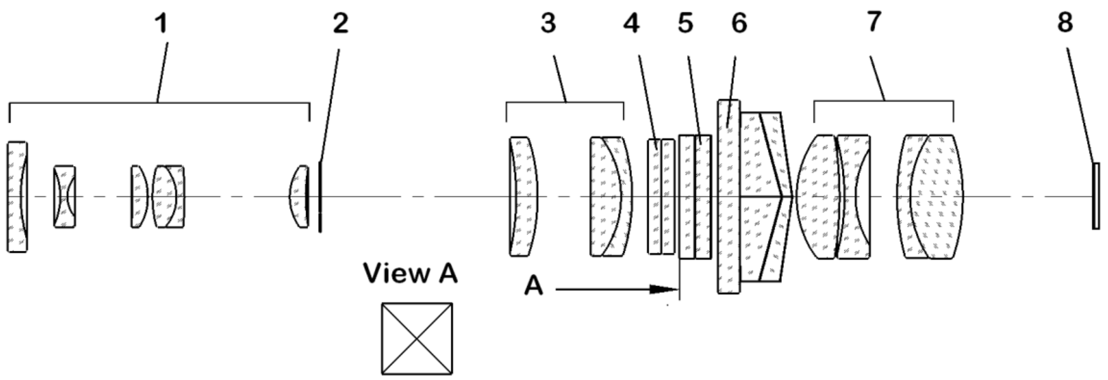

Figure 5. The optical system (a) of the MSIP (one channel). A – view to input window with four film sector polarizes. In the four-sector analyzer of the MSIP polarimeter orientation of the polarizing elements are (see Figure 6): 1 – 0º, 2 – 90º, 3 – 45º, 4 – 135º.

Next group of elements in Figure 6 that impact on quality of the polarization scene retrievals are four film polarizers (analyzers). Each of the polarizers A1–A4 is described by the Mueller matrix of analyzer in the equation (4) and their azimuths are approximately $\alpha_{A1} = 0^o$, $\alpha_{A2} = 90^o$, $\alpha_{A3} = 45^o$, $\alpha_{A4} = 135^o$ with offsets $\varepsilon_1 - \varepsilon_4$ correspondingly due to the possible discrepancy of polarization axes from to exact angles 0º....135º.

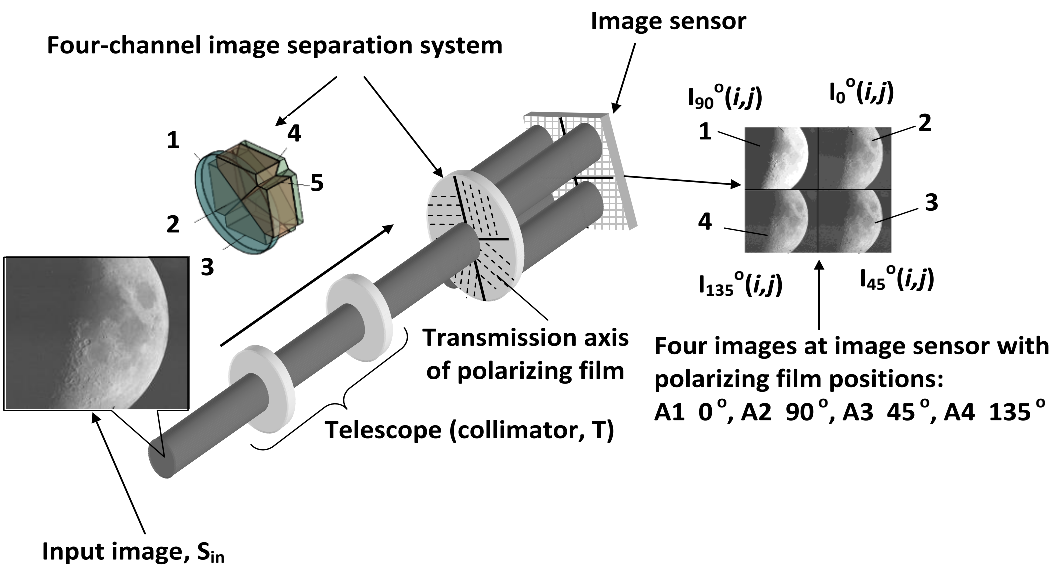

Figure 6. The main elements of the one polarization channel of the MSIP. Image separation system (at top left): 1– the base of separation unit, 2–4 achromatic wedges for four sub-channels in the one MSIP spectral channel. Set of four images (at right) with the position of polarizing element 0º, 90º, 45º, and 135º in the areas 1, 2, 3, and 4 at the image sensor. The four real Moon images in four polarization angles are shown as an example.

Similar to the ScanPol, the each of the MSIP intensity channels can be characterized by different gain. Values of "dark level" (zero bias) for camera's pixels may be different as well. The light intensity that is received by specific pixel in the corresponded film polarizer can be described as:

$$\begin{aligned}
I_{0^o} &= A\cdot(R_{0^o}-D_{0^o}),\\
I_{90^o} &= A\cdot K1\cdot(R_{90^o}-D_{90^o}),\\
I_{45^o} &= A\cdot K2\cdot(R_{45^o}-D_{45^o}),\\
I_{135^o} &= A\cdot K3\cdot(R_{135^o}-D_{135^o}).
\end{aligned} \tag{26}$$

where $D_{0^o,90^o,45^o,135^o}$ – zero value bias of pixel, measured for different polarizing segments of camera; $R_{0^o,90^o,45^o,135^o}$ – digital values of signal measured at the ADC output without compensation of zero bias; A – radiometric correction for measured intensity; $K1$–$K3$ – the gain coefficients in the corresponded channels of intensity.

Considering polarization parameters of the MSIP optical elements (Figure 6), we get the equations that determine signals $R_{0^o,90^o,45^o,135^o}$ from the ADC output for the input light polarization $\mathbf{S}_{scene}$:

$$\begin{aligned}
R_{0^o} &= A^{-1}\left[\mathbf{M}_{A1}(a_1,0^o+\varepsilon_1)\cdot\mathbf{M}_{T1}(\delta_{T1},\alpha_{T1})\cdot\mathbf{S}_{scene}\right]_0+D_{0^o},\\
R_{90^o} &= (AK1)^{-1}\cdot\left[\mathbf{M}_{A2}(a_2,90^o+\varepsilon_2)\cdot\mathbf{M}_{T2}(\delta_{T2},\alpha_{T2})\cdot\mathbf{S}_{scene}\right]_0+D_{90^o},\\
R_{45^o} &= (AK2)^{-1}\cdot\left[\mathbf{M}_{A3}(a_3,45^o+\varepsilon_3)\cdot\mathbf{M}_{T3}(\delta_{T3},\alpha_{T3})\cdot\mathbf{S}_{scene}\right]_0+D_{45^o},\\
R_{135^o} &= (AK3)^{-1}\cdot\left[\mathbf{M}_{A4}(a_4,135^o+\varepsilon_4)\cdot\mathbf{M}_{T4}(\delta_{T4},\alpha_{T4})\cdot\mathbf{S}_{scene}\right]_0+D_{135^o}.
\end{aligned} \tag{27}$$

Therefore, we need to determine parameters $K1-K3$, $a_{1-4},\varepsilon_{1-4}$, $\delta_{T1-4},\alpha_{T1-4}$ to retrieve the required Stokes parameters I,Q,U, DoLP and AoLP. In the equations (27) and below, we use also the scale factor a instead of extinction ratio e as argument of the polarizer's Mueller matrix, since it appears in the final equations. In addition, we separate the telescope parameters in polarizing channels to four independent considerations, because a spatial distribution of the telescope's polarization parameters in the image system should be also taken into account.

2.2 Polarimetric calibration of the MSIP channels

The general expression for the light intensity at the output of the birefringent telescope and imperfect polarizer sequence was given above by the equation (10) in the Section 1.2. There was also demonstrated that the given elements sequence effects on the input light intensity as the single imperfect polarizer with some new effective azimuth α_{AT} and depolarization factor a' (see Section 1.2). The relationship of these effective parameters with the actual parameters α_T , δ_T , α_A , and a was described in the equations (11)–(14). We consider also the possible way for the experimental determination of these effective parameters. An absence of the mirror system in the MSIP polarimeter eliminates needs to introduce additional distortional instrumental Stokes parameters. Let summarize the all factors that need to take into account during the MSIP ground-based calibration procedure for each spectral and polarimetric channels. They are: four zero bias factors $D_{0^o,90^o,45^o,135^o}$, four gain factors A and $K1-K3$, four offset factors ε'_{1-4}, and four depolarization factors a'_{1-4}. All of these factors are corresponded to each pixel of the image sensor.

Basically, the MSIP imager-polarimeter has four separated intensity channels with the different depolarization factors a'_{1-4} and offsets ε'_{1-4}. Therefore, we cannot use the equations (6) (see Section 1.1) for retrieving of the Stokes parameters. However, the raw intensities after the film polarizers and the separation system are related to the input intensities as

$$
\begin{aligned}
RD_{0^o} &= A^{-1} I\{1 + a_1'^{-1}[q\cos(2\varepsilon_1') + u\sin(2\varepsilon_1')]\}, \\
RD_{90^o} &= (AK1)^{-1} I\{1 - a_2'^{-1}[q\cos(2\varepsilon_2') + u\sin(2\varepsilon_2')]\}, \\
RD_{45^o} &= (AK2)^{-1} I\{1 - a_3'^{-1}[q\sin(2\varepsilon_3') - u\cos(2\varepsilon_3')]\}, \\
RD_{135^o} &= (AK3)^{-1} I\{1 + a_4'^{-1}[q\sin(2\varepsilon_4') - u\cos(2\varepsilon_4')]\},
\end{aligned}
\qquad (28)
$$

where $RD_{0^o,90^o,45^o,135^o} = R_{0^o,90^o,45^o,135^o} - D_{0^o,90^o,45^o,135^o}$. The dark signal D measurements to determine the zero bias coefficients $D_{0^o,90^o,45^o,135^o}$ are carried out by signal measurements when the aperture of the MSIP channel is closed. To determine the transmission coefficients *A*, and *K1–K3*, we will use the moving CCD image sensor, which can shift from area 1 consecutively to areas 2, 3 and 4 (see Figure 6). In conditions of exact overlap of areas and knowing zero bias coefficients the transition coefficients *K1–K3* are:

$$\begin{aligned} K1_{i,j} &= \left(RD1_{90^o i,j} / RD2_{90^o i,j} \right); \\ K2_{i,j} &= \left(RD1_{45^o i,j} / RD3_{45^o i,j} \right); \\ K3_{i,j} &= \left(RD1_{135^o i,j} / RD4_{135^o i,j} \right). \end{aligned} \tag{29}$$

where $RDn_{\alpha,i,j} = Rn_{\alpha,i,j} - Dn_{\alpha,i,j}$ are zero compensated intensity signals detected by pixels of *n* area of image sensor when it position after film polarizer correspond to axes azimuth α. There no specific requirements to polarization of input light in that case.

For determination of the offsets ε'_{1-4} and parameters depolarization factors a'_{1-4}, we use the highly depolarized light source with extended homogeneous field (for example, the integrating sphere) that cover the MSIP field-of-view. Then we insert the rotated Glan prism polarizer on the way of light between source and input of the MSIP polarimeter. The polarizer should overlap the MSIP FOV. In this case the signal at output of the MSIP polarizers will be in general:

$$RD(t) = K^{-1} I\{1 + a'^{-1}[\cos(2\Omega t)\cos(2(\alpha_A + \varepsilon')) + \sin(2\Omega t)\sin(2(\alpha_A + \varepsilon'))]\} \tag{30}$$

The equation (30) is the similar for all pixels of the CCD matrix when instead of K, a' and ε' we introduce $A_{i,j}$, $K1_{i,j}$–$K3_{i,j}$, $a'_{1-4,i,j}$ and $\varepsilon'_{1-4,i,j}$, respectively, to specify the certain pixel of the CCD matrix. Instead of the parameter α_A, we insert 0°, 45°, 90° or 135° angles for different film polarizers, respectively. According to the equation (30) and similar to equations (13)–(15), the calibration parameter for each pixel can be assessed as:

$$\alpha_A + \varepsilon' = \frac{1}{2}\arctan(\mathrm{b}_2 / \mathrm{a}_2) - ; \;\; a'^{-1} = \sqrt{\mathrm{a}_2{}^2 + \mathrm{b}_2{}^2} \,/\, \mathrm{a}_0 \, ; \qquad (31)$$

$$K1 = \mathrm{a}_0\big|_{\alpha_A = 0^o} / \, \mathrm{a}_0\big|_{\alpha_A = 90^o} \, ; \; K2 = \mathrm{a}_0\big|_{\alpha_A = 0^o} / \, \mathrm{a}_0\big|_{\alpha_A = 45^o} \, ; \; K3 = \mathrm{a}_0\big|_{\alpha_A = 0^o} / \, \mathrm{a}_0\big|_{\alpha_A = 135^o} \, . \qquad (32)$$

where $\mathrm{a}_0 = IK$ – the value of the Fourier series zero harmonic, and $\mathrm{a}_2 = IK\, a'^{-1}\cos(2(\alpha_A + \varepsilon'))$, $b_2 = IK\, a'^{-1}\sin(2(\alpha_A + \varepsilon'))$ – the values of the second harmonics by cosine and sine correspondingly. Radiometric correction *A* should be determined in series of radiometric calibrations with non-polarized source with the well-known intensity.

To retrieve the Stokes parameters from the MSIP imager-polarimeter, we cannot use the equations (6) because of the MSIP, in contrast to the ScanPol polarimeter, has four separated intensity channels with different depolarization factors a'_{1-4} and different offsets ε'_{1-4}. To derive the new suitable equations, we rewrite the equation (28) in the matrix form:

$$\begin{bmatrix} I_{0^o} \\ I_{90^o} \\ I_{45^o} \\ I_{135^o} \end{bmatrix} = \begin{bmatrix} 1 & a_1'^{-1}\cos(2\varepsilon_1') & a_1'^{-1}\sin(2\varepsilon_1') \\ 1 & -a_2'^{-1}\cos(2\varepsilon_2') & -a_2'^{-1}\sin(2\varepsilon_2') \\ 1 & -a_3'^{-1}\sin(2\varepsilon_3') & a_3'^{-1}\cos(2\varepsilon_3') \\ 1 & a_4'^{-1}\sin(2\varepsilon_4') & -a_4'^{-1}\cos(2\varepsilon_4') \end{bmatrix} \cdot \begin{bmatrix} I \\ Q \\ U \end{bmatrix} \qquad (33)$$

where $I_{0^o} = A\,RD_{0^o}$, $I_{90^o} = AK1RD_{90^o}$, $I_{45^o} = AK2\,RD_{40^o}$, $I_{135^o} = AK3RD_{135^o}$.

Than the retrieving expressions are:

$$\begin{bmatrix} I \\ Q \\ U \end{bmatrix} = \begin{bmatrix} 1 & a_1'^{-1}\cos(2\varepsilon_1') & a_1'^{-1}\sin(2\varepsilon_1') \\ 1 & -a_2'^{-1}\cos(2\varepsilon_2') & -a_2'^{-1}\sin(2\varepsilon_2') \\ 1 & -a_3'^{-1}\sin(2\varepsilon_3') & a_3'^{-1}\cos(2\varepsilon_3') \\ 1 & a_4'^{-1}\sin(2\varepsilon_4') & -a_4'^{-1}\cos(2\varepsilon_4') \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_{0^o} \\ I_{90^o} \\ I_{45^o} \\ I_{135^o} \end{bmatrix} \qquad (34)$$

The power (–1) upon the matrix of imperfections in the equation (34) determines pseudo inversion of the rectangular matrix. The equations (34) allows retrieving the absolute Stokes parameters by the least square approach. However the equations (34) is unsuitable to provide the intercalibration procedure. Therefore it is reasonable to use the relations of the raw measurements for the further simplicity of the ScanPol and the MSIP polarimeters intercalibration. Furthermore, given case are more suitable to avoid the using the absolute intensity and the radiometric calibration parameters.

There are no specific criteria to choose the specific pair of the raw data to build relations mentioned above. Moreover, we would to use all possible combination of the relations to average the values of the normalized Stokes parameters. Below we consider one of the possible cases. In particular, from the equation (28) we can write:

$$\frac{RD_{0^o}}{RD_{90^o}} = K1\frac{1+a_1'^{-1}(q\cos(2\varepsilon_1')+u\sin(2\varepsilon_1'))}{1-a_2'^{-1}(q\cos(2\varepsilon_2')+u\sin(2\varepsilon_2'))},$$

$$\frac{RD_{45^o}}{RD_{135^o}} = \frac{K3}{K2}\frac{1-a_3'^{-1}(q\sin(2\varepsilon_3')-u\cos(2\varepsilon_3'))}{1+a_4'^{-1}(q\sin(2\varepsilon_4')-u\cos(2\varepsilon_4'))}. \quad (35)$$

The equations (35) we can rewrite in matrix form:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \mathbf{B}\begin{pmatrix} q \\ u \end{pmatrix} \quad (36)$$

where

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \dfrac{RD_{0^o}}{RD_{90^o}} - K1 \\ \dfrac{RD_{45^o}}{RD_{135^o}} - \dfrac{K3}{K2} \end{pmatrix};$$

$$\mathbf{B} = \begin{pmatrix} a_1'^{-1}K1\cos(2\varepsilon_1') + a_2'^{-1}\dfrac{RD_{0^o}}{RD_{90^o}}\cos(2\varepsilon_2') & a_1'^{-1}K1\sin(2\varepsilon_1') + a_2'^{-1}\dfrac{RD_{0^o}}{RD_{90^o}}\sin(2\varepsilon_2') \\ -a_3'^{-1}\dfrac{K3}{K2}\sin(2\varepsilon_3') - a_4'^{-1}\dfrac{RD_{45^o}}{RD_{135^o}}\sin(2\varepsilon_4') & a_3'^{-1}\dfrac{K3}{K2}\cos(2\varepsilon_3') + a_4'^{-1}\dfrac{RD_{45^o}}{RD_{135^o}}\cos(2\varepsilon_4') \end{pmatrix}.$$

(37)

From the equation (36) we find the required Stokes parameters:

$$\begin{pmatrix} q \\ u \end{pmatrix} = \mathbf{B}^{-1} \begin{pmatrix} X \\ Y \end{pmatrix} \quad (38)$$

The equation (38) is the final equation for determination of the normalized Stokes polarization parameters of scene in the pixel area. The equation includes all calibration parameters described above and is a general equation for all pixels of the scene image where each parameter is the individual characteristic for image field in area of each specific pixel. Absolute value of the intensity can be retrieved from every of the equation (28). For example, from (28) it is easy to obtain the value $I = RD_{0^o} A / \{1 + a_1'^{-1}[q\cos(2\varepsilon_1') + u\sin(2\varepsilon_1')]\}$.

3 ScanPol and MSIP polarimeters intercalibration

The ScanPol includes the calibration units, which provides control of the measurements reliability and specifying several calibration parameters on orbit. However, the MSIP polarimeter has not includes onboard calibration due to the technical difficulties to design theirs. The both polarimeters are combined in single mechanical unit (Figure 7), adapted to the YuzhSat satellite platform). The FOVs of the ScanPol and MSIP polarimeters have the parts of the scene that are the same (see Figure 8). We use this feature for intercalibration of the MSIP on the orbit using the ScanPol data in the same parts field-of-view. For calibration, we consider two possible cases. In the first case, we determine the parameters of the radiometric calibration and the gain factors *A*, *K*1–*K*3 when values $D_{0^o,90^o,45^o,135^o}$, a'_{1-4} and ε'_{1-4} are stable. In the second case, we calculate the depolarization factors a'_{1-4} when the parameters gain *A*, *K*1-*K*3, bias $D_{0^o,90^o,45^o,135^o}$, and offsets ε'_{1-4} are stable. For the MSIP calibration, we need to know the precise calibrated values of absolute intensity I_{cal} and

Stokes parameters q_{cal} and u_{cal} of scene that is observed by the MSIP polarimeter. These values are provided by the calibrated ScanPol measurements. Then, using the equation (28), we can obviously update the MSIP radiometric and gain factors as

$$A = \frac{I_{cal}\{1 + a_1'^{-1}[q_{cal}\cos(2\varepsilon_1') + u_{cal}\sin(2\varepsilon_1')]\}}{RD_{0^o}};$$

$$K1 = \frac{RD_{0^o}}{RD_{90^o}}\frac{\{1 - a_2'^{-1}[q_{cal}\cos(2\varepsilon_2') + u_{cal}\sin(2\varepsilon_2')]\}}{\{1 + a_1'^{-1}[q_{cal}\cos(2\varepsilon_1') + u_{cal}\sin(2\varepsilon_1')]\}},$$

$$K2 = \frac{RD_{0^o}}{RD_{45^o}}\frac{\{1 - a_3'^{-1}[q_{cal}\sin(2\varepsilon_3') - u_{cal}\cos(2\varepsilon_3')]\}}{\{1 + a_1'^{-1}[q_{cal}\cos(2\varepsilon_1') + u_{cal}\sin(2\varepsilon_1')]\}},$$

$$K3 = \frac{RD_{0^o}}{RD_{135^o}}\frac{\{1 + a_4'^{-1}[q_{cal}\sin(2\varepsilon_4') - u_{cal}\cos(2\varepsilon_4')]\}}{\{1 + a_1'^{-1}[q_{cal}\cos(2\varepsilon_1') + u_{cal}\sin(2\varepsilon_1')]\}} \qquad (39)$$

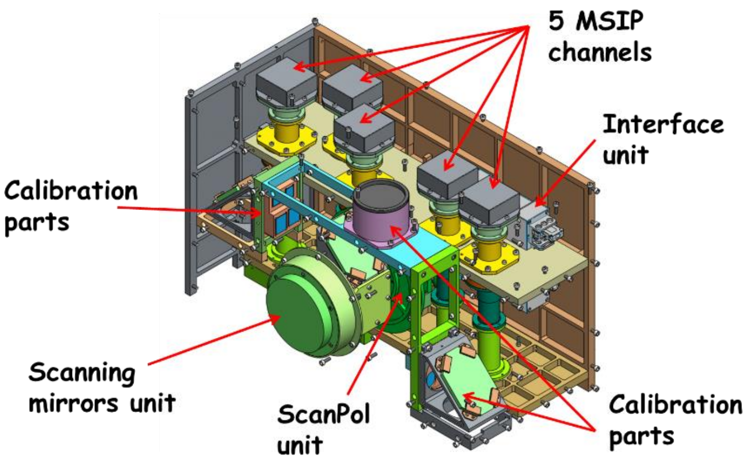

Figure 7. The ScanPol and MSIP polarimeters combined in the single unit adapted to the YuzhSat satellite platform.

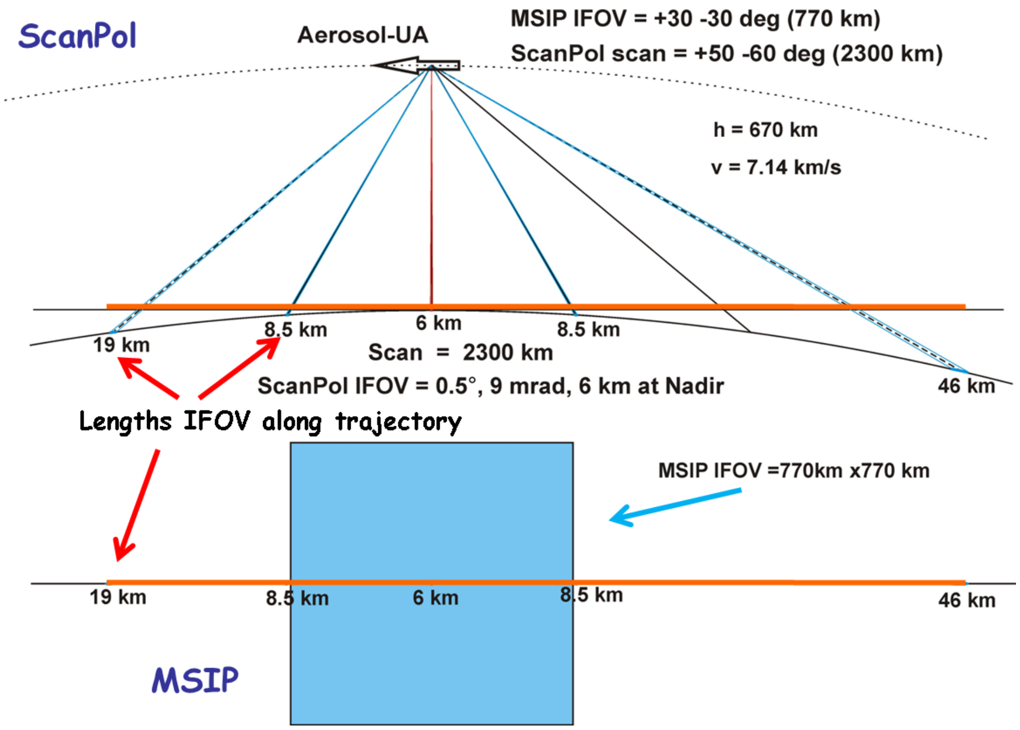

Figure 8. Intersection of the ScanPol and MSIP polarimeters fields of view.

The corrections of the depolarization factors we obtain from the equation (28) as:

$$\begin{aligned}
&\frac{I_{cal}(q_{cal}\cos(2\varepsilon_1')+u_{cal}\sin(2\varepsilon_1'))}{ARD_{0^o}-I_{cal}}=a_1'\,;\\
&-\frac{I_{cal}(q_{cal}\cos(2\varepsilon_2')+u_{cal}\sin(2\varepsilon_2'))}{AK1RD_{90^o}-I_{cal}}=a_2'\\
&-\frac{I_{cal}(q_{cal}\sin(2\varepsilon_3')-u_{cal}\cos(2\varepsilon_3'))}{AK2RD_{45^o}-I_{cal}}=a_3'\\
&\frac{I_{cal}(q_{cal}\sin(2\varepsilon_4')-u_{cal}\cos(2\varepsilon_4'))}{AK3RD_{135^o}-I_{cal}}=a_4'
\end{aligned}\tag{40}$$

In the equations (39) and (40) the intensity I_{cal} and Stokes parameters q_{cal} and u_{cal} are calibrated parameters provided by the ScanPol for the corresponding pixels of the MSIP CCD matrix.

4. Discussion and conclusions

The space mission Aerosol-UA instruments calibration methods and procedures are discussed. The polarimetric models are developed and described using self-consistent Mueller-Stokes formalism for the ScanPol scanning polarimeter and the multi-spectral imager polarimeter MSIP. The models describe the transformation of polarization and intensity of light at input of ScanPol MSIP polarimeters during it propagation trough the channels of polarimeters with polarization imperfect optics. Basing on the developed models, the corresponding calibration procedures were proposed. The calibration procedures allow compensating the zero bias of polarimeter channels, the channels' gain difference, orientation offsets and finite extinction ratio of the polarizers, birefringence inserted by telescopes and additional instrumental polarizations produced by mismatches in the scanning mirror system of the ScanPol polarimeter. The ground-based and on orbit calibration approaches are described as well.

The numerical experiments were carried out to validate the calibration procedures. During experiments it was established that the calibration procedure allows effective compensation of the imperfections of the polarimeters optical units. Particularly, the excellent compensation of the ScanPol and MSIP polarization imperfections by ground-based calibration procedures was established. Thought the potentially reached level of the compensation of the imperfections, that level is restricted only by quality of reference sources and signal-to-noise ratio. For on orbit calibrations the numerical experiment predicts the DoLP values determination from the ScanPol polarimeter with standard error (SE) ~0.0008 (correspondent to the relative error (RE) ~0.08%, since RE<0.15% is required), which not depends on the absolute values of DoLP or AoLP if the error of the polarizer positions increased up to 0.02 degrees. The standard error of the AoLP value does not depend on the

AoLP absolute value, but depends on the DoLP absolute value by the relation shown in Figure 9.

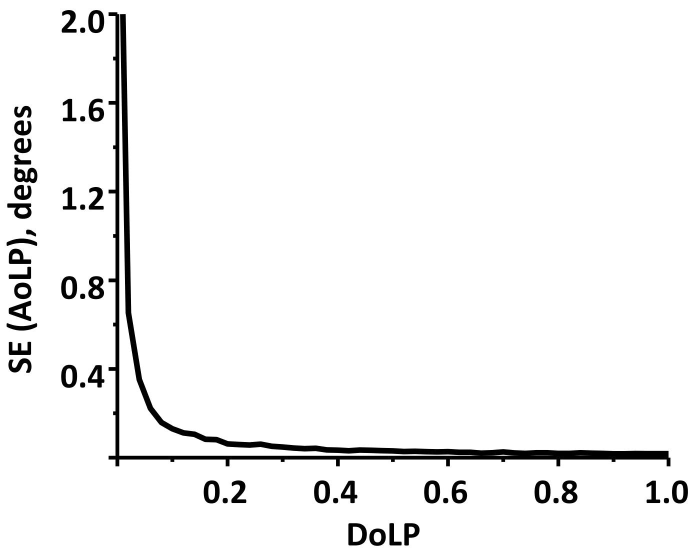

Figure 9. Dependence of the AoLP standard error on the DoLP absolute value retrieved from numerical calibration experiment.

The ScanPol polarimeter calibration on orbit will serve to control the polarizations and radiometric measurements quality. The data corrections will be made using the reference calibration measurements (see the equations (20)–(22)) in assumption that the polarization characteristics of reference units are more stable then polarization characteristics of the ScanPol optics. The ScanPol optics will experience the degradation or/and the contamination during the mission orbital period and the measurements quality may be reduced. Therefore, the polarized light calibration reference will serve as critical point at the DoLP = 1 for the ScanPol measurements interpretation.

The MSIP calibration procedure is more effective in theory, in comparison to the ScanPol calibration, because in the MSIP polarimeter does not included scanning mirror system, which introduce the additional instrumental polarization. In practice, we understand that the lower extinction ratio of the polarizing films, inhomogeneity and lower durability (in comparison with prism polarizers) of the polarizing films, the images mismatches in specific pixels, geometrical and spatial imperfections of optics and many other distortions, which are usual for the imaging systems, will significantly reduce accuracy of the polarimetric measurements. However, we expect that the MSIP polarimeter will provide the DoLP measurements within accuracy when RE $\leq 0.5\%$ and an error of the AoLP measurements SE $\leq 1^{o}$. The possible degradation of the MSIP optics during the mission period on the orbit should be partially compensated by the intercalibration procedure using the ScanPol data. We expect that degradation processes in the MSIP optics will have the homogeneous spatial distribution. Therefore the corrections, which will be applied to the MSIP data using corresponded ScanPol measurement, can be used for all other CCD matrix pixels.

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