

# **Section 2: Definition & Preliminaries**

# A Retrieval-based LM: Definition

A language model (LM) that uses  
an external datastore at test time

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**A language model (LM) that uses  
an external datastore at test time**

# A language model (LM)

$$P(x_n | x_1, x_2, \dots, x_{n-1})$$

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Language model (Transformers)

The capital city of Ontario is

$x_1$

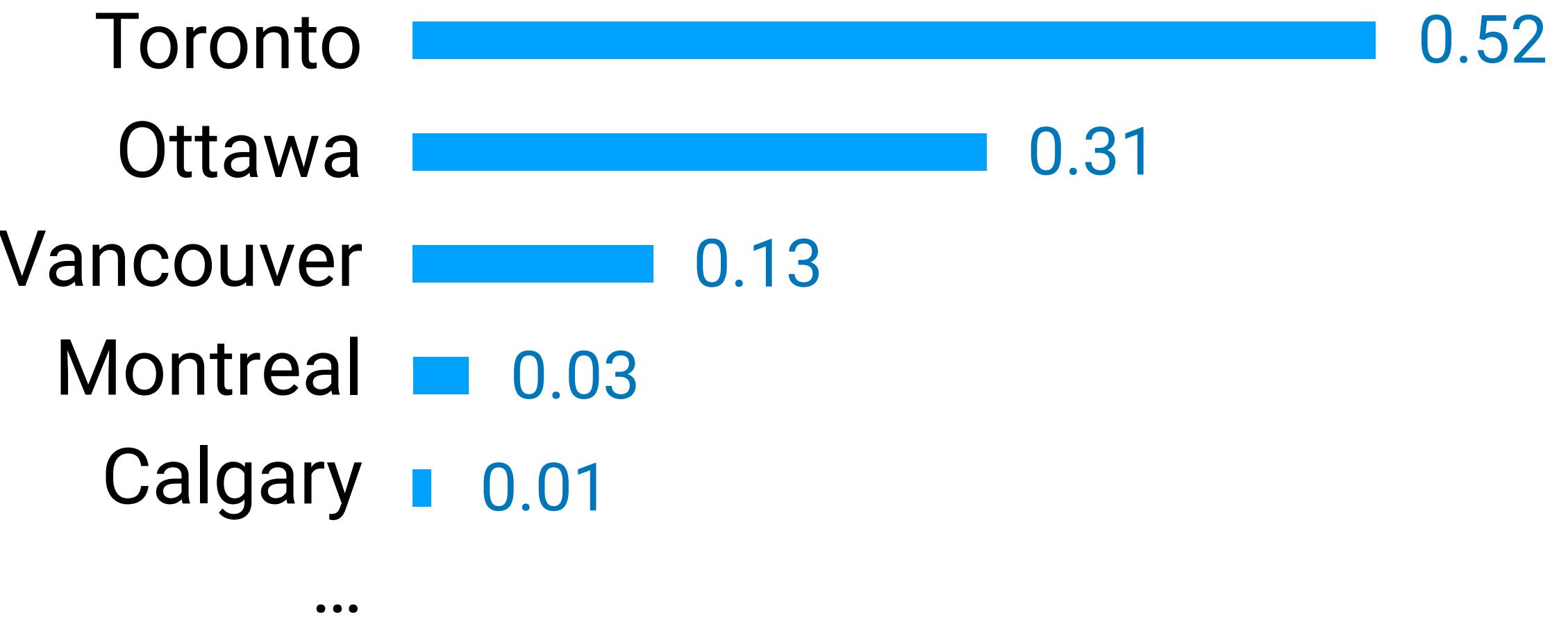
$x_2$

...

$x_{n-1}$

# A language model (LM)

$$P(x_n | x_1, x_2, \dots, x_{n-1})$$



Language model (Transformers)

The capital city of Ontario is

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# A language model (LM): Categories

Toronto

Autoregressive LM

The capital city of Ontario is \_\_\_\_\_

# A language model (LM): Categories

Toronto

Autoregressive LM

The capital city of Ontario is \_\_\_\_\_

vs

capital

Ontario

Masked LM

The \_\_\_\_\_ city of \_\_\_\_\_ is Toronto

# A language model (LM): Categories

Toronto

Autoregressive LM

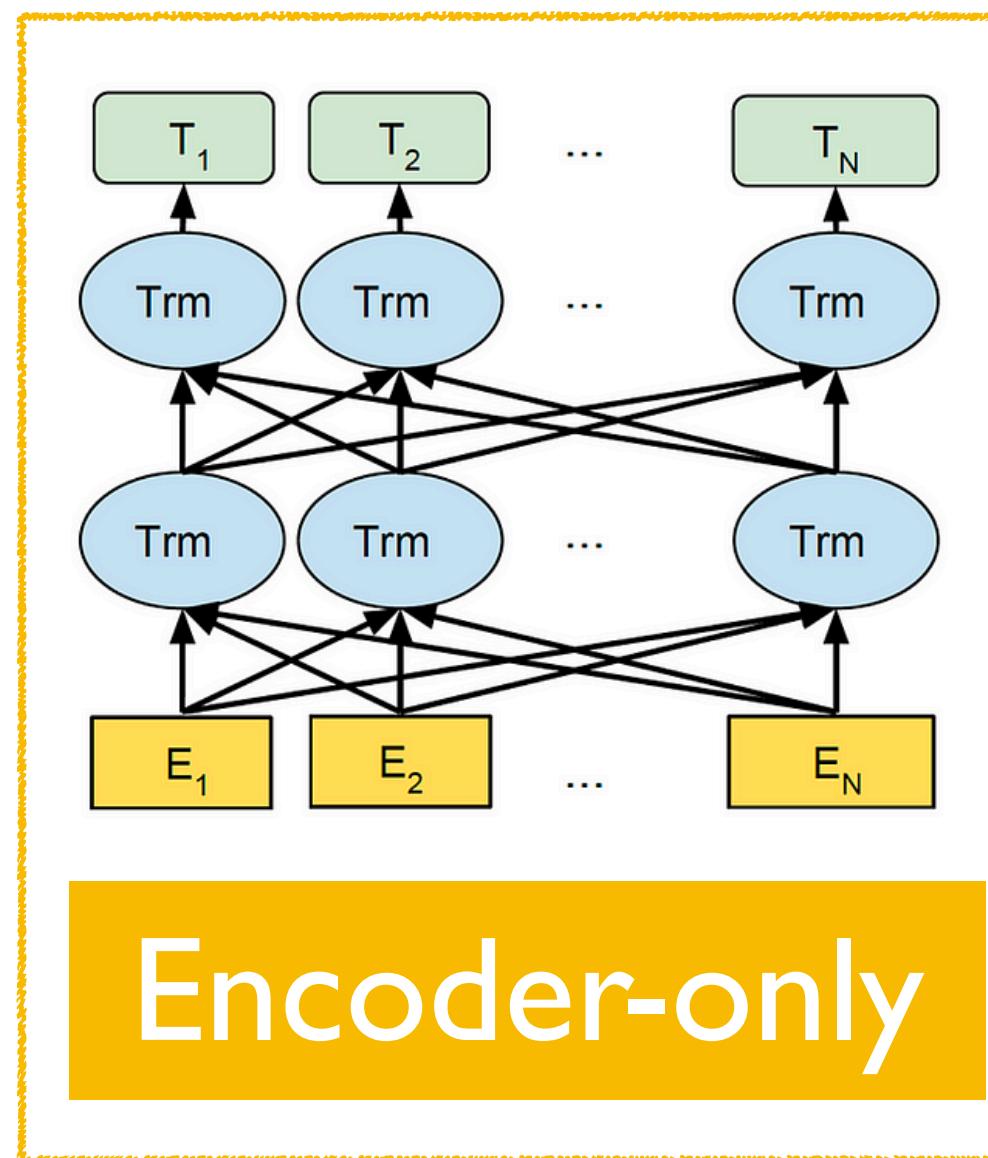
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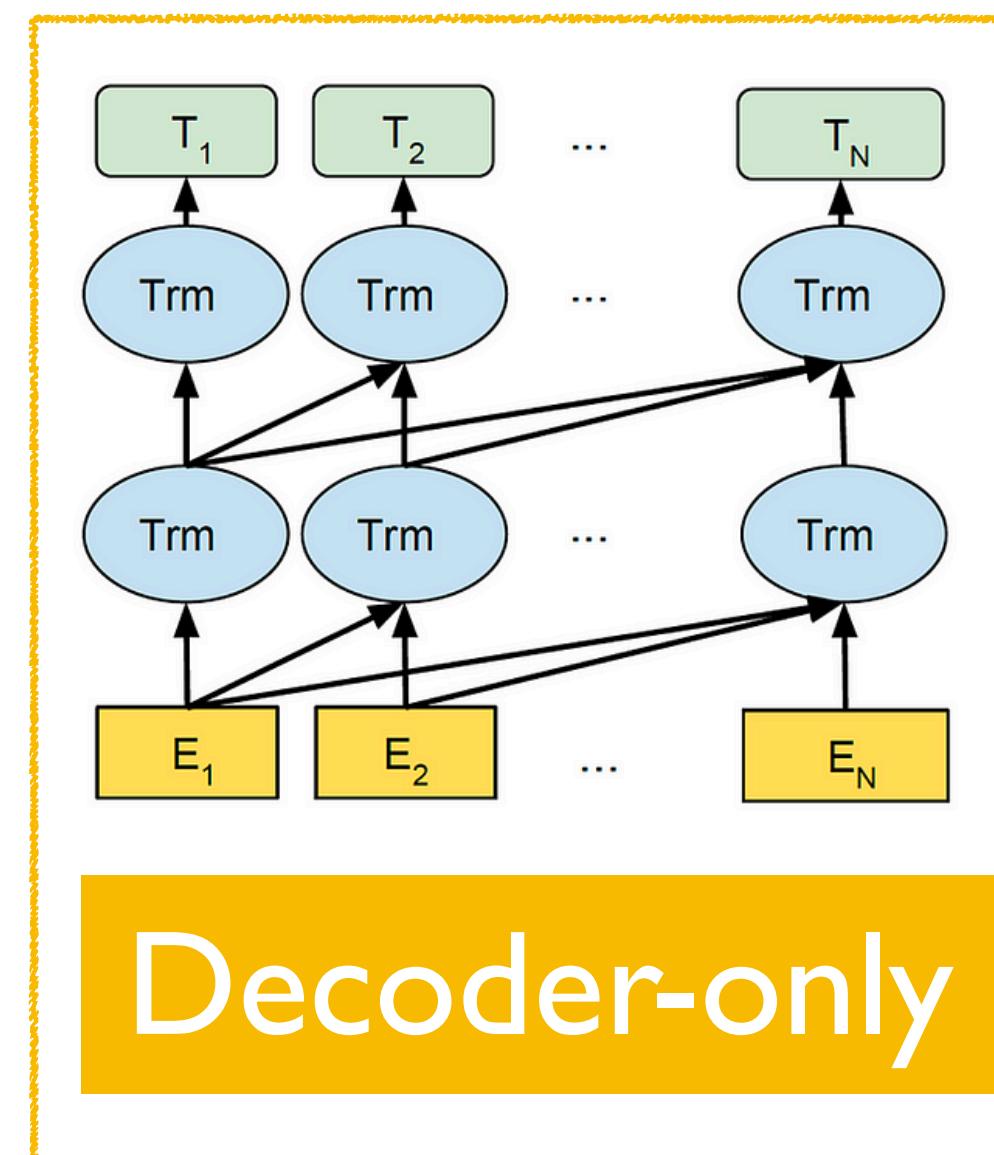
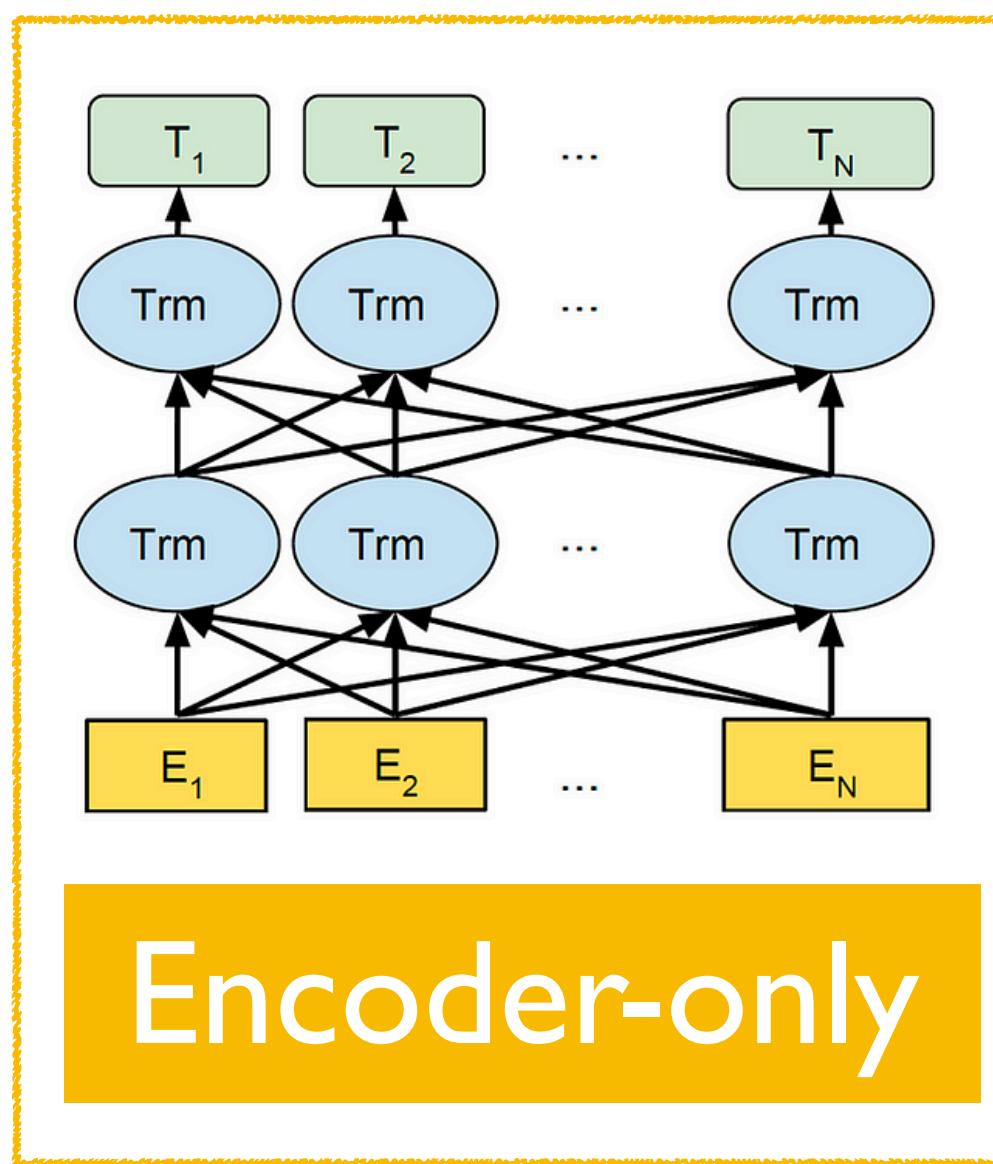
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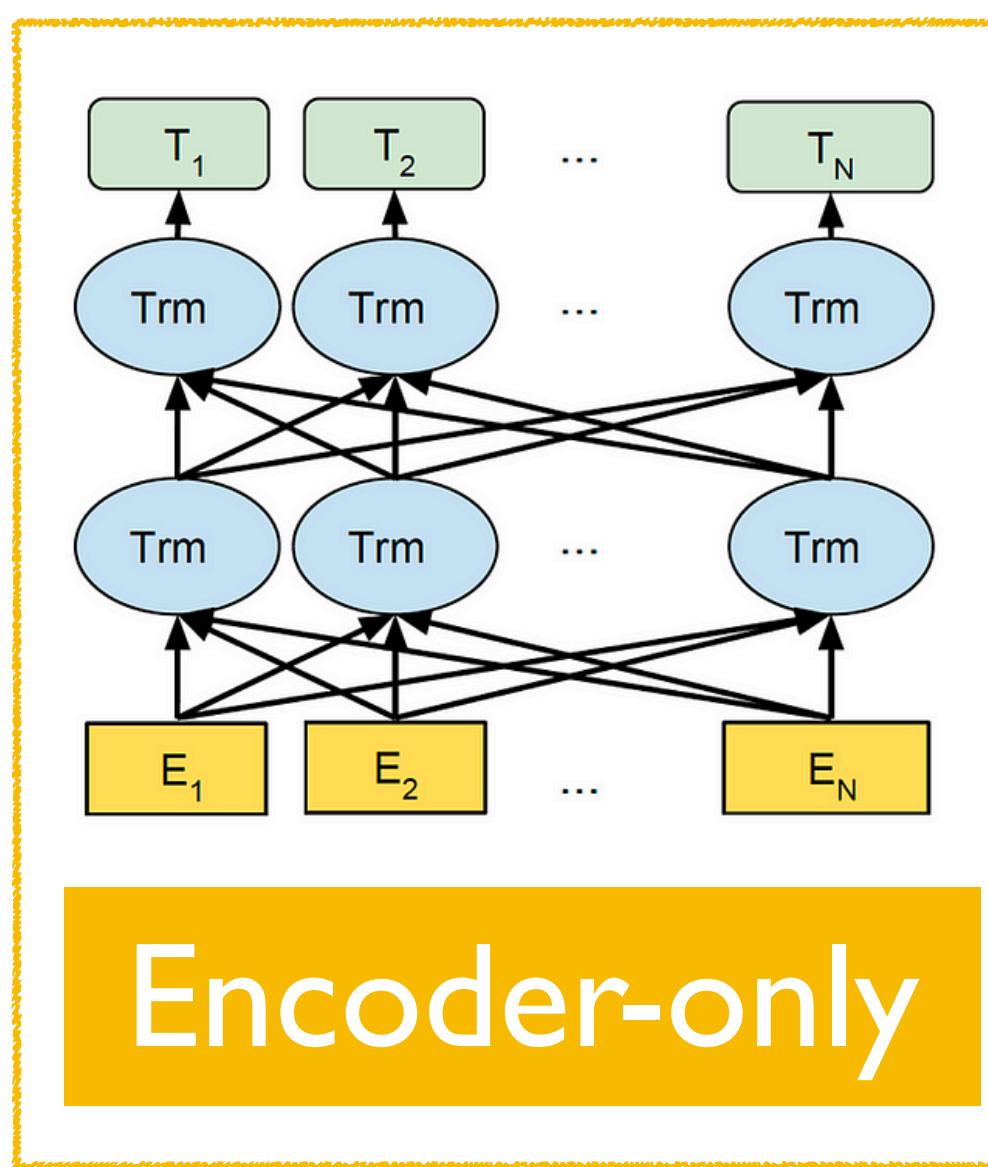
## Autoregressive LM

The capital city of Ontario is \_\_\_\_\_

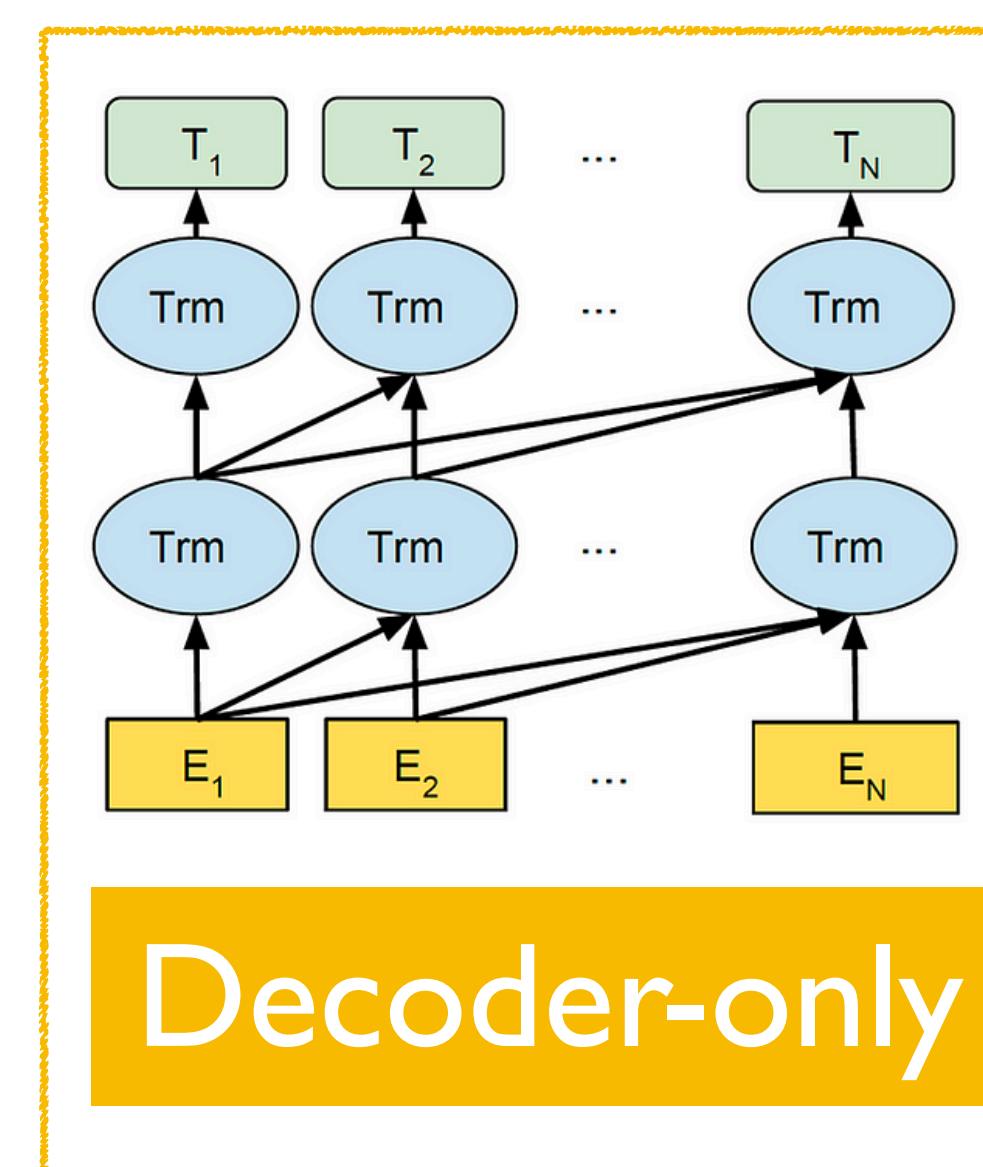
capital      Ontario

## Masked LM

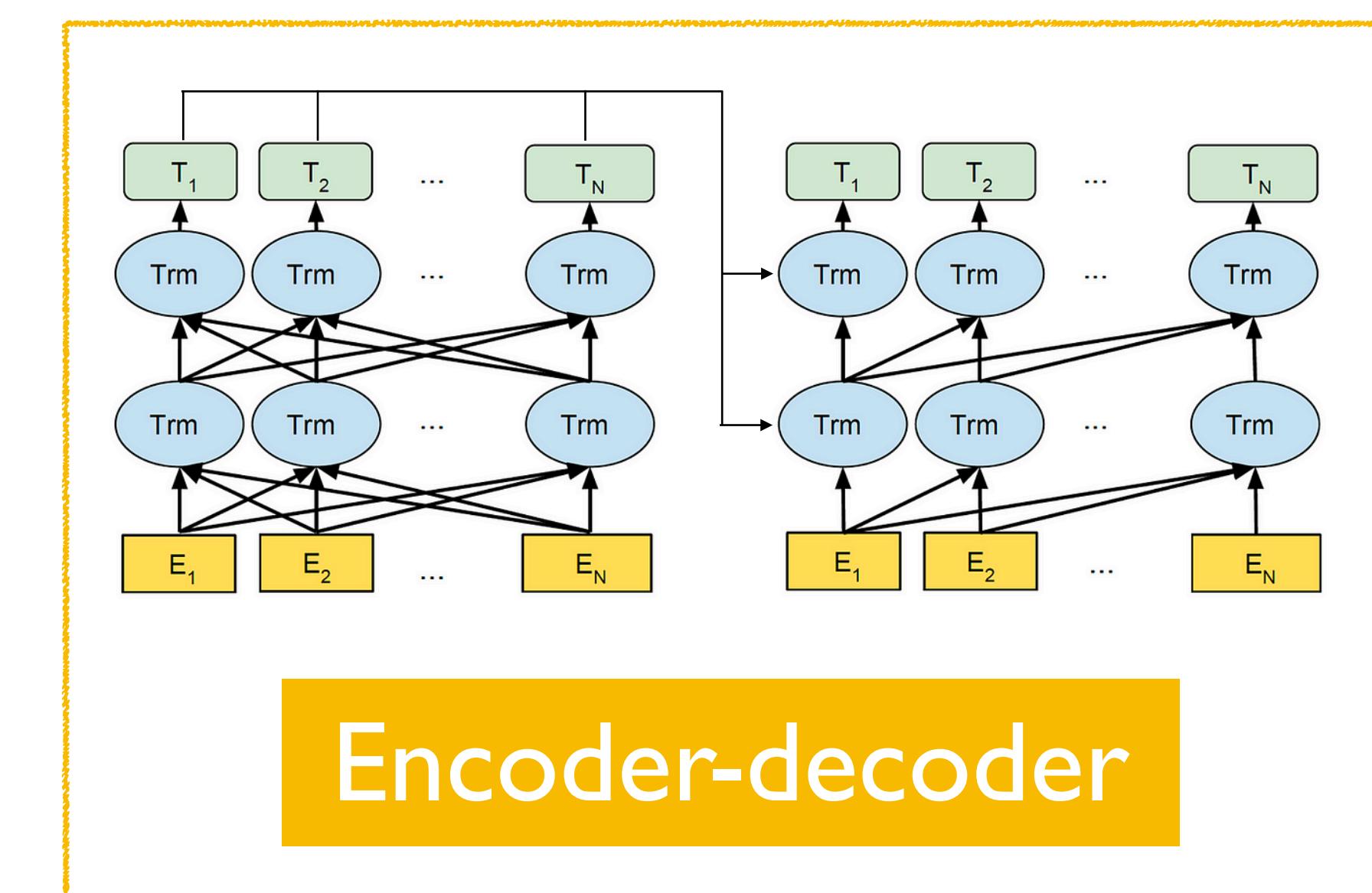
The \_\_\_\_\_ city of \_\_\_\_\_ is Toronto



Encoder-only



Decoder-only

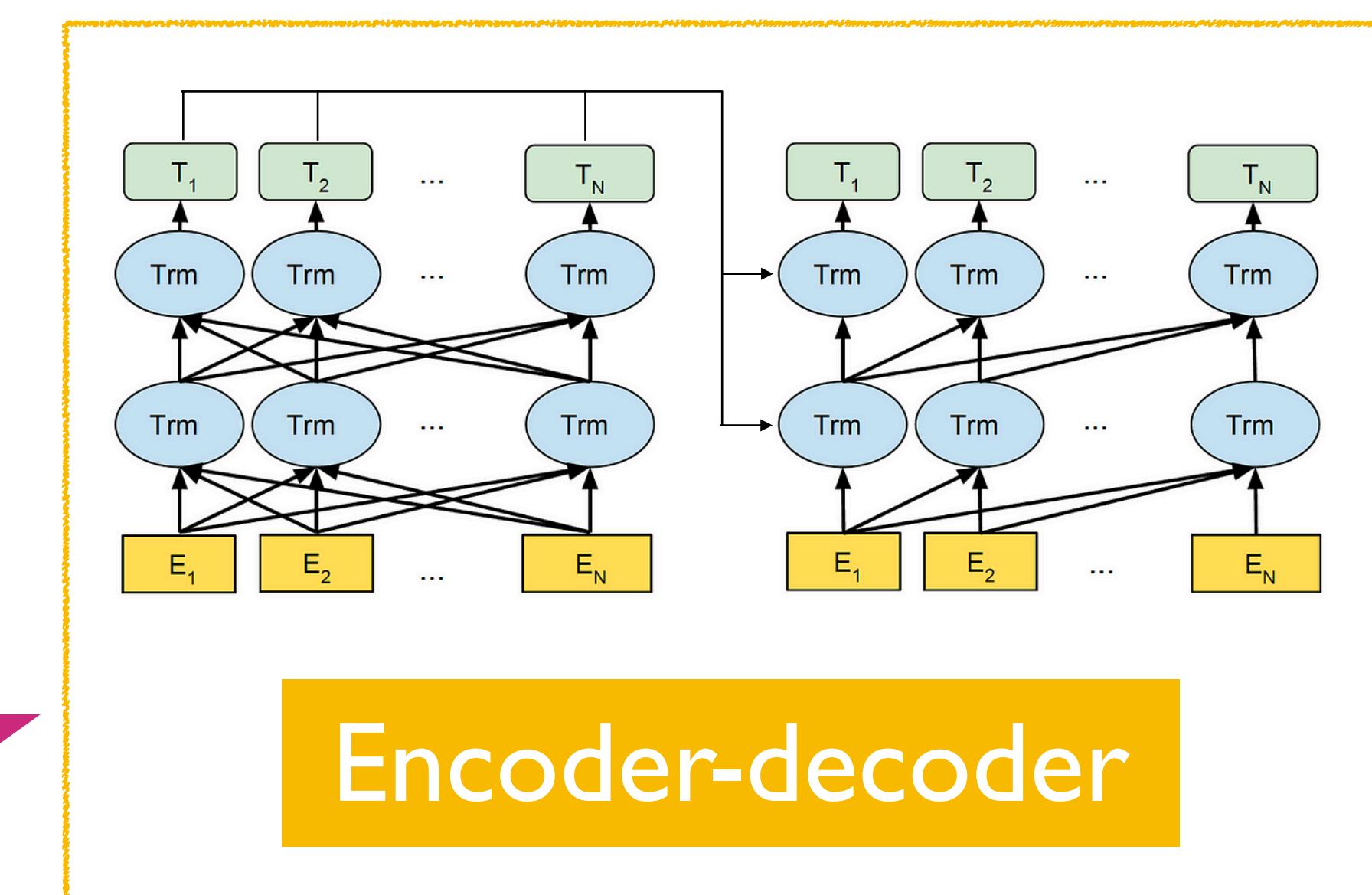
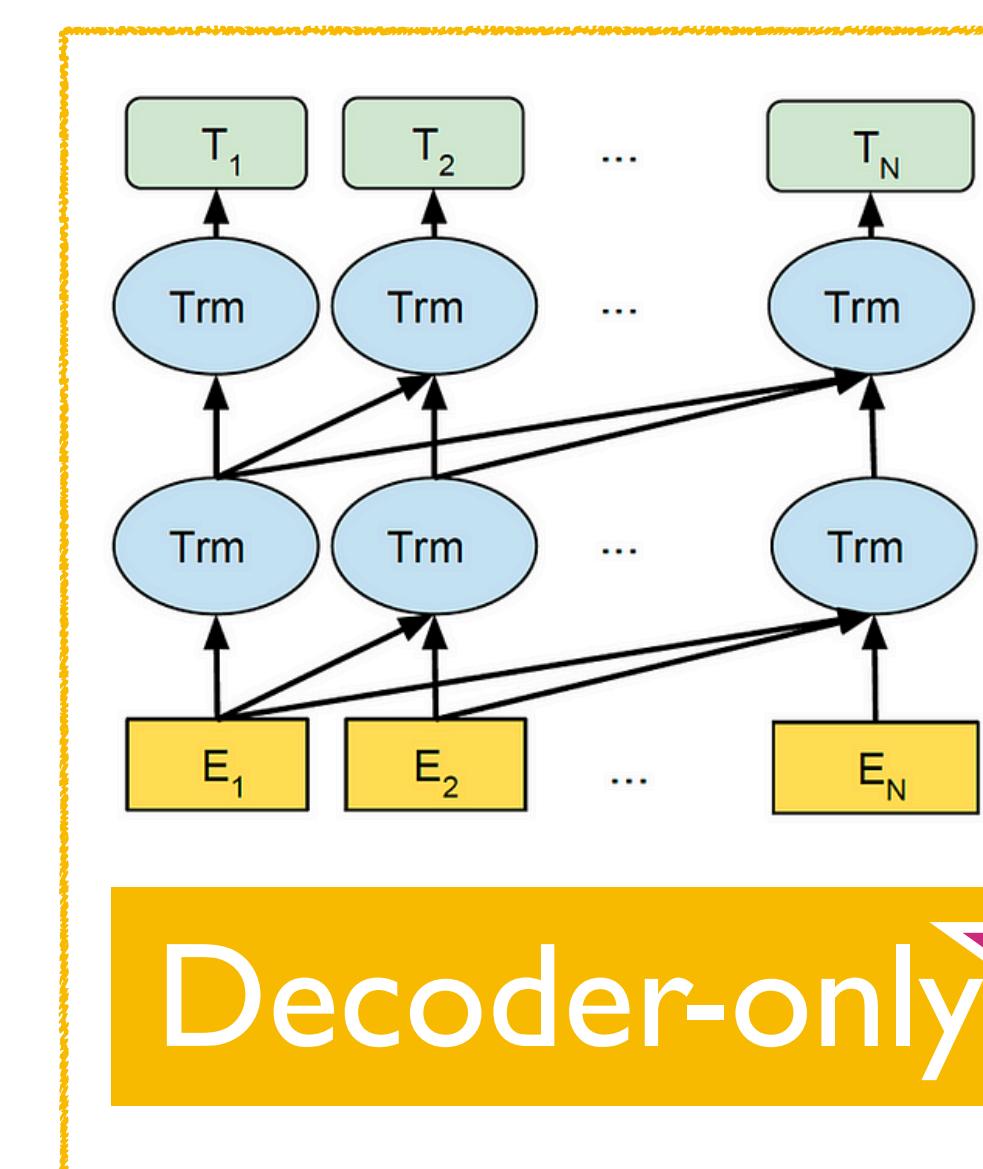
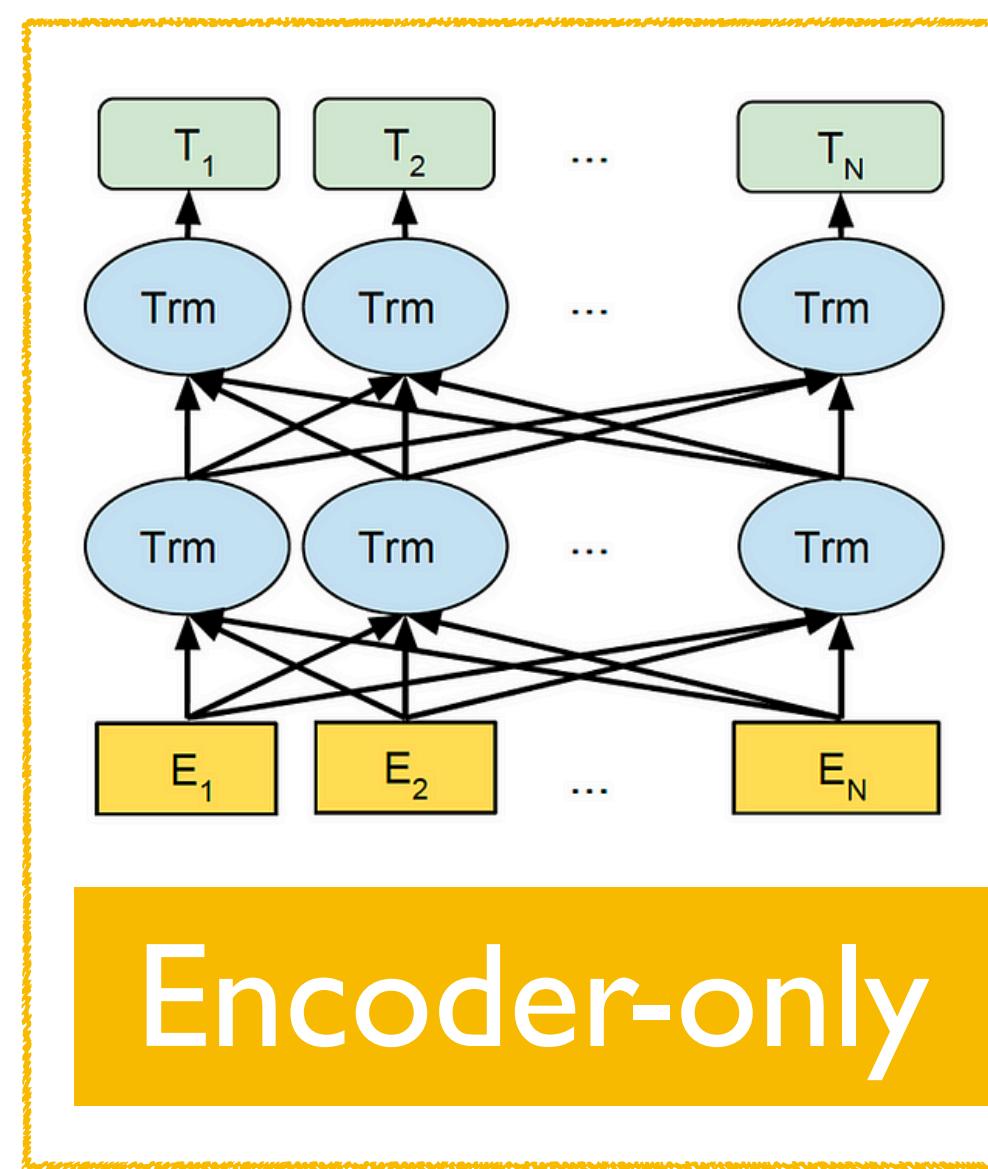


Encoder-decoder

# A language model (LM): Categories

Toronto  
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# A language model (LM): Prompting

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The capital city of Ontario is

LM

Toronto

Fact probing

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The capital city of Ontario is

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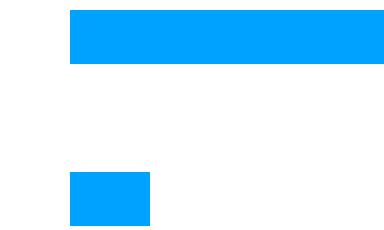
Toronto

Fact probing

Cheaper than an iPod. It was

LM

great  
terrible



Sentiment analysis

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Sentiment  
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“Hello” in French is

LM

Bonjourno

Translation

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Translation

I'm good at math.  $5 + 8 \times 12 =$

LM

101

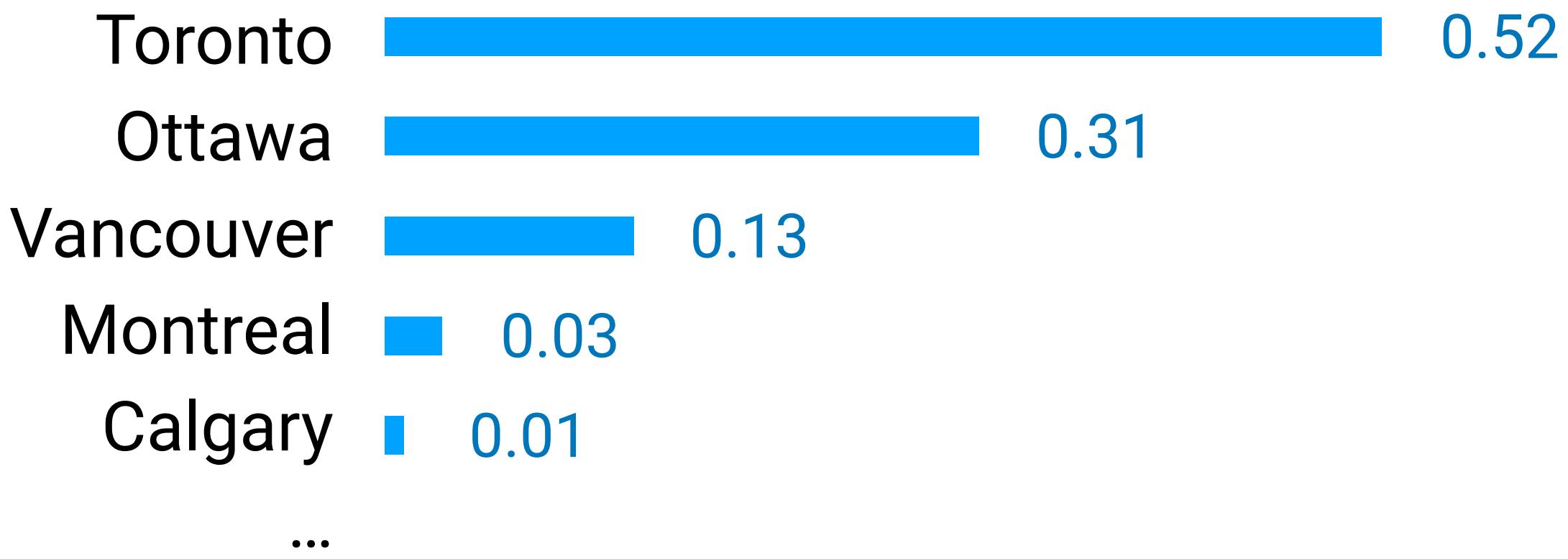
Arithmetic

# A language model (LM)

Often evaluated with

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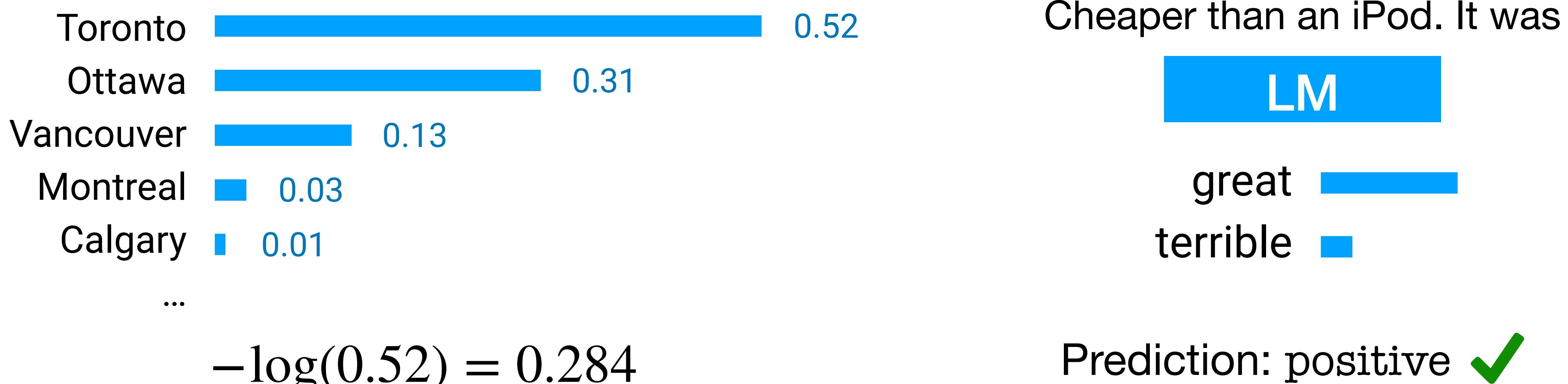


$$-\log(0.52) = 0.284$$

**Perplexity**

# A language model (LM)

Often evaluated with



**Perplexity**

**Downstream accuracy**

(Zero-shot or few-shot in-context learning,  
or fine-tuning)

(More in Section 5)

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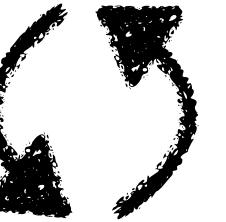
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# Typical LMs



The capital city of Ontario is **Toronto**



LM

# Training time

The capital city of Ontario is \_\_\_\_\_



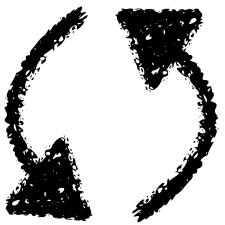
LM

# Test time

# Retrieval-based LMs



The capital city of Ontario is **Toronto**



LM

# Training time



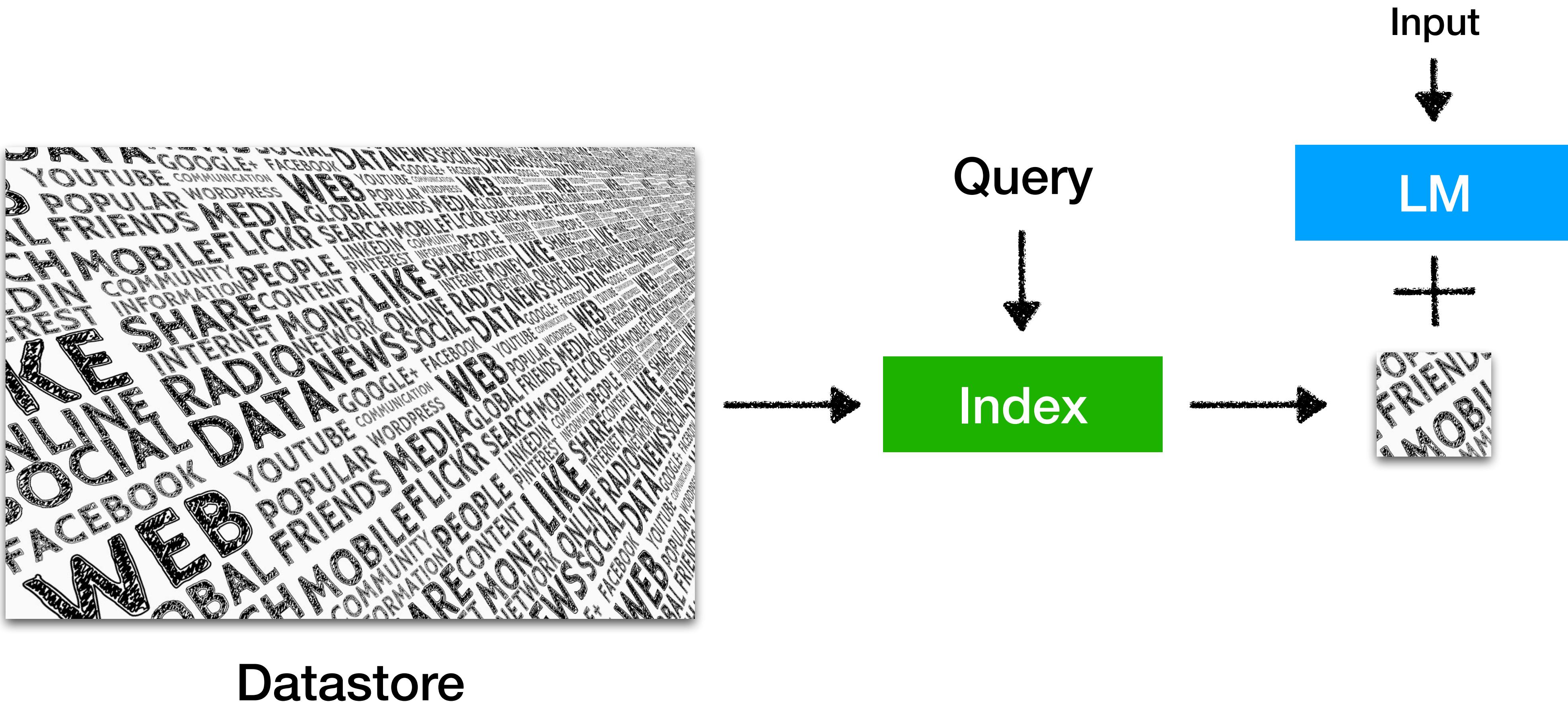
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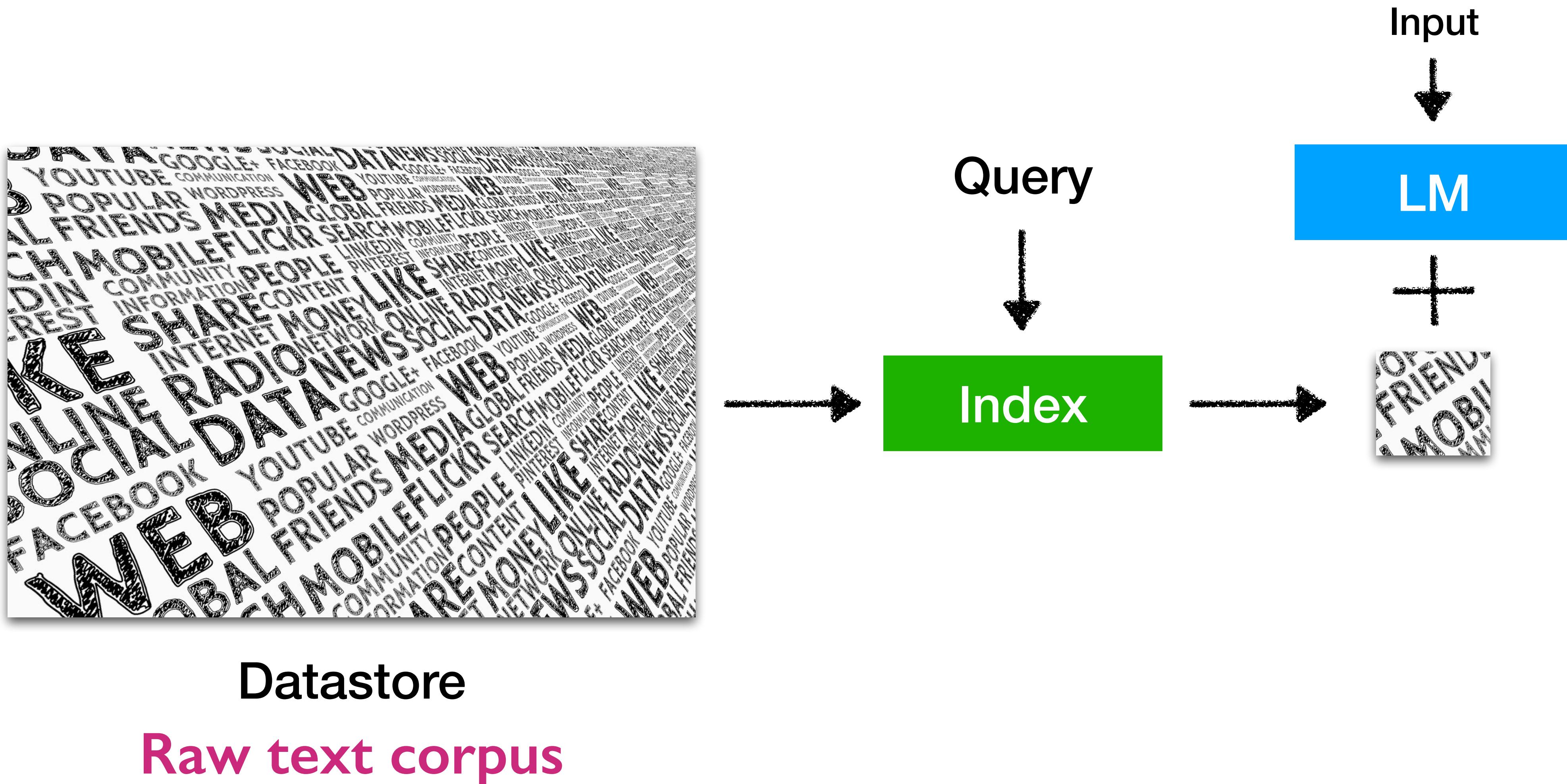
LM

# Test time

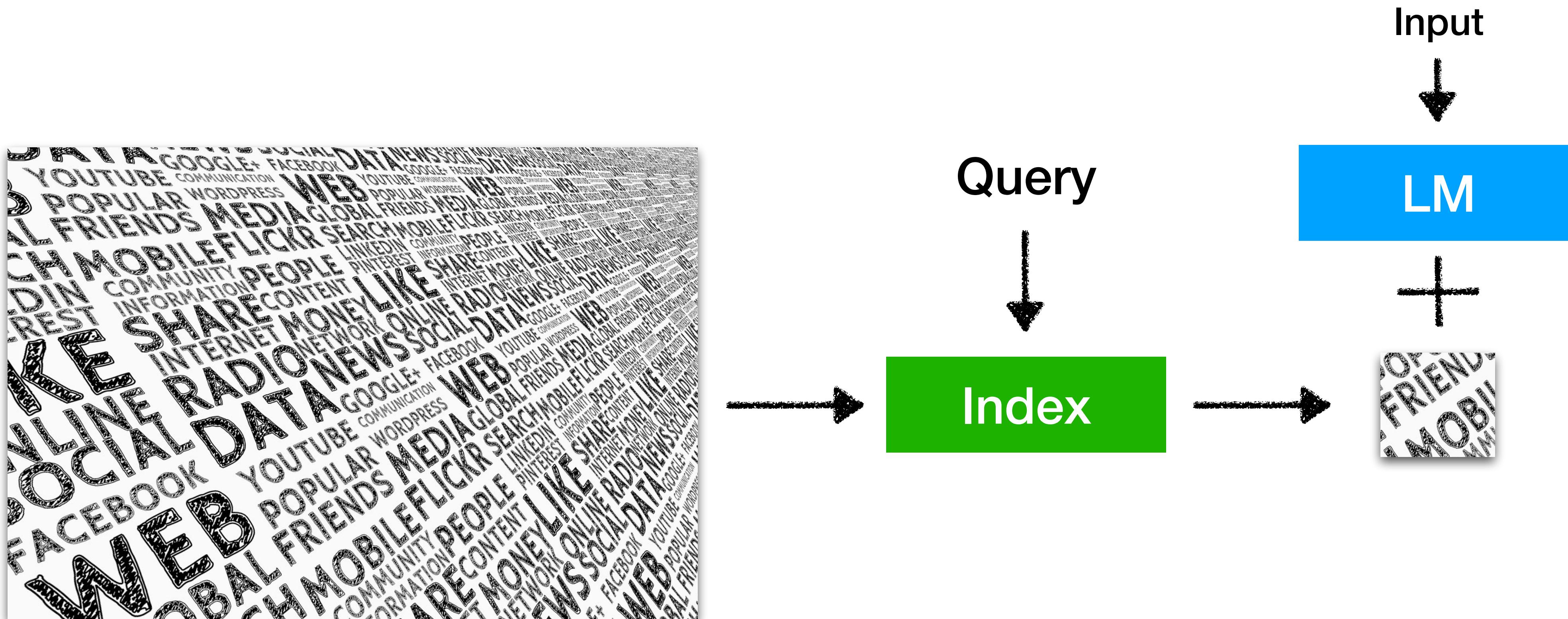
# Inference



# Inference: Datastore



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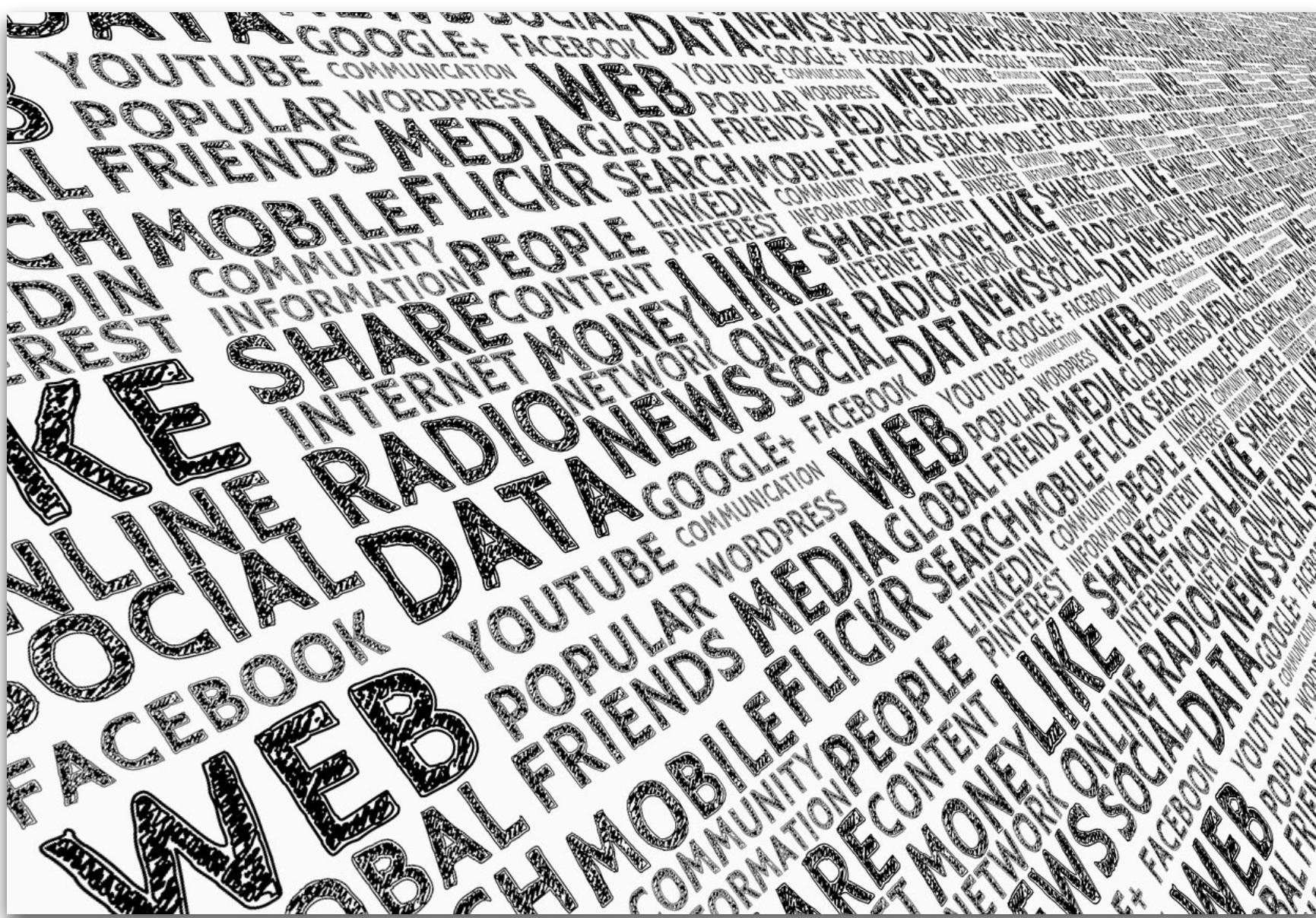


# Datastore

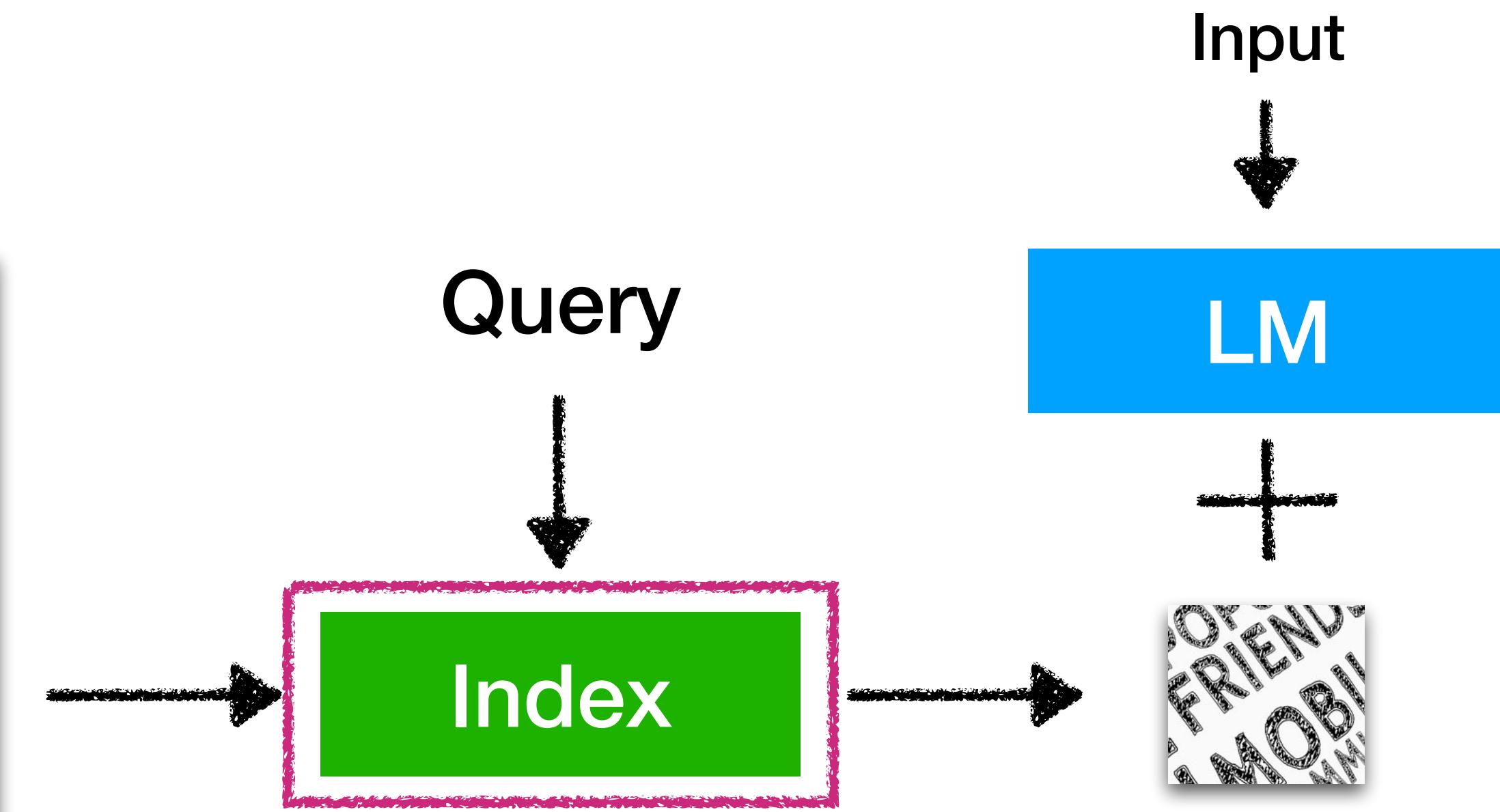
# Raw text corpus

At least billions~trillions of tokens  
Not labeled datasets  
Not structured data (knowledge bases)

# Inference: Index



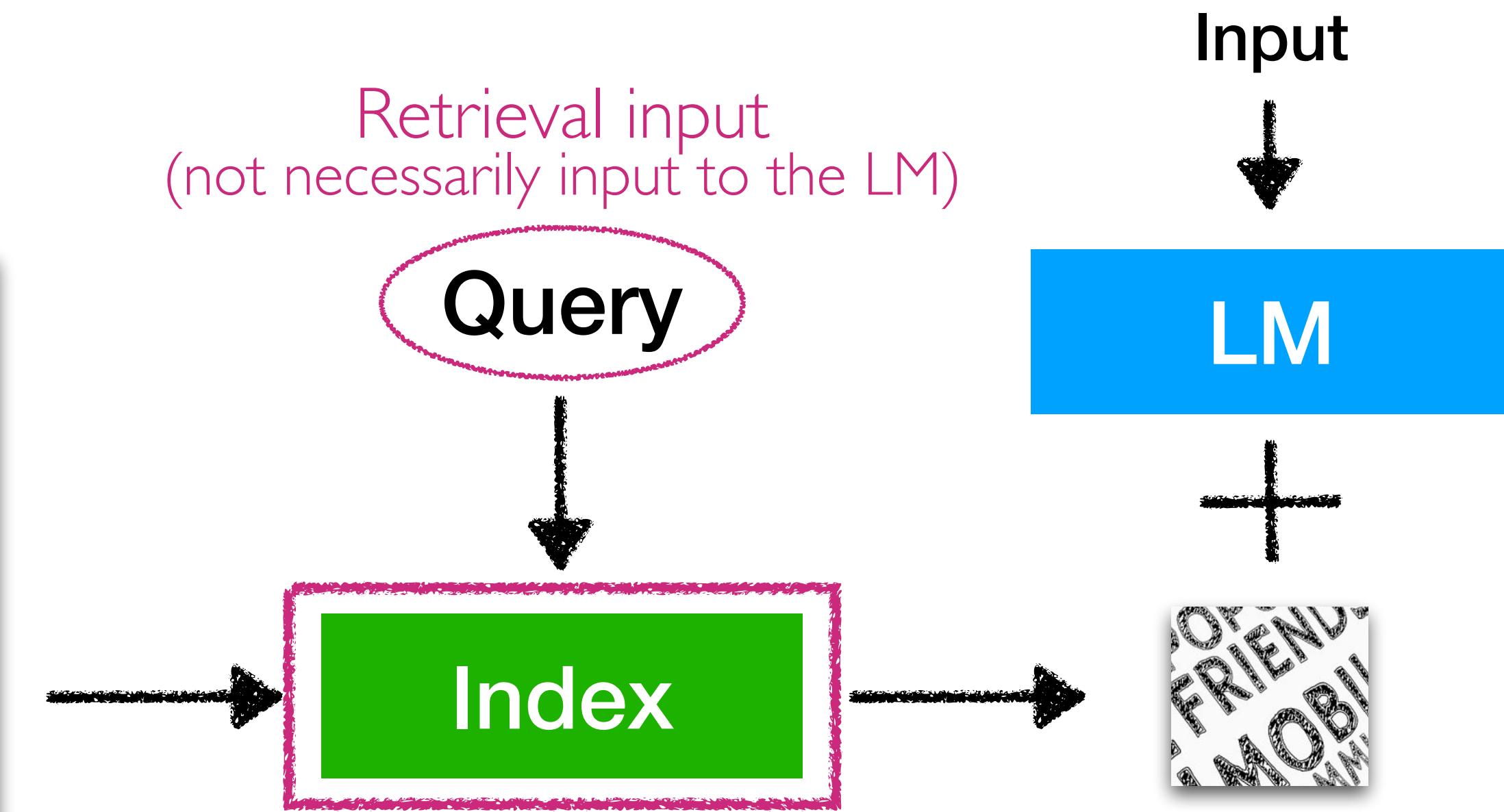
Datastore



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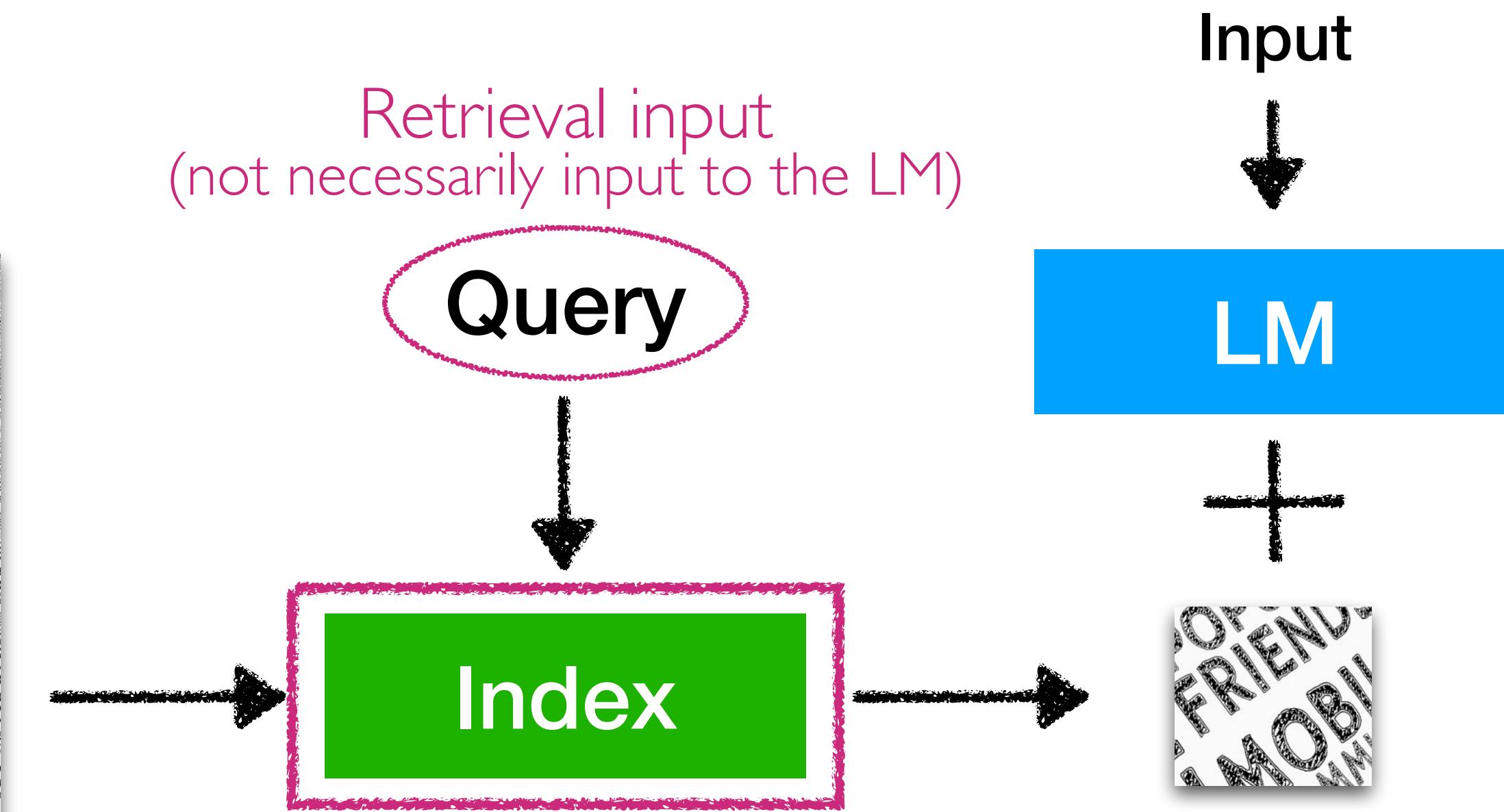
Datastore



# Inference: Index



Datastore



Find a small subset of elements in a datastore  
that are the most similar to the query

# Inference: Index

Goal: find a small subset of elements in a datastore  
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**sim**: a similarity score between two pieces of text

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Example

$$\text{sim}(i, j) = \frac{\text{tf}_{i,j} \times \log \frac{N}{\text{df}_i}}{\# \text{ of occurrences of } i \text{ in } j}$$

# of total docs  
# of docs containing  $i$

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Example  $\text{sim}(i, j) = \underline{\text{Encoder}(i)} \cdot \underline{\text{Encoder}(j)}$

Maps the text into an  $h$ -dimensional vector

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Example

$$\text{sim}(i, j) = \underline{\text{Encoder}(i)} \cdot \underline{\text{Encoder}(j)}$$

Maps the text into an  $h$ -dimensional vector

An entire field of study on how to get (or learn) the similarity function better  
(We'll see some in Section 4)

# Inference: Index

Goal: find a small subset of elements in a datastore  
that are the most similar to the query

**sim**: a similarity score between two pieces of text

**Index**: given  $q$ , return  $\arg \text{Top-}k_{d \in \mathcal{D}} \text{sim}(q, d)$  through fast nearest neighbor search

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$k$  elements from a datastore

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Goal: find a small subset of elements in a datastore  
that are the most similar to the query

**sim**: a similarity score between two pieces of text

Can be a totally separate research area on  
how to do this fast & accurate

---

**Index**: given  $q$ , return  $\arg \text{Top-}k_{d \in \mathcal{D}} \text{sim}(q, d)$  through fast nearest neighbor search

---

$k$  elements from a datastore

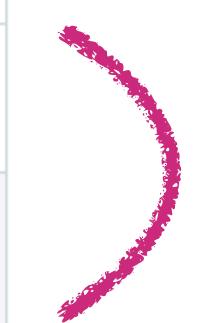
Software: FAISS, Distributed FAISS, SCaNN, etc...

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Method	Class name	<code>index_factory</code>	Main parameters	Bytes/vector	Exhaustive	Comments
Exact Search for L2	<code>IndexFlatL2</code>	"Flat"	<code>d</code>	<code>4*d</code>	yes	brute-force
Exact Search for Inner Product	<code>IndexFlatIP</code>	"Flat"	<code>d</code>	<code>4*d</code>	yes	also for cosine (normalize vectors beforehand)
Hierarchical Navigable Small World graph exploration	<code>IndexHNSWFlat</code>	"HNSW,Flat"	<code>d, M</code>	<code>4*d + x * M * 2 * 4</code>	no	
Inverted file with exact post-verification	<code>IndexIVFFlat</code>	"IVFx,Flat"	<code>quantizer, d, nlists, metric</code>	<code>4*d + 8</code>	no	Takes another index to assign vectors to inverted lists. The 8 additional bytes are the vector id that needs to be stored.
Locality-Sensitive Hashing (binary flat index)	<code>IndexLSH</code>	-	<code>d, nbits</code>	<code>ceil(nbites/8)</code>	yes	optimized by using random rotation instead of random projections
Scalar quantizer (SQ) in flat mode	<code>IndexScalarQuantizer</code>	"SQ8"	<code>d</code>	<code>d</code>	yes	4 and 6 bits per component are also implemented.
Product quantizer (PQ) in flat mode	<code>IndexPQ</code>	"PQx", "PQ" "M" "x" "nbites"	<code>d, M, nbits</code>	<code>ceil(M * nbites / 8)</code>	yes	
IVF and scalar quantizer	<code>IndexIVFScalarQuantizer</code>	"IVFx,SQ4" "IVFx,SQ8"	<code>quantizer, d, nlists, qtype</code>	<code>SQfp16: 2 * d + 8, SQ8: d + 8 or SQ4: d/2 + 8</code>	no	Same as the <code>IndexScalarQuantizer</code>
IVFADC (coarse quantizer+PQ on residuals)	<code>IndexIVFPQ</code>	"IVFx,PQ" "y" "x" "nbites"	<code>quantizer, d, nlists, M, nbits</code>	<code>ceil(M * nbites/8)+8</code>	no	
IVFADC+R (same as IVFADC with re-ranking based on codes)	<code>IndexIVFPQR</code>	"IVFx,PQy+z"	<code>quantizer, d, nlists, M, nbits, M_refine, nbits_refine</code>	<code>M+M_refine+8</code>	no	

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Exact Search

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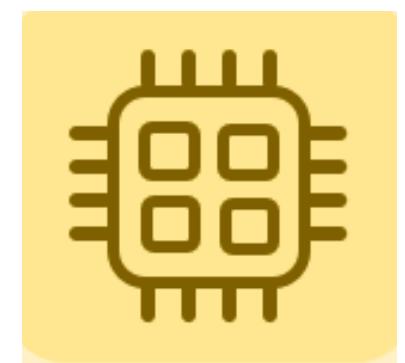
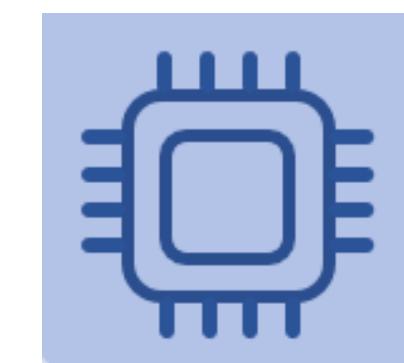
Exact Search

Approximate Search  
(Relatively easy to scale to ~1B elements)

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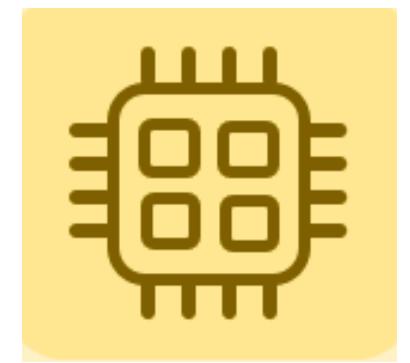
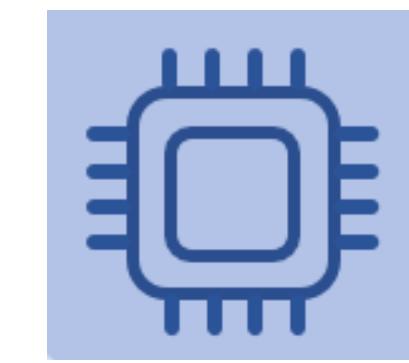
CPU vs. GPU

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Locality-Sensitive Hashing (binary flat index)	<code>IndexLSH</code>	-	<code>d, nbits</code>	<code>ceil(nbites/8)</code>	yes	optimized by using random rotation instead of random projections
Scalar quantizer (SQ) in flat mode	<code>IndexScalarQuantizer</code>	"SQ8"	<code>d</code>	<code>d</code>	yes	4 and 6 bits per component are also implemented.
Product quantizer (PQ) in flat mode	<code>IndexPQ</code>	"PQx", "PQM"x"nbits"	<code>d, M, nbits</code>	<code>ceil(M * nbites / 8)</code>	yes	
IVF and scalar quantizer	<code>IndexIVFScalarQuantizer</code>	"IVFx,SQ4" "IVFx,SQ8"	<code>quantizer, d, nlists, qtype</code>	<code>SQfp16: 2 * d + 8, SQ8: d + 8 or SQ4: d/2 + 8</code>	no	Same as the <code>IndexScalarQuantizer</code>
IVFADC (coarse quantizer+PQ on residuals)	<code>IndexIVFPQ</code>	"IVFx,PQ"y"x"nbits"	<code>quantizer, d, nlists, M, nbits</code>	<code>ceil(M * nbites/8)+8</code>	no	
IVFADC+R (same as IVFADC with re-ranking based on codes)	<code>IndexIVFPQR</code>	"IVFx,PQy+z"	<code>quantizer, d, nlists, M, nbits, M_refine, nbits_refine</code>	<code>M+M_refine+8</code>	no	

Exact Search



CPU vs. GPU

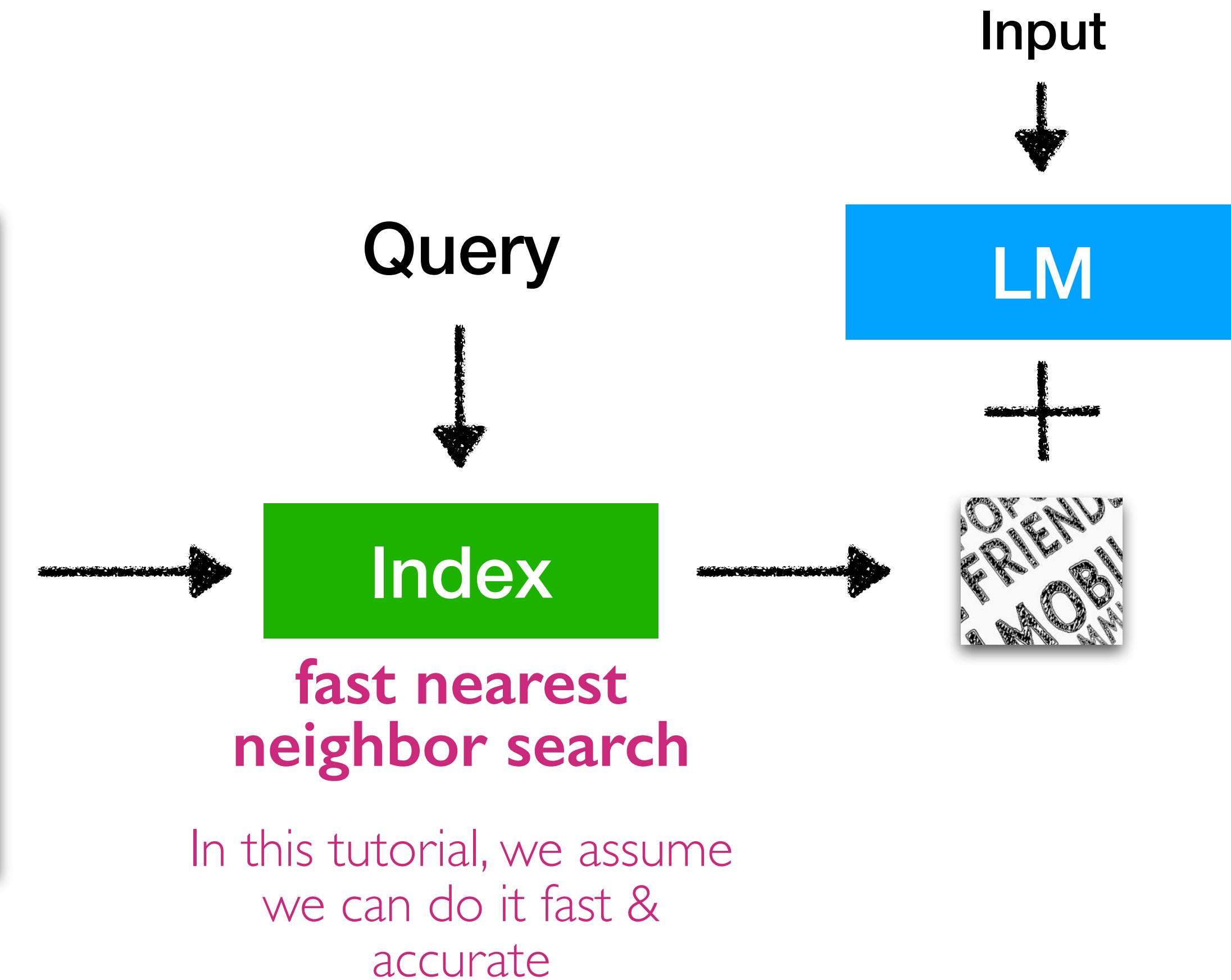
Approximate Search  
(Relatively easy to scale to ~1B elements)

More info: <https://github.com/facebookresearch/faiss/wiki>

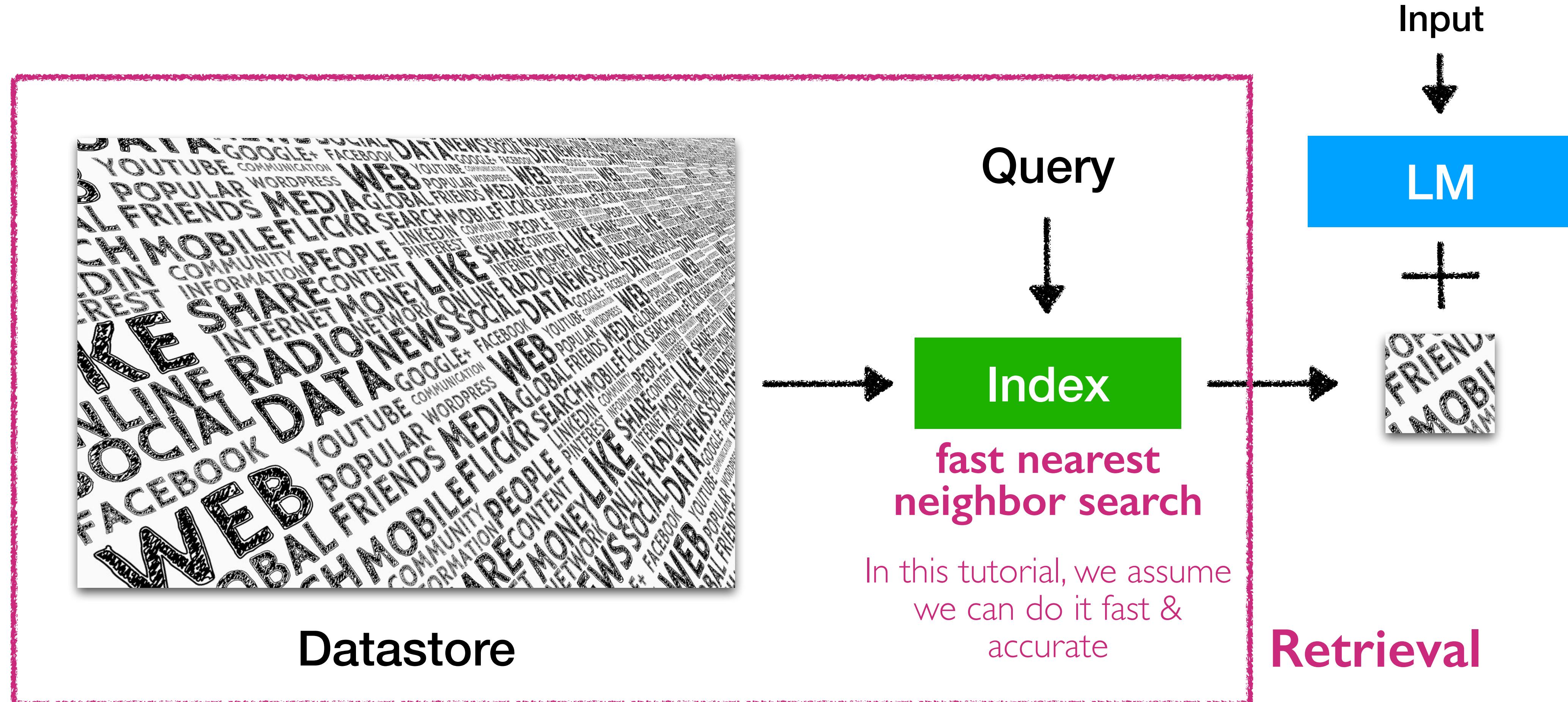
# Inference: Search



Datastore



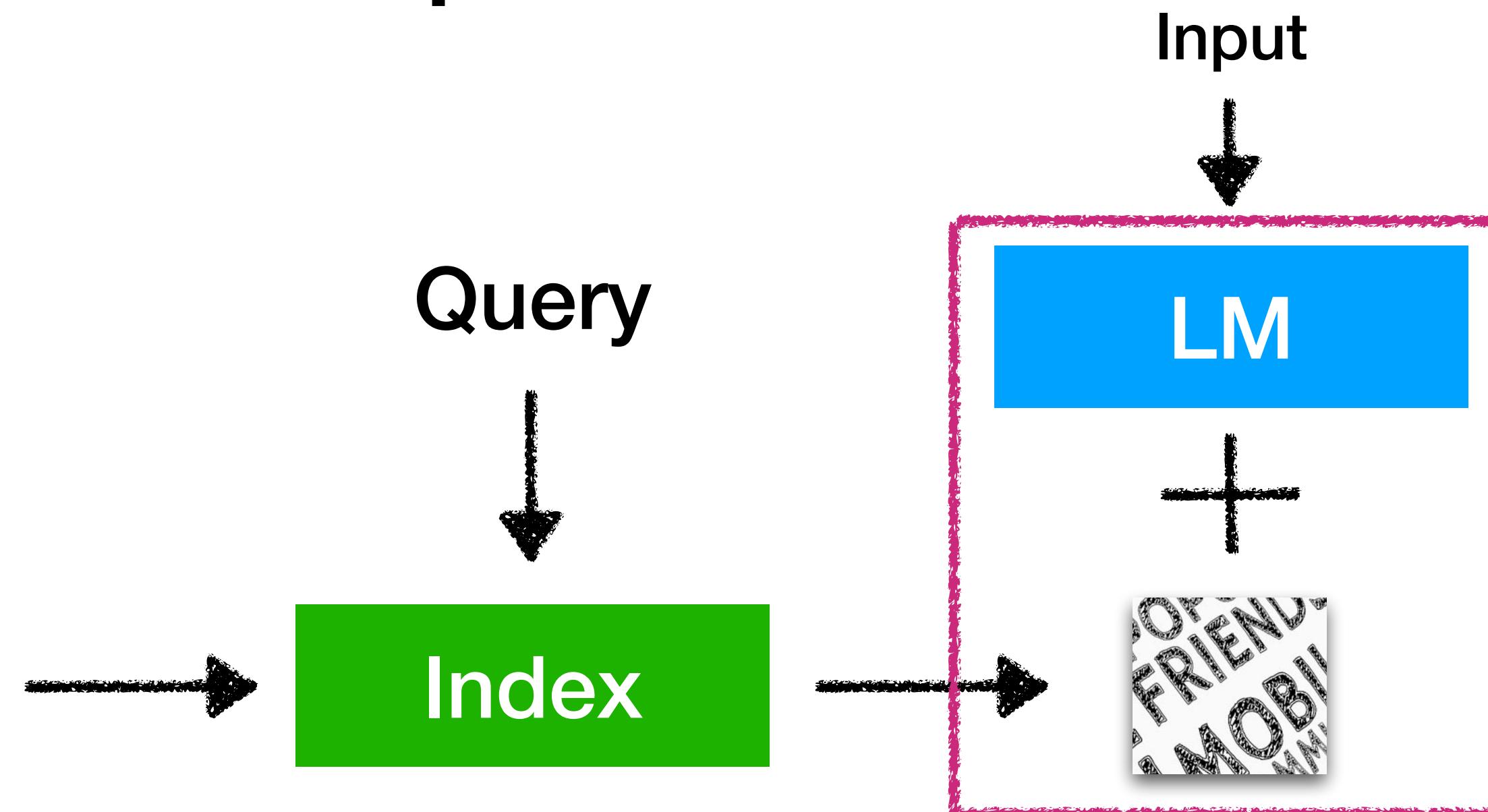
# Inference: Search



# Inference: Incorporation



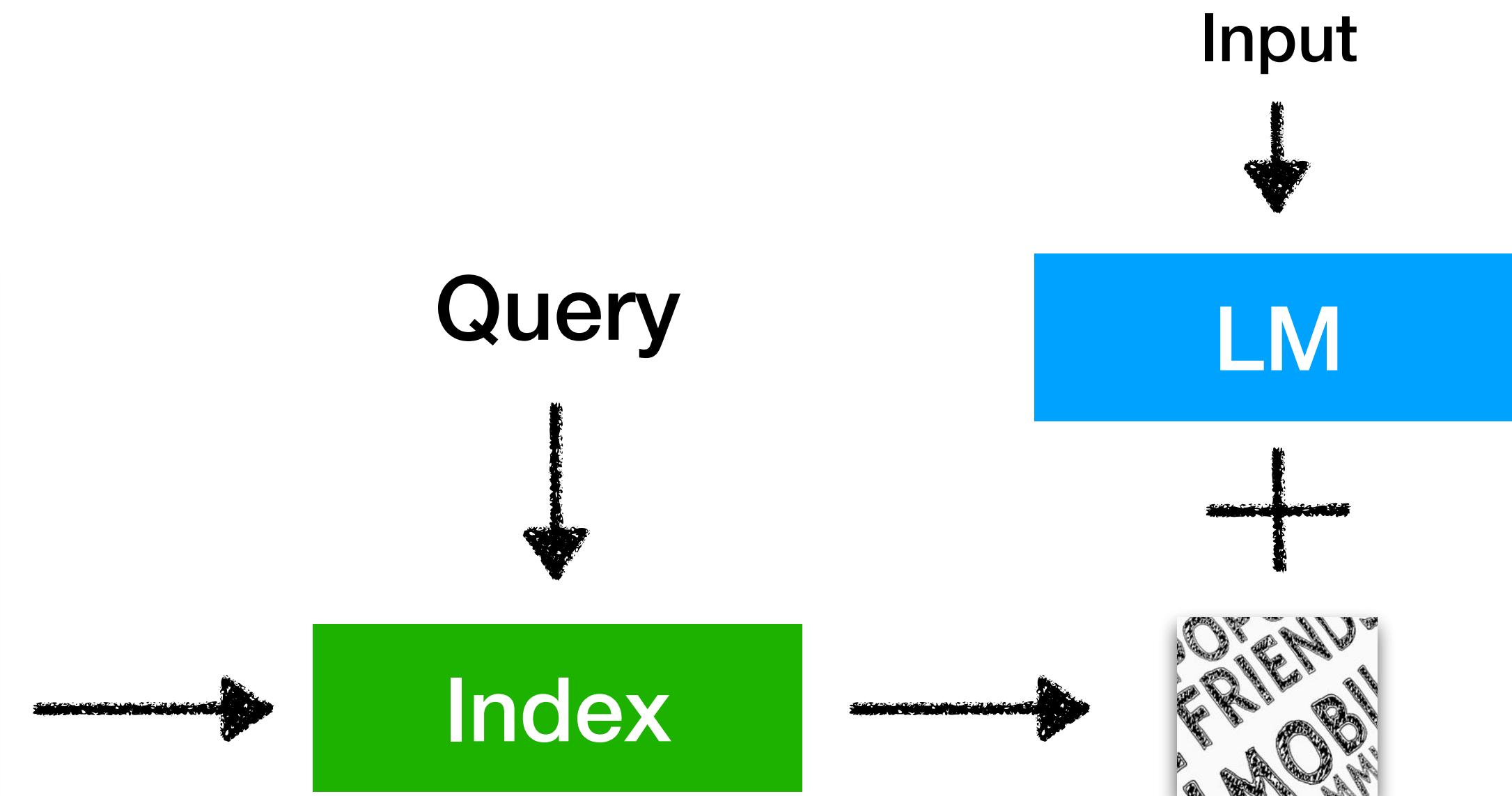
Datastore



# Questions to answer



Datastore



# Questions to answer

What's the query &  
when do we retrieve?

Input

LM

Query

+

Index

FOR  
FRIENDS  
MOBILE



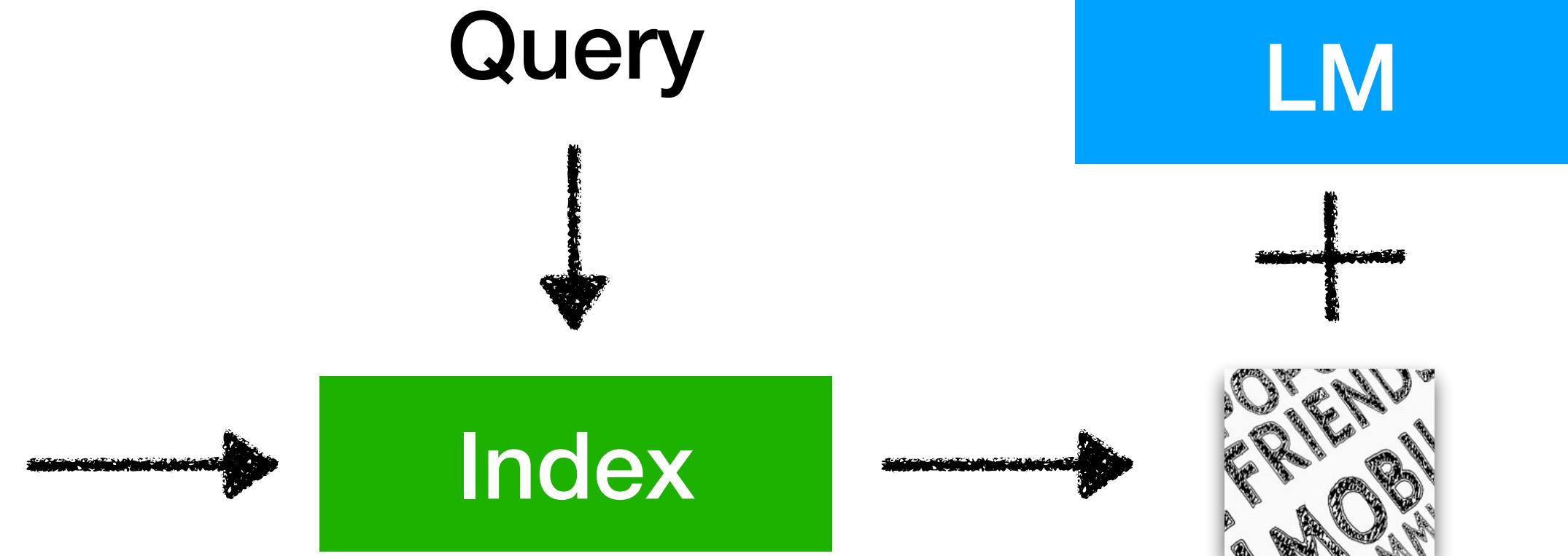
Datastore

# Questions to answer



Datastore

What's the query &  
when do we retrieve?



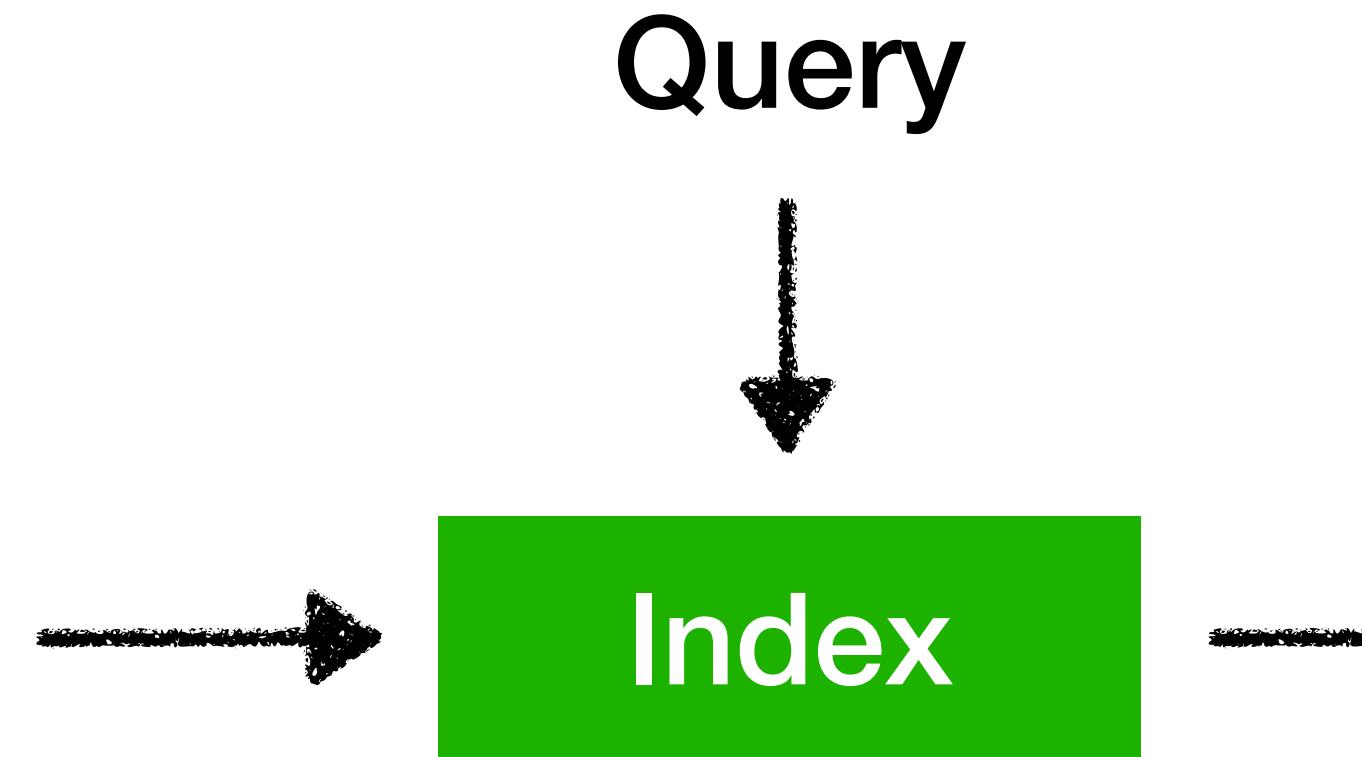
What do we  
retrieve?

# Questions to answer

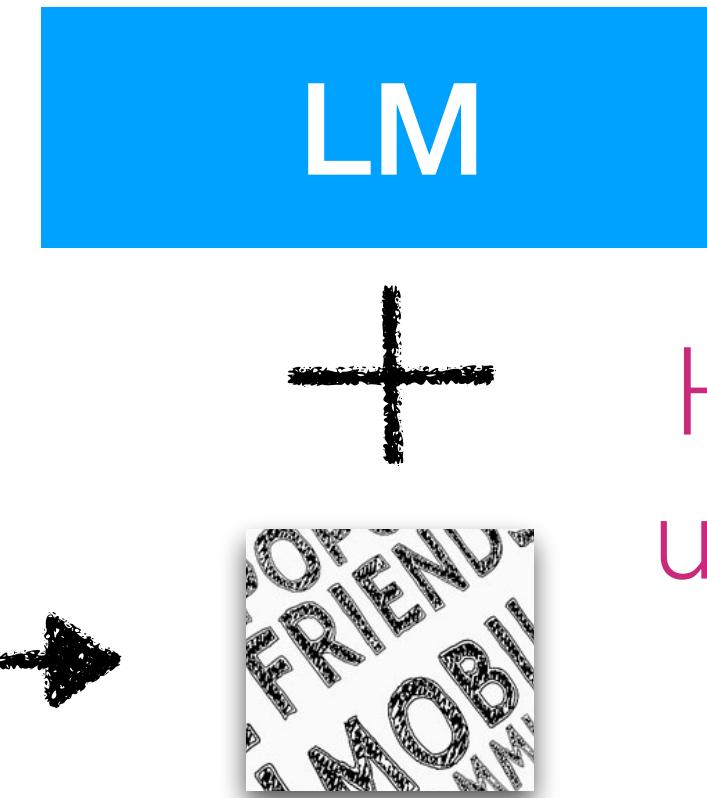


Datastore

What's the query &  
when do we retrieve?



Input



How do we  
use retrieval?

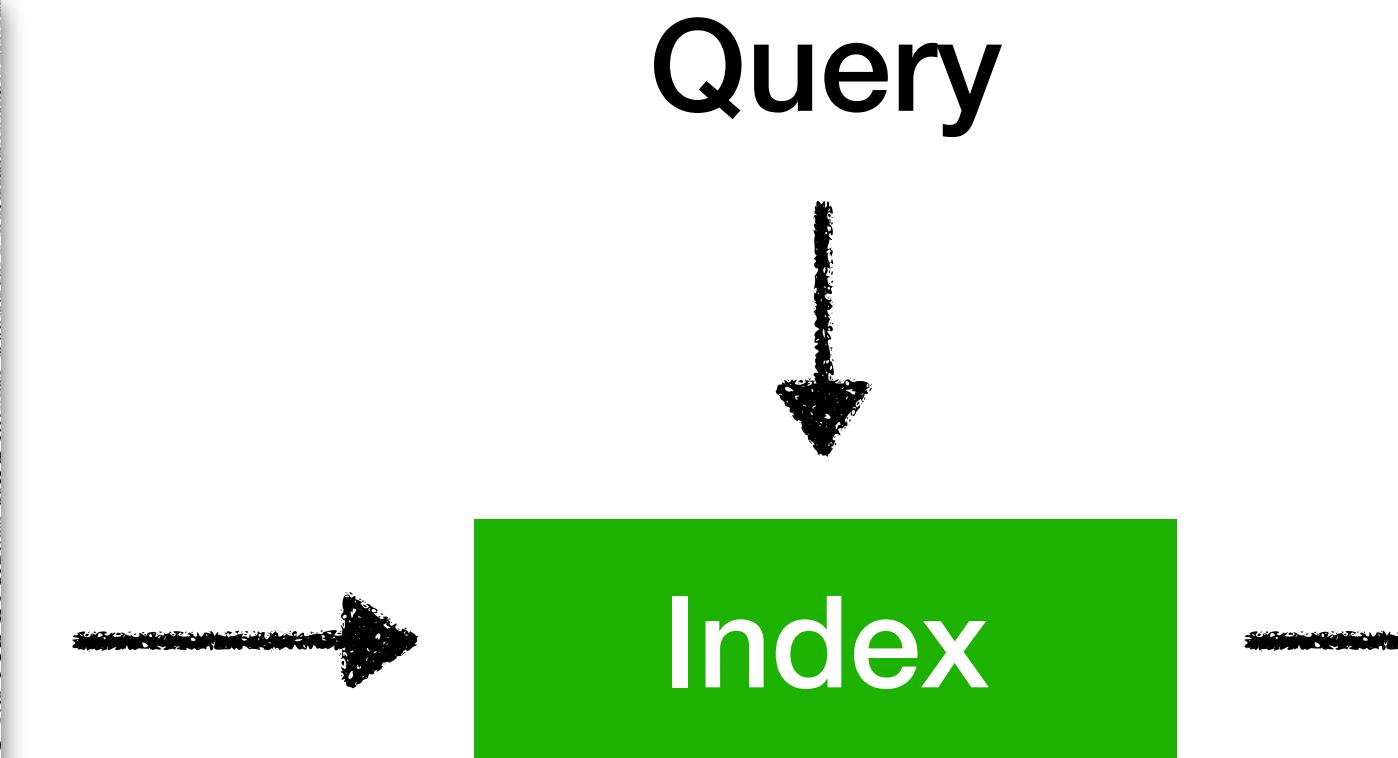
What do we  
retrieve?

# Questions to answer



Datastore

What's the query &  
when do we retrieve?



Input

LM

+  
+ FRIENDS  
+ MOBILE

How do we  
use retrieval?

What do we  
retrieve?

We'll answer these questions in Section 3!

# Notations



# Datastore

D

