Date :

Assignment 5

Q1) Let the scene be marked by spatial variables (x,y) (coordinates)

$g_1(x,y)$ and $g_2(x,y)$ are the final images.
outside in focus reflection in focus.

Given equations: $g_1(x,y) = f_1(x,y) + h_2(x,y) * f_2(x,y)$
$g_2(x,y) = f_2(x,y) + h_1(x,y) * f_1(x,y)$

'h_1', 'h_2' are 'blurring kernels' acting on the actual scene/reflection, and are known to us.

To solve for $f_1(x,y)$ and $f_2(x,y)$, take the Fourier transforms of both equations. (Denote by capital letters. Eg: $\mathcal{F}(k(x,y)) = K(u,v)$

$\therefore$ $G_1(x,y) = F_1(x,y) + \mathcal{F}(h_2 * f_2)$ $((x,y) \Rightarrow (u,v))$
$G_2(x,y) = F_2(x,y) + \mathcal{F}(h_1 * f_1)$

Using the convolution theorem $\mathcal{F}(f*g) = \mathcal{F}(f).\mathcal{F}(g)$

~~$G_1(x,y) = F_1(x,y) + H_2(x,y) F_2(x$~~

$G_1(u,v) = F_1(u,v) + H_2(u,v) F_2(u,v)$ — 1

$G_2(u,v) = F_2(u,v) + H_1(u,v) F_1(u,v)$ — 2

$$\therefore \quad F_2(u,v) = \frac{H_1(u,v)\,G_1(u,v) - G_2(u,v)}{H_1(u,v)\,H_2(u,v) - 1}$$

$\therefore$ After isolating $F_2(u,v)$, take the inverse fourier transform.

$$\therefore \quad f_2(x,y) = \mathcal{F}^{-1}\left(F_2(u,v)\right)$$

$$\boxed{f_2(x,y) = \mathcal{F}^{-1}\left(\frac{H_1(u,v)\,G_1(u,v) - G_2(u,v)}{H_1(u,v)\,H_2(u,v) - 1}\right)} \quad (3)$$

Similarly, $H_2(u,v) \times (2) - 1$:

we obtain the following solution by symmetry :

$$f_1(x,y) = \mathcal{F}^{-1}\left(F_1(u,v)\right)$$

$$\Rightarrow \quad \boxed{f_1(x,y) = \mathcal{F}^{-1}\left(\frac{H_2(u,v)\,G_2(u,v) - G_1(u,v)}{H_1(u,v)\,H_2(u,v) - 1}\right)} \quad (4)$$

In these equations, 3 and 4, we observe the presence of the term "$H_1(u,v)H_2(u,v) - 1$" in the denominator. We know that $h_1(x,y)$ and $h_2(x,y)$ are blurring kernels, so they remove edges and sharp features.

Edges are 'high-frequency' components of the image. This implies that the blurring kernels h_1 and h_2 are actually low-pass filters in the fourier domain.

For that reason, the low frequencies in the image pass through, and may tend to a value of unity (1) when evaluating $H_1(u,v)$ and $H_2(u,v)$. Hence, the denominator may have very small values, and the term within the argument of $\mathcal{F}^{-1}$ will blow up.

This is not an issue if the images are exact, we will still get the solution using our formula (assuming the equations $g_1(x,y) = f_1(x,y) + h_2(x,y) * f_2(x,y)$ and the other one) are valid, f_1 and f_2 are obtained. However, if we observe even small perturbations / noise in the image parts corresponding to low frequencies, the Fourier inverse argument is noisy, susceptible to change, and hence the ~~orig~~ actual scenes f_1 and f_2 may be noisy in their reconstruction.