CS663 Assignment 5

Q2) The 'h' kernel in spatial domain will be a 1 dimensional Sobel Operator $[-1, 0, 1]$. This performs differentiation of intensities by giving $g(x) = f(x+1) - f(x-1)$ for $x \in [1, N]$ with proper boundary conditions (i.e $f(0) = f(N+1) = 0$ or cyclic continuity or similar such boundary condition).

Now, DFT of g is $G(u) = \left(e^{\frac{j2\pi u}{N}} - e^{-\frac{j2\pi u}{N}}\right) F(u)$ which gives

$$F(u) = \frac{G(u)}{e^{\frac{j2\pi u}{N}} - e^{\frac{-j2\pi u}{N}}} = \frac{G(u)\, e^{\frac{j2\pi u}{N}}}{\left(e^{\frac{j4\pi u}{N}} - 1\right)}.$$

This expression leads to problem when denominator is 0. Thus for $u=0$, that is DC component of F, is not defined with this formula. Thus some assumption or pre defined DC component needs to be set. This was caused since we had originally assumed $f(0)$ and $f(N+1)$ values.

For 2D image, similarly $h(u,v)$ will be convolution of 1D versions. Similiar to above situation, if we have I_x and I_y's DFT, it is not easy to get back image $f(x,y)$. DFT of I_x is

$F_x(u,v) = \left(e^{\frac{j2\pi u}{N}} - e^{\frac{-j2\pi u}{N}}\right) F(u,v)$ and of I_y is

$F_y(u,v) = \left(e^{\frac{j2\pi v}{N}} - e^{\frac{-j2\pi v}{N}}\right) F(u,v)$. For $F_x(u,v)$ frequency components with $u=0$ causes problem and for $F_y(u,v)$, $v=0$ causes the problem. Even if individually these conditions were known, for DC component $u=v=0$, the problem would remain. Here too, $F(0,0)$ needs to be known as differentiating constant will anyways set it to 0 and there is no way of knowing constant from the gradients.