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Q2 We have to maximise $f^t C f$ subject to constraints:

- $f^t e = 0$
- $f^t f = 1$

$\therefore$ Using the technique of Lagrange Multipliers:

$$E(f) = f^t C f - \lambda(f^t f - 1) - \mu(f^t e)$$

$\rightarrow$ Taking derivative wrt f and setting to 0

$$2C\vec{f} - \lambda(2\vec{f}) - \mu\vec{e} = 0 \quad \text{— i)}$$

$\rightarrow$ Multiplying RHS & LHS by e^t:

$$2e^t C\vec{f} - \lambda(2e^t f) - \mu e^t e = 0$$

$$\rightarrow 2e^t C f - \mu = 0 \quad \text{— ii)}$$

$\rightarrow$ Now since e is an eigenvector of S ($S = (N-1)C$)

$\rightarrow Se = \lambda_1 e$

$\rightarrow f^t Se = f^t \lambda_1 e = \lambda_1 f^t e = 0$

$\rightarrow f^t (Se) = (Se)^t f = e^t S f = 0$

Since $S = (N-1)C$, we have $e^t C f = 0$

$\therefore$ From ii), we have $\underline{\mu = 0}$

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→ Hence from i)

$$C\vec{f} = \lambda \vec{f}$$

i.e. $\vec{f}$ is an eigenvector of C

→ $f^t C \vec{f} = \lambda f^t f = \lambda$

Hence maximizing $f^t C f$ is equivalent to maximising the eigenvalue of C and choosing the eigenvector corresponding to it.

→ Thus, we choose f to be the eigenvector which maximizes the eigenvalue of C, among all eigenvectors which are perpendicular to e.

→ But e already corresponds to the eigenvector of C with the maximum eigenvalue

→ Since all eigenvalues are distinct:
f is the eigenvector of C with the <u>2nd largest eigenvalue</u>.