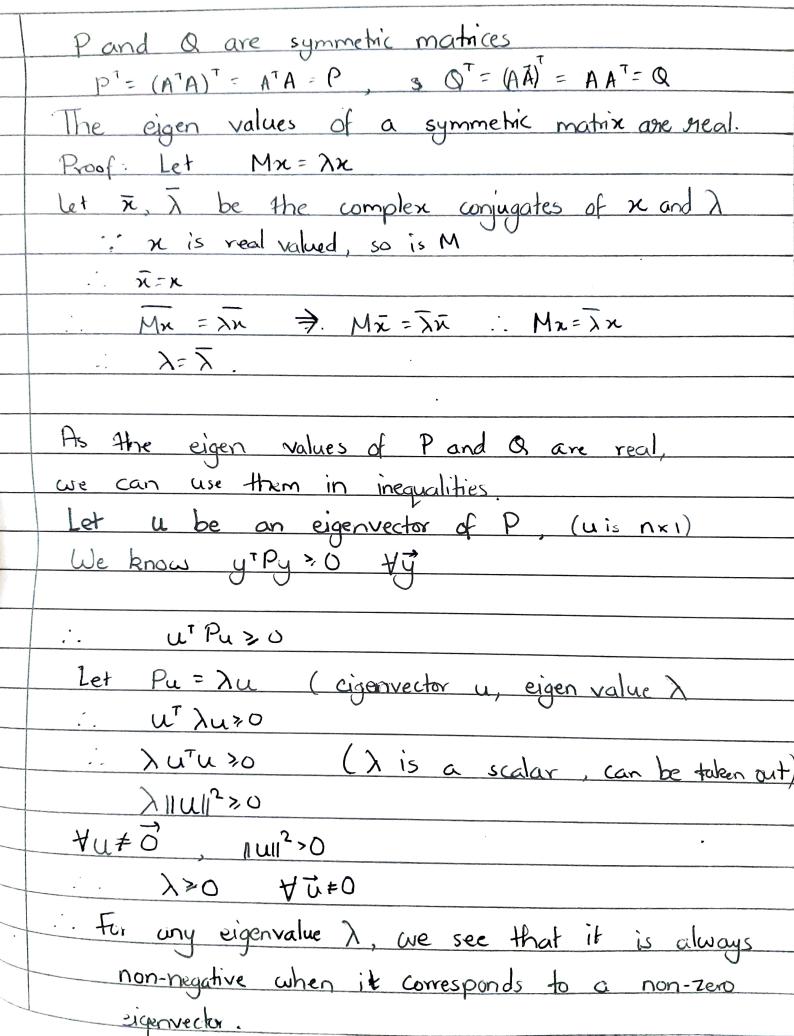
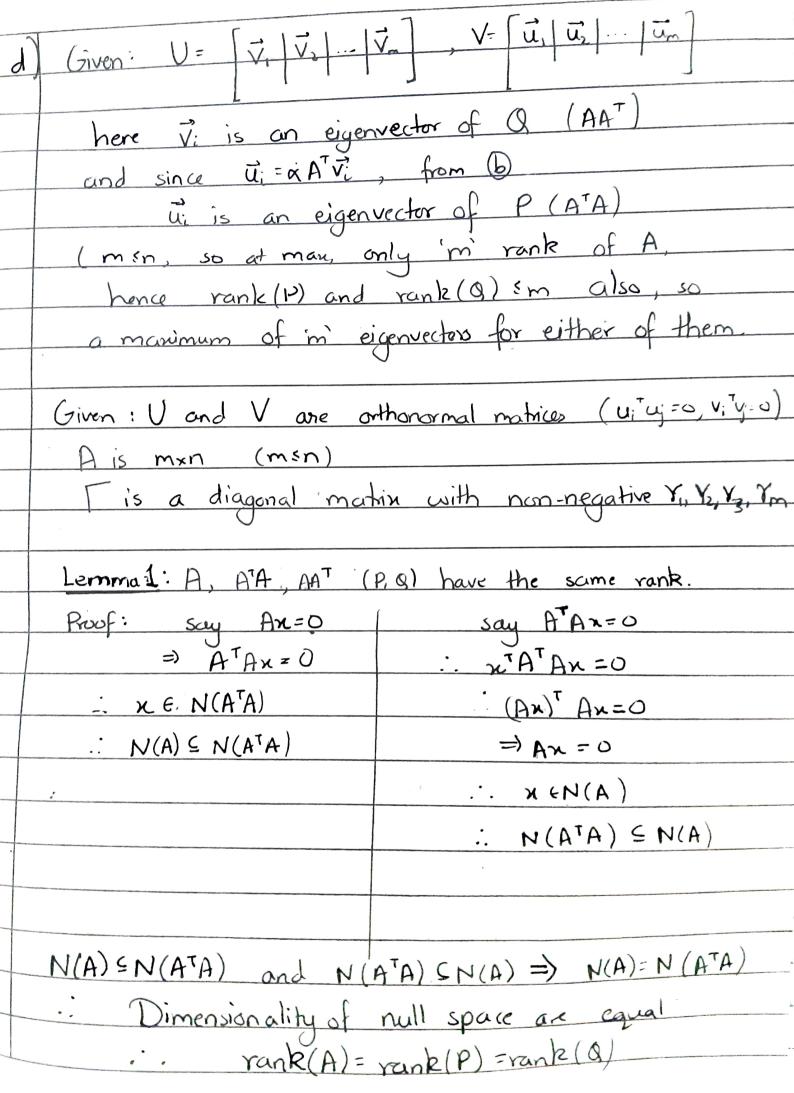
	Date :
	Assignment 4
3)	A=min matin (min).
1	P=ATA. Q=AAT (all values real).
	$(n \times n)$ $(m \times m)$
a)	Let y be an nxl vector.
	: To prove, y'Py > 0
	We have upy
	We have ytpy = yt (ATA)y = ytATAy
	$= (Ay)^{T}(Ay)$
	Now, fly is an mx1 vector.
	$(Ay)^{T}(Ay) = \ Ay\ ^{2}$
,	:- (Ay) T (Ay) = Ay ² Ay ² >0 (sum of squares of real numbers)
And the second s	y Py >0, Hence proved
	Let z be an mx1 vector.
	We have z'Qz (a scalar).
	$= z^{T} A A^{T} z$
	$= (A^{T}z)^{T}(A^{T}z)$
	Now, Atz is an nxl vector
	$\frac{(A^{T}z)^{T}(A^{T}z)}{(A^{T}z)^{T}(A^{T}z)} = \frac{\ A^{T}z\ _{2}^{2}}{\ A^{T}z\ _{2}^{2}}$
	11 Azll2 >0 (sum of squares of real numbers)
	$z^{\dagger}Qz>0$
	2 42 7 0



we know: z'Qz>0 \Z -. let V be an eigenvector of Q with eigenvalue (Qv=mv) ZTQZ30 => ZTA: VTQJ30 ·: V TUV > 0 .. M VTV >0 => M 11V112 >0 $\int \vec{v} + \vec{0} \qquad ||v||^2 > 0$ 0 x M ... All the eigenvalues of a are also non-negative

b) i) Pc	$u = \lambda u$ (P:n×n, $u = n \times 1$)
	$A^{T}Au = \lambda u$
	multiply by A (valid, as we have nx1 vectors
	on both sides)
.:	AATAU = A(Xu)
	$AA^{T}Au = \lambda Au$ (as λ is a scalar.
	: AAT=Q, Au is an mx1 vector, say u'
	$Qu' = \lambda u'$
	· · U' is an eigenvector of Q with value)
	··· U' is an eigenvector of Q with value > Au is an eigenvector of Q, eigenvalue >.
ii) (Qv = uv (Q = mxm, V = mx1)
1	AATV = UV
1	nuttiply by AT (valid as we have mx) vectors on
1	outh sides and AT is (nxm)
1	ATAATV = ATUV
	(ATA) (ATV) = u (ATV) (u is scalar)
	ATA = P and ATV is an nx1 vector
	$P(A^{T}V) = \mu(A^{T}V)$
	ATV is an eigenvector of P with eigenvalue u.
	11 V 13 an eigenverse of the eigenverse jest
	i has 'n' elements
1	
	Thas in elements.

c)	Given: Vi is an eigenvector of Q
	Let QVi= uivi (Mi is a scalar)
	·
	Given: ui = ATVi => ui is an 1x) vector
	11 ATVILL2 (nxmxmx))
	Premultiply by mxn matrix A:
	Au: = AATVi = Qvi
	11 ATVilla
	Aui = ui Vi
	II A Ville
	from part @, u:>0, as eigenvalues, of P&Q are
	non-negative.
	11 ATVIII2 > 0, as it is the sum of squares of
	real numbers
	\therefore Let $\aleph_i = \mu_i$
	リダンに
	Au; = Xivi, where Y; >0
	Hence Proved
and the second	



1	
	We know that rank(p), rank(q) = m, because
	there exist in distinct, orthonormal eigenvectors.
	: We have, rank (A)= m also (full row rank)
	Hence, we have: Ui = ATVi \iti \iti \inm
	11ATVI12
	(non of the values are O).
	Vi is mxl and u; is nxl.
_	We have the system of 'm' equations:
-	ui = ATVi ie 1m.
ndama.	NA"V; II
	.: la .: ATVi= ui ATVi 2
_	Writing these in matrix form
	ATU = VI
	(Dxm, x (mxm)= (nxm)x(mxm)
	VIIV2 Vm are columns of U
	U,, U Um are columns of V
	Is a diagonal matrix with non-negative
	Values 11A Villa (norms are non-negative)
	(let these be the Yi's required).
	Vis and Ui's are unit vectors.
_	U, V are orthonormal.
	$U^{T}U = \left\langle v_{i}, v_{i}, v_{i} \right\rangle = I$
- Name	
	$U^{T} = U^{T}$

ATU = VT Taking transpose: (ATU)T = (VI)T .. UTATT = CTVT UTA = TVT (as T is symmetric; T= TT) · Premultiply by U UUTA = UIVT Bet UT=U-1 · UUTA = UTVT IA=UTVT Hence, there exists a singular value decomposition for the matrix A, with the required

matrices U, T and V.