Introduction:

- Considering simple formulation for stabilizing an inverted pendulum where the state space is defined as.

$$x(t) = [x(t), \dot{x}(t), \theta(t), \dot{\theta}(t)]^T$$, where

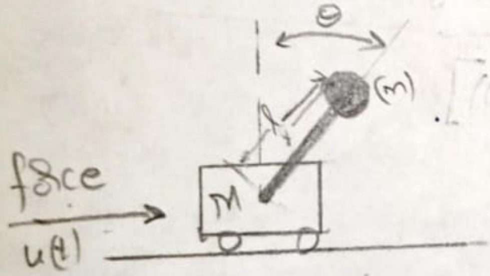
$x(t)$ - displacement of the cart.

$\dot{x}(t)$ - velocity of the cart

$\theta(t)$ - angle of the pendulum w.r.t vertical

$\dot{\theta}(t)$ - angular velocity of the pendulum

t - time instant.



where

m - mass of pendulum

M = mass of cart

l = length of the pendulum

The control input for the system is defined by $u(t)$ ~~where~~
which the force applied on the cart.

$u(t) = a(t)$. , $a(t)$ = Boost acceleration.

The dynamic of the system is give by as follows:

$$\cancel{x(t+1)} \quad x(t+1) = x(t) + \Delta x(t).$$

$$x(t+1) = x(t) + \dot{x}(t) \cdot \Delta t$$
$$\dot{x}(t+1) = \dot{x}(t) + \ddot{x}(t) \cdot \Delta t$$ where Δt = Frame time.
$$\theta(t+1) = \theta(t) + \dot{\theta}(t) \cdot \Delta t$$
$$\dot{\theta}(t+1) = \dot{\theta}(t) + \ddot{\theta}(t) \cdot \Delta t$$

$\Rightarrow \ddot{x}(t)$ = Acceleration of the cart

$$= \frac{m l \dot{\theta}^2(t) \sin[\theta(t)] + g \cdot m \cdot \sin(\theta(t)) \cos(\theta(t)) + u(t)}{M + m \sin^2[\theta(t)]}$$

$\Rightarrow \ddot{\theta}(t)$ = Angular acceleration of the cart

$$= \frac{-m l [\dot{\theta}(t)]^2 \sin(\theta(t)) \cdot \cos(\theta(t)) - g (M+m) \sin(\theta(t)) - u(t) \cdot \sin\theta(t)}{l [M + m \sin^2(\theta(t))]}.$$

$$\Rightarrow x(t+1) = x(t) + \dot{x}(t) \Delta t.$$
$$\dot{x}(t+1) = \dot{x}(t) + \left[\frac{m l [\dot{\theta}(t)]^2 \sin[\theta(t)]}{deno} + g \cdot \frac{m \sin(\theta(t)) \cos(\theta(t))}{deno} + u(t) \frac{1}{deno} \right] \Delta t$$

$$\theta(t+1) = \theta(t) + \dot{\theta}(t) \Delta t$$
$$\dot{\theta}(t+1) = \dot{\theta}(t) + \frac{-m l [\dot{\theta}(t)]^2 \sin(\theta(t)) \cos(\theta(t))}{l * deno} - g \cdot \frac{(M+m) \sin(\theta(t))}{l * deno} - \frac{u(t) \cdot \sin(\theta(t))}{l * deno}$$

where, $deno = M + m \sin^2[\theta(t)]$

→ Matrix form of dynamic:

$$\rightarrow x(t+1) = \underbrace{\begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{step\_mat}} x(t) + \Delta t \left[\underbrace{\frac{ml(\dot{\theta}(t))^2}{deno} \begin{bmatrix} 0 \\ \sin\theta \\ 0 \\ -\frac{\sin\theta\cos\theta}{l} \end{bmatrix}}_{\text{delta1}} + \underbrace{g \begin{bmatrix} 0 \\ m\sin(\theta(t))\cos(\theta(t)) \\ 0 \\ -\frac{(M+m)\sin(\theta(t))}{l} \end{bmatrix}}_{\text{delt2}} + \underbrace{u(t) \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{\sin(\theta(t))}{l} \end{bmatrix}}_{\text{delta 3}} \right]$$

$$x(t+1) = (\text{step\_mat})(x(t)) + \Delta t\,[\text{delta1} + \text{delta2} + \text{delta3}]$$

where, delta1 = is change due to dynamic

delta2 = is change due to gravity

delta3 = is change due to application of force.

→ Objective function:

Since we are try to stabilize the inverted pendulum we are try to minimize the velocity of the cart ($\dot{x}(t)$), angle ($\theta(t)$) and angular velocity ($\dot{\theta}(t)$) of the pendulum w.r.t any position $x(t)$.

$$\rightarrow \min \| loss \|^2$$

$$= \min \left[\left([\dot{x}(t)]^2 + [\theta(t)]^2 + [\dot{\theta}(t)]^2 \right) / 3 \right]$$