Introduction:

- Considering simple formulation for stabilizing an inverted pendulum where the state space is defined as.

$x(t) = [x(t), \dot{x}(t), \theta(t), \dot{\theta}(t)]^T$, where

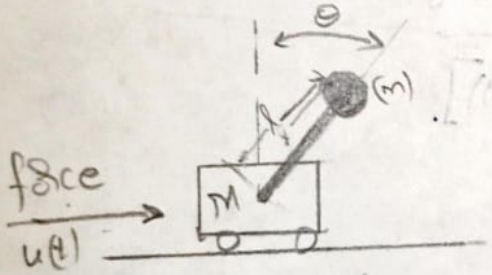
$x(t)$ - displacement of the cart.

$\dot{x}(t)$ - velocity of the cart

$\theta(t)$ - angle of the pendulum w.r.t vertical

$\dot{\theta}(t)$ - angular velocity of the pendulum

t - time instant.



where

m - mass of pendulum

M = mass of cart

l = length of the pendulum

The control input for the system is defined ~~by~~ $u(t)$ ~~where~~ which the force applied on the cart.

$u(t) = a(t)$., $a(t)$ = Boost acceleration.

The dynamic of the system is give ~~by~~ as follows:

~~$x(t)$~~ $x(t+1) = X(t) + \Delta X(t)$.

$$\Rightarrow x(t+1) = x(t) + \dot{x}(t) \cdot \Delta t$$

$$\dot{x}(t+1) = \dot{x}(t) + \ddot{x}(t) \cdot \Delta t$$

$$\theta(t+1) = \theta(t) + \dot{\theta}(t) \cdot \Delta t$$

$$\dot{\theta}(t+1) = \dot{\theta}(t) + \ddot{\theta}(t) \cdot \Delta t$$

where Δt = Frame time.

$\Rightarrow \ddot{x}(t)$ = Acceleration of the cart

$$= \frac{ml\dot{\theta}^2(t)\sin[\theta(t)] + g \cdot m \cdot \sin(\theta(t))\cos(\theta(t)) + u(t)}{M + m\sin^2[\theta(t)]}$$

$\Rightarrow \ddot{\theta}(t)$ = Angular acceleration of the cart

$$= \frac{-ml[\dot{\theta}(t)]^2 \sin(\theta(t)) \cdot \cos(\theta(t)) - g(M+m)\sin(\theta(t)) - u(t) \cdot \sin\theta(t)}{l[M + m\sin^2(\theta(t))]}$$

$$\Rightarrow x(t+1) = x(t) + \dot{x}(t)\,\Delta t$$

$$\dot{x}(t+1) = \dot{x}(t) + \left[\frac{ml[\dot{\theta}(t)]^2 \sin[\theta(t)]}{\text{deno}} + g \cdot \frac{m\sin(\theta(t))\cos(\theta(t))}{\text{deno}} + u(t)\frac{1}{\text{deno}}\right]\Delta t$$

$$\theta(t+1) = \theta(t) + \dot{\theta}(t)\,\Delta t$$

$$\dot{\theta}(t+1) = \dot{\theta}(t) + \frac{-ml[\dot{\theta}(t)]^2 \sin(\theta(t))\cos(\theta(t))}{l \cdot \text{deno}} - g \cdot \frac{(M+m)\sin(\theta(t))}{l \cdot \text{deno}} - \frac{u(t) \cdot \sin(\theta(t))}{l \cdot \text{deno}}$$

where, $\text{deno} = M + m\sin^2[\theta(t)]$

→ Matrix form of dynamic:

$$\rightarrow X(t+1) = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} X(t) + \Delta t \left[\frac{ml(\dot{\theta}(t))^2}{deno} \begin{bmatrix} 0 \\ \sin\theta \\ 0 \\ \frac{\sin\theta\cos\theta}{l} \end{bmatrix} + g \begin{bmatrix} 0 \\ m\sin(\theta(t))\cos(\theta(t)) \\ 0 \\ \frac{(M+m)\sin(\theta(t))}{l} \end{bmatrix} \right] + u(t) \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{-\sin\theta(t))}{l} \end{bmatrix}$$

step_mat — delta1 — delta2 — delta 3

$$X(t+1) = (\text{step\_mat})(x(t)) + \Delta t \left[\text{delta1} + \text{delta2} + \text{delta3} \right]$$

where, delta1 - is change due to dynamic

delta2 = is change due to gravity

delta3 = is change due to application of force.

→ Objective function:

Since we are try to stabilize the Inverted pendulum we are try to minimize the velocity of the cart ($\dot{x}(t)$), angle ($\theta(t)$) and angular velocity ($\dot{\theta}(t)$) of the pendulum w.r.t any position $x(t)$.

→ $\min \| \text{loss} \|^2$

$= \min \left[\left([\dot{x}(t)]^2 + [\theta(t)]^2 + [\dot{\theta}(t)]^2 \right) / 3 \right]$

⇒ Project 2: MPC Inverted Pendulum:

⇒ The dynamics of inverted pendulum are explained in the project 1. which is also attached to this code.

Here, deno = $M + m \sin^2(\theta(t))$

$$A = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ Boost\_Accel * dt / deno \\ 0 \\ Boost\_Accel * dt * \left(\frac{-l}{l * deno}\right) \end{bmatrix}$$

$$C = \begin{bmatrix} 0 \\ g * dt * m * s * c / deno \\ 0 \\ -(g * dt * (m+M) * s) / l * deno \end{bmatrix}$$