

# Adjusting Manual Rates to Own Experience: Comparing the Credibility Approach to Machine Learning

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## 1 Introduction

The use of market data as an aid for setting own company rates has been a common practice in the Insurance Industry. External data, as provided by Insurance Rating Bureaus, [reinsurers](#) or Advisory Organizations, may supplement internal company's ones that may be scarce or unreliable because of a non-representative and/or a too short history, or, non-existing at all, e.g. when entering in a new business lines or territory. The importance of external data to both support adequate rates that preserve company solvency and in easing the entrance of new players has been historically recognized by regulator, e.g. granting a partial antitrust – law exception in the US jurisdiction (Danzon 1983).

When the [insurer](#) takes into account its own experience in order to enhance the credibility of its rates, [he](#) need to [benchmark](#) its portfolio experience [compared](#) the market one. The actuarial profession traditionally used techniques based on Bayesian statistics and non-parametric credibility to optimally combine the market and insurer's portfolio experience in the technical rates. [From this point of view, the contribution of market data makes it possible to satisfy the two classic approaches addressed by the theory of credibility, the](#)

limited fluctuation credibility theory and the greatest accuracy credibility theory (Norberg 2004). The former refers to need of incorporate individual experience into the rate calculation in order to stabilize the level of individual rates. The second approach corresponds to the application of modern credibility theory and consists in combining both individual and collective experience to predict individual rates by mean square error minimization. Credibility theory is extensively used in non-life insurance and early models were not based on policyholders' rate-making variables, see e.g. Bühlmann and Gisler (2006) for a review. Yet some advanced regression credibility models have been proposed in the actuarial literature, such as the so-called Hachemeister model (Hachemeister and others 1975). On the contrary, rates based on Generalized Linear Models (GLM), the current gold standard in personal rates pricing (Goldburd, Khare, and Tevet 2016), are only based on the impact of ratemaking factors, giving no credit to the individual policy experience. Nevertheless, mixed effects GLM allow to incorporate policyholders' experience within GLM tariff structure ("Bayesian credibility for GLMs" 2018; Antonio and Beirlant 2007) but they are not widespread used. All these regressions approaches are enable to incorporate individual risk profile covariates into a credibility model. The structure of insurance data, notably the distinction between own experience and market experience, is dealt with the use of the hierarchic credibility model of Bühlmann and Straub (1970).

In some situations, the use of company data is not possible at all and only a tariff at market level is reliable. This is the case for instance in France for fire insurance of commercial lines. During the 1980s, a first pricing structure was considered with a Buehlmann-Straub model with 24 risk classes, Cohen, Dupin, and Levi (1986). In 2004, a more complex pricing structure was proposed by the French association of private insurers (FFSA) with a two-level hierarchical model with 10 business types and 141 business classes for fire and business interruption insurance, Douville (2004).

Credibility theory is also largely used in life insurance applications for modeling mortality risks. A first attempt for stabilizing mortality rates by combining the mortality data of the small population with the average mortality of the neighboring populations is proposed by Ahcan et al. (2014). Regarding this issue of limited mortality data (small population or short historical period of observations), Li and Lu (2018) introduce a Bayesian non-parametric model for benchmarking a small population compared to a reference population, and Bozikas and Pitselis (2019) focus on a credible regression framework to efficiently forecast populations with a short-base-period. In order to improve mortality forecasting, some recent contributions have been done in the literature for combining usual mortality models, such as the (Lee and Carter 1992) model and Bühlmann credibility theory, see C. Tsai and Lin (2017), C. C.-L. Tsai and Zhang (2019) and C. C.-L. Tsai and Wu (2020) among other.

The recent widespread/massive usage of Machine Learning has provided many more techniques to the practitioner actuaries' toolkit, see @blierwongmachine2021 for a review in non-life insurance. **TO DO : summarize the main contributions of ML for pricing and reserving..** Recently, @diaoregression2019 combine the use of credibility and regression tree models. In these publications, the ultimate goal of the use of Machine Learning is to improve the usual regression setup in actuarial science based on the GLM. However, these techniques, such as the Gradient Boosting Models (GBM) and the Deep Learning (DL), can also be used in a manner that permits to "transfer" what the model has learned on a much bigger data set (as the market data, MKT) to a smaller set (the portfolio data of the company, CPN). "Transfer learning" (henceforth TFL) is typically in computer vision DL modes to fine tune standard architecture on specific recognition tasks. An "initial score" can be provided to GBMs to take into account the known effect of exposure or a priori modeled estimate before "boosting" the prediction. In this paper, our aim is to exploit the advantages of ML for easily handling complex non-linear relationship compared to standard credibility or GLMs based approaches to assess the policyholders' risk more precisely. Hence, our work contrast with traditional methods to ML ones in the task of blending market data to individual portfolio experience.

The rest of this paper is organized as follows. After a brief business and methodological introduction, we apply in Section XX on a (properly anonymized) data set comprised by both market and own portfolio experience relative to a European country non – life business line. The final comparisons will consider not only on the predictive performance, but also in the ease of practical application in term of computational request, ease of understanding and interpretability of the results.

## 2 Overview of the use of external experience in own business in the insurance industry

The history of the Insurance Industry is not new to the practice of cooperation between entities to set rates. Two are the main reasons supporting this practice, the first one purely technical, the second one of socio-economic nature.

The first one is that setting rates that adequately assess the risk being underwritten requires credible and homogeneous historical data. The quality and amount of data increases as much as the risks are complex. This is still an issue in the 21st century, but it was certainly even more prominent in the past two centuries, characterized by data collection and actuarial modelling procedures archaic. Secondly, in the past a stronger regulation of insurance business to prevent excessive competition (as well as excessive rates) and to avoid insolvencies was socially accepted. According to Porter (Porter, Chartered Property Casualty Underwriters, and America 2008), pools of insurers existed already in the second part of the 19th century in the US that supported their members in setting rates through data collection and standardized policy forms. In the first part of the 20th century the McCarran Fergusson Act partially exempted the Insurance industry from the US federal antitrust regulation, thus NAIC model laws explicitly allowed cooperation in setting rates specifying the role of rating organization.

According to IRMI ("Rating Bureau," n.d.), a Rating Bureau is "an organization that collects statistical data (such as premiums, exposure units, and losses), computes advisory rating information, develops standard policy forms, and files information with regulators on behalf of insurance companies that purchase its services." The use of their services has become progressively less compulsory in recent decades, and their activities have become increasingly consultative: single carriers may purchase their data collection service and decide whether and how to use them. In the US market, the most relevant rating bureau are: NCCI (for Worker Compensation, WC), the Insurance Service Office (ISO), that serves most personal and commercial lines; the Surety Association of America (SAA) that operates in the surety and crime insurance and the American Association of Insurance Services (AAIS) specialized in many commercial lines different from WC.

Regarding other countries, we can provide few examples from the Italian and France markets. The Italian Association of Insurers, ANIA, aggregates data from the motor lines (but only pure premiums with few covariates), the Long Term Care insurance for health, while it collects and extensive statistical plan (with many covariates) for crop insurance. In France, ...

- Role of reinsurance as providing advice and support in pricing.
- deduce the portfolio experience of the market by extracted the rates published by competitors.

Apart from the references cited in introduction, I have no particular knowledge on this subject. In France, it seems that this practise is less developed. I suggest to briefly describe these practises and then to describe more precisely how our data set is organized. In order to fit within the framework of credibility theory, it is important to explain that:

- the market data set is much bigger than the company data set and is thus almost equal to the collective premium when combining the data set.
- the process on which we focus is recursive and consists in predicting rates of year  $n$  based both on the market and the company experiences accumulated on the claim history from year 0 to  $n - 1$ .

## 3 Hierarchical credibility-based models with covariates

*TODO*

- notation and assumptions.
- insert the hierarchical tree structure.
- explain how integrate market data.
- briefly recall the estimators related the hierarchic Bühlmann credibility theory and introduce the use of covariates as in the hachemeister model.

## 4 Modeling approach with machine learning

This research aims to compare the predictive power of traditional [credibility](#) and machine learning methods that use an initial estimate of loss costs, e.g. from [MKT](#) experience, to predict those of a smaller portion (the CPN one) in a subsequent period (the test set). Therefore, [following the idea of the greatest accuracy credibility theory](#), our modeling process aims to predict the losses on the last available year (the test set) by training models on the experience of the previous years, eventually split into a train and validation test. [Then, we focus on models that permits to use an initial estimate of losses performed on another set \(transfer learning\)](#). While the paper explores the use of such approach applying ML methods, traditional GLM may be used as well. For instance, under a log-linear regression framework and initial log-estimate of either the frequency, the severity of the pure - premium may be set as an offset for a subsequent model (Yan et al. 2009).

The empirical data available for the study regards a risk for which year to year loss cost may materially fluctuate due to external condition (systematic variability) much more than the portfolios' risks heterogeneity composition. At this regards, the performance assessment has considered not only the discrepancy between the actual and predicted losses, but the ability of the model to rank risks [What do you mean by "rank risks" and how ML can achieve this purpose](#).

The losses are the number of damaged units while the exposure are the number of insured units. Therefore, only the frequency component has been modeling, choosing either a [Binomial](#) or a [Poisson](#) loss function. Henceforth losses in this paper shall be considered as synonym of claim number.

[Machine learning techniques I suggest to move this paragraph into the introduction](#) ML methods have been acquiring increasing attention by actuarial practitioners especially. Beginning from the analysis of policyholders' behavior (Spedicato, Dutang, and Petrini 2018), several applications have sprung also for risk pricing. An application of boosting techniques to estimate the frequency and the severity of an MTPL dataset [define MTPL](#) can be found in [@noll2020case](#), while [@schelldorfer2019nesting](#) present a joint model that boosts GLM performance using Deep Learning.

ML methods used in insurance pricing are strongly non - linear and are able to automatically find interactions among ratemaking factors and exclude non relevant features. In particular two techniques are acquiring widespread importance: Boosting and Deep learning. Both techniques allow the use of an initial estimate of loss / exposure to risk [to train the model on last observations](#). ~~the be feed to the model to be fine tuned for the current dataset.~~

[Brief overview](#)

### 4.1 Boosting techniques

The boosting approach [\[@friedman2001greedy\]](#) can be synthesized by the following formula:

$$F_t(x) = F_{t-1}(x) + \eta * h_t(x)$$

that is, the prediction at the  $t$ -it step is given by the contribution, to the prediction of the previous step, of a weak predictor  $h_t(x)$ , properly weighted by a learning factor  $\eta$ , being  $x$  the covariate vector. The most common choice for the weak predictor  $h_t(x)$  lies in the classification and regression trees family, from which the Gradient Boosted Tree (GBT) models. It can be shown that "boosting" weak predictors lead to very strong predictive models (Elith, Leathwick, and Hastie 2008). Almost all winning solutions of data science competitions held by Kaggle are at least partially based on XGBoost (Chen and Guestrin 2016), the most famous GBT model. More recent and interesting alternatives to be tested are: LightGbm (Ke et al. 2017), which is particularly renowned for its speed, and Catboost (Prokhorenkova et al. 2017), which has introduced an efficient solution for handling categorical data.

[I suggest to describe the LightGbm method \(notations, loss function and the algorithm\)](#). Maybe it could be interesting to explain why you prefer LGBM on the selected data set compared to other boosting methods. [Do you have tested other approaches before and select the LGBM as the winner?](#)

A set of hyperparameter defines a boosted model and even more define a GBT one. The core hyperparameters that influence the boosting part are the number of models (trees),  $t = 1, 2, \dots, T$  (typically between 100 and 1000) and the learning rate  $\eta$ , whose typical values lies between 0.2 and 0.001.  $h_t(x)$  can be, when it belongs to the CART family, the maximum depth, the minimum number of observation in final leafs, the fraction of observation (rows or columns) that are considered when growing each tree. The optimal combination of hyperparameters is learn using either a grid search approach or a more refined one (e.g. bayesian optimization).

When applied to claim frequency prediction, they are fit to optimize a Poisson log-loss function. In addition, to handle uneven risk exposure, the log - measure of exposure risk is given (in log scale) as an init-score ( $F_t(x)$ ) to initialize the learning process. The init-score (or base margin) in the boosting approach has the same role of the traditional GLM offset term (Goldburd, Khare, and Tevet 2016).

## 4.2 Deep Learning

An artificial network is a mathematical structure that applies a non linear function to a linear combination of inputs, say  $\phi(\bar{x}_i^T \times \bar{w} + \beta)$ , being  $\bar{w}$  and  $\beta$  the weights and bias respectively. A neural network consists in one of more layer of interconnected neurons, that receives a (possibly multivariate) input set and retrieves and output set [Cite a general reference for NN](#). Modern Deep Neural Networks are constructed by many (deep) layers of neurons. Deep Learning has been knowing a hype in interest for a decade, thanks to the availability of huge amount of data, computing power (in particular GPU computing) and the development of newer approaches to reduce the overfitting that had halted the widespread adoption of such techniques in previous decades (Goodfellow, Bengio, and Courville 2016). Different architectures has reached state of the art performances in many fields; e.g. convolutionary neural networks achieved top performance in computer vision (e.g. image classification and object detection) [Cite a general reference](#), while recurrent neural networks, see e.g. [@hochreiter1997long for Long Short Term Memory ones](#), provides excellent results in Natural language processing tasks like sequence-to-sequence modeling (translation) and text classification (sentiment analysis). [For applications in actuarial science, we refer to the recent review of @blierwongmachine2021, and to the the work of @richmanai2021 and @richmanai20211 for deep neural network.](#)

Simpler structure are needed for a claim frequency regression, the multi-layer perception (MLP) architectures that basically consist in stacked simple neurons layers, from the input one to the single output cell one. This structure is dealt to handle the relation between the relation between the ratemaking factors and the frequency (the structural part). To handle the different exposures, the proposed architecture is based on the solution presented by (Ferrario, Noll, and Wuthrich 2020; Schelldorfer and Wuthrich 2019). A separate branch collects the exposure, applies a log-transformation, then this exposure is added in a specific layer just before the final one (that has a dimension of one).

Training a DL model consists in providing batches of data to the network, evaluate the loss performance and updating the weights in the direction that minimize the training (backpropagation). The whole data set is provided to the fitting algorithms many times (epochs) split in batch. One of the common practice to avoid overfitting is to use a validation set where the loss is scored at each epoch. When it starts to systematically diverge, the training process is stopped (early stopping).

[I suggest to described here with more the NN structure and the activation function used in our application, as well as the algorithm used.](#)

*TODO*

## 5 Numerical application

[In this study, the analysis is performed on two real and anonymized data sets](#), the CMP and MKT, preprocessed and split into train, validation and test set as previously discussed. Then, the models [is fitted](#) on the train set and the predictive performance [is](#) assessed on the test set. The validation set was used in DL and BST models to avoid overfitting. Finally, the models are compared in terms of predictive accuracy, using the (The actual / predicted ratio) and risk classification performance, using the Normalized Gini index (Frees, Meyers,

and Cummings 2014). The latter index has become quite popular in the actuarial academia and practitioners to compare competing risk models.

Describe the computer environment and cite softwares used.

## 5.1 The structure of the dataset

Two (anonymized) dataset were provided, one for the marketwide ("mkt\_anonymized\_data.csv") and one for the company ("mkt\_anonumized\_data.csv"), henceforth MKT and CMP datasets. These datasets share the same structure, as each company provides its data in the same format to the Pool, that aggregates individual filings into a marketwide file, that is provided back to the companies. The dataset contains the exposures and claim numbers, aggregated by some categorical variables. Variable names, levels and numeric variable distribution have been masked and anonymized for privacy and confidentiality purposes.

maybe a graphic could be done? yes: display the schematic of the process at training/testing stages.

Would it be possible to display summary statistics for describing the volume of data, the effect of categorical variables and the distribution of continuous variables? Maybe, it could be interesting to compare the contribution of the company into the market data set in terms of claim numbers.

The following variables are contained in the provided data set: Make a table, and if possible with some summary statistics (mean, median, Q1, Q3, sd, number of levels, ...)

- *exposure*: the insurance exposure measures by classification group, on which the rate is filled (aggregated outcomes) Define "classification group";
- *claims*: the number of claims by classification group (aggregated outcomes);
- *ID*: unique row number (helper variable); Could be removed ?;
- *zone\_id*: territory (aggregating variable);
- *year*: filing year (aggregating variable);
- *group*: random partition of the dataset into train, valid and test set.
- *cat1*: categorical variable 1, available in the original file (aggregating variable); suggestion : renames 'cat1' and 'cat2' as 'risk class 1' and 'risk class 2'
- *cat2*: categorical variable 2, available in the original file (aggregating variable);
- *cat3*: categorical variable 3, available in the original file (aggregating variable);
- *cat4-cat8*: categorical variables related to the territory (joined to the original file by zone\_id);
- *cont1-cont12*: numeric variables related to the territory (joined to the original file by zone\_id);

Categorical and continuous variables have been anonymized by label encoding and scaling (calibrated on market data). In addition, the last available year (2008) has been used as test set, while data from precedent years have been randomly split between train and validation sets on a 80/20 basis. Market data is available for 11 years, while company data for the last five one. Also, the number of exposures is widely dependent by the cat1 variable.

## 5.2 ML techniques Implementation details

### 5.2.1 Boosting approach

The `lightgbm` model has been used to apply boosted trees on the provided data sets, minimizing Poisson deviance. As for most modern ML methods, a `lightgbm` model is fully defined by a set of many hyperparameters for which default values may not be optimal for the given data and there is no closed formula to identify the best combination for the given data.

Therefore an hyperparameter optimization step is performed. For each hyperparameter a range of variation is set, then a 100-run trial was performed using a Bayesian Optimization approach performed by the `hyperopt Python` library (Bergstra, Yamins, and Cox 2013). Under the BO approach, each subsequent iteration is performed toward the point that minimize the loss to be optimized, being the loss distribution by hyperparameter updated each iteration using a bayesian approach.



As suggested by boosting trees practitioners [Add reference](#), the number of boosted models [is](#) not estimated under the BO approach but determined by early stopping. The loss [is](#) scored under the validation set and the number of trees chosen is that beyond which the loss stop to decrease and start diverging up.

The CMP and MKT [models use](#) the standard exposure (in logarithm base) as init score. The TRF model instead uses as init score the “a-priori” prediction of the MKT model on the CMP data.

- [Cite used packages](#)
- [Describe the computation complexity and the run time](#)

### 5.2.2 Deep Learning

The chose DL architecture was set by several trials, based on previous experiments and [practitioners](#) architecture found in the literature for tabular data analysis. Unfortunately, the hyperparameter space of a DL architecture is very vaste, comprising not only fitting level degrees of freedom (the [optimizer](#), the number of epochs, the batch size) but the whole layers’ architecture: the number of layers, the numbers of neuron within etc. . . At this regard it is common among [practitioners](#) to starts with a [knowing](#) working architecture in a [similar](#) field and perform moderate changes. While more [systematizing](#) DL architecture optimization approaches are being developed (e.g. the Neural Architecture Search) the use of such techniques [is](#) out of the scope of the paper.

[suggestion: gather the all the cat variables in one box and do the same with cont variables.](#)

[suggestion: this figure is difficult to read for me.](#)

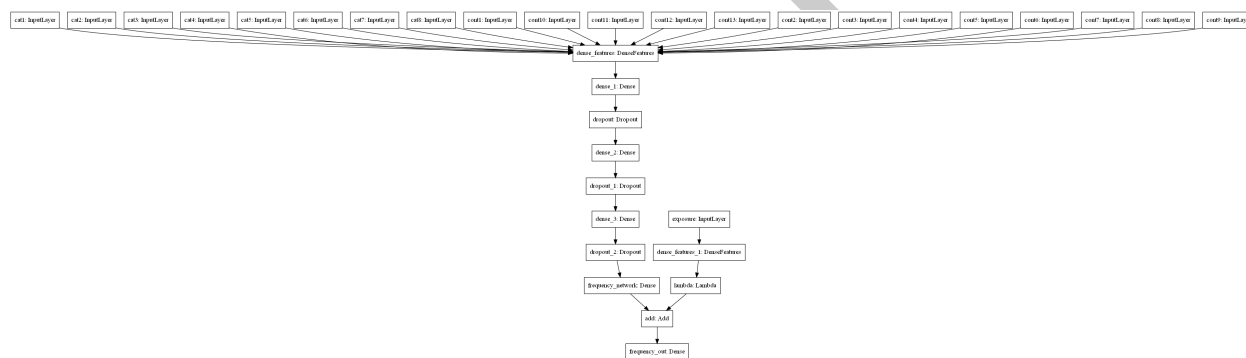


Figure 1: DL model structure

The same model shown above [is](#) used for both the CMP, MKT and TRF models. A dense layer collects the inputs, where the categorical variables have been handled using embedding. Three hidden layers perform the feature engineering and knowledge extraction from the input; Dropout layers [is added to increase the robustness of the process](#). As anticipated in the methodological section, the exposure part is separately handled in another branch and then merged in the final layer.

Overfitting [is](#) controlled using an EarlyStopping callback scoring the loss on the validation test and stop learning over next epochs [cite reference](#) if the loss did not improve for more than 20 epochs.

The TRF model has been build using the pre-trained weights calculated on the market data and continuing the [training](#) process on the CPN data.

- [Cite used packages](#)
- [Describe the computation complexity and the run time](#)

### 5.2.3 Credibility [model](#)

- [Cite used packages](#)
- [Explain how we consider continous variables](#)

- Explain how we define the hierarchic structure
- Explain data imputation in the test sample

#### 5.2.4 Assessment of performance

- Describe how are constructed the training and test samples.
- Define the metrics considered for assesing the performance. If we directly predict the number for claims, it could be usefull to compute the RMSE, the MAE and/or the MAPE.

### 5.3 Forecasting performance

For a better understanding of the results, I suggest to assess the importance of each explanatory variable.

The table below reports the predictive performance, evaluated on the company test set, for the DL and Bst models' families. The columns approach indicates whether the model is trained on market-only (MKT), company-only (CPN) or company data using a transfer learning approach (TRF).

Table 1: Models comparison

model	approach	normalized_gini	actual_predicted_ratio	mean_actual_pred_ratio	MeanAE	MedianAE	RMSE
dl	mkt	0.921	0.924	0.919	207.725	14.395	3194.502
dl	cpn	0.909	0.775	1.113	269.216	13.039	4976.958
dl	trf	0.925	0.967	0.944	201.802	13.291	3368.277
bst	mkt	0.939	0.975	0.833	187.390	10.096	3066.079
bst	cpn	0.924	0.841	0.860	249.489	11.450	4912.252
bst	trf	0.940	1.052	0.831	179.091	9.972	3198.866
cred1	cpn	0.880	0.882	1.808	260.369	12.407	5651.256
cred1	mkt	0.937	0.975	0.951	183.917	9.775	2966.997
cred1	trf	0.898	1.061	1.369	233.687	14.345	5053.139

First, we see that the actual/predicted ratio is between 0.9 - 1.1 for all models, but company ones - is the worse. We anticipate that as the test set considers a year different from the train and validation pools, the predictions may be structurally biased as the insured risk depends by the year's climate and that frequency trending is not consider in the modeling framework at all. The transfer learning approach offers higher predictive accuracy when measured by the NG index and the predictive performance is the highest among all competitive approaches except for the boosting approach. Regarding the predictive accuracy, on the other hand, and especially for the DL models, we cannot rule out that the superiority of TRF approaches holds for all possible MLP architectures. Also, it is likely to happen that as far as the company data increases, the advantage of the TRF approach decreases with respect to a model trained on company data only.

TODO

## 6 Conclusion

We presented an application of TF that can be resembled to the traditional “credibility” approach to transfer the experience applied on a different, but similar, book of business to a newer one. We saw that ML approach may provides interesting results and may be worth to try with.

Finally, we performed our empirical analysis transferring loss experience from an external insurance bureau to a specific company portfolio. This “transfer of experience” may be also performed within the same company for example when new products, taylored for niche books of business, are created. Initial losses estimates may be performed on the initial product and then applied as initial scores on the newer portfolio.

TODO



## 7 Appendix

### 7.1 Code

The modeling has been performed using both R (R Core Team 2021) and (Van Rossum and Drake 2009). The following files has been provided:

0. preprocess and anonymize.py: work on original data, anonymizing column and internal datasets and split data across train, validation and test set; the config\_all.py file provides ancillary functions to perform this step;
1. analysis\_neural\_network.py: implement the MLP approach on the dataset;
2. analysis\_lightgbm.py: apply the LightGbm on the dataset;
3. compare\_predictions.R: contrasts the different predictive approaches on the company test data set,

### 7.2 Data Preparation and Anonymization

The market and company data file were loaded. An initial renaming of the variable has been performed, conventionally naming the continuous one as cont\_x while the categorical one as cat\_x, being x a number from one up to the number of variables of such category. The following criterion have been used to filter out anomalous observations: presence of missing values in any of the observations, zero exposures.

Then, the available data has been split threefold: the last available year has been set to the test set, while the remaining years have been split into a train / validation set using a 80/20 ratio. Therefore we have available three dataset for the marked data, and another three for the company one.

### 7.3 Acknowledgments

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