In [1]:

```
from prob140 import Table
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
```

In [2]:

```
def add(num1, num2):
    return num1 + num2

dice = [1, 2, 3, 4, 5, 6]
X = set()
Y = set()
all_poss = []

for d1 in dice:
    for d2 in dice:
        all_poss.append( str(d1)+str(d2) )
        X.add( max(d1,d2) )
        Y.add( add(d1, d2) )
```

In [3]:

```
print("All possibilities are", all_poss)
print("Random Variable X:", X)
print("Random Variable Y:", Y)
```

```
All possibilities are ['11', '12', '13', '14', '15', '16', '21', '22', '23', '24', '25', '26',
'31', '32', '33', '34', '35', '36', '41', '42', '43', '44', '45', '46', '51', '52', '53', '54',
'55', '56', '61', '62', '63', '64', '65', '66']
Random Variable X: {1, 2, 3, 4, 5, 6}
Random Variable Y: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
```

In [4]:

```
def str_max(str_poss):
    num1 = int(str_poss[0])
    num2 = int(str_poss[1])
    return max(num1,num2)

def str_add(str_poss):
    num1 = int(str_poss[0])
    num2 = int(str_poss[1])
    return num1 + num2


def calc_prob(num, all_poss, fun):
    c = 0

    for poss in all_poss:
        if fun(poss) == num:   # If this poss satisfies the Random Variable's individual then increase count
er
            c = c + 1

    prob = c/len(all_poss)
#     print(num, "occurs", c, "with", fun, "having", prob)
    return prob


# Calculating probs of all values of Random Variables
X_probs = {}
for x in X:
    X_probs[x] = calc_prob(x, all_poss, str_max)
# print()
Y_probs = {}
for y in Y:
    Y_probs[y] = calc_prob(y, all_poss, str_add)

print(round(sum(X_probs.values())) == 1)
print(round(sum(Y_probs.values())) == 1)
print(X_probs, '\n')
print(Y_probs)
```

```
True
True
{1: 0.027777777777777776, 2: 0.08333333333333333, 3: 0.1388888888888889, 4: 0.19444444444444445
, 5: 0.25, 6: 0.3055555555555556}

{2: 0.027777777777777776, 3: 0.05555555555555555, 4: 0.08333333333333333, 5: 0.1111111111111111
, 6: 0.1388888888888889, 7: 0.16666666666666666, 8: 0.1388888888888889, 9: 0.1111111111111111,
10: 0.08333333333333333, 11: 0.05555555555555555, 12: 0.027777777777777776}
```

In [5]:

```
res = []   # Calculating probs of all cartesian pairs of X and Y
for x in X:
    for y in Y:
        prob = X_probs[x] * Y_probs[y]
#         print(x,"with",X_probs[x], y, "with", Y_probs[y], "has", prob)
        res.append(prob)
```

In [6]:

```
print(round(sum(res)) == 1)
print(len(res))
```

```
True
66
```

In [7]:

```
dist = Table().domain("X", X, "Y", Y)
print(dist, '\n')
print(dist.shape)
```

```
X    | Y
1    | 2
1    | 3
1    | 4
1    | 5
1    | 6
1    | 7
1    | 8
1    | 9
1    | 10
1    | 11
... (56 rows omitted)

X    | Y
[66] | [66]
```

```
/home/muhammadammarabid/anaconda3/lib/python3.8/site-packages/datascience/tables.py:222: Future
Warning: Implicit column method lookup is deprecated.
  warnings.warn("Implicit column method lookup is deprecated.", FutureWarning)
```

In [8]:

```
dist = dist.probabilities(res)
```

In [9]:

```
dist
```

Out[9]:

	X=1	X=2	X=3	X=4	X=5	X=6
Y=12	0.000772	0.002315	0.003858	0.005401	0.006944	0.008488
Y=11	0.001543	0.004630	0.007716	0.010802	0.013889	0.016975
Y=10	0.002315	0.006944	0.011574	0.016204	0.020833	0.025463
Y=9	0.003086	0.009259	0.015432	0.021605	0.027778	0.033951
Y=8	0.003858	0.011574	0.019290	0.027006	0.034722	0.042438
Y=7	0.004630	0.013889	0.023148	0.032407	0.041667	0.050926
Y=6	0.003858	0.011574	0.019290	0.027006	0.034722	0.042438
Y=5	0.003086	0.009259	0.015432	0.021605	0.027778	0.033951
Y=4	0.002315	0.006944	0.011574	0.016204	0.020833	0.025463
Y=3	0.001543	0.004630	0.007716	0.010802	0.013889	0.016975
Y=2	0.000772	0.002315	0.003858	0.005401	0.006944	0.008488

In [10]:

```
dist.both_marginals()
```

Out[10]:

	X=1	X=2	X=3	X=4	X=5	X=6	Sum: Marginal of Y
Y=12	0.000772	0.002315	0.003858	0.005401	0.006944	0.008488	0.027778
Y=11	0.001543	0.004630	0.007716	0.010802	0.013889	0.016975	0.055556
Y=10	0.002315	0.006944	0.011574	0.016204	0.020833	0.025463	0.083333
Y=9	0.003086	0.009259	0.015432	0.021605	0.027778	0.033951	0.111111
Y=8	0.003858	0.011574	0.019290	0.027006	0.034722	0.042438	0.138889
Y=7	0.004630	0.013889	0.023148	0.032407	0.041667	0.050926	0.166667
Y=6	0.003858	0.011574	0.019290	0.027006	0.034722	0.042438	0.138889
Y=5	0.003086	0.009259	0.015432	0.021605	0.027778	0.033951	0.111111
Y=4	0.002315	0.006944	0.011574	0.016204	0.020833	0.025463	0.083333
Y=3	0.001543	0.004630	0.007716	0.010802	0.013889	0.016975	0.055556
Y=2	0.000772	0.002315	0.003858	0.005401	0.006944	0.008488	0.027778
Sum: Marginal of X	0.027778	0.083333	0.138889	0.194444	0.250000	0.305556	1.000000

In [11]:

```
X_marginal_dist = dist.marginal_dist("X")
print(X_marginal_dist)
```

```
Value | Probability
1     | 0.0277778
2     | 0.0833333
3     | 0.138889
4     | 0.194444
5     | 0.25
6     | 0.305556
```

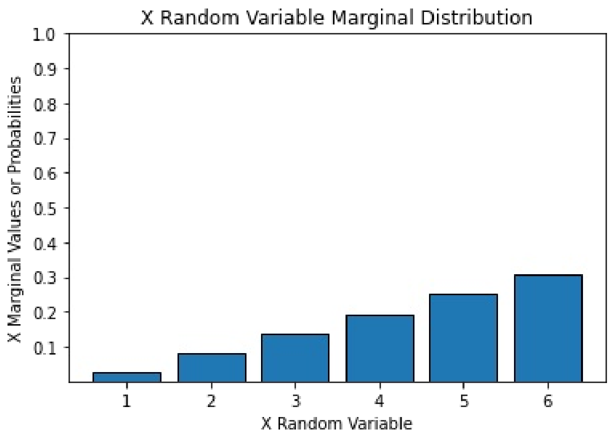
In [12]:

```
X_vals, X_probs = X_marginal_dist[0], X_marginal_dist[1]
print( X_vals, X_probs)
```

```
[1 2 3 4 5 6] [0.02777778 0.08333333 0.13888889 0.19444444 0.25       0.30555556]
```

In [13]:

```
def plot_X_marginal_dist(X_vals, X_probs):
    plt.bar(X_vals, X_probs)
    plt.yticks(np.arange(0.1, 1.10, 0.10))
    plt.xlabel("X Random Variable")
    plt.ylabel("X Marginal Values or Probabilities")
    plt.title("X Random Variable Marginal Distribution")
    plt.show()

plot_X_marginal_dist(X_vals, X_probs)
```



In [15]:

```
Y_marginal_dist = dist.marginal_dist("Y")
print(Y_marginal_dist)
```

```
Value | Probability
2     | 0.0277778
3     | 0.0555556
4     | 0.0833333
5     | 0.111111
6     | 0.138889
7     | 0.166667
8     | 0.138889
9     | 0.111111
10    | 0.0833333
11    | 0.0555556
... (1 rows omitted)
```

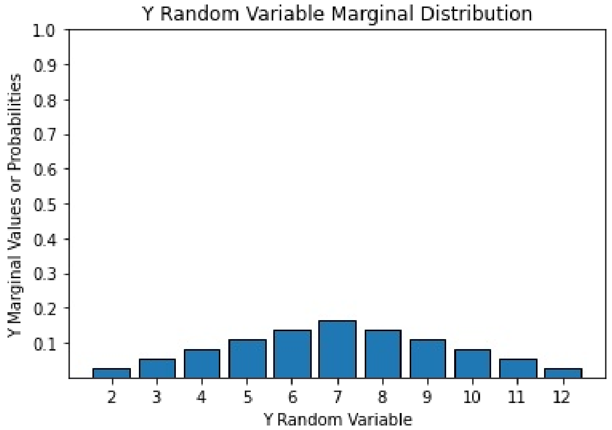
In [16]:

```
Y_vals, Y_probs = Y_marginal_dist[0], Y_marginal_dist[1]
print(Y_vals, Y_probs)
```

```
[ 2  3  4  5  6  7  8  9 10 11 12] [0.02777778 0.05555556 0.08333333 0.11111111 0.13888889 0.16
666667
 0.13888889 0.11111111 0.08333333 0.05555556 0.02777778]
```

In [17]:

```
def plot_Y_marginal_dist(Y_vals, Y_probs):
    plt.bar(Y_vals, Y_probs)
    plt.xticks(np.arange(2, 13, 1))
    plt.yticks(np.arange(0.1, 1.10, 0.10))
    plt.xlabel("Y Random Variable")
    plt.ylabel("Y Marginal Values or Probabilities")
    plt.title("Y Random Variable Marginal Distribution")
    plt.show()

plot_Y_marginal_dist(Y_vals, Y_probs)
```



Let's Try Challenge Question

In [18]:

```
first_half_prob = 0.4/3   # because 0.4 is prob of 1-3 but then prob of each would be 0.4/3
second_half_prob = 0.6/3  # because 0.6 is prob of 4-6 but then prob of each would be 0.6/3
unfair_dice = [(1, first_half_prob), (2, first_half_prob), (3, first_half_prob),
               (4, second_half_prob), (5, second_half_prob), (6, second_half_prob)]
all_poss = []

for d1 in unfair_dice:
    for d2 in unfair_dice:
        all_poss.append( (str(d1[0])+str(d2[0]), d1[1]*d2[1]) )

print("All possibilities are", all_poss)
print("Random Variable X:", X)
print("Random Variable Y:", Y)
print( round(sum(p[1] for p in all_poss)) == 1 )
```

```
All possibilities are [('11', 0.017777777777777778), ('12', 0.017777777777777778), ('13', 0.017
777777777777778), ('14', 0.026666666666666665), ('15', 0.026666666666666665), ('16', 0.02666666
6666666665), ('21', 0.017777777777777778), ('22', 0.017777777777777778), ('23', 0.0177777777777
77778), ('24', 0.026666666666666665), ('25', 0.026666666666666665), ('26', 0.026666666666666665
), ('31', 0.017777777777777778), ('32', 0.017777777777777778), ('33', 0.017777777777777778), ('
34', 0.026666666666666665), ('35', 0.026666666666666665), ('36', 0.026666666666666665), ('41',
0.026666666666666665), ('42', 0.026666666666666665), ('43', 0.026666666666666665), ('44', 0.039
999999999999994), ('45', 0.039999999999999994), ('46', 0.039999999999999994), ('51', 0.02666666
6666666665), ('52', 0.026666666666666665), ('53', 0.026666666666666665), ('54', 0.0399999999999
99994), ('55', 0.039999999999999994), ('56', 0.039999999999999994), ('61', 0.026666666666666665
), ('62', 0.026666666666666665), ('63', 0.026666666666666665), ('64', 0.039999999999999994), ('
65', 0.039999999999999994), ('66', 0.039999999999999994)]
Random Variable X: {1, 2, 3, 4, 5, 6}
Random Variable Y: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
True
```

In [19]:

```
# This logic could be used for above case too, it would give same results
def calc_prob(num, all_poss, fun):
    total_prob = 0

    for poss in all_poss:
        # Summing the probs of the possibilities which satisfy Random Variable's individual
        if fun(poss[0]) == num:
            total_prob = total_prob + poss[1]

    return total_prob
```

In [20]:

```
X_probs = {}
for x in X:
    X_probs[x] = calc_prob(x, all_poss, str_max)
# print()
Y_probs = {}
for y in Y:
    Y_probs[y] = calc_prob(y, all_poss, str_add)

print(round(sum(X_probs.values())) == 1)
print(round(sum(Y_probs.values())) == 1)
print(X_probs, '\n')
print(Y_probs)
```

```
True
True
{1: 0.017777777777777778, 2: 0.05333333333333333, 3: 0.08888888888888889, 4: 0.2, 5: 0.27999999
999999997, 6: 0.35999999999999993}

{2: 0.017777777777777778, 3: 0.035555555555555556, 4: 0.05333333333333333, 5: 0.08888888888888
88, 6: 0.12444444444444444, 7: 0.16, 8: 0.14666666666666667, 9: 0.1333333333333333, 10: 0.11999
999999999998, 11: 0.07999999999999999, 12: 0.039999999999999994}
```

In [21]:

```python
# Probs of all cartesian pairs
res = []
for x in X:
    for y in Y:
        prob = X_probs[x] * Y_probs[y]
#         print(x,"with",X_probs[x], y, "with", Y_probs[y], "has", prob)
        res.append(prob)

print()
print(round(sum(res)) == 1)
print(len(res))
```

```
True
66
```

In [22]:

```python
dist = Table().domain("X", X, "Y", Y)
print(dist, '\n')
print(dist.shape)
dist = dist.probabilities(res)
```

```
X    | Y
1    | 2
1    | 3
1    | 4
1    | 5
1    | 6
1    | 7
1    | 8
1    | 9
1    | 10
1    | 11
... (56 rows omitted)

X    | Y
[66] | [66]

/home/muhammadammarabid/anaconda3/lib/python3.8/site-packages/datascience/tables.py:222: Future
Warning: Implicit column method lookup is deprecated.
  warnings.warn("Implicit column method lookup is deprecated.", FutureWarning)
```

In [23]:

```python
dist
```

Out[23]:

	X=1	X=2	X=3	X=4	X=5	X=6
Y=12	0.000711	0.002133	0.003556	0.008000	0.011200	0.0144
Y=11	0.001422	0.004267	0.007111	0.016000	0.022400	0.0288
Y=10	0.002133	0.006400	0.010667	0.024000	0.033600	0.0432
Y=9	0.002370	0.007111	0.011852	0.026667	0.037333	0.0480
Y=8	0.002607	0.007822	0.013037	0.029333	0.041067	0.0528
Y=7	0.002844	0.008533	0.014222	0.032000	0.044800	0.0576
Y=6	0.002212	0.006637	0.011062	0.024889	0.034844	0.0448
Y=5	0.001580	0.004741	0.007901	0.017778	0.024889	0.0320
Y=4	0.000948	0.002844	0.004741	0.010667	0.014933	0.0192
Y=3	0.000632	0.001896	0.003160	0.007111	0.009956	0.0128
Y=2	0.000316	0.000948	0.001580	0.003556	0.004978	0.0064

In [24]:

```
dist.both_marginals()
```

Out[24]:

	X=1	X=2	X=3	X=4	X=5	X=6	Sum: Marginal of Y
Y=12	0.000711	0.002133	0.003556	0.008000	0.011200	0.0144	0.040000
Y=11	0.001422	0.004267	0.007111	0.016000	0.022400	0.0288	0.080000
Y=10	0.002133	0.006400	0.010667	0.024000	0.033600	0.0432	0.120000
Y=9	0.002370	0.007111	0.011852	0.026667	0.037333	0.0480	0.133333
Y=8	0.002607	0.007822	0.013037	0.029333	0.041067	0.0528	0.146667
Y=7	0.002844	0.008533	0.014222	0.032000	0.044800	0.0576	0.160000
Y=6	0.002212	0.006637	0.011062	0.024889	0.034844	0.0448	0.124444
Y=5	0.001580	0.004741	0.007901	0.017778	0.024889	0.0320	0.088889
Y=4	0.000948	0.002844	0.004741	0.010667	0.014933	0.0192	0.053333
Y=3	0.000632	0.001896	0.003160	0.007111	0.009956	0.0128	0.035556
Y=2	0.000316	0.000948	0.001580	0.003556	0.004978	0.0064	0.017778
Sum: Marginal of X	0.017778	0.053333	0.088889	0.200000	0.280000	0.3600	1.000000

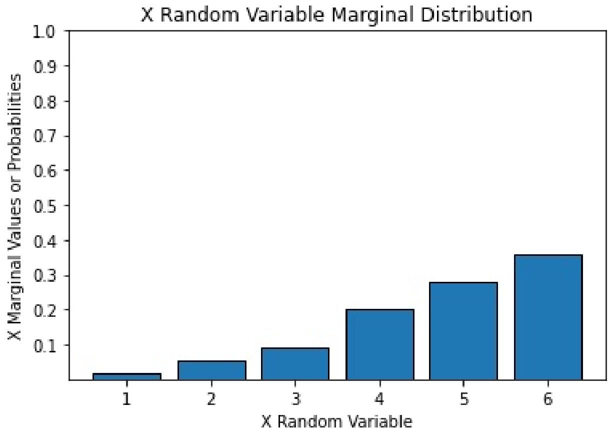
In [25]:

```
X_marginal_dist = dist.marginal_dist("X")
X_vals, X_probs = X_marginal_dist[0], X_marginal_dist[1]
print(X_vals, X_probs, '\n')

Y_marginal_dist = dist.marginal_dist("Y")
Y_vals, Y_probs = Y_marginal_dist[0], Y_marginal_dist[1]
print(Y_vals, Y_probs)
```

```
[1 2 3 4 5 6] [0.01777778 0.05333333 0.08888889 0.2        0.28       0.36      ]

[ 2  3  4  5  6  7  8  9 10 11 12] [0.01777778 0.03555556 0.05333333 0.08888889 0.12444444 0.16
 0.14666667 0.13333333 0.12       0.08       0.04      ]
```

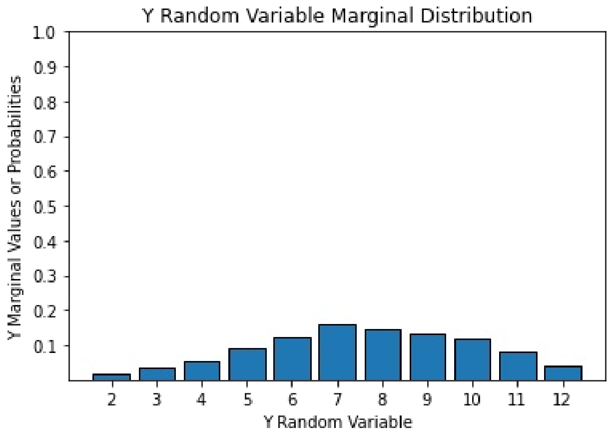
In [26]:

```
plot_X_marginal_dist(X_vals, X_probs)  # Difference btw 1-3 and 4-6 is little bit more compared to fair one
```



In [27]:

```
plot_Y_marginal_dist(Y_vals, Y_probs) # Change after 7 compared to last one
```



In [ ]:

```

```