张鑫源/16611
7606600l
作业14

9.(1). $T \vdash A_t^x \to \exists x A$

$A_1 = \forall x \neg A \to \neg A_t^x$

$A_2 = \neg\neg A_t^x \to \neg \forall x \neg A$

$A_3 = A_t^x \to \neg\neg A_t^x$

$A_4 = A_t^x \to \neg \forall x \neg A$

$A_5 = A_t^x \to \exists x A$

(2). $T \vdash \forall x(A \to B) \to (\exists x A \to \exists x B)$

$A_1 = \exists x A \to A$

$A_2 = \forall x(A \to B) \to (A \to B)$

$A_3 = B \to \exists x B$

$A_4 = (A \to B) \to (A \to \exists x B)$

$A_5 = (A \to \exists x B) \to (\exists x A \to \exists x B)$

$A_6 = (A \to B) \to (\exists x A \to \exists x B)$

$A_7 = \forall x(A \to B) \to (\exists x A \to \exists x B)$

(3). $T \vdash \forall x(A \wedge B) \leftrightarrow (\forall x A \wedge \forall x B)$

先证 $T \vdash \forall x(A \wedge B) \to (\forall x A \wedge \forall x B)$

再证 $T \vdash (\forall x A \wedge \forall x B) \to \forall x(A \wedge B)$

则 $T \vdash \forall x(A \wedge B) \leftrightarrow (\forall x A \wedge \forall x B)$

$T \vdash \forall x(A \wedge B) \to (\forall x A \wedge \forall x B)$

$A_1 = \forall x(A \wedge B) \to A \wedge B$

$A_2 = A \wedge B \to A$

$A_3 = A \to \forall x A$

$A_4 = A \wedge B \to \forall x A$

$A_5 = A \wedge B \to B$

$A_6 = B \to \forall x B$

$A_7 = A \wedge B \to \forall x B$

$A_8 = \forall x(A \wedge B) \to \forall x A$

$A_9 = \forall x(A \wedge B) \to \forall x B$

$A_{10} = \forall x(A \wedge B) \leftrightarrow (\forall x A \wedge \forall x B)$

$T \vdash (\forall x A \wedge \forall x B) \to \forall x (A \wedge B)$

$A_1 = \forall x A \wedge \forall x B \to \forall x A$

$A_2 = \forall x A \to A$

$A_3 = \forall x A \wedge \forall x B \to A$

$A_4 = \forall x A \wedge \forall x B \to \forall x B$

$A_5 = \forall x B \to B$

$A_6 = \forall x A \wedge \forall x B \to B$

$A_7 = \forall x A \wedge \forall x B \to A \wedge B$

$A_8 = A \wedge B \to \forall x (A \wedge B)$

$A_9 = \forall x A \wedge \forall x B \to \forall x (A \wedge B)$

(4). $T \vdash \exists x (A \vee B) \leftrightarrow (\exists x A \vee \exists x B)$

引理 若 $T \vdash A \leftrightarrow B$, 则 $T \vdash \forall x A \leftrightarrow \forall x B$

$A_1 = A \leftrightarrow B$

$A_2 = (A \leftrightarrow B) \to (A \to B)$

$A_3 = (A \to B) \to (\forall x A \to \forall x B)$

$A_4 = (A \leftrightarrow B) \to (\forall x A \to \forall x B)$

$A_5 = (A \leftrightarrow B) \to (B \to A)$

$A_6 = (A \to B) \to (\forall x B \to \forall x A)$

$A_7 = (A \leftrightarrow B) \to (\forall x B \to \forall x A)$

$A_8 = (A \leftrightarrow B) \to \forall x A \leftrightarrow \forall x B$

$A_1 = \neg (A \vee B) \leftrightarrow \neg A \wedge \neg B$

$A_2 = \forall x \neg (A \vee B) \leftrightarrow \forall x (\neg A \wedge \neg B)$

$A_3 = \forall x (\neg A \wedge \neg B) \leftrightarrow (\forall x \neg A \wedge \forall x \neg B)$

$A_4 = \forall x \neg (A \vee B) \leftrightarrow (\forall x \neg A \wedge \forall x \neg B)$

$A_5 = (\forall x \neg A \wedge \forall x \neg B) \leftrightarrow \forall x \neg (A \vee B)$

$A_6 = \neg \forall x \neg (A \vee B) \leftrightarrow \neg (\forall x \neg A \wedge \forall x \neg B)$

$A_7 = \neg (\forall x \neg A \wedge \forall x \neg B) \leftrightarrow \neg \forall x A \vee \neg \forall x \neg B$

$A_8 = \neg \forall x \neg (A \vee B) \leftrightarrow \neg \forall x A \vee \neg \forall x \neg B$

$A_9 = \exists x (A \vee B) \leftrightarrow (\exists x A \vee \exists x B)$。

(5). $T \vdash \exists x (A \wedge B) \to (\exists x A \wedge \exists x B)$

$A_1 = \exists x (A \wedge B) \to A \wedge B$

$A_2 = A \wedge B \to A$

$A_3 = A \to \exists x A$

$A_4 = A \wedge B \to B$

$A_5 = B \to \exists x B$

$A_6 = A \wedge B \to \exists x A \wedge \exists x B$

$A_7 = \exists x (A \wedge B) \to \exists x A \wedge \exists x B$

张钰厚/160611
7606001

(6). $T \vdash \forall x A \leftrightarrow \neg\exists x \neg A$

$A_1 = A \leftrightarrow \neg\neg A$

$A_2 = \forall x A \leftrightarrow \forall x \neg\neg A$

$A_3 = \forall x A \leftrightarrow \neg\neg\forall x \neg\neg A$

$A_4 = \forall x A \leftrightarrow \neg\exists x \neg A$

(7). $T \vdash \forall x (A \to B) \to (\exists x A \to B)$

$A_1 = \exists x A \to A$

$A_2 = (A \to B) \to (\exists x A \to B)$

$A_3 = \forall x (A \to B) \to (A \to B)$

$A_4 = \forall x (A \to B) \to (\exists x A \to B)$

(8). $T \vdash \exists x \forall y A \to \forall y \exists x A$

$A_1 = \exists x \forall y A(x,y) \to \forall y A(c,y)$

$A_2 = \forall y A(c,y) \to A(c,y)$

$A_3 = A(c,y) \to \exists x A(x,y)$

$A_4 = \forall y \exists x A(x,y)$。