

$$\begin{aligned}
 a) f(x) &= (2+ke^x)^3 \\
 &= c_0^3(2)^3 + c_1^3(2)^2(ke^x) + c_2^3(2)^1(k e^x)^2 + c_3^3(k e^x)^3 \\
 &= 8 + 12ke^x + 6k^2e^{2x} + k^3e^{3x} \\
 &= 8 + 12k\left(1+x+\frac{x^2}{2}+\dots\right) + 6k^2\left(1+2x+2x^2\right) + k^3\left(1+3x+4.5x^2+\dots\right)
 \end{aligned}$$

Coefficient of $x = 12k + 12k^2 + 3k^3$

Coefficient of $x^2 = 6k + 12k^2 + 4.5k^3$ or $\frac{9}{2}$

b) from (a), we have coefficient of $x = 3k^3 + 12k^2 + 12k$

$$\begin{aligned}
 3k^3 + 12k^2 + 12k + 9 &= 0 \\
 (k+3)(3k^2 + 3k + 3) &= 0
 \end{aligned}$$

$$\begin{aligned}
 k &= -3 \text{ or } k^2 + k + 1 = 0 \\
 \Delta &= (1)^2 - 4(1)(1) = -3 < 0
 \end{aligned}$$

i.e. no real roots

$$\therefore k = -3$$

Coefficient of x^2

$$= 6(-3) + 12(-3)^2 + 4.5(-3)^3$$

$$= -31.5 \text{ or } \frac{-63}{2}$$