

$$a) f(x) = (2 + ke^x)^3$$

$$= C_0^3(2)^3 + C_1^3(2)^2(ke^x) + C_2^3(2)(ke^x)^2 + C_3^3(ke^x)^3$$

$$= 8 + 12ke^x + 6k^2e^{2x} + k^3e^{3x}$$

$$= 8 + 12k\left(1 + x + \frac{x^2}{2} + \dots\right) + 6k^2(1 + 2x + 2x^2) + k^3(1 + 3x + 4.5x^2 + \dots)$$

$$\text{Coefficient of } x = 12k + 12k^2 + 3k^3$$

$$\text{Coefficient of } x^2 = 6k + 12k^2 + 4.5k^3 \quad \text{or } \frac{9}{2}$$

b) from (a), we have coefficient of $x = 3k^3 + 12k^2 + 12k$

$$3k^3 + 12k^2 + 12k + 9 = 0$$

$$(k+3)(3k^2 + 3k + 3) = 0$$

$$k = -3 \text{ or } k^2 + k + 1 = 0$$

$$\Delta = (1)^2 - 4(1)(1) = -3 < 0$$

\therefore no real roots

$$\therefore k = -3$$

$$\text{Coefficient of } x^2$$

$$= 6(-3) + 12(-3)^2 + 4.5(-3)^3$$

$$= -31.5 \quad \text{or } \frac{-63}{2}$$