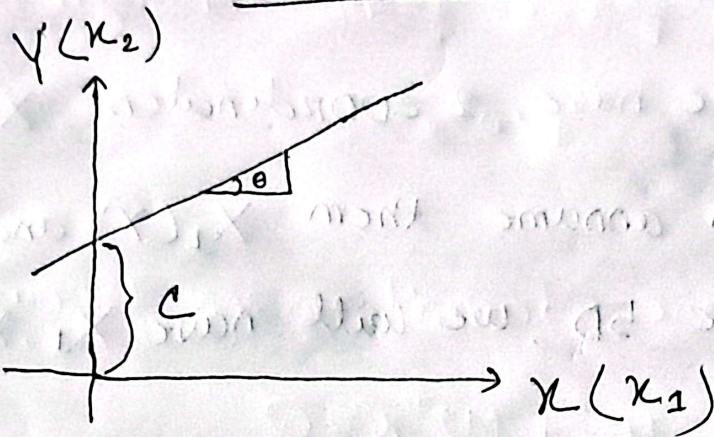


Hyper Plane on N Dimension



The equation of that line,

$$y = mx + c \quad \text{--- (1)}$$

m = slope of the line

c = distance from origin on y axis

The general form of this equation is,

$$ax + by + c = 0 \quad \text{--- (2)}$$

$$\Rightarrow by = -ax - c$$

$$\therefore y = -\frac{a}{b}x - \frac{c}{b} \quad \text{--- (3)}$$

If we compare the equation (3) with (1), we get

$$m = -\frac{a}{b}, \quad c = -\frac{c}{b}$$

For 2D, we have 2 coordinates, x and y ,
 or we can assume them $x_1(x)$ and $x_2(y)$.
 If we have 5D, we will have x_1, x_2, \dots, x_5 .

$$\textcircled{1} \Rightarrow ax + by + c = 0$$

$$\Rightarrow w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\Rightarrow \frac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n + w_0}{w_0 = 0}$$

Assume

$$a = w_1$$

$$b = w_2$$

$$c = w_0$$

$$\Rightarrow \sum_{i=1}^N w_i x_i + w_0 = 0$$

$$\Rightarrow w_0 + \sum_{i=1}^N w_i x_i = 0$$

$$\Rightarrow w_0 + \underbrace{[w_1, w_2, w_3, \dots, w_n]}_{w_{1 \times n}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}}_{n \times 1} = 0$$

$$w \cdot x = w^T x$$

$$\textcircled{3} \quad w, x = \text{vector}$$

By default vectors are column vector.

Means

$$W = W_{n \times 1} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix}, \quad X = X_{n \times 1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

For Dot Matrix, $W \cdot X = W^T X$, Means, $[w_1, w_2, \dots, w_n] = W^T$.

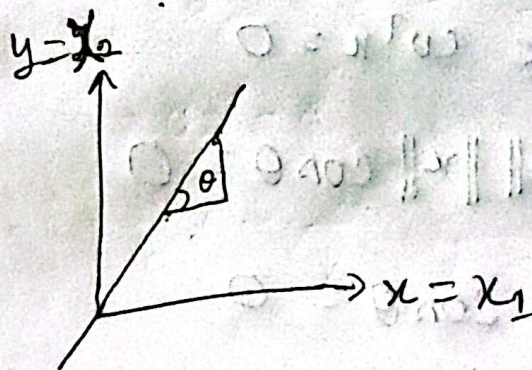
Dot Product sign

Equation (3) \Rightarrow

$$w_0 + w^T x = 0 \quad \text{--- (4)}$$

Hyperplane equation for any n Dimensions.

But what if:



Here the slope went through center (origin).

$$c = 0$$

Since $c = w_0$, equation (4) \Rightarrow

$$0 + w^T x = 0$$

$$\therefore w^T x = 0$$

∴ If a plane pass through origin,

$$\omega^T x = 0$$

If a plane doesn't pass through origin,

$$\omega_0 + \omega^T x = 0$$

Question: What is ω ?

$$\omega^T x = \omega \cdot x = \|\omega\| \|x\| \cos \theta$$

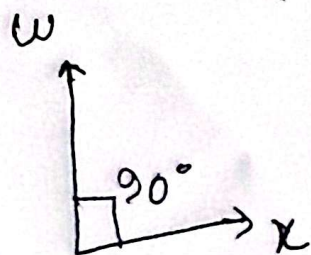
$$\text{Since } \omega^T x = 0$$

$$\therefore \|\omega\| \|x\| \cos \theta = 0$$

$$\Rightarrow \cos \theta = 0$$

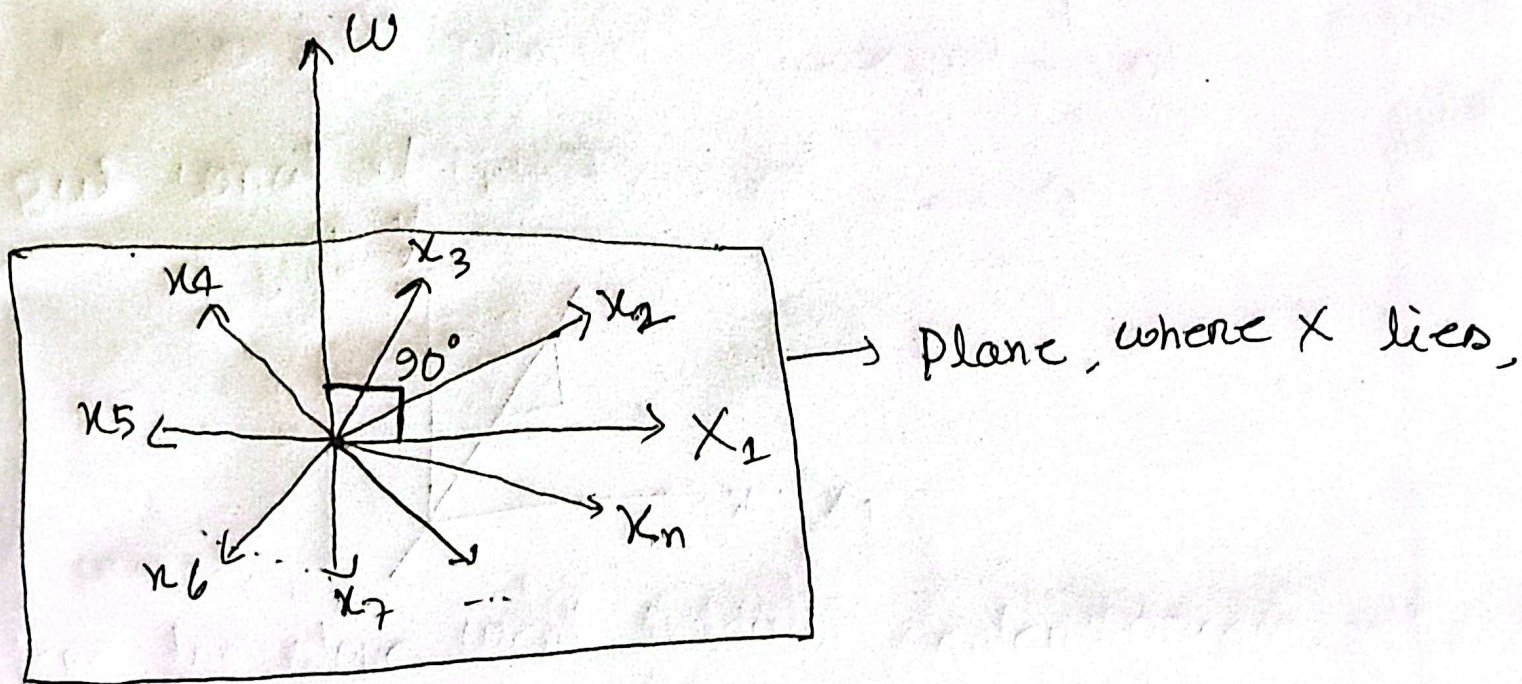
$$\theta = \cos^{-1}(0) = 90^\circ$$

∴ $\|\omega\| \|x\| \cos \theta$ will be 0 if θ between ω and x is 90° .



$$\omega \perp x$$

Here w and x are vectors, $w = [w_1, w_2 \dots w_n]$ and $x = [x_1, x_2 \dots x_n]$. So x is the coordinates / axes of N Dimension and since $w \perp x$, w is perpendicular of all the axis of N Dimension.



More general image:

