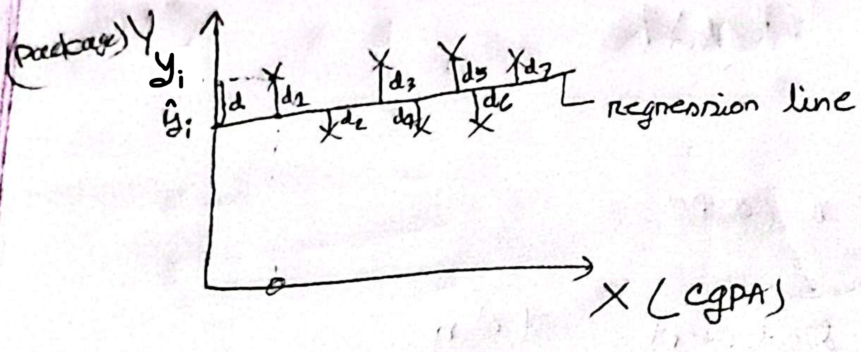


x Input	y Output
cgPA	package
3.80	5.8 lakh
2.50	7.7 lakh

$$\text{Total Error} = E = d_1 + d_2 + d_3 + \dots + d_n$$

$$= d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2$$

$$\Rightarrow E = \sum_{i=1}^{n} d_i^2$$

$$\Rightarrow E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$d = y_i - \hat{y}_i$
= Actual Value - Predicted Value

A regression line is created ~~to~~ calculating the value of m (slope) and b intercepts when input and output columns are given. Now, we have to ~~calculate~~ create a line i.e. a m and b, for what the total error i.e. E of that line will be minimized. y_i is the actual value for x_i which is created by that line but we have to predict $\hat{y}_i$ for that x_i.

E can only be minimized SELECTING UPON THE RIGHT m and b.

$$\therefore \hat{y}_i = mx_i + b$$

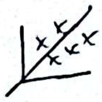
E is also called LOSS FUNCTION. So it can also be denoted with L.

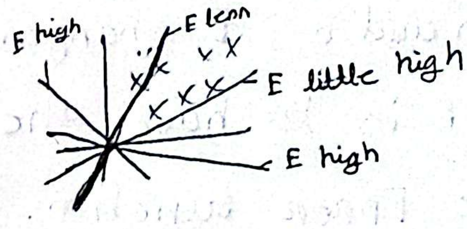
$$\therefore E(m,b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

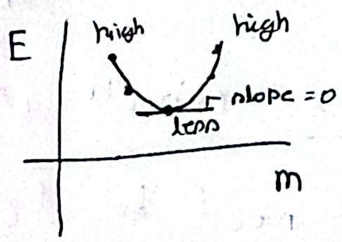
[Read right side's explanation].

Let's find m an b now.

For m :

Assume $b=0$ i.e. 





If we spin the line ~~at~~ on center (since $b=0$), our E in terms of m changes somewhat like parabola.

Also $E \propto m^2$. So, the graph of E and m^2 will be PARABOLA either way.

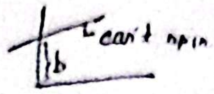
[redacted]

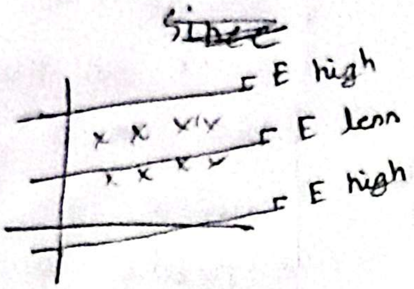
Here x_i is constant, which is given in the dataset. We can only change the value of m and b to create whatever line we want because that line may pass through the points or not for our experiment purpose.

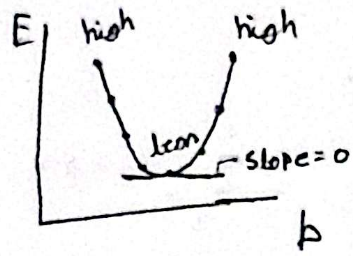
E(m,b) is now a function which depends on the value of m and b as they are not constant.

Yes m and b will be constant after calculating them but before that we can draw it ANYWHERE we want it but we need to find such m and b for what line ~~total~~ error will be minimized.

ion b

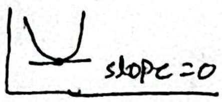
Assume m=1 i.e. constant . i.e. 





So for the changes of b, my E is changing like parabola.

From the loss function, E, $E \propto b^2$.
So, the graph of E and b^2 will be PARABOLA either way.

So in terms of both m and b, E changes like parabola. Our target is to have the 'less' point to minimize the Error function. The slope at 'less' is 0, 

which is obvious. And to calculate slope we need to do derivative and equal it to 0 since 'slope=0 on 'less'.

↳ derivative == slope

And how do we calculate slope? The changes of y for the changes of x / a particular point of x e.g. $\frac{dy}{dx}$.

· For our case, we want the error function be minimized for both m and b. So our slopes, $\frac{\partial E}{\partial m}$ and $\frac{\partial E}{\partial b}$. But since the error function is minimized on 'less' point where slope ($\frac{\partial E}{\partial m}$ for E, m graph, $\frac{\partial E}{\partial b}$ for E, b graph) is 0.

$$\therefore \frac{\partial E}{\partial m} = 0, \quad \frac{\partial E}{\partial b} = 0$$

This is what we need to do now to minimize the E in terms of both m and b which is our main target in Linear Regression.

For b:

$$\frac{\partial E}{\partial b} = 0$$

$$\Rightarrow \frac{\partial}{\partial b} \sum_{i=1}^{n} (y_i - mx_i - b)^2 = 0$$

$$\Rightarrow \sum_{i=1}^{n} \frac{\partial}{\partial b} (y_i - mx_i - b)^2 = 0$$

$$\Rightarrow \sum_{i=1}^{n} 2(y_i - mx_i - b) \cdot \frac{\partial}{\partial b} (y_i - mx_i - b) = 0$$

$$\Rightarrow \sum_{i=1}^{n} 2(y_i - mx_i - b) \cdot (0 - 0 - 1) = 0$$

$$\Rightarrow \sum_{i=1}^{n} -2(y_i - mx_i - b) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (y_i - mx_i - b) = 0$$

$$\Rightarrow \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} mx_i - \sum_{i=1}^{n} b = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{n} y_i}{n} - \frac{\sum_{i=1}^{n} mx_i}{n} - \frac{\sum_{i=1}^{n} b}{n} = 0$$

$$\Rightarrow \bar{y} - m\bar{x} - \frac{\cancel{n} b}{\cancel{n}} = 0$$

[m, b is constant here,
b+b+b=3b, b+b...+b=nb]

$$\Rightarrow \bar{y} - m\bar{x} - b = 0$$

$$\boxed{\therefore\ b = \bar{y} - m\bar{x}}$$

For m :

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$\therefore E = \sum (y_i - mx_i - \bar{y} + m\bar{x})^2 \quad \Big| \quad \sum = \sum_{i=1}^{n}$$

Now,

$$\frac{\partial E}{\partial m} = 0$$

$$\Rightarrow \frac{\partial}{\partial m} \sum (y_i - mx_i - \bar{y} + m\bar{x})^2 = 0$$

$$\Rightarrow \sum \frac{\partial}{\partial m} (y_i - mx_i - \bar{y} + m\bar{x})^2 = 0$$

$$\Rightarrow \sum . 2(y_i - mx_i - \bar{y} + m\bar{x}) . (0 - x_i - 0 + \bar{x}) = 0$$

$$\Rightarrow \sum 2(y_i - mx_i - \bar{y} + m\bar{x}) . (-x_i + \bar{x}) = 0$$

$$\Rightarrow \sum -2 \left[(y_i - \bar{y}) - m(x_i - \bar{x})\right] (x_i - \bar{x}) = 0$$

$$\Rightarrow \sum \left[(y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2\right] = 0$$

$$\Rightarrow \sum (y_i - \bar{y})(x_i - \bar{x}) - m \sum (x_i - \bar{x})^2 = 0$$

$$\Rightarrow m \sum (x_i - \bar{x})^2 = \sum (y_i - \bar{y})(x_i - \bar{x})$$

$$\Rightarrow \boxed{m = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

So Simple Linear Regression is all about : Draw that regression line(the Predicted values for input column) in a way with such m and b, so the Total Error gets minimized.