Formula :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

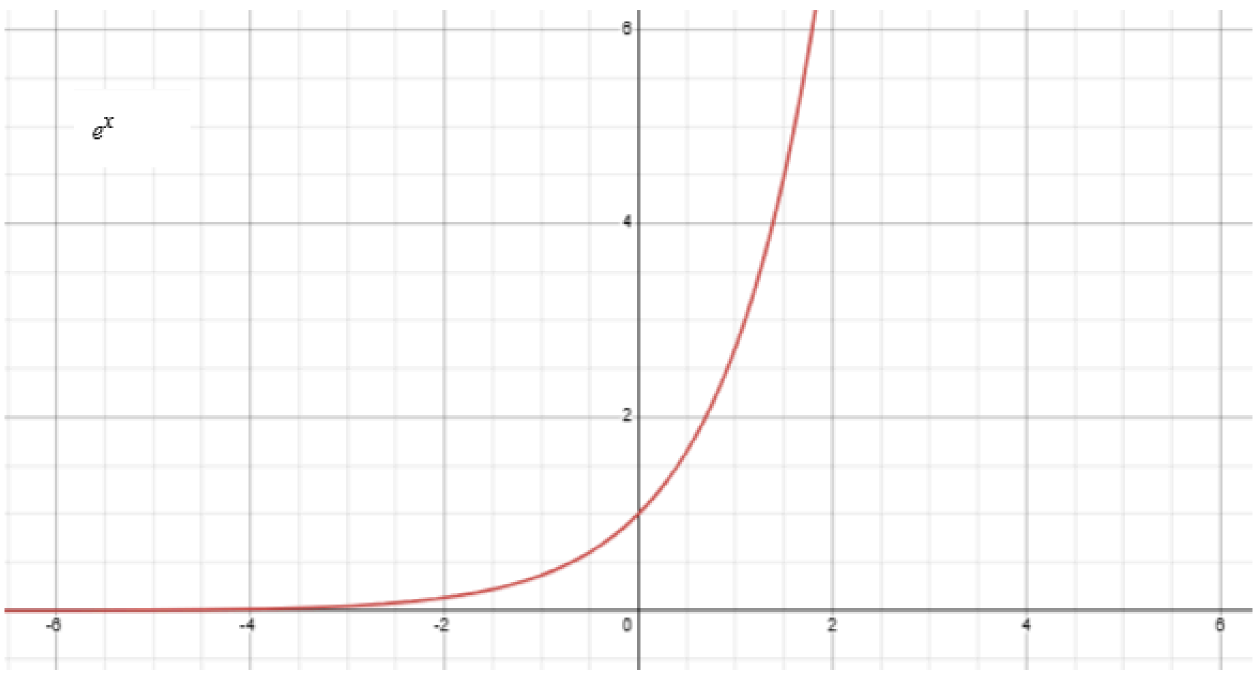
$f(x)$ = probability density function

σ = standard deviation

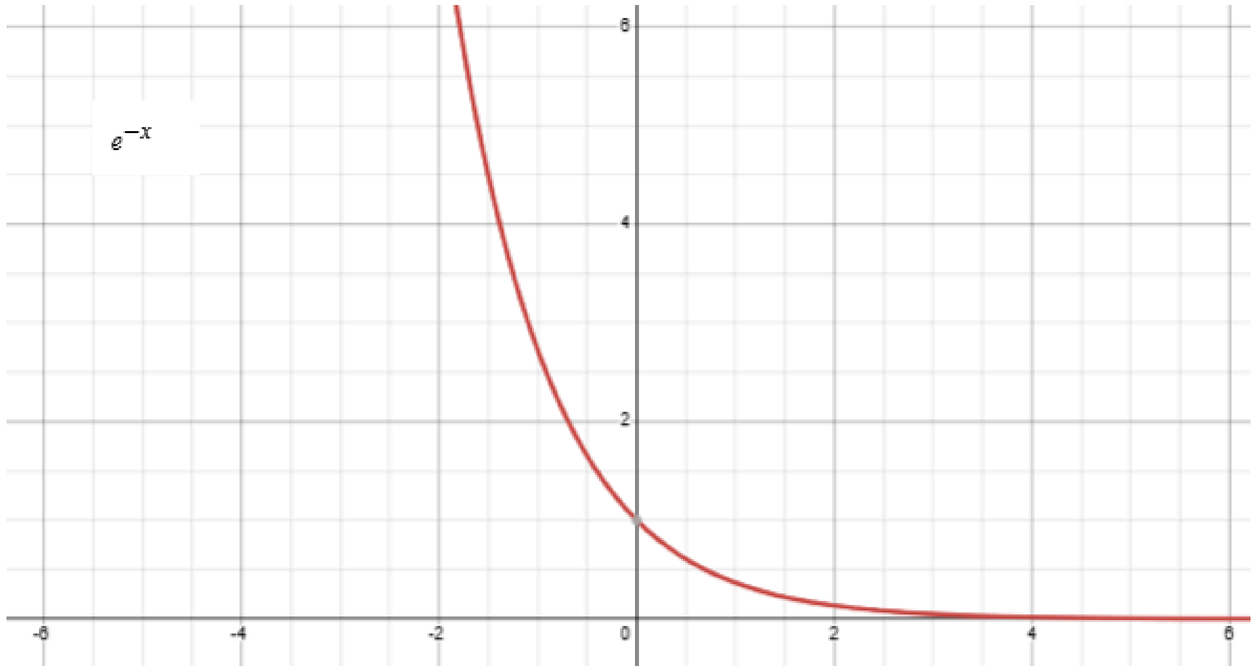
μ = mean

The steps to derive that formula :

1) e^x



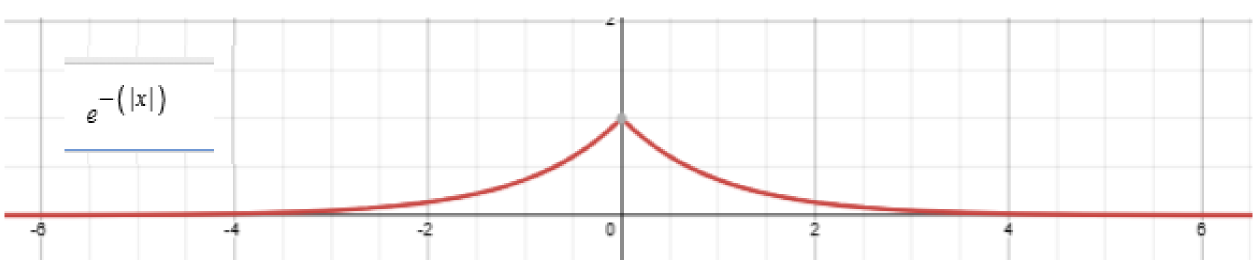
2) e^{-x}



3) $e^{-|x|}$: Why is there '-' before |x|?

i) e^0 = 1 and we want the maximum value on Y Axis is 1 according to **Normal Distribution**.

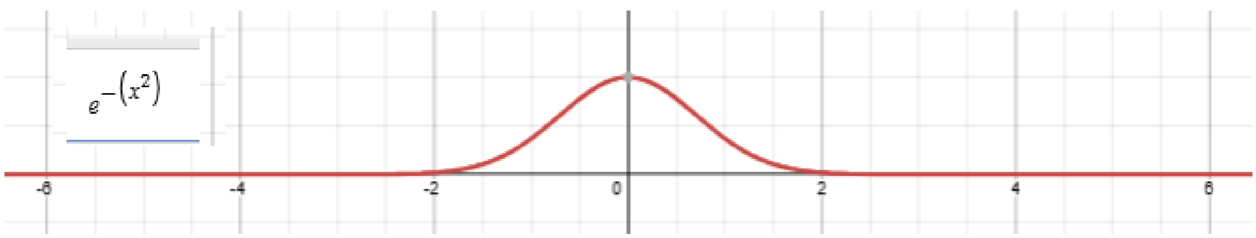
ii) $e^{any\_negative\_value}$ is < 1. So if we take |x| which turns -ve value into +ve and then put '-' before |x| to get <1 values for all x!=0 values, then we will have a graph which kind of resembles Normal Distribution but not, because e^0 = 1, $e^{0.1}$ = 0.9, $e^{0.2}$ = 0.8, $e^{0.3}$ = 0.7 etc where the outputs are continuously decreasing that doesn't make it Normal Distribution and $e^{-(x^2)}$ solves this issue. How? See the next step.



4) for x = 0.1, $e^{-(0.1\text{^}2)} = e^{-0.01} = 0.99$,
for x = 0.2, $e^{-(0.2\text{^}2)} = e^{-0.02} = 0.98$,
for x = 0.3, $e^{-(0.3\text{^}2)} = e^{-0.03} = 0.97$…

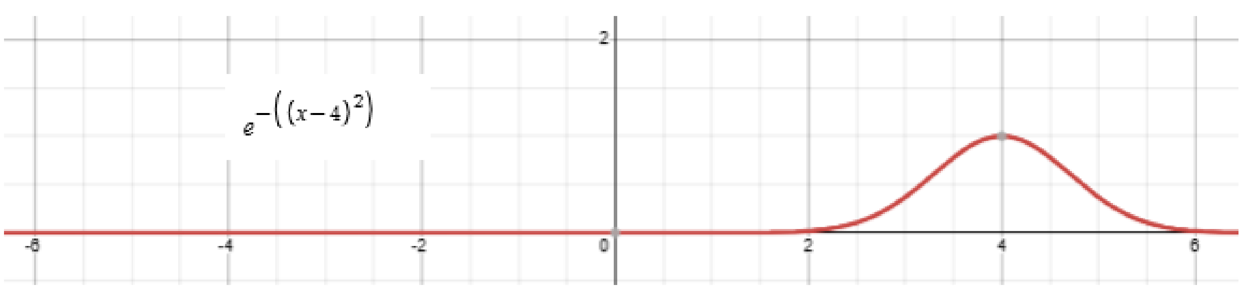
This time the values are not directly decreasing from 0.9 to 0.8 to 0.7 and hence we will have “swelling” **around the mean** like we see in **Normal Distribution**.

But why x^2? $0.1^2 = 0.01$ and 0.01 is almost 0 and that’s why for x = 0.1, $e^{-0.01}$ gives us a value veryyyyyyyyyy close to 1. And since 0.01(x=0.1), 0.02(x=0.2), 0.03(x=0.3) .... are almost 0, so they all give us 0.9_ which creates **SWELL around the MEAN.**

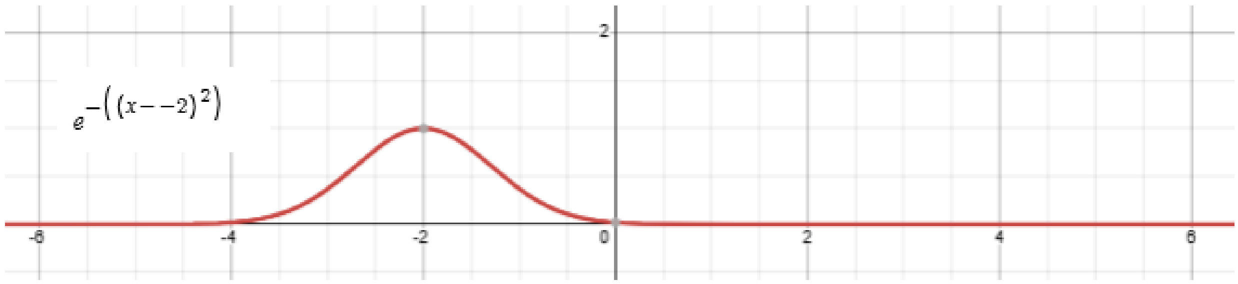


5) The Middle_Line/Mean is 0 here.
Now, how can we shift the graph towards the Positive Side e.g. we want the Middle_Line/Mean on 4. That means on 4 the value on Y Axis must be 1. But only $e^0 = 1$, not e^4. So how can we treat 4 as 0? By subtracting 4 from 4.

$$e^{-(x-4)\text{^}2} = e^{-(4-4)\text{^}2} = e^0 = 1$$



Similarly to shift the graph towards the negative side e.g. on -2 :



So final equation so far :

$$e^{-(x-\mu)^2}$$

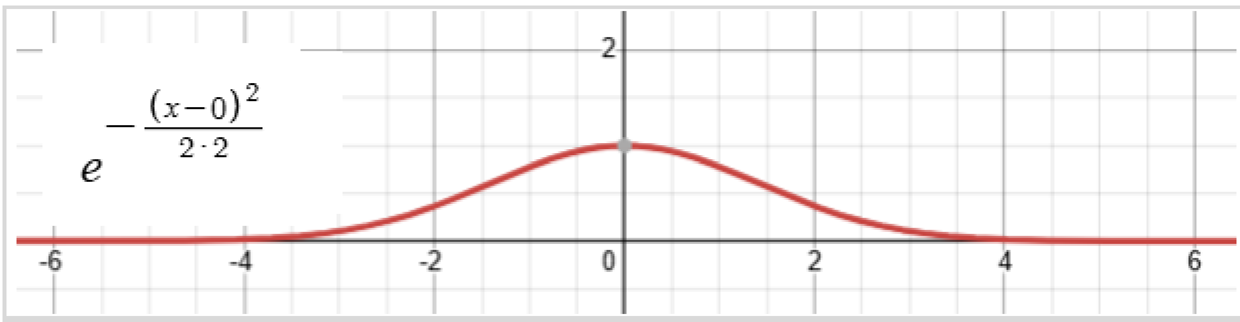
, where **μ** = Mean

6) We have done **shifting**. Now how can we make it look Tall/Fat i.e. **scale**_it/control_its_**deviation**? Divide by **Variance**(σ^2) i.e. STD^2 :

$$e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Why variance(σ^2) but not STD(σ)? Both GPT and deepseek said "At above we have done square, so at below we will do square as well".

Why is 2 multiplied with variance(σ^2)? So that both sides of the MEAN get properly distributed.



So final equation so far :

$$e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

7) If we integrate the above equation, we will get :

$$\sigma\sqrt{2\pi}$$

Now since the Area Under The Graph is 1, so we have to divide 1 with that result :

$$\frac{1}{\sigma\sqrt{2\pi}}$$

Now multiply this with the final equation in step 6. So FINAL EQUATION :

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where σ is the standard deviation and μ the mean

Note : Step **7** is not the main point to understand here, but until step **6** where we learnt how to shift the Graph on X Axis for any MEAN(**μ**) and how to scale(**σ**) it.

e^{-\frac{(x-0)^{2}}{4}}