Homework 3 in EL2450 Hybrid and Embedded Control Systems

March 4, 2016

Task 1

The robot is modelled as follows

$$\dot{x} = Ru_{\omega}cos\theta \tag{1a}$$

$$\dot{y} = Ru_{\omega}sin\theta \tag{1b}$$

$$\dot{\theta} = \frac{R}{L}u_{\psi} \tag{1c}$$

We can write them as

$$u_r = u_{\omega} + \frac{u_{\psi}}{2} \tag{2}$$

and

$$u_l = u_{\omega} - \frac{u_{\psi}}{2} \tag{3}$$

Task 2

The true values of `R` and `L` are $R_{true} = 0.10014$ and $L_{true} = 0.505286$. We're solving R from the differential equation for $\dot{x}$ as

$$R = \frac{\dot{x}}{u_{\omega}cos(\theta)} \tag{4}$$

where $u_{\omega} = 200$. We get the derivative of x by fitting a line to the data for x in `forward.csv`, then get R by taking the mean of equation 4. The result is $R = 0.00100574$, which obviously is off by two orders of magnitude. From there we apply the same solution to $\dot{\theta}$ to get $L = 0.50769$ although we used our R multiplied with the factor 100.

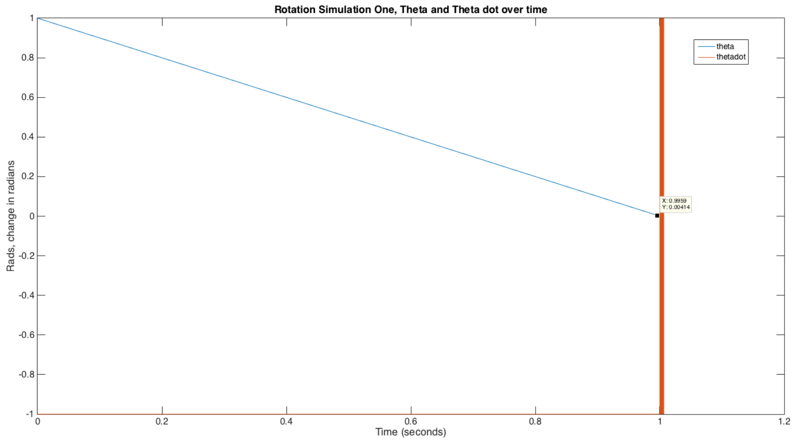
Task 3

The control system in this question can be assumed to be a bang bang or on-off controller operating in a relay manner. Zeno behaviour is known as an infinite number of steps of a variable within a finite amount of time and can often be difficult to control within a hybrid system.

When simulating a system with such a behaviour in Matlab it will cause a lot of "chatter" or oscillation around the operating point, which leads to the solver being unable to solve the system. This is observed in this simulation at around 1 second and therefore the system cannot be observed to be stable within the simulation.

Judging by the later questions, i.e. question 4, it is likely that the system has an extremely small error oscillation around zero and is not asymptotically stable in theory.

In practice this will be an extremely small value.

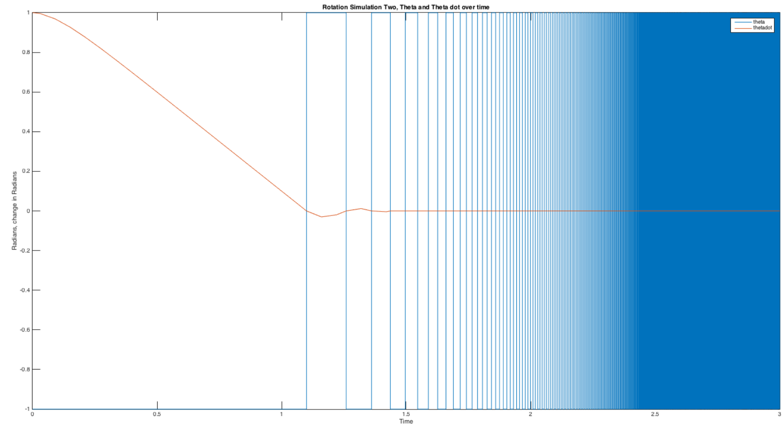


Task 4

The control system in this question can be assumed to be a bang bang or on-off controller operating in a relay manner with a time delay. The system appears to converge to zero, the time delay added in this section now means that the system can be simulated unlike the first rotation simulation.

The downside to this is that Zeno-like behaviour again develops, with continuous switching between control input of -1 and 1. Asymptotic stability cannot be shown under close observation as the output fluctuates between extremely small positive and negative numbers. This is not a montonically decreasing function so it cannot have finite value as time goes to infinity.

This is not an infinite number of steps, condition for Zeno behaviour, though clearly as the solver can now solve the simulation. This is, however, extremely hard on the actuators and is behaviour that should be avoided.

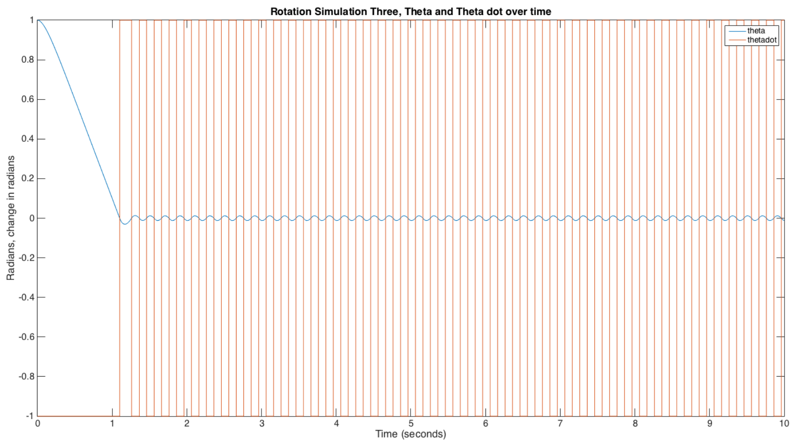


Task 5

The control system in this question can be assumed to be a bang bang or on-off controller operating in a relay manner with a time delay and also a zero order hold representing sampling of the system. The system is not asymptotically stable in that there is continued oscillation at a fixed amplitude and frequency of theta but it appears to be stable.

This system does at least avoid the extremely quick oscillation of the control signal that was observed in previous simulations. Now the control signal only changes at the frequency of the sampler, i.e. the ZOH module.

Unlike the previous question the frequency of the control signal oscillation does not increase, which was certainly a problem that could be observed.



Task 6

Discretizing the given system using the Euler forward method with sampling time T_s leads to the following state equations for the discrete system:

$$x[k+1] = x[k] + T_s R u_\omega[k] \cos(\theta[k]) \tag{5}$$

$$y[k+1] = y[k] + T_s R u_\omega[k] \sin(\theta[k]) \tag{6}$$

$$\theta[k+1] = \theta[k] + T_s \frac{R}{L} u_\Psi[k] \tag{7}$$

Task 7

We got the controller

$$u_\psi[k] = K_\psi(\theta^R - \theta[k]) \tag{8}$$

inserting this into equation (7), using $n = k + 1$ and z-transforming gives us the closed loop system from the reference to θ yields

$$\frac{\theta}{\theta^R} = \frac{RT_sK_\psi}{zL - L + RT_sK_\psi}. \tag{9}$$

In order for it to be asymptotically stable the pole has to be inside the unit circle, i.e. $|z| < 1$. We solve the characteristic equation of equation (9) for z and get

$$z = 1 - \frac{RT_sK_\psi}{L} \tag{10}$$

using aforementioned inequality and solving for K_ψ we get

$$0 < K_\psi < \frac{2L}{RT_s} \tag{11}$$

which then is the maximum gain for the proportional controller.

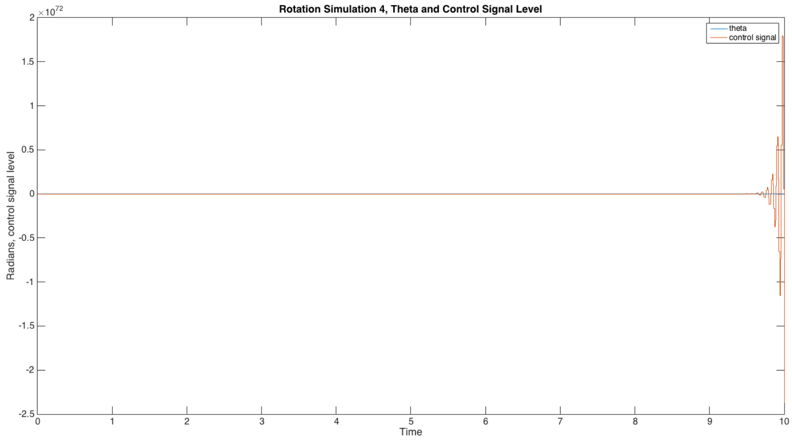
MUST MOTIVATE CHOICE OF K

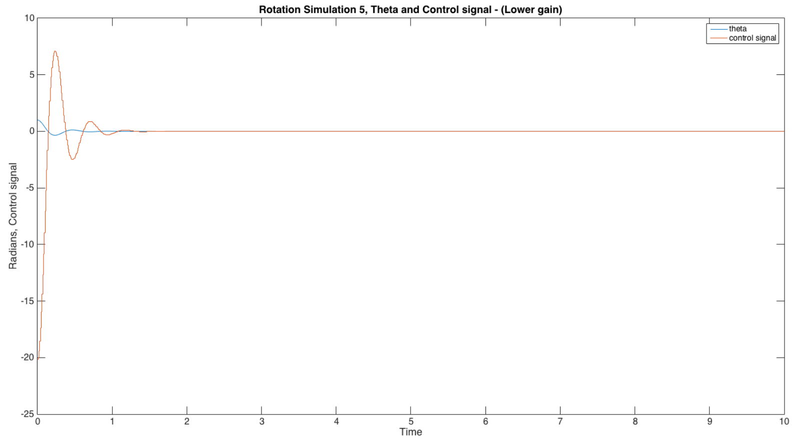
Task 8

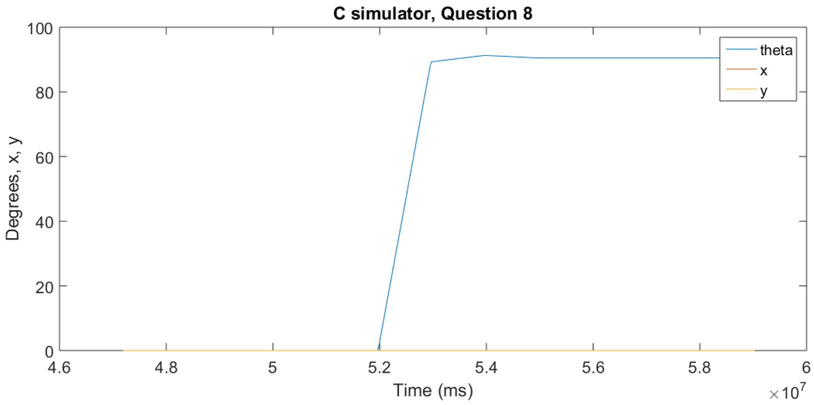
When implementing the controller in Matlab, as per the first figure below, performance was only reasonable if using a gain value around 50x less than specified in Task 7, see figure y. The controller was unstable if using the values specified in Task 8 and up to around 5x less, as you can see in the second figure below.

This is from a missing conversion factor at this point in time, however, it is also perfectly normal for the maximum theoretical value of gain to not be completely accurate when it comes to simulation of the system.

When using the c code version of this algorithm in the robot simulator it was found that oscillation would occur using the calculated values. Adding a conversion factor of 1/100 gave reasonable performance in the rotation controller, showing that the Matlab simulation gave the valuable insight that a conversion factor was required.
The plot of this rotation is shown in the third figure below. There appears to be no slipping in position in the simulator as one might expect from a real turn, despite having no x and y compensation enabled in the controller during this question. It moves relatively quickly to its goal angle of 90 degrees theta.







During the lab we found out that we were limited to using one second sampling time for the controller, which led to us looking at our answers for this part again. In the lab a K psi of L/(R*Ts) was used, and worked relatively well, which was equivalent to the 2*50*L/(R*Ts*100) that was used previously in the question. 50 coming from the difference between 1 second, the lab, and 0.02 second, the initial assumed sampling time in the controller code and Matlab simulation. Therefore the factor for K psi and relevant plots were correct. This also shows us that our theoretical calculated maximum for the gain was quite accurate, but that half this value gave better performance.

Further testing proved this to be the case by using the final controller files, with small modifications to replicate part 8.

Task 9

We're given

$$u_\omega[k] = K_\omega d_0[k] \tag{12}$$

where

$$d_0[k] = v_c[k]^\mathrm{T}\Delta_0[k] \tag{13}$$

where

$$v_c[k] = [\cos(\theta[k]), \sin(\theta[k])]^\mathrm{T} \tag{14}$$

and where

$$\Delta_0[k] = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} x[k] \\ y[k] \end{bmatrix} \tag{15}$$

From equation (1a) and euler forward we have

$$x = \frac{T_s}{z-1} R u_\omega \cos\theta \tag{16}$$

and likewise for equation (1b)

$$y = \frac{T_s}{z-1} R u_\omega \sin\theta \tag{17}$$

which jointly can be written, with equation (14), as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{T_s}{z-1} R u_\omega v_c[k]. \tag{18}$$

The transfer function from u_ω to x and y respectively is

$$G = \frac{T_s}{z-1} R v_c[k]. \tag{19}$$

The closed loop system with controller $F = K_\omega v_c[k]^\mathrm{T}$ and $v_c[k]^\mathrm{T} v_c[k] = 1$, is

$$H = \frac{K_\omega T_s R}{z - 1 + K_\omega T_s R} \tag{20}$$

where the characteristic equation together with $|z| < 1$ gives us the following inequality

$$\mid 1 - K_\omega T_s R \mid < 1 \tag{21}$$

which gives us the condition

$$0 < K_\omega < \frac{2}{T_s R} \tag{22}$$

With constant θ we get

$$d_0[k] = \cos(\theta)(x[0] - x[k]) + \sin(\theta)(y[0] - y[k]) \tag{23}$$

Task 10

When implementing the controller, as per question 8, performance was only reasonable if using a gain value around 50 times less than specified in Task 9. The controller was unstable if using the values specified in Task 9. This was only for the case of starting at some x and y values and moving to a reference value though, which is a different case as compared to the c robot simulator.

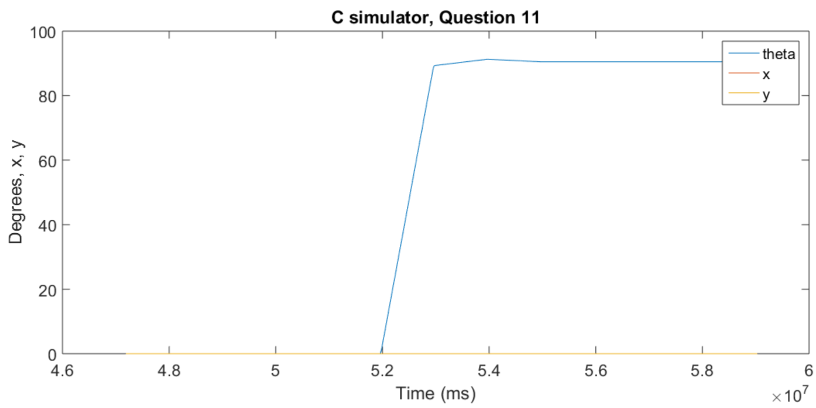
When using the c code version of this algorithm in the robot simulator it was found that the robot does not move, which makes sense considering it is starting at x0 and y0 and the aim of this part of the controller is to not let the robot move from its x and y coordinates when it is rotating.

Moving into the next task it was decided to set the constant K omega similarly to the rotational constant from question 8 but at around 1/2 the value. Seeing as one would only be trying to cancel out small movements off the position in the real experiment.

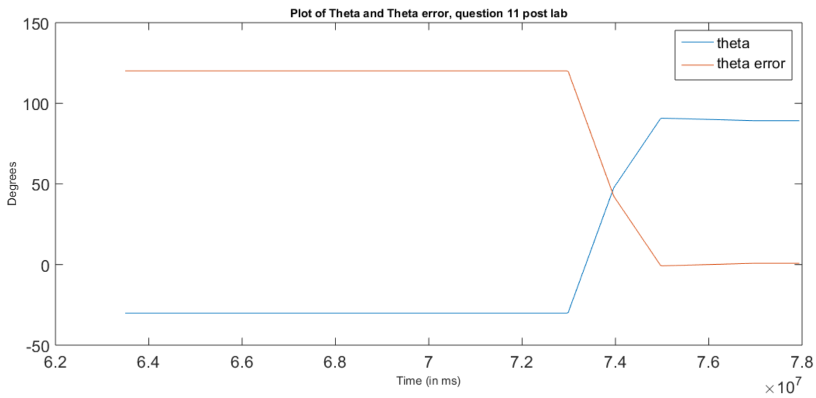
Similarly to question 8, for question 10, we ended up actually having calculated the correct value for the maximum theoretical K omega in question 9. It was also 1/2 of this maximum value in the end. Redoing the simulation had no effect of course, because this part of the controller is not particularly useful in the simulation as remarked earlier. If it had an effect in the simulator it may have made some difference because we set it in this part of the simulation initially to 1/2 of the value we ended up using in the laboratory.

Task 11

The result from the C controller is much like in q8, as the robot does not move off the spot. This seems like an overly perfect result, due to it being a simulation.
When plotted the results are as follows.



Redoing this part after the lab also had no effect because d0 was zero as stated previously. See a plot of theta and theta error over time below, d0 was was zero so could not be plotted.



Task 12

We're given the controller

$$u_\omega[k] = K_\omega d_g[k] \tag{24}$$

where

$$d_g[k] = v_g^{\mathrm{T}} \Delta_g[k] \tag{25}$$

and where

$$v_g = [\cos(\theta_g), \sin(\theta_g)]^{\mathrm{T}} \tag{26}$$

and

$$\Delta_g[k] = \begin{bmatrix} x_g \\ y_g \end{bmatrix} - \begin{bmatrix} x[k] \\ y[k] \end{bmatrix} \tag{27}$$

We get, with a constant θ_g

$$d_g[k] = \cos(\theta_g)(x[0] - x[k]) + \sin(\theta_g)(y[0] - y[k]). \tag{28}$$

By inserting equation (24) into equations (5) and (6) and by shifting one time step we can create

$$X[n] = X[n-1] + T_s R K_\omega \left(X_g - X[n-1]\right) \tag{29}$$

where

$$X[n] = \begin{bmatrix} x[n] \\ y[n] \end{bmatrix} \tag{30}$$

and similarly with X_g.

By z-transformation we get the transfer function from X_g to $X(z)$ as

$$H = \frac{z T_s R K_w}{z - 1 + T_s R K_w} \tag{31}$$

with the usual condition, the poles inside unit disc we get the limits for K_w to be

$$0 < K_w < \frac{2}{T_s R} \tag{32}$$

For the simulations, K_ω was set to the calculated middle value.

Task 13

With `K_omega` $= K_{\omega,mean}$, it stopped without any overshoot at 0.996 when set to 1.0. Furthermore, the theory supports that there should be a steady state error when using a P-controller. In order to arrive exactly at the goal without overshoot (or luck) we'll have to implement a PID.

On the real robot, one is probably not going to be able to use the full pwm range for the motors, so the steady state error will probably be larger.

Task 14

We're given the controller

$$u_\psi[k] = K_\psi d_p[k] \tag{33}$$

where

$$d_p[k] = v_{g\perp}^{\mathrm{T}} v_p[k] \tag{34}$$

and where

$$v_{g\perp}^{\mathrm{T}} = \begin{bmatrix} \sin\theta_g \\ -\cos\theta_g \end{bmatrix} \tag{35}$$

and

$$v_p[k] = \begin{bmatrix} x[k] + p\cos\theta[k] - x_0 \\ y[k] + p\sin\theta[k] - y_0 \end{bmatrix} \tag{36}$$

where $p > 0$.

We're also told we can make the approximation

$$d_p[k] \approx p(\theta[k] - \theta_g) \tag{37}$$

In the same say as with the previous tasks we get the transfer function as

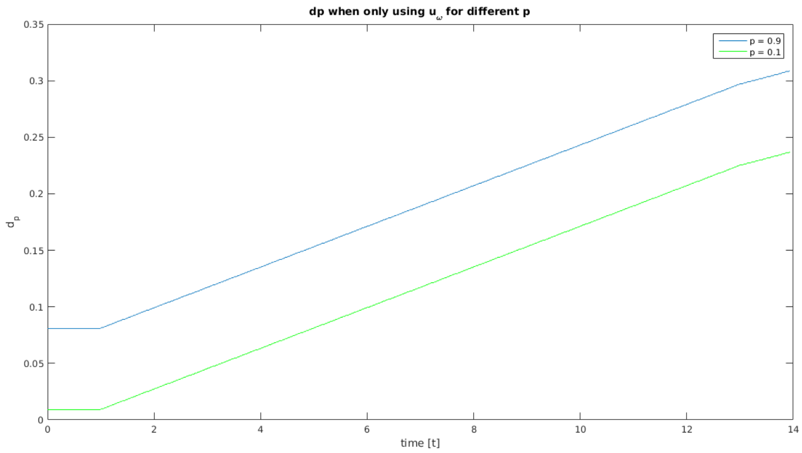
$$\theta = -\frac{zT_s\frac{R}{L}K_\psi p}{z - 1 - T_s\frac{R}{L}K_\psi p} \tag{38}$$

with poles inside the unit circle we get

$$-\frac{2L}{RT_s p} < K_\psi < 0 \tag{39}$$

Task 15

With a larger p we tolerate less of a deviation from the line we're following. A too large p will make it oscillate around the line while a too small p will possibly have it diverging from the line, especially if you hit some grit with one wheel while driving, throwing the robot off its course. This is supported in the figure below, where the greater error factor is had with the greater p. The figure is created with $u_\psi = 0$ in order to visualize it better.

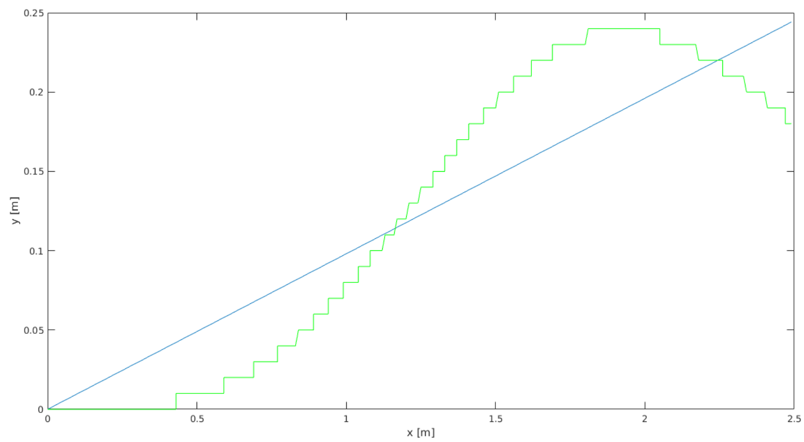


Task 16

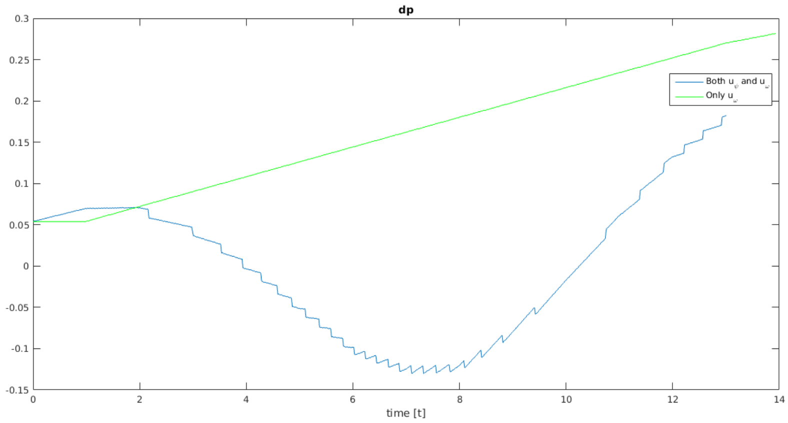
When setting a goal with $\theta_g = \theta[k]$, the robot is still and the same when setting it to point from the origin to $[x, y] = [2.55, 0.23]$, i.e node 6.

Task 17

With $p = 0.6$ and $K_\psi = K_{\psi,mean}$ and when setting the path from the origin to $[x, y] = [2.55, 0.23]$, i.e node 6, the robot adjusts nicely but with a slight overshoot and end on the node. Below we can see the x and y position together with the straight path.



Below we see d_p for when the angle correcting part of the line following controller is in use.



Task 18

The hybrid automaton is defined by the 8-tuple $H = (Q, X, Init, f, D, E, G, R)$, whereat Q is the set of states, X the continous state space, Init the initial states $\subseteq Q \times X$, f the vector fields $Q \times X \to x$, D the domain $Q \to 2^x$, E the set of edges $\subseteq Q \times Q$, G the guards $E \to 2^x$ and R the resets $E \times X \to 2^x$. In our case, we defined the automaton like the following:

$$Q = \{q_p, q_g, q_i\} \tag{40}$$

$$X = \mathbb{R}^3 \tag{41}$$

$$X = \mathbb{R}^3 \tag{42}$$

$$f : \begin{cases} f(q_p, [x, y]^T) = 0 \\ f(q_p, \theta) = \frac{R}{L} * u_\Psi \\ f(q_g, [x, y]^T) = R * u_\omega * [\cos\theta, \sin\theta]^T \\ f(q_g, \theta) = 0 \\ f(q_i, [x, y]^T) = 0 \\ f(q_i, \theta) = 0 \end{cases} \tag{43}$$

$$D : \begin{cases} D(q_p) = \{\theta \in [-\pi, \pi] : \theta \neq \theta^R, [x, y]^T \in \mathbb{R}^2\} \\ D(q_g) = \{\theta \in [-\pi, \pi] : \theta = \theta^R, [x, y]^T \in \mathbb{R}^2 : [x, y]^T \neq [x_g, y_g]^T\} \\ D(q_i) = \{\theta \in [-\pi, \pi] : \theta = \theta^R, [x, y]^T \in \mathbb{R}^2 : [x, y]^T = [x_g, y_g]^T\} \end{cases} \tag{44}$$

$$E = \{(q_p, q_g), (q_g, q_i), (q_i, q_p)\} \tag{45}$$

$$G : \begin{cases} G((q_p, q_g)) = \{\theta \in [-\pi, \pi] : \theta = \theta^R\} \\ G((q_g, q_i)) = \{[x, y]^T \in \mathbb{R}^2 : [x, y]^T = [x_g, y_g]^T\} \\ G((q_i, q_p)) = \{\theta \in [-\pi, \pi] : \theta \neq \theta^R\} \end{cases} \tag{46}$$

The initial state is the turning state q_p as predefined in the assignment paper. The robot will rotate until it is pointing towards the goal with an error that is smaller than

2°. Once the system has switched from turning to line following state q_g the robot will start driving towards the goal. The line following state is active until the robot is less than 5 cm away from the goal. A distance less than 5 cm to the goal is considered as having arrived the desired goal. Once the robot has arrived at the goal it will switch to the idle state q_i, where it does remains standing still. In this way an oscillatory behaviour is prevented, where the robot would travel back and forth over the goal point. The robot will stand still at the goal until new goal coordinates are entered and the distance to the goal is larger than 5 cm again. This criteria is the guard condition to go to the initial state q_p again, where the robot starts rotating until it is pointing into the direction of the new goal. The behaviour of the Automaton allows only one direction of transition. It will only transit 'forward' but never back to a previous state.

Task 19

Task 20

This task intentionally left blank.

Task 21

When driving manually, you send a fixed PWM to the motors for a short time per click. This is in contrary to to the case of automatic/autonomous driving where the robot slows down when approaching the target. In order to drive accurately you have to have a very slow feedback from your eyes to your control finger in order not to drive too far or turn too much. This is obviously hard without implementing a more sophisticated tele-op interface, such as proportional throttle.

Task 22

As expected, the simulation results were far from identical to the results when trying it out practically. Two major issues limited our performance when applying the control strategy to the nexus robot. One was that we hadn't calibrated p in equation (36) properly, causing it to not follow a path but just driving straight and thus being very vulnerable to bad starting angles. The second issue was our stopping conditions. In the simulation it worked well with the small tolerances we used, this didn't work as well in practice. First of all, we forced the robot to be within a certain distance from the goal both in x and y direction. In the simulation case the robot had reached its goal if it was at most 5 cm from the goal in x and y. A smarter solution would be to pay attention to the angle to the goal. For instance, if the robot drives from $[x, y] = [0.0, 0.0]$ to $[x, y] = [1.0, 0.0]$, it should rather stop at $[x, y] = [1.0, 0.1]$ than $[x, y] = [0.95, 0.05]$, trying to modify its position. When stopping at $[x, y] = [1.0, 0.1]$, since this case used straight corridors, it won't matter when driving to the next point which probably will be π radians off. These are of course just example tolerances and attention has to be paid to the width of corridors and such so that the robot is still able to continue forward.

In the end, differences between simulation and real life usually involves that some events weren't modelled. Easy things like measurement noise, wheel size discrepancies and motor/wheel non-linearities from friction or battery voltage. But also stochastic events such as floor slip. In order to make your robot work well you always have to tune your controller on the real system and not spend too much time in simulation.