Vibration Testing of Small Satellites

This series of papers provides a tutorial along with guidelines and recommendations for vibration testing of small satellites. Our aim with these papers is to help you (a) ensure the test meets its objectives in demonstrating flight worthiness and (b) avoid test failures, whether associated with a design deficiency or with excessive loading during test. Addressed are sine-burst testing, random vibration testing, and low-level diagnostic sine sweeps. Although much of the guidance provided in this series applies to CubeSats, the series is primarily aimed at satellites in the 50 – 500 lb (23 – 230 kg) range. Most of the guidance applies to larger satellites as well if they will be tested on a shaker.

The plan is for this series to include seven parts, each of which will be released when completed:

The most recent versions of these papers are available for free download at

http://instarengineering.com/vibration_testing_of_small_satellites.html.

Part 6: Notching and Force Limiting

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Rev. A, July 19, 2017

In the context of testing on a shaker, ***notching*** means reducing the input at selected natural frequencies of the test article in order to avoid excessive loading. To implement notching properly, we need to be able to recognize situations in which loading would be excessive without notching.

As noted in Part 1 of this series, testing on a shaker is not truly representative of the vibration that a small satellite or a component experiences in flight. Uniform acceleration across the entire mounting interface is one aspect of a shaker that tends to be more severe than actual flight vibration. But even more severe can be the fact that energy imparted by the shaker is limited only by the force rating of the shaker, whereas the vibration energy during launch is usually much more limited.

If we mount a mass-spring system to a vibrating machine that has limited energy, some of the energy moves into the mass at frequencies near and equal to the system's natural frequency. The result is a

natural reduction (notch) in acceleration at the base of the mass-spring system at that frequency. The same happens when we mount a spacecraft to a launch vehicle. When we derive a random vibration environment from flight data, we envelop the peaks, as shown in Fig. 6-1, rather than match any notches because the frequency for any notches will be different when we launch a different payload.
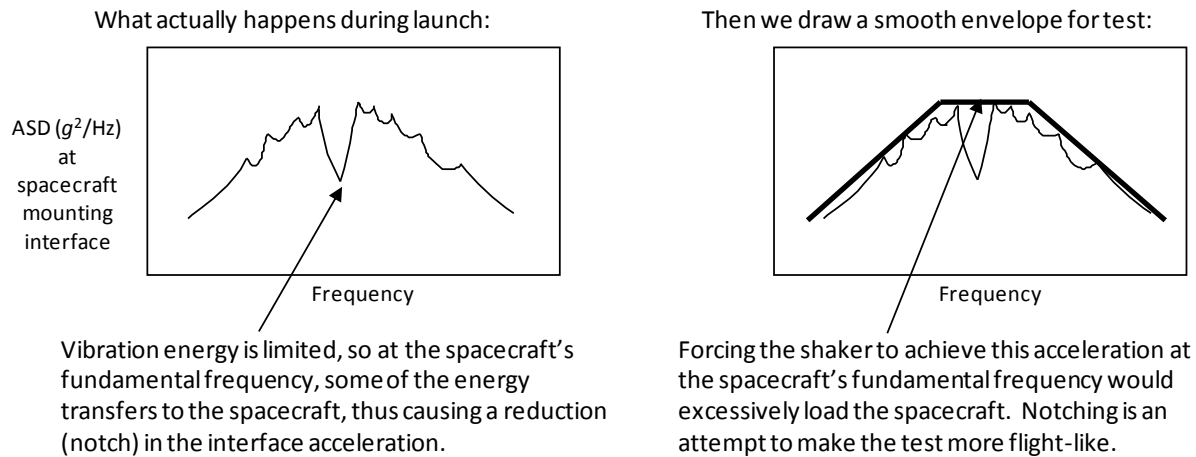
What actually happens during launch: Then we draw a smooth envelope for test:

ASD (g^2/Hz) at spacecraft mounting interface

Frequency   Frequency

Vibration energy is limited, so at the spacecraft's fundamental frequency, some of the energy transfers to the spacecraft, thus causing a reduction (notch) in the interface acceleration.

Forcing the shaker to achieve this acceleration at the spacecraft's fundamental frequency would excessively load the spacecraft. Notching is an attempt to make the test more flight-like.

Fig. 6-1. Justification for Notching.

Three methods of notching used in the space industry are

- Force limiting
- Response limiting
- Manual notching

These methods are individually discussed below. Before we start that discussion, though, we need to make the following point:

> **Regardless of the method, notching should be used only with valid technical rationale!**
>
> **Wanting to keep component responses during system-level testing from exceeding predicted capability or environments to which the components were tested is NOT valid technical rationale!**

Remember, the purpose of the test is to stress the hardware at least as severely as it will be stressed during launch. No one wants the risk of flying hardware that has been inadequately tested—especially not the people flying other satellites on the same launch! So, before notching the test environment, we must convince ourselves and other stakeholders that the test will envelop launch random vibration. The importance of doing so is illustrated by the following case history:

> In 1997, the Space Test Experiments Platform 4 (STEP-4) spacecraft was successfully placed into orbit by a Pegasus XL launch vehicle. However, the mission was a failure because no contact could be made with the spacecraft after launch. The failure investigation[1] concluded that the most likely cause of failure was vibration during the captive-carry phase of the mission during which Pegasus is carried by an aircraft, prior to

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[1] "AFI 51-503 Space Accident Investigation Report for Space Test Experiments Platform Mission 4 (STEP-4)". June 29, 1998. Air Force Materiel Command,.

dropping and firing its engines. STEP-4 had been under-tested in the frequency range (40 – 50 Hz) at which the vibration levels peaked during flight. Test levels at the integrated spacecraft level had been reduced in order to keep components from seeing levels that exceeded those they had been tested to previously. There was no technical justification for doing so and, rather than finding the hardware deficiency during ground testing, the STEP-4 program incurred mission failure.

In our opinion, unless there is evidence that the specified test environment is unnecessarily conservative, any notching during a vibration test, regardless of method, should be based on the force-limiting principles and methods described in NASA-HDBK-7004C [1]. With this methodology for a random vibration test, the applicable test acceleration spectral density (ASD) is reduced as needed to keep the force between the test article and the shaker (*base force*) from exceeding a justifiable limit. For random vibration testing from 20 to 2000 Hz, the force limit is described as a function of frequency: a force spectral density (FSD) in units of lb^2/Hz or N^2/Hz, from 20 to 2000 Hz. We refer to this as a ***force-limit function***.

Ideally, the test is performed with force gages between the test article and the shaker, and test control switches from accelerometers to the force gages for any frequency at which the base force otherwise would exceed the prescribed force-limit function. Such a test is referred to as a ***force-limited test***.

Without force gages in the setup, force-limiting methods still should be used, but with other indicators of base force.   For example, if acceleration at a particular location on the test article corresponds well with base force within the applicable frequency range for the fundamental mode, we might use ***response limiting*** within that range. For that accelerometer, we would derive a limit ASD that keeps the predicted base FSD from exceeding a justifiable limit, and then have test control switch automatically to that channel if needed to keep response below the limit. An example of response limiting appears near the end of this paper.

Manual notching also may be used to approximate true force limiting. With this approach, we derive a notched input ASD that can be entered directly into the control system. The notch should be based initially on force-limiting methodology and then adjusted (shifted in frequency and modified in depth) based on test data from incremental low-level runs (e.g., -6 dB, then -3 dB). This process is described in detail at the end of this paper.

With manual notching and response limiting, it is typically acceptable only to notch or limit response for the fundamental mode because both methods are dependent on accuracy of the pretest finite element model (FEM). In some cases, we also can notch a well-defined, high-mass second mode with confidence.

Let's take a close look at the principles and practice of force limiting; then we'll revisit response limiting and manual notching.

Force Limiting

Force limiting, when used properly, is the most commonly accepted method of notching within the space industry. In most cases, it's the only justifiable method of notching for higher-order modes of vibration, those other than the first mode or two that show significant response in a particular test axis.

The premise of force limiting is that (a) the modes with the most effective mass in the test axis are the ones that should be notched, and (b) the best indication of modal effective mass is base force. Thus, if we can identify an appropriate upper limit on that force—which is in actual force units for a sinusoidal vibration test and in terms of FSD for a random vibration test—we can justify controlling the shaker to

the force-limit function instead of to base acceleration within frequency ranges in which the base force otherwise would be exceeded. In other words, using random vibration testing as an example, the control system switches between controlling the test to the specified ASD, as measured by accelerometers at the base, to controlling off of force gages at the base, as needed to ensure a derived FSD is not exceeded.

When a payload is mounted on a vibrating host, vibration levels at the payload base are reduced by the presence of the payload, with the extent of reduction at any frequency depending on the payload's interface **impedance** (resistance to—or force caused by—acceleration) at that frequency relative to the impedance of the host. Impedance varies with frequency, as indicated by modal effective mass. Force-limit functions can be derived either by calculating impedance on both sides of the interface or by performing system-level response analysis (such as vibro-acoustic analysis) to determine the maximum interface force. Other methods have evolved as well, including the semi-empirical method discussed later in this paper.

Force limiting is a relatively new technology, first put in use at the Jet Propulsion Laboratory in 1990. Following is a brief history, excerpted from NASA Reference Publication RP-1403 [2], Sec. 1.0:

> *The practice of limiting the shaker force in vibration tests was instigated at the NASA Jet Propulsion Laboratory (JPL) in 1990 after the mechanical failure of an aerospace component during a vibration test. Now force limiting is used in almost every major vibration test at JPL and in many vibration tests at NASA Goddard Space Flight Center (GSFC) and at many aerospace contractors. The basic ideas behind force limiting have been in the literature for several decades, but the piezo-electric force transducers necessary to conveniently implement force limiting have been available only in the last decade. In 1993, funding was obtained from the NASA headquarters Office of Chief Engineer to develop and document the technology needed to establish force limited vibration testing as a standard approach available to all NASA centers and aerospace contractors.*

Section 2.0 of [2] provides additional history of the idea and theory of force limiting dating back to the 1950s.

Section 4 of NASA-HDBK-7004C provides excellent advice on use of force gages in a force-limited test—clearly lessons learned from many years of experience. In summary,

a. The gages should be the piezoelectric ring-shaped types, with the mounting bolt passing through and preloading the ring. These gages are very stiff.

b. If possible and practical, the gages should be placed directly between the test article and the fixture that adapts to the shaker in order to get an accurate measure of force on the test article. Whether doing so is possible depends on spacing of the test article's mounting bolts.

c. Sandwiching force gages between fixtures (Fig. 6-2) is also possible, with the test article mounted to the upper fixture, but acceleration of the upper fixture's mass adds to the force reading. For the test article's first mode of vibration, we can account for this additional force, increasing the force limit by fixture mass times acceleration, but, for higher-order modes, the mass of the fixture absorbs some of the force from the test article so that the force readings in the gages are actually lower than the test article sees. If the mass of the upper fixture is less than about 10% of the test article's mass, the above effects may be considered negligible. In many test configurations, it's not possible to meet this constraint while also keeping the fixture stiff enough to avoid

unacceptably affecting the test article's modes of vibration in the test frequency range. When using a heavy upper fixture, we typically can force limit only the first mode. Figure 6-3 shows an example of a force-limiting assembly for ESPA[2]-class satellites.

d. Preload for the bolts passing through the ring gages must be high enough to prevent gapping (the gages must be used in compression) and to keep the surfaces from slipping in shear within clearance holes, overcoming friction.

e. A junction box should be used to convert individual force readings into total force at the interface. Total moment also can be calculated.
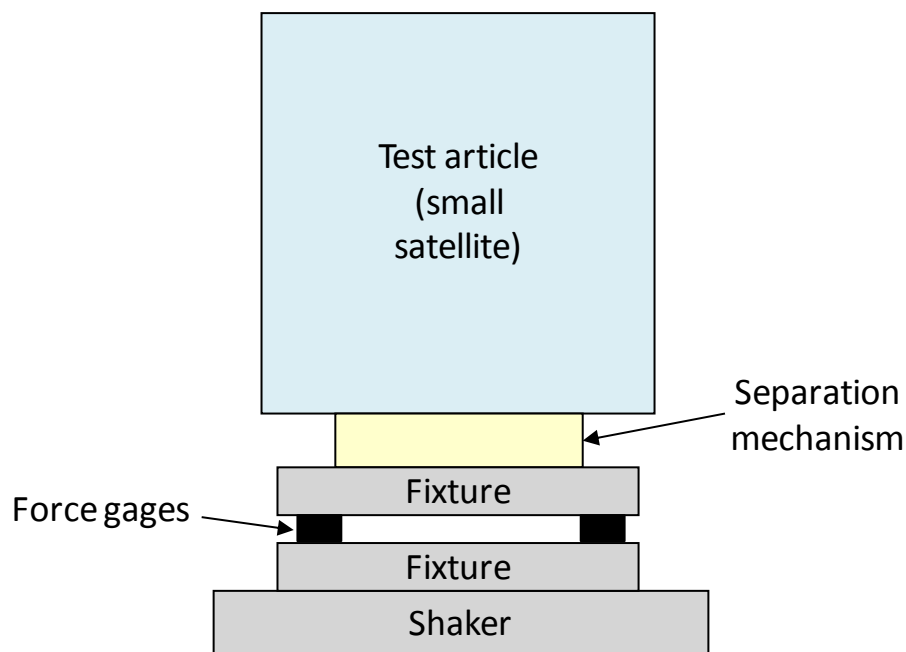
Fig. 6-2. Alternate Test Configuration for Force Limiting, with Force Gages Sandwiched Between Test Fixtures.
The preferred approach, when space is available, is to put the gages directly between the test article and the lower fixture shown here, without use of an upper fixture.

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[2] EELV Secondary Payload Adapter, where EELV is Evolved Expendable Launch Vehicle. An ESPA-class satellite weighs no more than 400 lb and uses a separation mechanism with 24 bolts on a 15"-diameter bolt circle to mate to the satellite and also to the ESPA.
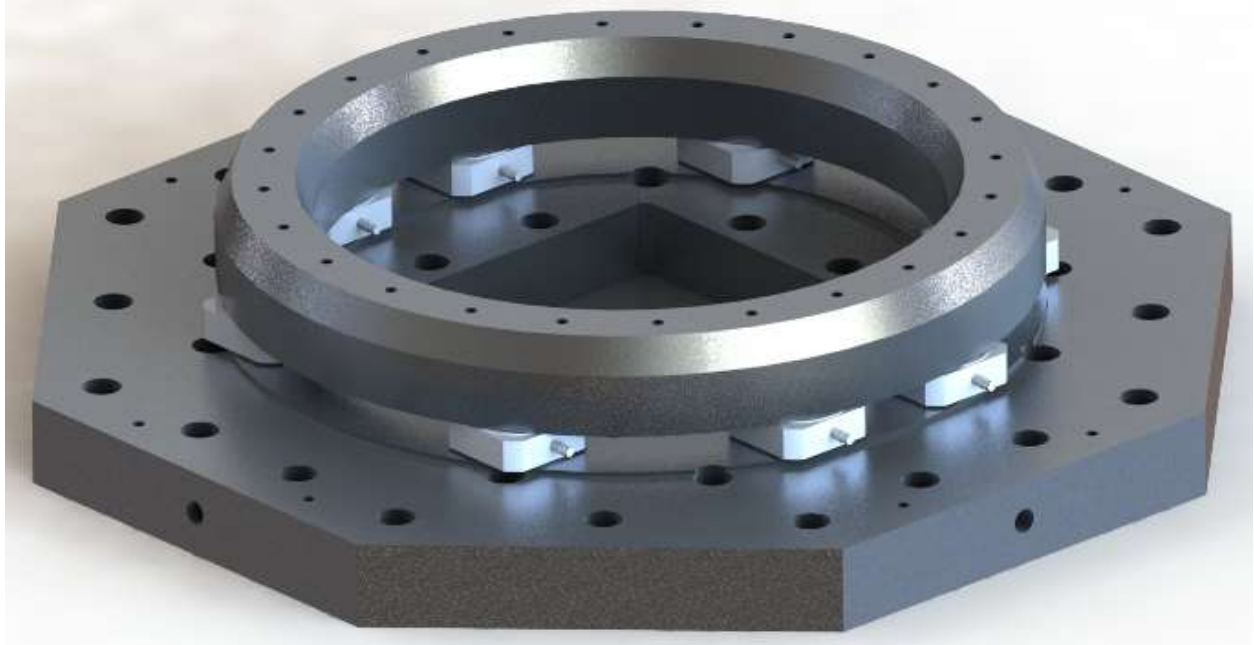
Fig. 6-3. Example Force Limiting Assembly for ESPA-class Satellites. Eight force gages are sandwiched between fixtures. The lower fixture adapts to the shaker; the upper fixture provides for 24 bolts to a 15"-diameter separation mechanism. Putting the force gages on the same circle as the standard interface allows the upper fixture to be relatively light, only about 15 lb in this case. The upper fixture's thickness is needed to enable a relatively uniform load distribution between the 24 bolts to the separation mechanism.

The most commonly used method of deriving force-limit functions is the ***semi-empirical method*** (SEM) described in NASA-HDBK-7004C. This method is so popular because, with certain assumptions, it does not require calculation of impedance on both sides of the interface; in most cases, the small-satellite developer does not have access to the math models necessary for calculating impedance of the launch vehicle. For random vibration testing, the force-limit function is in two parts:

$$\text{For } f < f_b, \quad S_{FF}(f) = C^2 M_0^2 S_{AA}(f) \tag{6.1}$$

$$\text{For } f \geq f_b, \quad S_{FF}(f) = C^2 M_0^2 S_{AA}(f)\left(\frac{f_b}{f}\right)^{2n} \tag{6.2}$$

where f is frequency, f_b is the break frequency (see below), S_{FF} is the force limit in lb^2/Hz, C and n are configuration-dependent constants without units, M_0 is the mass of the test article in lb (lb is used as the mass unit when g is the unit of acceleration), and S_{AA} is the applicable ASD for the test in g^2/Hz. If S_{AA} is constant between 20 – 2000 Hz (not usually the case), the resulting FSD limit looks something like the plot in Fig. 6-4. C^2 sets the height of the low-frequency plateau, and n sets the slope at frequencies above f_b.
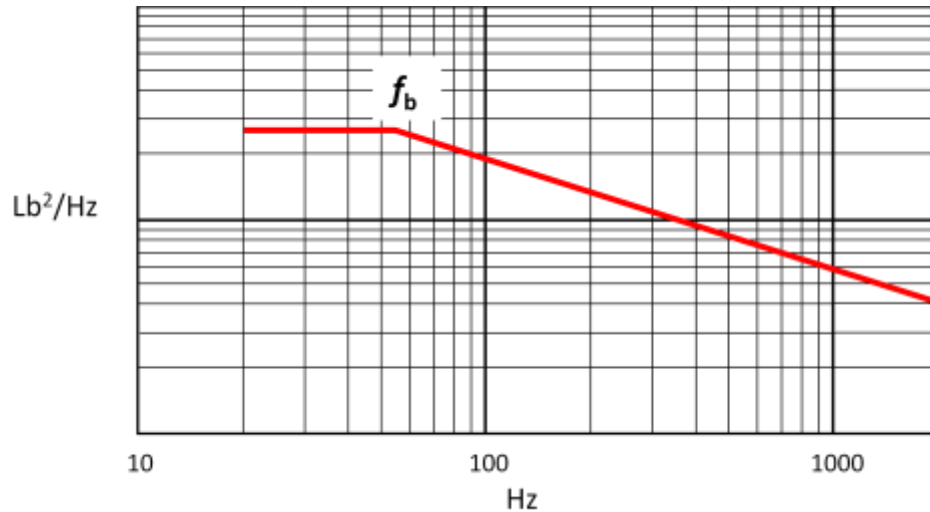
Fig. 6-4. Example of a Derived FSD Limit.

As a simple example of how to use the SEM, consider a base-driven mass on a spring, shown in Fig. 6-5 along with the transmissibility curve.
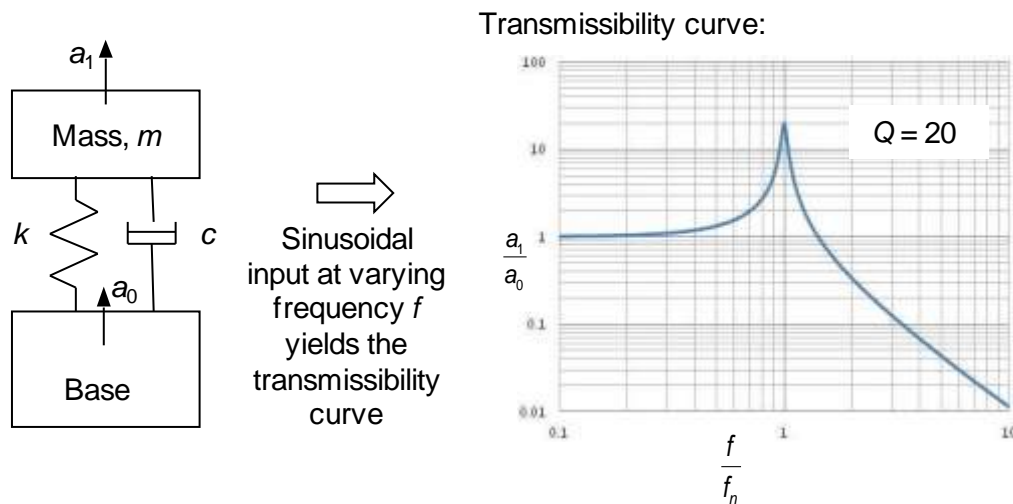

Fig. 6-5. Transmissibility Curve for a Base-driven Mass on a Spring. k is the spring stiffness; c is the damping factor; a_0 and a_1 are the peak sinusoidal accelerations of the base and the mass, respectively; f is the input frequency; f_n is the natural frequency of the mass-spring system; and Q is the quality factor, assumed in this case to be 20.

We'll set the weight of the mass equal to 100 lb and the natural frequency when the spring is grounded at its base to 80 Hz, and we'll again assume $Q = 20$. Multiplying the transmissibility function, $T_R(f/f_n)$, by the mass, M, yields the **apparent mass**, $M_A(f)$, which is the spring force caused by one-g sinusoidal base acceleration that is slowly swept through a given frequency range.

$$M_A(f) = M T_r(f / f_n) \tag{6.3}$$

Note that, when using g as the unit of acceleration, we use weight, in force units (e.g., lb), in place of mass for equations such as the above; then, conveniently, the unit g drops out of the equation $F = MA$. Figure 6-6 shows the apparent mass for this spring-mass system.
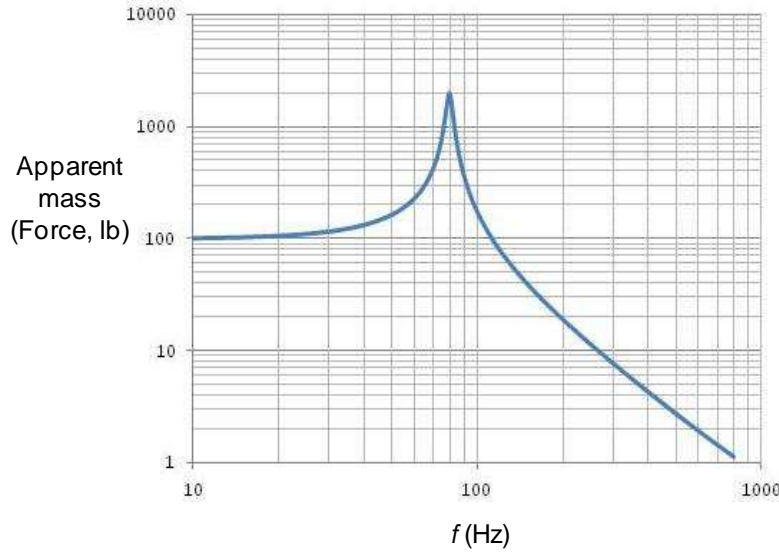

Fig. 6-6. Apparent Mass for a Base-driven Mass on a Spring. 100-lb mass, 80-Hz system, with Q = 20.

Now let's see how force limiting with the SEM applies to this spring-mass system. For sinusoidal input, the SEM defines the force-limit function, $F_L(f)$, as

$$\text{For } f < f_b, \quad F_L(f) = CM_0 a_0(f) \tag{6.4}$$

$$\text{For } f \geq f_b, \quad F_L(f) = CM_0 a_0(f)\left(\frac{f_b}{f}\right)^n \tag{6.5}$$

where M_0 is mass (again in force units), a_0 is the base acceleration (as a multiple of g), and C and n are constants. Arbitrarily setting $C = 2$ (establishing this value is discussed later in this paper), and selecting $n = 2$ to approximately match the slope of the force-limit function to the roll-off of the transmissibility curve, the force-limit function for unit-g sinusoidal input is shown in Fig. 6-7, as compared with the apparent mass. The break frequency, f_b, is set equal to the natural frequency, f_n.

Now let's assume the same spring-mass system is base driven with random vibration at a constant ASD, $S_{AA}(f)$, of 0.1 g^2/Hz from 20 – 800 Hz. The response acceleration causes an FSD, $S_F(f)$, at the base of the spring, calculated as

$$S_F(f) = M_A^2(f) S_{AA}(f) \tag{6.6}$$

where $M_A(f)$ is the apparent mass. Once again setting $C = 2$ ($C^2 = 4$) and $n = 2$, and assuming constant input ASD of 0.1 g^2/Hz, we use Eqs. 6.1 and 6.2 to derive the force-limit function (this time as an FSD) shown in Fig. 6-8, as compared with $S_F(f)$.

Input would be reduced (notched) here
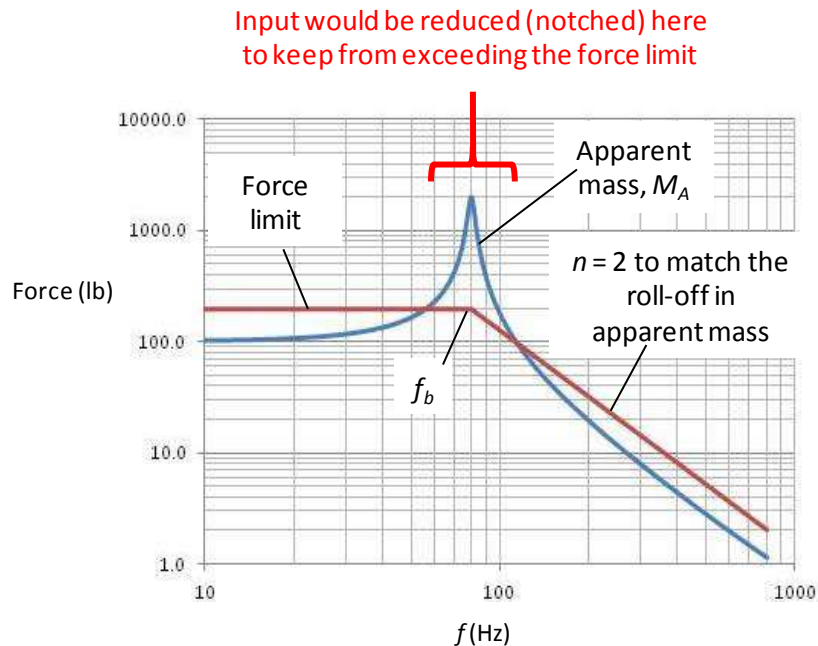to keep from exceeding the force limit

Fig. 6-7. Example Force-limit Function Compared with Apparent Mass for a Sinusoidally Base-driven Mass on a Spring. 100-lb mass, 80-Hz system, $Q = 20$, with a one-g slowly swept sinusoidal input.
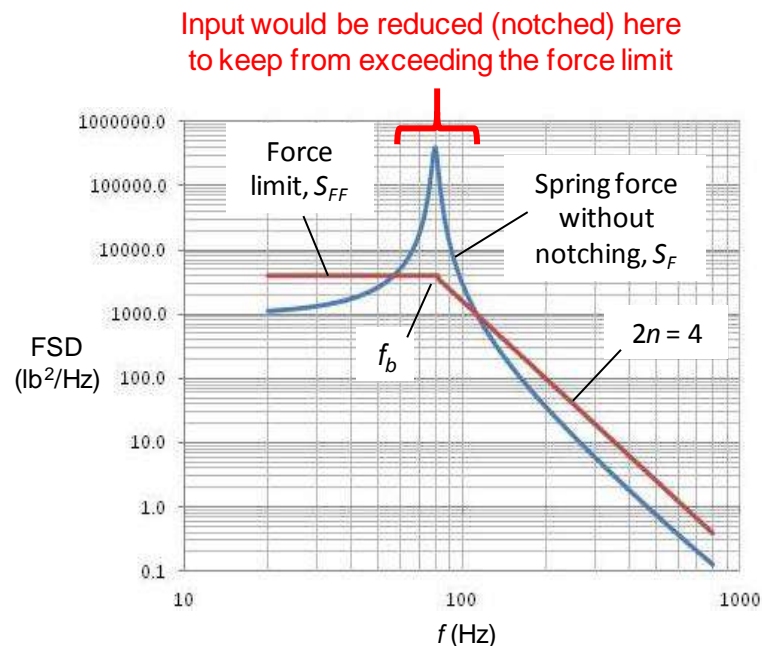
Input would be reduced (notched) here
to keep from exceeding the force limit

Fig. 6-8. Example Force-limit Function for Base-driven Random Vibration for a Mass on a Spring. 100-lb mass, 80-Hz system, $Q = 20$, with a 0.1 g^2/Hz input from 20 – 800 Hz.

Proper use of the SEM for tests of actual hardware requires proper selection of the variables C^2, n, and f_b. NASA-HDBK-7004C gives guidance on how to do this; here we summarize that guidance and add to it.

The break frequency, f_b, is often set equal to the fundamental frequency of the test article in the given test axis, but it's appropriate to use that frequency only if the fundamental mode has most of the modal effective mass. NASA-HDBK-7004C says that, if the interface FSD (or apparent mass) exhibits a two or more nearly equal peaks, the break frequency should be set equal to the frequency at which the asymptote of the apparent mass rolls off, i.e., the highest frequency for the nearly equal peaks. Examples appear later in this paper.

As explained above, the constant n sets the slope of the force-limit plot above the break frequency, with the intention of approximating the roll-off in apparent mass. Setting n requires judgment, as there is no single "right" value. NASA-HDBK-7004C says n "is often equal to unity" but goes on to say this assumption should be confirmed with the actual measured base force in a low-level, non-force-limited test run. Essentially, what we are confirming is that there will be no excessive, broad-band notches at higher frequencies, balanced with the desire to avoid unrealistic over-test.

A good way to determine an appropriate value of n is to derive the ***asymptotic apparent mass*** (a.k.a. ***asymptotic mass***), which is a smooth-line average of the peaks and valleys in the apparent-mass curve. The slope n should be set to approximate the roll-off in asymptotic mass. Start with $n = 1$, and see how well it matches. For pretest analysis with a FEM, the asymptotic mass can be calculated by setting $Q = 1$ in the analysis. Examples appear below.

NASA-HDBK-7004C says C^2 should be based on relative impedance between the test article and its flight mounting structure, and provides several methods for attempting to quantify this effect. Unfortunately, without access to a FEM of the mounting (launch vehicle) structure, it's hard to become confident when selecting a value for C^2. We would need to know the apparent-mass function at the interface not only for the satellite but also for the launch vehicle (LV).

Fortunately, for a small satellite, the LV contractor typically defines flight limit load factors[3] that envelop the maximum expected loads on the satellite's primary structure and mounting interface from all launch sources. When this is the case, and when the LV FEM is not available—and assuming a separate sine-burst or other test will be done to verify strength of the primary structure—we recommend setting C^2 equal to the value that keeps the predicted 3-sigma base force (and moment) at or slightly below the flight limit base load, as determined from the defined limit load factor in the test axis. Any lower value of C^2, which would further limit the base force, is subject to debate and thus may become an issue with other mission stakeholders.

Even for a protoflight or protoqualification test, 3 dB above maximum expected, we recommend force limiting to ensure the 3-sigma base force and moment do not exceed the flight limit loads when a separate strength test is performed. (See the 4th paper in this series, "Sine-Burst Testing.") If the random vibration test is to be the strength test, the 3-sigma base force and moment should be as high as the target values established for strength verification.

As an aside, for a satellite heavier than about 50 lb, the random vibration test should not be the strength test of the primary structure because it would unnecessarily introduce fatigue damage. Strength testing with random vibration is more applicable for smaller satellites because most of the load in the primary structure during launch is typically caused by random vibration; as a satellite gets larger, quasi-static and transient loads become larger percentages of the total launch load.

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[3] A ***load factor*** is a multiple of weight on Earth, representing the inertia load that resists uniform acceleration, a load we refer to as a ***quasi-static load***.

See Soucy [3] for additional practical guidance on use of the SEM.

Examples of Force Limiting with the Semi-Empirical Method

Figure 6-9 shows an example (Example 1) of base-force spectral density (FSD) for un-notched base-acceleration input. In this case, the FSD was predicted with a FEM as opposed to measured during test. A FEM prediction is typically used to derive an initial force-limit function, which is then modified in test if the prediction doesn't agree well with test data.
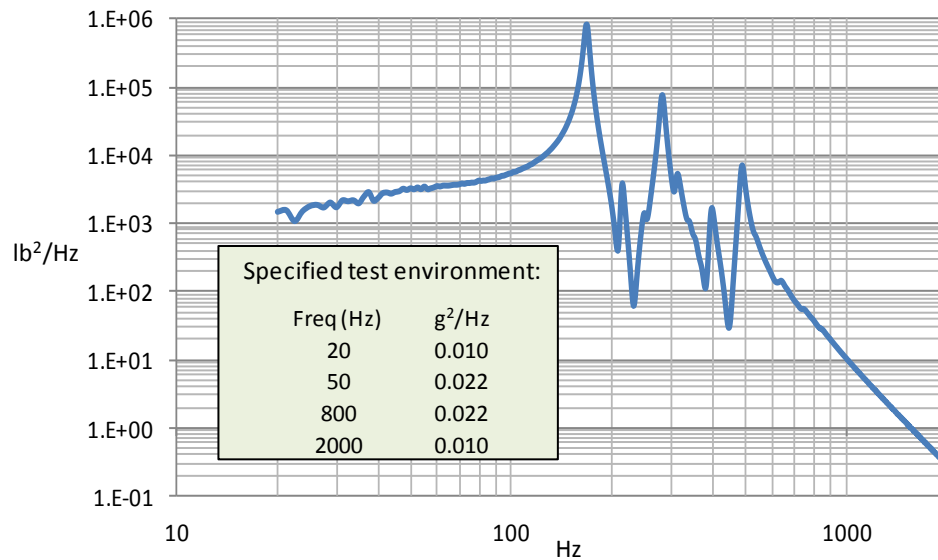

Specified test environment:

Freq (Hz)	g^2/Hz
20	0.010
50	0.022
800	0.022
2000	0.010

Fig. 6-9. Example 1—Predicted FSD Without Force Limiting. The RMS base force is 3161 lb, making the 3-sigma base force equal to 9483 lb.

In this example, the test article's weight is 362 lb, and, in the test axis, the specified limit load factor for launch is 8.5, making the limit base force 3080 lb. Because the predicted 3-sigma base force is 9483 lb, force limiting is required to prevent an unrealistically severe test for the primary structure. The target RMS force is 3080/3 = 1027 lb.

To derive a force-limit function, we first calculate the apparent mass from Fig. 6-9 using Eq. (6.6). The apparent mass for this example is shown in Fig. 6-10. Because the first peak is much higher than the others, we set the break frequency equal to the fundamental frequency, 169 Hz.

Figure 6-10 also shows an approximation of the asymptotic apparent mass, which , in this case, uses a straight line up to the break frequency and a unit slope above that ($n = 1$). A unit slope is most commonly assumed, but it's not always appropriate. In this case, it appears to match pretty well the average of the decreasing apparent mass with frequency. This analysis included modes of vibration only up to about 600 Hz.
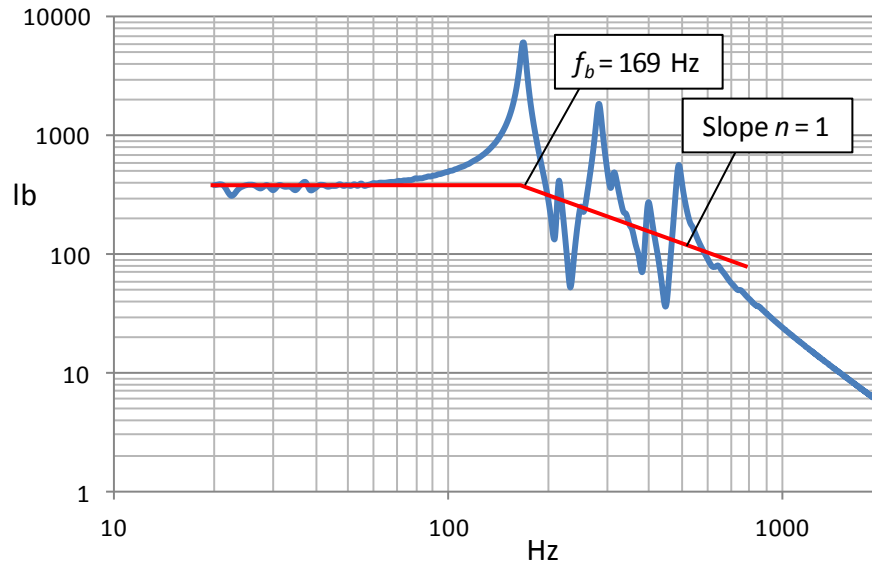
Fig. 6-10. Example 1—Apparent Mass (blue) and Approximate Asymptotic Apparent Mass (red).

With f_b and n established, next we must determine an appropriate value of C^2. Doing so entails integrating the force-limited FSD and aiming for the target RMS force. Recall that the RMS is the square root of the area under the power spectrum. As noted in the caption for Fig. 6-9, the RMS of the predicted FSD without force limiting is 3161 lb, and the target RMS is 1027 lb. Thus, we try to find the value of C^2 that makes the square root of the integral of the force-limited FSD approximately equal to 1027 lb. Numerical integration with a spreadsheet or a tool such as MatLab enables iteration with a "guess and check" approach to finding the desired C^2 value.

For Example 1, after iteration we decided on $C^2 = 2$, which, with Eqs. (6.1) and (6.2), gives the force-limit function shown in Fig. 6-11. The RMS of the force-limited (truncated) FSD shown in Fig. 6-12 is 1080 lb, which is a bit higher than the target value. The overshoot is acceptable because this is a pretest force-limit function only; we would modify it as needed based on actual test data to stay below the 1027 RMS value, as discussed below. Note that, if this were a test in one of the satellite's lateral axes, we also would predict the RMS moment and, if necessary, reduce the value of C^2 to keep from exceeding the flight limit moment at 3 sigma (3 times the RMS).

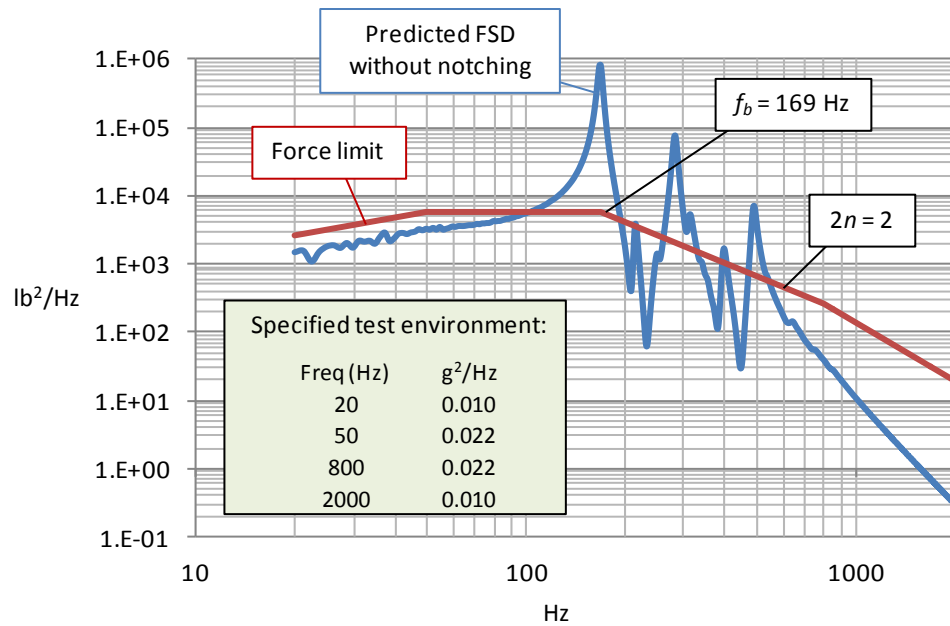Figure 6-13 shows the predicted notches in the input ASD corresponding to force limiting.

Fig. 6-11. Example 1—Force-limit Function Overlaid on Top of the FSD Predicted without Force Limiting
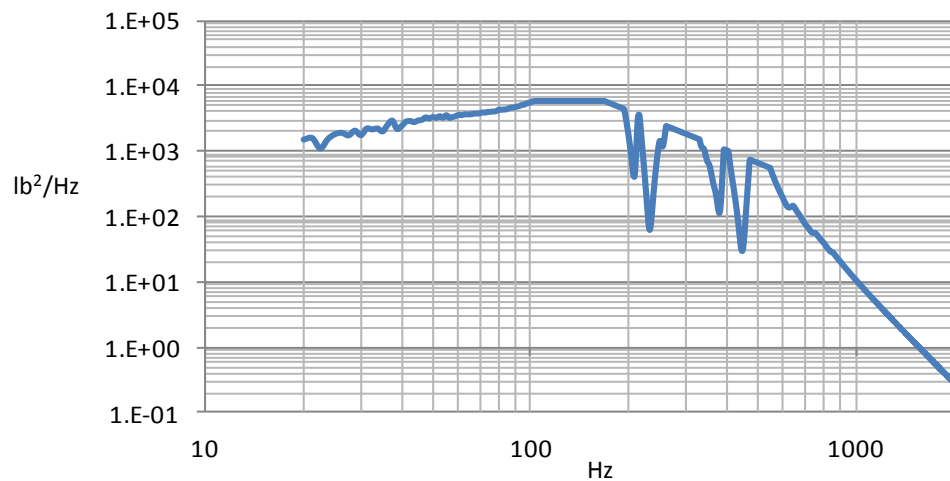

Fig. 6-12. Example 1—FSD after Force Limiting. The controller reduces input acceleration as needed during the test to keep the base force from exceeding the prescribed force-limit function. The RMS force is 1080 lb.
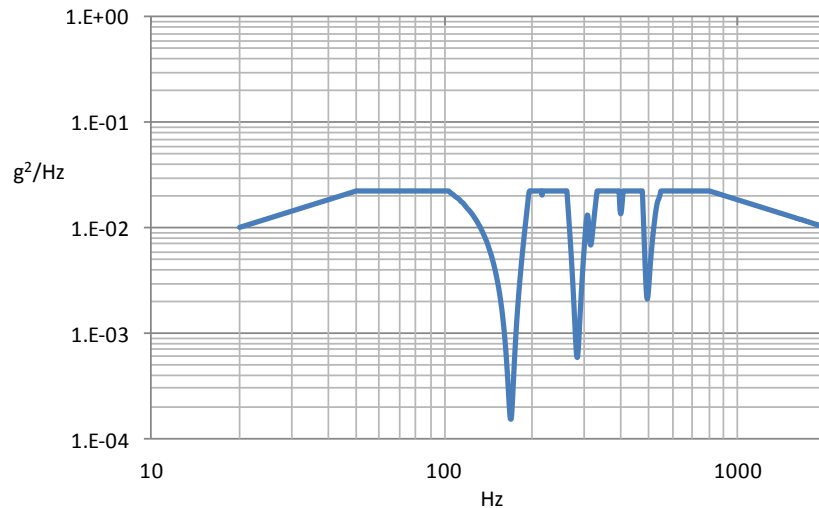
Fig. 6-13. Example 1—Notches Corresponding to the Force Limiting in Fig. 6-12.

Example 2 (Fig. 6-14) is a case in which the break frequency for force limiting is set to be higher than the fundamental frequency. This example is for lateral testing of a small satellite on its separation mechanism. The first peak in the FSD corresponds to a rocking mode, and the second corresponds to a mode in which the separation mechanism distorts in shear. Because both peaks are nearly the same, using the rocking (fundamental) frequency as the break frequency would lead to a massive, wide notch for the second mode, which indicates that the satellite may not be adequately tested for the launch environment. Instead, by using 231 Hz as the break frequency, both modes will be limited with relatively narrow notches, as shown in Fig. 6-15.
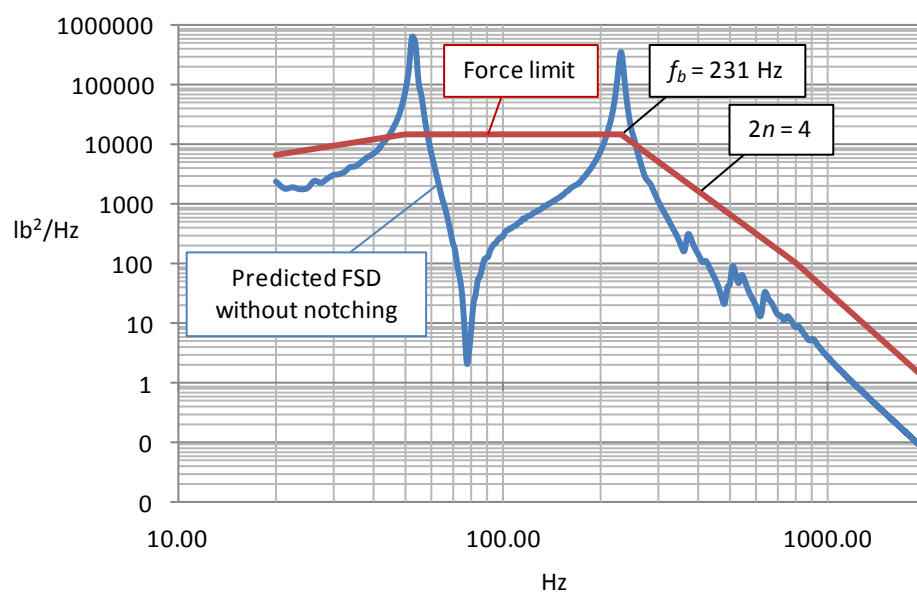

Fig. 6-14.   Example 2—Use of the SEM for a Test Article that Has Two Modes with High Effective Mass. In this case, the break frequency is set equal to the frequency of the second mode.
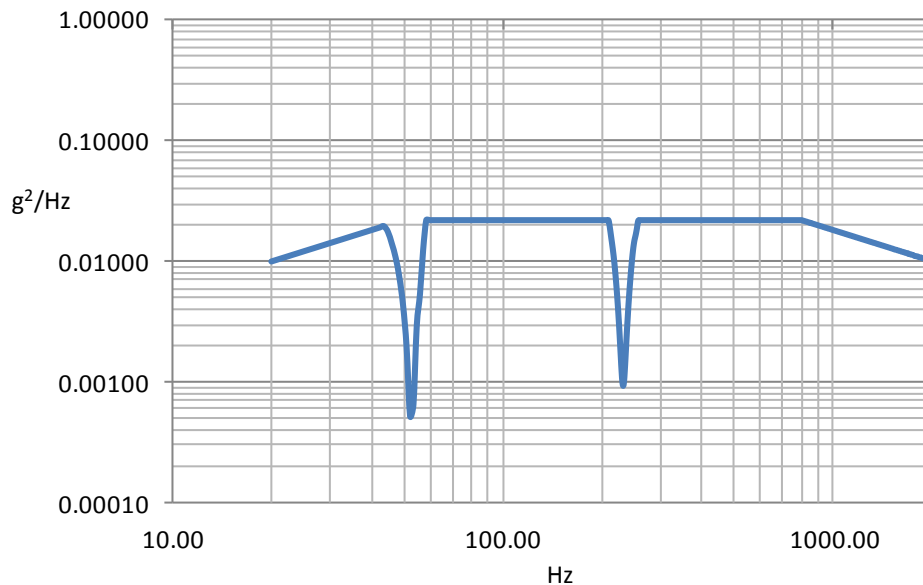
Fig. 6-15. Example 2—Notches Corresponding to the Force Limiting in Fig. 6-14.

In Example 1, we approximated the asymptotic mass by eye (Fig. 6-10). Example 3 (Fig. 6-16) shows the asymptotic mass as calculated by a FEM, although this time the FSD and asymptotic mass are shown in lb^2/Hz units. Both the FSD and the asymptotic mass were calculated as responses to constant 0.1 g^2/Hz input[4]; the FSD was based on the actual estimated damping for the test article, and the asymptotic mass is based on the assumption that the quality factor, Q, is unity, i.e., no dynamic gain. To establish the slope of the roll -off to be used in force limiting ($2n$ for a power spectrum), we enveloped the rounded peaks in the asymptotic mass. We took this approach rather than trying to match the average slope of the asymptotic-mass curve to stay on the conservative side (conservative meaning to ensure the test is adequate) in recognition that FEM accuracy drops off for higher-order modes of vibration. In other words, we didn't believe the model above 400 Hz or so! This approach to establishing the slope of the roll-off is useful when it's difficult to approximate the asymptotic mass by eye, as is the case in this example.

Pretest Analysis and Modification During the Test

As noted, each of the above examples used FEM predictions, which we would use prior to a test to derive a preliminary force-limit function. Because judgment is often needed to derive such force limits, it's important to distribute a summary of the pretest analysis to the other mission stakeholders for approval before starting the test.

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[4] Because the plots in Fig. 6-16 are in lb^2/Hz units, and the input is 0.1 g^2/Hz, the red curve is not really the asymptotic mass. We can, however, use it as asymptotic mass when deriving a value for $2n$.
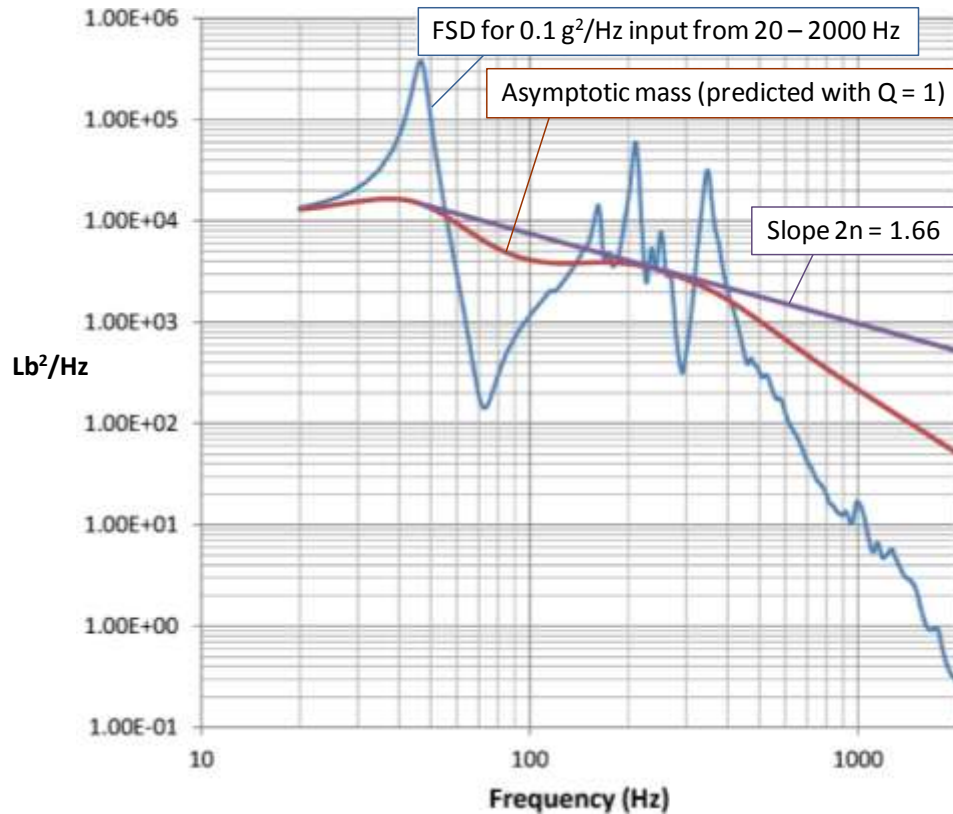
Fig. 6-16. Example 3—Notches Corresponding to the Force Limiting in Fig. 6-14.

During the test, the approved pretest force-limit functions are then subject to modification if the results vary significantly from the predicted responses. (Explain this also to the other stakeholders.) Following is some guidance for such modification, using base forces measured during a low-level sine sweep or low-constant-level random vibration test without force limiting (be sure the input levels are low enough not to over-stress the hardware):

- Break frequency, f_b: Move it to match the applicable modal frequency found in test. If using the fundamental frequency, check to confirm that no other base-force peaks are about the same or higher than the peak for the fundamental mode; if so, move the break frequency to the second peak. (See Example 2, above.)

- Roll-off slope, $2n$: Check by eye that the pretest value matches fairly well the average roll-off in the apparent mass. If not, adjust accordingly.

- C^2: Run a force-limited random vibration test to the specified environment -12 dB, with a force-limit function based on f_b and $2n$, as modified per the above guidance, and the pretest C^2 value. The force-limit function also should be scaled down 12 dB. Upon acquiring data, stop the test for assessment. If the proper C^2 value is used, the RMS base force should be approximately 25% of the target RMS value at full test levels. If not, estimate the amount C^2 should be raised or lowered, and then use the modified value in the next-higher-level test (-9 dB).

- Incrementally ramp up the test, and keep an eye on the measured FSD regarding the above guidance. Stop and adjust the force-limiting variables if needed.

Note that force limiting is not an exact science. Structural assemblies don't have perfectly linear response; as input increases, natural frequencies and dynamic gains tend to drop. So don't expect that, at full test levels, the RMS base force will be within a few percent of any target value. Such target values are mostly for pretest analysis. They serve more as upper limits during test as we try to protect the satellite structure from excessive loading. That said, however, we should adjust the variables if needed to stay close to the target in order to ensure an adequate test.

Response Limiting

As an example of response limiting used to approximate force limiting, consider the test article shown in Fig. 6-17 along with its first two modes in one of the lateral axes. Example 2, above (Fig. 6-14), is for a small satellite with modes such as these. Figure 6-17 shows two accelerometer locations, with lateral accelerations denoted as a_1 and a_2. Within the frequency range in which response is dominated by the first mode, there's a strong relationship between base force and a_1; for the second mode, the same applies for base force and a_2. If we have high confidence in the FEM for these two modes, we can derive response limits for these two accelerometers to simulate force limiting.
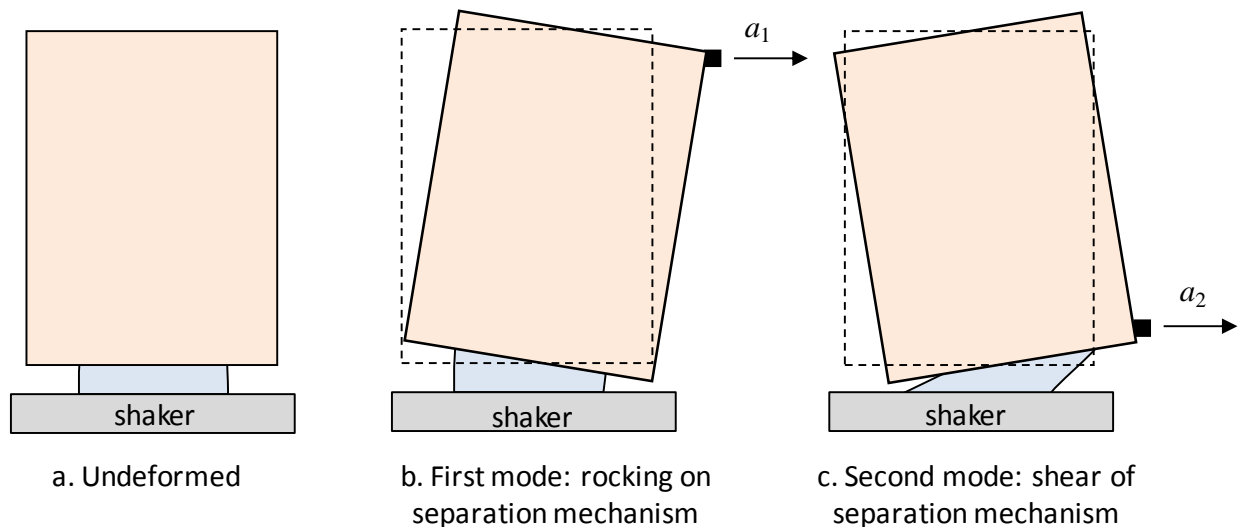

Fig. 6-17. Modes of Vibration for Example 2 (Fig. 6-14).

Figure 6-18 shows the predicted responses a_1 and a_2 to the test environment up to 600 Hz for Example 2 without notching. Note the peaks—a_1 for the first mode and a_2 for the second—look much like those in Fig. 6-14.
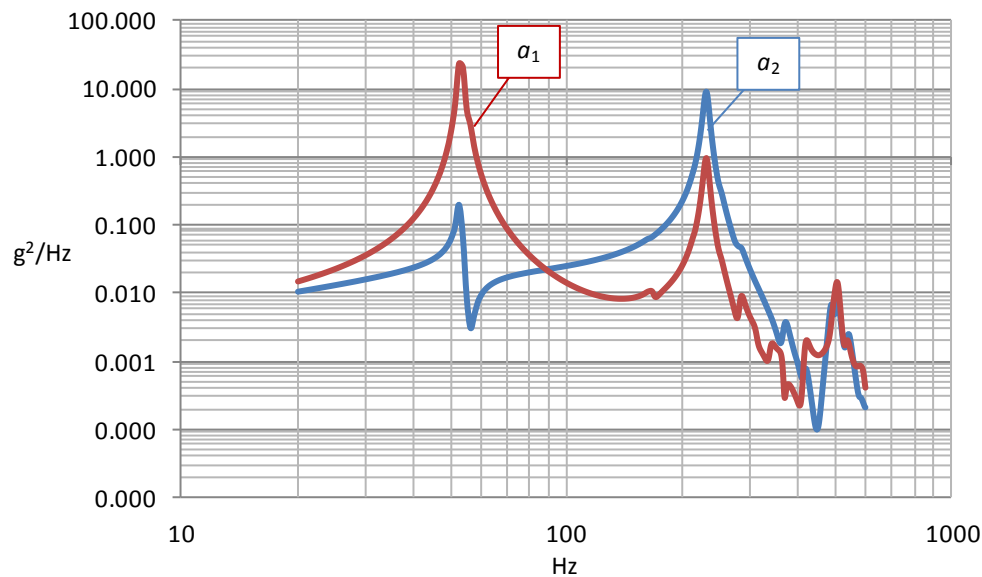
Fig. 6-18. Response Accelerations Without Force Limiting.

Figure 6-19 shows how these responses would change if we were to force limit the test as shown in Figs. 6-14 and 6-15. If base force and acceleration—a_1 for the first mode and a_2 for the second—were truly proportional within the target frequency bands, Fig. 6-19 would show the peaks for a_1 and a_2 to be truncated with the same shapes as the FSD peaks, or nearly flat, as indicated in Fig. 6-14. This is not the case, but it's close enough to being true for us to make an approximation that will simplify our response limiting. (Recall, we only use response limiting when force gages will not be present during test to allow force limiting.)
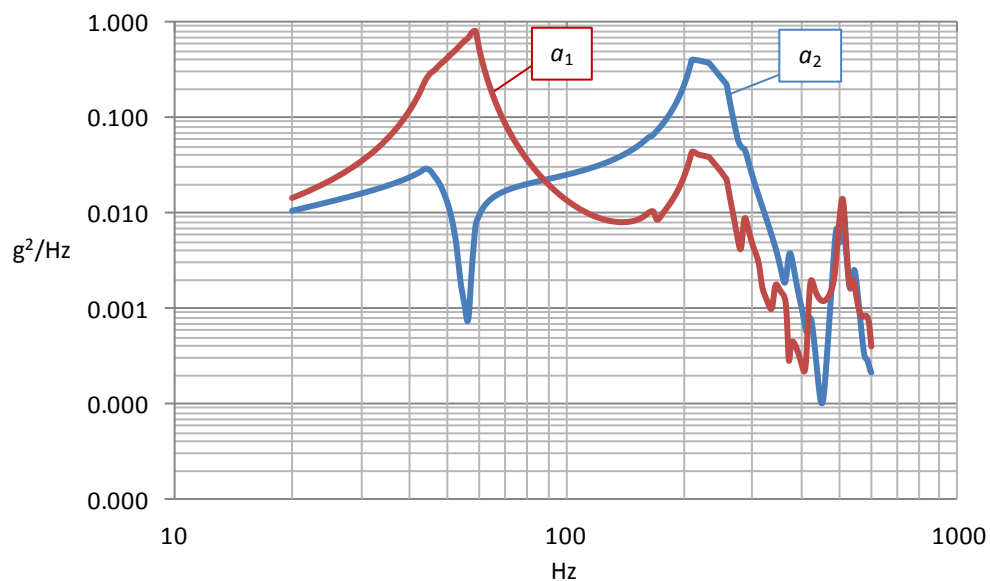

Fig. 6-19. Response Accelerations With Force Limiting.

Figure 6-20 shows predicted responses with response limits derived to be constant, providing a close approximation to force limiting. The limits are as follows: 0.50 g^2/Hz for a_1 between 20 – 100 Hz and 0.35 g^2/Hz for a_2 between 150 – 300 Hz. We derived these limits by trial and error, starting with on the Fig. 6-19 data, aiming for the same RMS base force and moment predicted for the force limiting shown in Fig. 6-14. We used a spreadsheet for the calculations, scaling the FEM predictions.
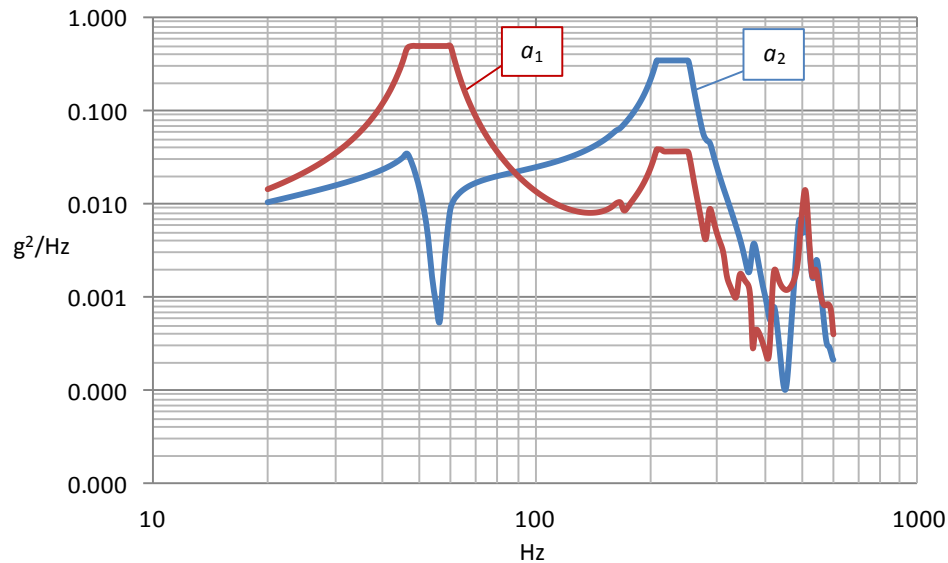

Fig. 6-20. Response Limiting Strategy—Nearly Equivalent to Force Limiting.

To see how well we simulated force limiting with this approach, we compared the resulting notches in the input ASD (Fig. 6-21). The force limiting (blue) plot is the same as shown in Fig. 6-15 up to 600 Hz. The response-limiting (red) curve nearly obscures the blue curve, which means the notches are nearly equivalent.
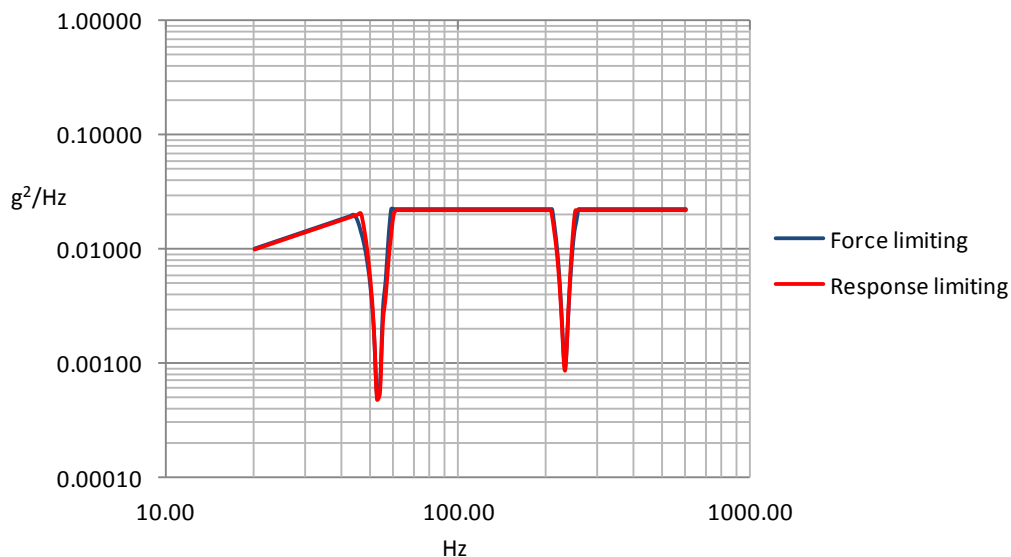

Fig. 6-21. Comparison of Notched Input ASD for Force Limiting and Response Limiting.

As stated above, response limiting should be used only when there is high confidence in the FEM for the target modes. To minimize error for a given mode, the response channel being limited should be the one that shows the highest response for that mode. In the above example, we used response limiting for two well-defined modes; in many cases, we would have confidence using this technique only for the first (fundamental) mode. When using response limiting in this manner, be sure to confirm accuracy of the FEM for mode shape during low-level test runs, comparing predicted responses to actual responses at multiple locations in the test article. To ensure an adequate test when the model does not correlate well with test data, we recommend increasing the RMS target base force as appropriate to account for uncertainty.

Manual Notching

Some of the older shakers and control systems don't have capability to force limit or response limit a test, and some test-lab personnel don't have experience with these techniques. If this is the case, you may decide to employ manual notching. With this approach, the pretest analysis is the same as for force limiting. Here's the full process:

1. Derive force limits per NASA-HDBK-7004C and the above guidance.

2. Identify the target mode(s) for notching (usually the fundamental mode only, but, as in the example above for response limiting, you may be able to notch a second high-mass mode as well).

3. Derive a notch of a type that can be controlled by the shaker (see below), that is centered on the notch predicted for force limiting, and that is predicted to result in an RMS base force that hits the target level. Use the shape of the notch corresponding with force limiting as a guide.

4. In preparation for test,

   a. Select an indicator accelerometer channel, such as a_1 in the above example on response limiting.

   b. Use the FEM to derive the transfer function between the indicator acceleration and the base force for the target mode. (Or determine a single coefficient that relates RMS base force to RMS acceleration.)

   c. Prepare a spreadsheet or other such tool that allows the measured acceleration to be transformed into base force during test.

5. During test,

   a. Perform a run at low, safe levels (say, -15 dB from full levels), without notches. Collect data and power down.

   b. Calculate the RMS base force using the tool from step 4c.

   c. Scale the base force up to full levels, assuming response is proportional to input level.

   d. Assuming the predicted full-level base force exceeds the target level, use the tool from step 4c to tailor a notch, starting with the pretest predicted notch, that causes the full-level RMS base force to hit the target level.

   e. Do a test run with the designed notch, 3 dB higher than the previous run, and repeat step 5c. Revise the notch as needed to hit the target.

   f. Continue iterating in this manner until the full-level test is performed.

As you can see, manual notching is more labor intensive than the other notching methods discussed, and it makes for more hours in the test lab. Make sure you prepare spreadsheets and dry-run the data transmittal with the lab ahead of time.

Regarding step 3, above, the shaker won't be able to control a sharply pointed notch, as shown in Figs. 6-13 and 6-15. The notch you define will need a wider, flattened bottom. Also, depending on the test equipment, the slope of the notch probably can't be greater than perhaps 30 dB/octave, depending on the test equipment. Check with the test lab in advance.

Figure 6-22 shows manual notches for Example 2 superimposed onto the notches from force limiting in Fig. 6-15. We derived these manual notches per step 3, above.
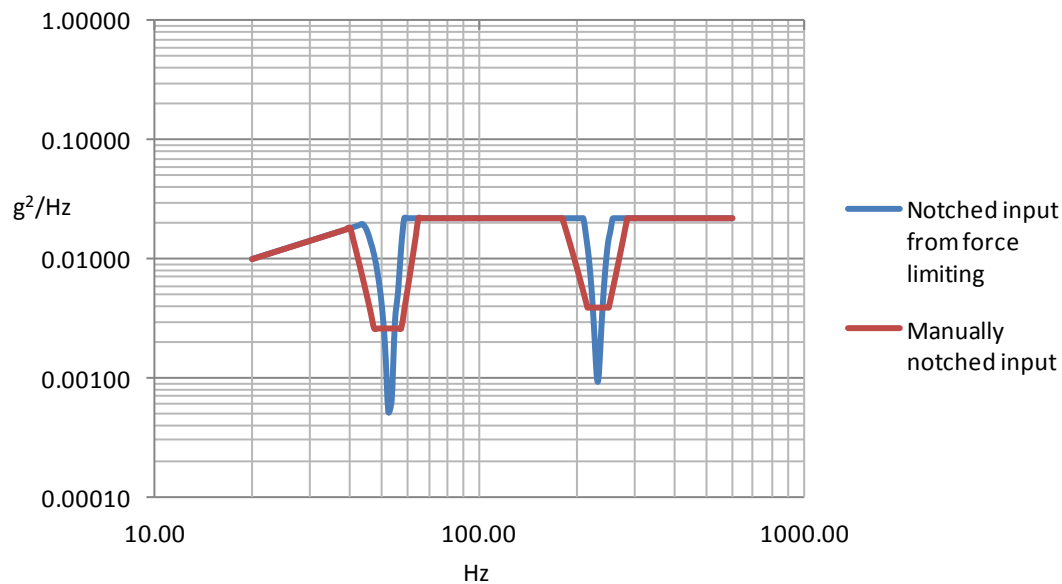

Fig. 6-22. Comparison of Notched Input ASD for Force Limiting and Manual Notching.

Figure 6-22 highlights a disadvantage of manual notching vs. force limiting. Because the manual notches are wider, there's a chance of under-testing part of the assembly that has a modal frequency near that of the target mode for notching. Thus, force limiting is the preferred approach when practical.

References

1. NASA-HDBK-7004C. "Force Limited Vibration Testing." November 30, 2012.

2. NASA Reference Publication RP-1403. "Force Limited Vibration Testing Monograph." T. D. Scharton. May 1997.

3. Soucy, Yvan. "On Force Limited Vibration for Testing Space Hardware." Proceedings of the IMAC XXIX Conference, Jan. 31 – Feb. 3, 2011.

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