2 RG&TC-Code

In[54]:=
```
xCoord = {t, χ, ϴ, φ};
g = {
    {-x y, 0, 0, 0},
    {0, x y t, 0, 0},
    {0, 0, z, 0},
    {0, 0, 0, x t}
   };
RGtensors[g, xCoord]
```

$$gdd = \begin{pmatrix} -x\,y & 0 & 0 & 0 \\ 0 & t\,x\,y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & t\,x \end{pmatrix}$$

$$\text{LineElement} = -x\,y\,d[t]^2 + z\,d[\Theta]^2 + t\,x\,d[\varphi]^2 + t\,x\,y\,d[\chi]^2$$

$$gUU = \begin{pmatrix} -\frac{1}{x\,y} & 0 & 0 & 0 \\ 0 & \frac{1}{t\,x\,y} & 0 & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{t\,x} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[56]=

All tasks completed in 0.

In[57]:=
```
(* Ricci Scalar *)
```

In[58]:=
```
R
```

Out[58]=

$$-\frac{1}{2\,t^2\,x\,y}$$

In[59]:=
```
(* Einstein Tensor *)
```

In[60]:=
```
EUd
```

Out[60]=

$$\left\{\left\{-\frac{1}{4\,t^2\,x\,y}, 0, 0, 0\right\}, \left\{0, \frac{1}{4\,t^2\,x\,y}, 0, 0\right\}, \left\{0, 0, \frac{1}{4\,t^2\,x\,y}, 0\right\}, \left\{0, 0, 0, \frac{1}{4\,t^2\,x\,y}\right\}\right\}$$

In[61]:=
```
(* Christoffel Symbol *)
```

In[62]:=
```
GUdd // MatrixForm
```

Out[62]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2\,y} \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{2\,t} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2\,t} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2\,t} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2\,t} \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[63]:=
```
Part[GUdd, 1, 2, 2]
Part[GUdd, 2, 2, 1]
```

Out[63]=

$$\frac{1}{2}$$

Out[64]=

$$\frac{1}{2\,t}$$

In[65]:=
```
(* Riemann tensor *)
```

In[66]:=
```
RUddd
```

Out[66]=

$$\left\{\left\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\},\right.$$
$$\left\{\left\{0,-\frac{1}{4\,t},0,0\right\},\left\{\frac{1}{4\,t},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left.\left\{\left\{0,0,0,-\frac{1}{4\,t\,y}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4\,t\,y},0,0,0\right\}\right\}\right\},$$
$$\left\{\left\{\left\{0,-\frac{1}{4\,t^2},0,0\right\},\left\{\frac{1}{4\,t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\right.$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left.\left\{\{0,0,0,0\},\left\{0,0,0,\frac{1}{4\,t\,y}\right\},\{0,0,0,0\},\left\{0,-\frac{1}{4\,t\,y},0,0\right\}\right\}\right\},$$
$$\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\},$$
$$\left\{\left\{\left\{0,0,0,-\frac{1}{4\,t^2}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4\,t^2},0,0,0\right\}\right\},\right.$$
$$\left\{\{0,0,0,0\},\left\{0,0,0,-\frac{1}{4\,t}\right\},\{0,0,0,0\},\left\{0,\frac{1}{4\,t},0,0\right\}\right\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left.\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\}$$

In[67]:=
```
(* Ricci Tensor *)
```

In[68]:=
```
Rdd
```

Out[68]=

$$\left\{\left\{\frac{1}{2\,t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\}$$

In[69]:=
```
Part[Rdd, 1, 1]
```

Out[69]=

$$\frac{1}{2\,t^2}$$

In[70]:=
```
xCoord = {t, r, θ, φ};
g = {-Exp[2 * a[r]], 0, 0, 0,
  {0, Exp[2 * b[r]], 0, 0, 0},
  {0, 0, r^{2} * Sin[θ], 0},
 }
```

Out[71]=

$$\left\{-e^{2\,a[r]},0,0,0,\left\{0,e^{2\,b[r]},0,0,0\right\},\left\{0,0,\left\{r^2\,\text{Sin}[\theta]\right\},0\right\},\text{Null}\right\}$$

1. Introduction

4. Solving for α and β

In[73]:=
```
xCoord = {t, r, θ, φ}
```
Out[73]=
$$\{t, r, \theta, \varphi\}$$

In[87]:=
```
g = {{-Exp[2 * a[r]], 0, 0, 0},
  {0, Exp[2 * b[r]], 0, 0},
  {0, 0, r^2, 0},
  {0, 0, 0, r^2 * (Sin[θ])^2}}
```
Out[87]=
$$\left\{\left\{-e^{2\,a[r]}, 0, 0, 0\right\}, \left\{0, e^{2\,b[r]}, 0, 0\right\}, \left\{0, 0, r^2, 0\right\}, \left\{0, 0, 0, r^2\,\text{Sin}[\theta]^2\right\}\right\}$$

In[74]:=
```
g
```

In[85]:=
```
{-e^(2 a[r]), 0, 0, 0, {0, e^(2 b[r]), 0, 0, 0}, {0, 0, {r^2 Sin[θ]}, 0}, Null}
```
Out[85]=
$$\left\{-e^{2\,a[r]}, 0, 0, 0, \left\{0, e^{2\,b[r]}, 0, 0, 0\right\}, \left\{0, 0, \left\{r^2\,\text{Sin}[\theta]\right\}, 0\right\}, \text{Null}\right\}$$

In[75]:=
```
GUdd
```
Out[75]=
$$\left\{\left\{\{0, 0, 0, 0\}, \left\{0, \frac{1}{2}, 0, 0\right\}, \{0, 0, 0, 0\}, \left\{0, 0, 0, \frac{1}{2\,y}\right\}\right\},\right.$$
$$\left\{\left\{0, \frac{1}{2\,t}, 0, 0\right\}, \left\{\frac{1}{2\,t}, 0, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\left.\left\{\left\{0, 0, 0, \frac{1}{2\,t}\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{\frac{1}{2\,t}, 0, 0, 0\right\}\right\}\right\}$$

In[88]:= **RGtensors[g, xCoord]**

$$\text{gdd} = \begin{pmatrix} -e^{2a[r]} & 0 & 0 & 0 \\ 0 & e^{2b[r]} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = e^{2b[r]}\, d[r]^2 - e^{2a[r]}\, d[t]^2 + r^2\, d[\theta]^2 + r^2\, d[\varphi]^2 \sin[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -e^{-2a[r]} & 0 & 0 & 0 \\ 0 & e^{-2b[r]} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\text{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[88]=

All tasks completed in 0.

In[89]:= **GUdd**

Out[89]=

$$\left\{\{\{0, a'[r], 0, 0\}, \{a'[r], 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},\right.$$
$$\{\{e^{2a[r]-2b[r]}\, a'[r], 0, 0, 0\}, \{0, b'[r], 0, 0\},$$
$$\{0, 0, -e^{-2b[r]}\, r, 0\}, \{0, 0, 0, -e^{-2b[r]}\, r \sin[\theta]^2\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, \frac{1}{r}, 0\}, \{0, \frac{1}{r}, 0, 0\}, \{0, 0, 0, -\text{Cos}[\theta] \sin[\theta]\}\},$$
$$\left.\{\{0, 0, 0, 0\}, \{0, 0, 0, \frac{1}{r}\}, \{0, 0, 0, \text{Cot}[\theta]\}, \{0, \frac{1}{r}, \text{Cot}[\theta], 0\}\}\right\}$$

In[90]:= **RUddd**

Out[90]=

$$\left\{\left\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\},\right.$$
$$\left\{\left\{0,-a'[r]^2+a'[r]\,b'[r]-a''[r],0,0\right\},\right.$$
$$\left.\left\{a'[r]^2-a'[r]\,b'[r]+a''[r],0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},$$
$$\left\{\left\{0,0,-e^{-2b[r]}\,r\,a'[r],0\right\},\{0,0,0,0\},\left\{e^{-2b[r]}\,r\,a'[r],0,0,0\right\},\{0,0,0,0\}\right\},$$
$$\left\{\left\{0,0,0,-e^{-2b[r]}\,r\,\text{Sin}[\theta]^2\,a'[r]\right\},\{0,0,0,0\},\right.$$
$$\left.\left.\{0,0,0,0\},\left\{e^{-2b[r]}\,r\,\text{Sin}[\theta]^2\,a'[r],0,0,0\right\}\right\}\right\},$$
$$\left\{\left\{\left\{0,-e^{2a[r]-2b[r]}\left(a'[r]^2-a'[r]\,b'[r]+a''[r]\right),0,0\right\},\right.\right.$$
$$\left.\left.\left\{e^{2a[r]-2b[r]}\left(a'[r]^2-a'[r]\,b'[r]+a''[r]\right),0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left\{\{0,0,0,0\},\left\{0,0,e^{-2b[r]}\,r\,b'[r],0\right\},\left\{0,-e^{-2b[r]}\,r\,b'[r],0,0\right\},\{0,0,0,0\}\right\},$$
$$\left\{\{0,0,0,0\},\left\{0,0,0,e^{-2b[r]}\,r\,\text{Sin}[\theta]^2\,b'[r]\right\},\right.$$
$$\left.\left.\{0,0,0,0\},\left\{0,-e^{-2b[r]}\,r\,\text{Sin}[\theta]^2\,b'[r],0,0\right\}\right\}\right\},$$
$$\left\{\left\{\left\{0,0,-\frac{e^{2a[r]-2b[r]}\,a'[r]}{r},0\right\},\{0,0,0,0\},\left\{\frac{e^{2a[r]-2b[r]}\,a'[r]}{r},0,0,0\right\},\{0,0,0,0\}\right\},\right.$$
$$\left\{\{0,0,0,0\},\left\{0,0,-\frac{b'[r]}{r},0\right\},\left\{0,\frac{b'[r]}{r},0,0\right\},\{0,0,0,0\}\right\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left\{\{0,0,0,0\},\{0,0,0,0\},\left\{0,0,0,e^{-2b[r]}\left(-1+e^{b[r]}\right)\left(1+e^{b[r]}\right)\text{Sin}[\theta]^2\right\},\right.$$
$$\left.\left.\left\{0,0,-e^{-2b[r]}\left(-1+e^{b[r]}\right)\left(1+e^{b[r]}\right)\text{Sin}[\theta]^2,0\right\}\right\}\right\},$$
$$\left\{\left\{\left\{0,0,0,-\frac{e^{2a[r]-2b[r]}\,a'[r]}{r}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{e^{2a[r]-2b[r]}\,a'[r]}{r},0,0,0\right\}\right\},\right.$$
$$\left\{\{0,0,0,0\},\left\{0,0,0,-\frac{b'[r]}{r}\right\},\{0,0,0,0\},\left\{0,\frac{b'[r]}{r},0,0\right\}\right\},$$
$$\left\{\{0,0,0,0\},\{0,0,0,0\},\left\{0,0,0,-e^{-2b[r]}\left(-1+e^{b[r]}\right)\left(1+e^{b[r]}\right)\right\},\right.$$
$$\left.\left\{0,0,e^{-2b[r]}\left(-1+e^{b[r]}\right)\left(1+e^{b[r]}\right),0\right\}\right\},$$
$$\left.\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\}\Big\}$$

In[91]:= **Rdd**

Out[91]=

$$\left\{\left\{\frac{e^{2a[r]-2b[r]}\left(2a'[r]+r\,a'[r]^2-r\,a'[r]\,b'[r]+r\,a''[r]\right)}{r},0,0,0\right\},\right.$$
$$\left\{0,-\frac{r\,a'[r]^2-2b'[r]-r\,a'[r]\,b'[r]+r\,a''[r]}{r},0,0\right\},$$
$$\left\{0,0,e^{-2b[r]}\left(-1+e^{2b[r]}-r\,a'[r]+r\,b'[r]\right),0\right\},$$
$$\left.\left\{0,0,0,e^{-2b[r]}\,\text{Sin}[\theta]^2\left(-1+e^{2b[r]}-r\,a'[r]+r\,b'[r]\right)\right\}\right\}$$