2 RG&TC-Code

In[54]:=
```
xCoord = {t, χ, θ, φ};
g = {
   {-x y, 0, 0, 0},
   {0, x y t, 0, 0},
   {0, 0, z, 0},
   {0, 0, 0, x t}
  };
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -x\,y & 0 & 0 & 0 \\ 0 & t\,x\,y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & t\,x \end{pmatrix}$$

$$\text{LineElement} = -x\,y\,d[t]^2 + z\,d[\theta]^2 + t\,x\,d[\varphi]^2 + t\,x\,y\,d[\chi]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{x\,y} & 0 & 0 & 0 \\ 0 & \frac{1}{t\,x\,y} & 0 & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{t\,x} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[56]=

All tasks completed in 0.

In[57]:= (* Ricci Scalar *)

In[58]:= R

Out[58]=

$$-\frac{1}{2\,t^2\,x\,y}$$

In[59]:= (* Einstein Tensor *)

In[60]:= EUd

Out[60]=

$$\left\{\left\{-\frac{1}{4\,t^2\,x\,y}, 0, 0, 0\right\}, \left\{0, \frac{1}{4\,t^2\,x\,y}, 0, 0\right\}, \left\{0, 0, \frac{1}{4\,t^2\,x\,y}, 0\right\}, \left\{0, 0, 0, \frac{1}{4\,t^2\,x\,y}\right\}\right\}$$

In[61]:= (* Christoffel Symbol *)

In[62]:=
```
GUdd // MatrixForm
```

Out[62]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2 y} \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{2 t} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2 t} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2 t} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2 t} \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[63]:=
```
Part[GUdd, 1, 2, 2]
Part[GUdd, 2, 2, 1]
```

Out[63]=

$$\frac{1}{2}$$

Out[64]=

$$\frac{1}{2 t}$$

In[65]:=
```
(* Riemann tensor *)
```

In[66]:= **RUddd**

Out[66]=

$$\left\{\left\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\right.\right.$$
$$\left\{\left\{0,-\frac{1}{4\,t},0,0\right\},\left\{\frac{1}{4\,t},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left.\left\{\left\{0,0,0,-\frac{1}{4\,t\,y}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4\,t\,y},0,0,0\right\}\right\}\right\},$$
$$\left\{\left\{\left\{0,-\frac{1}{4\,t^2},0,0\right\},\left\{\frac{1}{4\,t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\right.$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left.\left\{\{0,0,0,0\},\left\{0,0,0,\frac{1}{4\,t\,y}\right\},\{0,0,0,0\},\left\{0,-\frac{1}{4\,t\,y},0,0\right\}\right\}\right\},$$
$$\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\},$$
$$\left\{\left\{\left\{0,0,0,-\frac{1}{4\,t^2}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4\,t^2},0,0,0\right\}\right\},\right.$$
$$\left\{\{0,0,0,0\},\left\{0,0,0,-\frac{1}{4\,t}\right\},\{0,0,0,0\},\left\{0,\frac{1}{4\,t},0,0\right\}\right\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\left.\left.\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\}\right\}$$

In[67]:= **(* Ricci Tensor *)**

In[68]:= **Rdd**

Out[68]=

$$\left\{\left\{\frac{1}{2\,t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\}$$

In[69]:= **Part[Rdd, 1, 1]**

Out[69]=

$$\frac{1}{2\,t^2}$$

In[70]:= $$\boldsymbol{\frac{1}{2\,t^2}}$$

Out[70]=

$$\frac{1}{2\,t^2}$$

1. Surface of a Sphere

In[197]:=

```
xCoord = {θ, φ}
```

Out[197]=

$\{\theta, \varphi\}$

In[198]:=

```
g = {{1, 0}, {0, (Sin[θ])^2}}
```

Out[198]=

$\{\{1, 0\}, \{0, \mathrm{Sin}[\theta]^2\}\}$

In[199]:=

```
RGtensors[g, xCoord]
```

$\mathrm{gdd} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{Sin}[\theta]^2 \end{pmatrix}$

$\mathrm{LineElement} = d[\theta]^2 + d[\varphi]^2\,\mathrm{Sin}[\theta]^2$

$\mathrm{gUU} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{Csc}[\theta]^2 \end{pmatrix}$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Conformally Flat

Einstein computed in 0. sec

Einstein Space

Out[199]=

All tasks completed in 0.

In[200]:=

In[201]:=

```
RUddd
```

Out[201]=

$\{\{\{\{0, 0\}, \{0, 0\}\}, \{\{0, \mathrm{Sin}[\theta]^2\}, \{-\mathrm{Sin}[\theta]^2, 0\}\}\},$
$\{\{\{0, -1\}, \{1, 0\}\}, \{\{0, 0\}, \{0, 0\}\}\}\}$

In[202]:=

```
EUd
```

Out[202]=

$\{\{0, 0\}, \{0, 0\}\}$

In[78]:= **R**

Out[78]=

R

Hence, the Einstein tensor vanishes for this metric

Although the Ricci scalar is constant on the sphere, this is not a universal property of two-dimensional manifolds

In[79]:=
```
xCoord = {u, v};
g = {{(c + a * Cos[v]), 0},
  {0, a}}
```

Out[80]=

{{c + a Cos[v], 0}, {0, a}}

In[81]:= **RGtensors[g, xCoord]**

$$gdd = \begin{pmatrix} c + a\,\mathrm{Cos}[v] & 0 \\ 0 & a \end{pmatrix}$$

$$\mathrm{LineElement} = (c + a\,\mathrm{Cos}[v])\, d[u]^2 + a\, d[v]^2$$

$$gUU = \begin{pmatrix} \frac{1}{c + a\,\mathrm{Cos}[v]} & 0 \\ 0 & \frac{1}{a} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Conformally Flat

Einstein computed in 0. sec

Einstein Space

Out[81]=

All tasks completed in 0.

In[82]:= **R**

Out[82]=

$$\frac{2\,c\,\mathrm{Cos}[v] + 2\,a\,\mathrm{Cos}[v]^2 + a\,\mathrm{Sin}[v]^2}{2\,(c + a\,\mathrm{Cos}[v])^2}$$

Optional: Constant Negative Curvature Metric

In[83]:=
```
xCoord = {x, y};
g = {{1/y^2, 0}, {0, 1/y^2}}
```

Out[84]=

$$\left\{\left\{\frac{1}{y^2}, 0\right\}, \left\{0, \frac{1}{y^2}\right\}\right\}$$

In[85]:=
```
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} \frac{1}{y^2} & 0 \\ 0 & \frac{1}{y^2} \end{pmatrix}$$

$$\text{LineElement} = \frac{d[x]^2}{y^2} + \frac{d[y]^2}{y^2}$$

$$\text{gUU} = \begin{pmatrix} y^2 & 0 \\ 0 & y^2 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Conformally Flat

Einstein computed in 0. sec

Einstein Space

Out[85]=

All tasks completed in 0.

In[86]:=
```
R
```

Out[86]=

-2

2. Space Time Metrics

In[ ]:=

2.1 A Time Dependent Metric

In[87]:=
```
"Aborted after 0."
```
Out[87]=

Aborted after 0.

In[88]:=
```
xCoord = {t, x, y, z};
```
In[89]:=
```
g = {{-1, 0, 0, 0},
  {0, a[t]^2, 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
RGtensors[g, xCoord]
EUd
```
Out[89]=

$\{\{-1, 0, 0, 0\}, \{0, a[t]^2, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a[t]^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\text{LineElement} = -d[t]^2 + a[t]^2\, d[x]^2 + d[y]^2 + d[z]^2$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{a[t]^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[90]=

All tasks completed in 0.

Out[91]=

$$\left\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{0, 0, -\frac{a''[t]}{a[t]}, 0\right\}, \left\{0, 0, 0, -\frac{a''[t]}{a[t]}\right\}\right\}$$

In[92]:= **GUdd**

Out[92]=

$$\left\{\{\{0,0,0,0\},\{0,a[t]\,a'[t],0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\left\{0,\frac{a'[t]}{a[t]},0,0\right\},\left\{\frac{a'[t]}{a[t]},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\}$$

In[93]:=

In[94]:=

In[95]:=

In[96]:=

In[97]:=

In[98]:=

In[99]:=

In[100]:=

In[101]:=

In[102]:=

In[103]:=

In[104]:=

In[105]:=

In[106]:=

In[107]:=

In[108]:=

In[109]:=

2.2 Non-Constant Coefficients in Space

In[110]:=

```
xCoord = {t, x, y, z};
g = {{-a[x]^2, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Out[111]=

$\{\{-a[x]^2, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

In[112]:=

```
RGtensors[g, xCoord]
```

$$gdd = \begin{pmatrix} -a[x]^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\text{LineElement} = -a[x]^2 d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$

$$gUU = \begin{pmatrix} -\frac{1}{a[x]^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[112]=

All tasks completed in 0.

In[113]:=

```
GUdd
```

Out[113]=

$$\left\{\left\{\left\{0, \frac{a'[x]}{a[x]}, 0, 0\right\}, \left\{\frac{a'[x]}{a[x]}, 0, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\},\right.$$
$$\{\{a[x]\, a'[x], 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\left.\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\right\}$$

In[114]:=

```
"All tasks completed in 0."
```

Out[114]=

All tasks completed in 0.

In[115]:=

```
xCoord
```

Out[115]=

$\{t, x, y, z\}$

In[116]:=

In[117]:=

GUdd

Out[117]=

$$\left\{\left\{\left\{0, \frac{a'[x]}{a[x]}, 0, 0\right\}, \left\{\frac{a'[x]}{a[x]}, 0, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\}, \{\{a[x]\, a'[x], 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}, \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}, \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\right\}$$

In[118]:=

In[119]:=

xCoord

Out[119]=

{t, x, y, z}

In[120]:=

g

Out[120]=

$$\left\{\left\{-a[x]^2, 0, 0, 0\right\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\right\}$$

In[121]:=

```
g = {{-x^2, 0, 0, 0},
  {0, 1, 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[121]=

$$\left\{\left\{-x^2, 0, 0, 0\right\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\right\}$$

In[122]:=

RGtensors[g, xCoord]

$$\text{gdd} = \begin{pmatrix} -x^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = -x^2\, d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{x^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Flat Space!

Out[122]=

Aborted after 0.

In[123]:=

```
Rdddd
```

Out[123]=

```
{{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

In[124]:=

```
g = {{-x^4, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

RGtensors[g, xCoord]
```

Out[124]=

$\{\{-x^4, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

$$\text{gdd} = \begin{pmatrix} -x^4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\text{LineElement} = -x^4\, d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{x^4} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec
```

Out[125]=

```
All tasks completed in 0.
```

In[126]:=

```
RUddd
```

Out[126]=

$$\left\{\left\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\left\{0,-\frac{2}{x^2},0,0\right\},\left\{\frac{2}{x^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\},\left\{\left\{\left\{0,-2x^2,0,0\right\},\left\{2x^2,0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\},\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\},\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\}\right\}$$

In[127]:=

```
g = {{-1, 0, 0, 0},
  {0, Exp[-x^2 - y^2 - z^2], 0, 0},
  {0, 0, Exp[-x^2 - y^2 - z^2], 0},
  {0, 0, 0, Exp[-x^2 - y^2 - z^2]}}
```

Out[127]=

$$\left\{\{-1,0,0,0\},\left\{0,e^{-x^2-y^2-z^2},0,0\right\},\left\{0,0,e^{-x^2-y^2-z^2},0\right\},\left\{0,0,0,e^{-x^2-y^2-z^2}\right\}\right\}$$

In[128]:=

```
RGtensors[g, xCoord]
```

$$gdd = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{-x^2-y^2-z^2} & 0 & 0 \\ 0 & 0 & e^{-x^2-y^2-z^2} & 0 \\ 0 & 0 & 0 & e^{-x^2-y^2-z^2} \end{pmatrix}$$

$$LineElement = -d[t]^2 + e^{-x^2-y^2-z^2}\, d[x]^2 + e^{-x^2-y^2-z^2}\, d[y]^2 + e^{-x^2-y^2-z^2}\, d[z]^2$$

$$gUU = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{x^2+y^2+z^2} & 0 & 0 \\ 0 & 0 & e^{x^2+y^2+z^2} & 0 \\ 0 & 0 & 0 & e^{x^2+y^2+z^2} \end{pmatrix}$$

```
gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0.015 sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec
```

Out[128]=

```
All tasks completed in 0.015625
```

In[129]:=

RUddd

Out[129]=

$$\{\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\},$$
$$\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 2-z^2, y z\}, \{0, -2+z^2, 0, -x z\}, \{0, -y z, x z, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, y z, 2-y^2\}, \{0, -y z, 0, x y\}, \{0, -2+y^2, -x y, 0\}\}\},$$
$$\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, -2+z^2, -y z\}, \{0, 2-z^2, 0, x z\}, \{0, y z, -x z, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, -x z, x y\}, \{0, x z, 0, 2-x^2\}, \{0, -x y, -2+x^2, 0\}\}\},$$
$$\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, -y z, -2+y^2\}, \{0, y z, 0, -x y\}, \{0, 2-y^2, x y, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, x z, -x y\}, \{0, -x z, 0, -2+x^2\}, \{0, x y, 2-x^2, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\}\}$$

In[130]:=

RGtensors[g, xCoord]

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{-x^2-y^2-z^2} & 0 & 0 \\ 0 & 0 & e^{-x^2-y^2-z^2} & 0 \\ 0 & 0 & 0 & e^{-x^2-y^2-z^2} \end{pmatrix}$$

$$\text{LineElement} = -d[t]^2 + e^{-x^2-y^2-z^2}\, d[x]^2 + e^{-x^2-y^2-z^2}\, d[y]^2 + e^{-x^2-y^2-z^2}\, d[z]^2$$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{x^2+y^2+z^2} & 0 & 0 \\ 0 & 0 & e^{x^2+y^2+z^2} & 0 \\ 0 & 0 & 0 & e^{x^2+y^2+z^2} \end{pmatrix}$$

```
gUU computed in 0.016 sec
Gamma computed in 0. sec
Riemann(dddd) computed in 0. sec
Riemann(Uddd) computed in 0. sec
Ricci computed in 0. sec
Weyl computed in 0. sec
Einstein computed in 0. sec
```

Out[130]=

```
All tasks completed in 0.015625
```

In[131]:=

R

Out[131]=

$$-2\, e^{x^2+y^2+z^2} \left(-6 + x^2 + y^2 + z^2\right)$$

In[132]:=

In[133]:=

In[134]:=

In[135]:=

In[136]:=

In[137]:=

In[138]:=

Inverse: Argument {t, x, y, z} at position 1 is not a non–empty square matrix.

Inverse: Argument {t, x, y, z} at position 1 is not a non–empty square matrix.

Det: Argument {t, x, y, z} at position 1 is not a non–empty square matrix.

Det: Argument {t, x, y, z} at position 1 is not a non–empty square matrix.

Part: Part specification {t, x, y, z}⟦1, 1⟧ is longer than depth of object.

D: Multiple derivative specifier {{$-x^2$}, 0, 0, 0} does not have the form {variable, n}, where n is symbolic or a non−negative integer.

Part: Part specification {t, x, y, z}⟦1, 1⟧ is longer than depth of object.

D: Multiple derivative specifier {0, 1, 0, 0} does not have the form {variable, n}, where n is symbolic or a non−negative integer.

Part: Part specification {t, x, y, z}⟦1, 1⟧ is longer than depth of object.

General: Further output of Part::partd will be suppressed during this calculation.

D: Multiple derivative specifier {0, 0, 1, 0} does not have the form {variable, n}, where n is symbolic or a non−negative integer.

General: Further output of D::dvar will be suppressed during this calculation.

Part: Part 2 of Inverse[{t, x, y, z}] does not exist.

Part: Part 2 of Inverse[{t, x, y, z}] does not exist.

Part: Part 2 of Inverse[{t, x, y, z}] does not exist.

General: Further output of Part::partw will be suppressed during this calculation.

```
Gamma computed in 0.031 sec
Riemann(dddd) computed in 0.125 sec
```

In[139]:=

In[140]:=

2.3 Wave Solutions

A diagonal metric defined in terms of a function of x and t cannot satisfy the vacuum field equations as demonstrated by the non - vanishing Einstein tensor below

In[141]:=

```
xCoord = {t, x, y, z};
```

In[142]:=

```
g = {{-1 + f[x - t], 0, 0 , 0},
   {0, 1 + f[x - t], 0, 0},
   {0, 0, 1, 0},
   {0, 0, 0, 1}};
RGtensors[g, xCoord]
```

$$gdd = \begin{pmatrix} -1+f[-t+x] & 0 & 0 & 0 \\ 0 & 1+f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = d[y]^2 + d[z]^2 + d[t]^2\,(-1+f[-t+x]) + d[x]^2\,(1+f[-t+x])$$

$$gUU = \begin{pmatrix} \frac{1}{-1+f[-t+x]} & 0 & 0 & 0 \\ 0 & \frac{1}{1+f[-t+x]} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec
```

Out[143]=

```
All tasks completed in 0.
```

In[144]:=

```
RUddd
```

Out[144]=

$$\Big\{\Big\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\Big\{\Big\{0,-\frac{-f[-t+x]\,f'[-t+x]^2-f''[-t+x]+f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])},0,0\Big\},$$
$$\Big\{\frac{-f[-t+x]\,f'[-t+x]^2-f''[-t+x]+f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])},0,0,0\Big\},\{0,0,0,0\},$$
$$\{0,0,0,0\}\Big\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\Big\},$$
$$\Big\{\Big\{\Big\{0,\frac{-f[-t+x]\,f'[-t+x]^2-f''[-t+x]+f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2},0,0\Big\},$$
$$\Big\{-\frac{-f[-t+x]\,f'[-t+x]^2-f''[-t+x]+f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2},0,0,0\Big\},\{0,0,0,0\},$$
$$\{0,0,0,0\}\Big\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\Big\},$$
$$\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\},$$
$$\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},$$
$$\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\}\Big\}$$

In[145]:=

```
Part[RUddd, 2, 2]
```

Out[145]=

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[146]:=

```
Part[RUddd, 1, 2]
```

Out[146]=

$$\Big\{\Big\{0,-\frac{-f[-t+x]\,f'[-t+x]^2-f''[-t+x]+f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])},0,0\Big\},$$
$$\Big\{\frac{-f[-t+x]\,f'[-t+x]^2-f''[-t+x]+f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])},0,0,0\Big\},$$
$$\{0,0,0,0\},\{0,0,0,0\}\Big\}$$

In[147]:=

```
Part[RUddd, 1, 2, 2, 1]
```

Out[147]=

$$\frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])}$$

In[148]:=

```
Part[RUddd, 1, 2, 1, 2]
```

Out[148]=

$$-\frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])}$$

In[149]:=

```
Part[RUddd, 2]
```

Out[149]=

$$\Big\{\Big\{\Big\{0, \frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2}, 0, 0\Big\},$$
$$\Big\{-\frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2}, 0, 0, 0\Big\}, \{0, 0, 0, 0\},$$
$$\{0, 0, 0, 0\}\Big\}, \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\Big\}$$

In[150]:=

```
Part[RUddd, 2, 1, 1, 2]
```

Out[150]=

$$\frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2}$$

In[151]:=

```
Part[RUddd, 2, 1, 2, 1]
```

Out[151]=

$$-\frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2}$$

In[152]:=

In[153]:=

Introducing off diagonal elements to the metric as outlined in Section 2.3 would result in the vanishing of the Einstein tensor, thereby satisfying the vacuum field equations

In[154]:=

```
g = {{-1 + f[x - t], -f[x - t], 0, 0},
  {-f[x - t], 1 + f[x - t], 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[154]=

```
{{-1 + f[-t + x], -f[-t + x], 0, 0},
 {-f[-t + x], 1 + f[-t + x], 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

In[155]:=

```
RGtensors[g, xCoord]
```

$$gdd = \begin{pmatrix} -1+f[-t+x] & -f[-t+x] & 0 & 0 \\ -f[-t+x] & 1+f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$LineElement = d[y]^2 + d[z]^2 + d[t]^2 (-1 + f[-t+x]) - 2\, d[t] \times d[x] \times f[-t+x] + d[x]^2 (1 + f[-t+x])$$

$$gUU = \begin{pmatrix} -1-f[-t+x] & -f[-t+x] & 0 & 0 \\ -f[-t+x] & 1-f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec
```

Flat Space!

Out[155]=

```
Aborted after 0.
```

In[156]:=

```
Rdddd
```

Out[156]=

```
{{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

In[157]:=

```
gPeres = {{-1 + f[x - t, y, z], -f[x - t, y, z], 0, 0},
  {-f[x - t, y, z], 1 + f[x - t, y, z], 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[157]=

$$\{\{-1+f[-t+x,y,z],-f[-t+x,y,z],0,0\},\{-f[-t+x,y,z],1+f[-t+x,y,z],0,0\},\{0,0,1,0\},\{0,0,0,1\}\}$$

In[158]:=

```
RGtensors[gPeres, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -1+f[-t+x,y,z] & -f[-t+x,y,z] & 0 & 0 \\ -f[-t+x,y,z] & 1+f[-t+x,y,z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = d[y]^2+d[z]^2+d[t]^2\,(-1+f[-t+x,y,z]) - 2\,d[t]\times d[x]\times f[-t+x,y,z]+d[x]^2\,(1+f[-t+x,y,z])$$

$$\text{gUU} = \begin{pmatrix} -1-f[-t+x,y,z] & -f[-t+x,y,z] & 0 & 0 \\ -f[-t+x,y,z] & 1-f[-t+x,y,z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec
```

Out[158]=

```
All tasks completed in 0.
```

In[159]:=

```
Rdd
```

Out[159]=

$$\Big\{\Big\{\frac{1}{2}\left(-f^{(0,0,2)}[-t+x,y,z]-f^{(0,2,0)}[-t+x,y,z]\right),\ \frac{1}{2}\left(f^{(0,0,2)}[-t+x,y,z]+f^{(0,2,0)}[-t+x,y,z]\right),0,0\Big\},$$
$$\Big\{\frac{1}{2}\left(f^{(0,0,2)}[-t+x,y,z]+f^{(0,2,0)}[-t+x,y,z]\right),\ \frac{1}{2}\left(-f^{(0,0,2)}[-t+x,y,z]-f^{(0,2,0)}[-t+x,y,z]\right),0,0\Big\},\{0,0,0,0\},\{0,0,0,0\}\Big\}$$

In[160]:=

```
metricExample = {{-1 + x - t + y + z, - (x - t + y + z), 0, 0},
  {- (x - t + y + z), 1 + (x - t + y + z), 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[160]=

$$\{\{-1-t+x+y+z,t-x-y-z,0,0\},\{t-x-y-z,1-t+x+y+z,0,0\},\{0,0,1,0\},\{0,0,0,1\}\}$$

In[161]:=

```
xCoord
```

Out[161]=

```
{t, x, y, z}
```

In[162]:=

```
RGtensors[metricExample, xCoord]
```

$$gdd = \begin{pmatrix} -1-t+x+y+z & t-x-y-z & 0 & 0 \\ t-x-y-z & 1-t+x+y+z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LineElement =

$$(-1-t+x+y+z)\, d[t]^2 + 2\,(t-x-y-z)\, d[t] \times d[x] + (1-t+x+y+z)\, d[x]^2 + d[y]^2 + d[z]^2$$

$$gUU = \begin{pmatrix} -1+t-x-y-z & t-x-y-z & 0 & 0 \\ t-x-y-z & 1+t-x-y-z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0.016 sec

Flat Space!

Out[162]=

```
Aborted after 0.015625
```

In[163]:=

```
Rdddd
```

Out[163]=

```
{{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

In[164]:=

```
EUd
```

Out[164]=

```
EUd
```

In[165]:=
```
Rdd
```
Out[165]=
```
Rdd
```
In[166]:=

In[167]:=

In[168]:=

In[169]:=

In[170]:=

In[171]:=

In[172]:=

In[173]:=

In[174]:=

In[175]:=

In[176]:=

In[177]:=

In[178]:=

In[179]:=

In[180]:=

In[181]:=

In[182]:=