2 RG&TC-Code

In[54]:=
```
xCoord = {t, χ, θ, φ};
g = {
    {-x y, 0, 0, 0},
    {0, x y t, 0, 0},
    {0, 0, z, 0},
    {0, 0, 0, x t}
  };
RGtensors[g, xCoord]
```

$$gdd = \begin{pmatrix} -x\,y & 0 & 0 & 0 \\ 0 & t\,x\,y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & t\,x \end{pmatrix}$$

$$LineElement = -x\,y\,d[t]^2 + z\,d[\theta]^2 + t\,x\,d[\varphi]^2 + t\,x\,y\,d[\chi]^2$$

$$gUU = \begin{pmatrix} -\frac{1}{x\,y} & 0 & 0 & 0 \\ 0 & \frac{1}{t\,x\,y} & 0 & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{t\,x} \end{pmatrix}$$

```
gUU computed in 0. sec
Gamma computed in 0. sec
Riemann(dddd) computed in 0. sec
Riemann(Uddd) computed in 0. sec
Ricci computed in 0. sec
Weyl computed in 0. sec
Einstein computed in 0. sec
```

Out[∘]=
```
All tasks completed in 0.
```

In[57]:=
```
(* Ricci Scalar *)
```

In[58]:=
```
R
```

Out[∘]=

$$-\frac{1}{2\,t^2\,x\,y}$$

In[59]:=
```
(* Einstein Tensor *)
```

In[60]:=
```
EUd
```

Out[∘]=

$$\left\{\left\{-\frac{1}{4\,t^2\,x\,y}, 0, 0, 0\right\}, \left\{0, \frac{1}{4\,t^2\,x\,y}, 0, 0\right\}, \left\{0, 0, \frac{1}{4\,t^2\,x\,y}, 0\right\}, \left\{0, 0, 0, \frac{1}{4\,t^2\,x\,y}\right\}\right\}$$

In[61]:=
```
(* Christoffel Symbol *)
```

In[62]:=
```
GUdd // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2 y} \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{2 t} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2 t} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2 t} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2 t} \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[63]:=
```
Part[GUdd, 1, 2, 2]
Part[GUdd, 2, 2, 1]
```

Out[ ]=

$$\frac{1}{2}$$

Out[ ]=

$$\frac{1}{2 t}$$

In[65]:=
```
(* Riemann tensor *)
```

In[66]:= **RUddd**

Out[•]=

$$\left\{\left\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\left\{0,-\frac{1}{4t},0,0\right\},\left\{\frac{1}{4t},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\left\{0,0,0,-\frac{1}{4ty}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4ty},0,0,0\right\}\right\}\right\},\right.$$

$$\left\{\left\{\left\{0,-\frac{1}{4t^2},0,0\right\},\left\{\frac{1}{4t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\{0,0,0,0\},\left\{0,0,0,\frac{1}{4ty}\right\},\{0,0,0,0\},\left\{0,-\frac{1}{4ty},0,0\right\}\right\}\right\},$$

$$\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\},$$

$$\left.\left\{\left\{\left\{0,0,0,-\frac{1}{4t^2}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4t^2},0,0,0\right\}\right\},\left\{\{0,0,0,0\},\left\{0,0,0,-\frac{1}{4t}\right\},\{0,0,0,0\},\left\{0,\frac{1}{4t},0,0\right\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\}\right\}$$

In[67]:= **(* Ricci Tensor *)**

In[68]:= **Rdd**

Out[•]=

$$\left\{\left\{\frac{1}{2t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\}$$

In[69]:= **Part[Rdd, 1, 1]**

Out[•]=

$$\frac{1}{2t^2}$$

2. Maximally Symmetric Spaces

In[70]:=
```
xCoord = {r, θ, φ};
g = {{e^(2*b[r]), 0, 0},
  {0, r^2, 0},
  {0, 0, r^2*(Sin[θ])^2}}
```

Out[•]=

$$\left\{\left\{e^{2b[r]},0,0\right\},\left\{0,r^2,0\right\},\left\{0,0,r^2\,\mathrm{Sin}[\theta]^2\right\}\right\}$$

In[72]:= **g**

Out[•]=

$$\{\{e^{2\,b[r]}, 0, 0\}, \{0, r^2, 0\}, \{0, 0, r^2\,\mathrm{Sin}[\theta]^2\}\}$$

In[73]:= **RGtensors[g, xCoord]**

$$\mathrm{gdd} = \begin{pmatrix} e^{2\,b[r]} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

$$\mathrm{LineElement} = e^{2\,b[r]}\,d[r]^2 + r^2\,d[\theta]^2 + r^2\,d[\varphi]^2\,\mathrm{Sin}[\theta]^2$$

$$\mathrm{gUU} = \begin{pmatrix} e^{-2\,b[r]} & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Testing for 3-dim conformal flatness...

Outer: Heads Times and List at positions 3 and 2 are expected to be the same.

Einstein computed in 0. sec

Out[•]=

All tasks completed in 0.

In[74]:= **R**

Out[•]=

$$\frac{2\,e^{-2\,b[r]}\,\left(-1 + e^{2\,b[r]} + 2\,r\,\mathrm{Log}[e]\,b'[r]\right)}{r^2}$$

In[75]:= **Rdd**

Out[•]=

$$\left\{\left\{\frac{2\,\mathrm{Log}[e]\,b'[r]}{r}, 0, 0\right\}, \left\{0, e^{-2\,b[r]}\,\left(-1 + e^{2\,b[r]} + r\,\mathrm{Log}[e]\,b'[r]\right), 0\right\}, \left\{0, 0, e^{-2\,b[r]}\,\mathrm{Sin}[\theta]^2\,\left(-1 + e^{2\,b[r]} + r\,\mathrm{Log}[e]\,b'[r]\right)\right\}\right\}$$

In[76]:= **FullSimplify[Rdd]**

Out[•]=

$$\left\{\left\{\frac{2\,\mathrm{Log}[e]\,b'[r]}{r}, 0, 0\right\}, \left\{0, 1 + e^{-2\,b[r]}\,(-1 + r\,\mathrm{Log}[e]\,b'[r]), 0\right\}, \left\{0, 0, \mathrm{Sin}[\theta]^2\,\left(1 + e^{-2\,b[r]}\,(-1 + r\,\mathrm{Log}[e]\,b'[r])\right)\right\}\right\}$$

In[77]:= `DSolve[b'[r] - k * r * Exp[2 * b[r]] == 0, b[r], r]`

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[•]=

$$\left\{\left\{b[r] \to -\frac{1}{2}\,\mathrm{Log}\left[2\left(-\frac{k\,r^2}{2} - c_1\right)\right]\right\}\right\}$$

In[78]:= `R`

Out[•]=

$$\frac{2\,e^{-2\,b[r]}\left(-1 + e^{2\,b[r]} + 2\,r\,\mathrm{Log}[e]\,b'[r]\right)}{r^2}$$

In[79]:= `R / 6`

Out[•]=

$$\frac{e^{-2\,b[r]}\left(-1 + e^{2\,b[r]} + 2\,r\,\mathrm{Log}[e]\,b'[r]\right)}{3\,r^2}$$

In[80]:= $\mathrm{DSolve}\left[p'[r] == \frac{1}{\sqrt{1 - k * r^{\wedge}2}},\ p[r],\ r\right]$

Out[•]=

$$\left\{\left\{p[r] \to c_1 + \frac{k\,\mathrm{Log}\left[-\sqrt{-k}\;r + \sqrt{1 - k\,r^2}\right]}{(-k)^{3/2}}\right\}\right\}$$

In[81]:= $\mathrm{Integrate}\left[\frac{1}{\sqrt{1 - k * r^{\wedge}2}},\ r\right]$

Out[•]=

$$\frac{k\,\mathrm{Log}\left[-\sqrt{-k}\;r + \sqrt{1 - k\,r^2}\right]}{(-k)^{3/2}}$$

In[82]:= $\mathrm{Solve}\left[p == \frac{k\,\mathrm{Log}\left[-\sqrt{-k}\;r + \sqrt{1 - k\,r^2}\right]}{(-k)^{3/2}},\ r\right]$

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[•]=

$$\left\{\left\{r \to -\frac{i\,e^{-i\sqrt{k}\,p}\left(-1 + e^{2\,i\sqrt{k}\,p}\right)}{2\sqrt{k}}\right\}\right\}$$

In[83]:= $\mathrm{FullSimplify}\left[\left\{\left\{r \to -\frac{i\,e^{-i\sqrt{k}\,p}\left(-1 + e^{2\,i\sqrt{k}\,p}\right)}{2\sqrt{k}}\right\}\right\}\right]$

Out[•]=

$$\left\{\left\{r \to \frac{\mathrm{Sin}\left[\sqrt{k}\;p\right]}{\sqrt{k}}\right\}\right\}$$

In[84]:=
```
DSolve[p'[r] == 1/Sqrt[1 - k*r^2], p[r], r]
```

Out[•]=

$$\left\{\left\{p[r] \to c_1 + \frac{k \, \text{Log}\left[-\sqrt{-k}\ r + \sqrt{1 - k\, r^2}\right]}{(-k)^{3/2}}\right\}\right\}$$

3. The Geometry of Spacetime

In[85]:=
```
Integrate[1/Sqrt[1 - k*r^2], r]
```

Out[•]=

$$\frac{k \, \text{Log}\left[-\sqrt{-k}\ r + \sqrt{1 - k\, r^2}\right]}{(-k)^{3/2}}$$

In[86]:=
```
Solve[p == k Log[-Sqrt[-k] r + Sqrt[1 - k r^2]]/(-k)^(3/2), r]
```

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[•]=

$$\left\{\left\{r \to -\frac{i\, e^{-i \sqrt{k}\, p} \left(-1 + e^{2 i \sqrt{k}\, p}\right)}{2 \sqrt{k}}\right\}\right\}$$

In[87]:=
```
FullSimplify[-(I E^(-I Sqrt[k] p) (-1 + E^(2 I Sqrt[k] p)))/(2 Sqrt[k])]
```

Out[•]=

$$\frac{\text{Sin}\left[\sqrt{k}\ p\right]}{\sqrt{k}}$$

Christoffel Symbols for k = 1

In[88]:=
```
xCoord = {t, χ, θ, φ}
```

Out[•]=

{t, χ, θ, φ}

In[89]:=
```
g = {{-1, 0, 0, 0}, {0, q[t], 0, 0}, {0, 0, q[t] * Sin[χ]^2, 0},
   {0, 0, 0, q[t] * Sin[χ]^2}}
```

Out[•]=

$$\left\{\{-1, 0, 0, 0\}, \{0, q[t], 0, 0\}, \left\{0, 0, q[t]\, \text{Sin}[\chi]^2, 0\right\}, \left\{0, 0, 0, q[t]\, \text{Sin}[\chi]^2\right\}\right\}$$

In[90]:=
```
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & q[t] & 0 & 0 \\ 0 & 0 & q[t]\,\text{Sin}[\chi]^2 & 0 \\ 0 & 0 & 0 & q[t]\,\text{Sin}[\chi]^2 \end{pmatrix}$$

$$\text{LineElement} = -d[t]^2 + d[\chi]^2\,q[t] + d[\theta]^2\,q[t]\,\text{Sin}[\chi]^2 + d[\varphi]^2\,q[t]\,\text{Sin}[\chi]^2$$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{q[t]} & 0 & 0 \\ 0 & 0 & \frac{\text{Csc}[\chi]^2}{q[t]} & 0 \\ 0 & 0 & 0 & \frac{\text{Csc}[\chi]^2}{q[t]} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[∘]=

All tasks completed in 0.

In[91]:=

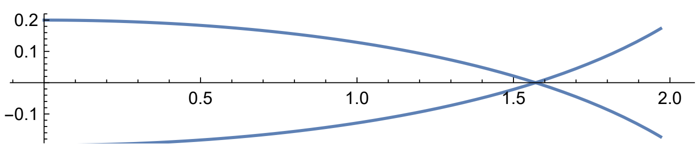
In[92]:=
```
GUdd = GUdd /. {χ → χ[λ], θ → θ'[λ], φ → φ[λ]};
derivs = {t'[λ], χ'[λ], θ'[λ], φ'[λ]};
geo1 =
  χ''[λ] + Sum[Part[GUdd, 2, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
geo2 = θ''[λ] + Sum[Part[GUdd, 3, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
geo3 = φ''[λ] + Sum[Part[GUdd, 4, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];

sol1 = NDSolve[{geo1 == 0, geo2 == 0, geo3 == 0, χ[0] == 0.2, θ[0] == π/2, φ[0] == π/2,
   χ'[0] == 0, θ'[0] == 0, φ'[0] == -1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0, 10}]
sol2 = NDSolve[{geo1 == 0, geo2 == 0, geo3 == 0, χ[0] == 0.2, θ[0] == π/2, φ[0] == -π/2,
   χ'[0] == 0, θ'[0] == 0, φ'[0] == 1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0, 10}]
Show[
 ParametricPlot[Evaluate[{χ[λ] * Cos[φ[λ]], χ[λ] * Sin[φ[λ]]} /. sol1], {λ, 0, 10}],
 ParametricPlot[Evaluate[{χ[λ] * Cos[φ[λ]], χ[λ] * Sin[φ[λ]]} /. sol2], {λ, 0, 10}],
 PlotStyle → Orange]
```

Out[•]=
```
{{χ[λ] → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][λ],
  θ[λ] → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][λ],
  φ[λ] → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][λ]}}
```

Out[•]=
```
{{χ[λ] → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][λ],
  θ[λ] → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][λ],
  φ[λ] → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][λ]}}
```

Out[•]=



In[100]:=

For k = -1

In[101]:=
```
xCoord = {t, χ, θ, φ}
g = {{-1, 0, 0, 0}, {0, 1, 0, 0},
  {0, 0, Sinh[χ]^2, 0}, {0, 0, 0, Sinh[χ]^2 * Sin[θ]^2}}
```
Out[•]=

{t, χ, θ, φ}

Out[•]=

$\{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, \text{Sinh}[\chi]^2, 0\}, \{0, 0, 0, \text{Sin}[\theta]^2\,\text{Sinh}[\chi]^2\}\}$

In[•]:=

In[•]:=

In[105]:=
```
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \text{Sinh}[\chi]^2 & 0 \\ 0 & 0 & 0 & \text{Sin}[\theta]^2\,\text{Sinh}[\chi]^2 \end{pmatrix}$$

$$\text{LineElement} = -\text{d}[t]^2 + \text{d}[\chi]^2 + \text{d}[\theta]^2\,\text{Sinh}[\chi]^2 + \text{d}[\varphi]^2\,\text{Sin}[\theta]^2\,\text{Sinh}[\chi]^2$$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \text{Csch}[\chi]^2 & 0 \\ 0 & 0 & 0 & \text{Csc}[\theta]^2\,\text{Csch}[\chi]^2 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

Out[•]=

All tasks completed in 0.

In[•]:= "Aborted after 0."

Out[•]=

Aborted after 0.

In[107]:=

```
GUdd
GUdd = GUdd /. {χ → χ[λ], θ → θ'[λ], φ → φ[λ]};
derivs = {t'[λ], χ'[λ], θ'[λ], φ'[λ]};
geo4 =
  χ''[λ] + Sum[Part[GUdd, 2, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
geo5 = θ''[λ] + Sum[Part[GUdd, 3, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
geo6 = φ''[λ] + Sum[Part[GUdd, 4, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
```

Out[∘]=

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0},
  {0, 0, 0, 0}, {0, 0, -Cosh[χ] Sinh[χ], 0}, {0, 0, 0, -Cosh[χ] Sin[θ]^2 Sinh[χ]}},
 {{0, 0, 0, 0}, {0, 0, Coth[χ], 0}, {0, Coth[χ], 0, 0}, {0, 0, 0, -Cos[θ] Sin[θ]}},
 {{0, 0, 0, 0}, {0, 0, 0, Coth[χ]}, {0, 0, 0, Cot[θ]}, {0, Coth[χ], Cot[θ], 0}}}
```

In[ ]:=

```
sol3 = NDSolve[{geo4 == 0, geo5 == 0, geo6 == 0, χ[0] == 0.2, θ[0] == π/2, φ[0] == π/2,
    χ'[0.2] == 0, θ'[0.2] == 0, φ'[0] == -1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0.2, 10}]
sol4 = NDSolve[{geo4 == 0, geo5 == 0, geo6 == 0, χ[0] == 0.2, θ[0] == π/2, φ[0] == -π/2,
    χ'[0.2] == 0.1, θ'[0] == 0, φ'[0] == 1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0, 10}]
```

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

General: Further output of Infinity::indet will be suppressed during this calculation.

NDSolve: Encountered non−numerical value for a derivative at λ == 0.`.

Out[ ]=

```
NDSolve[
 {-Cosh[χ[λ]] Sinh[χ[λ]] θ′[λ]^2 - Cosh[χ[λ]] Sin[θ′[λ]]^2 Sinh[χ[λ]] φ′[λ]^2 + χ′′[λ] == 0,
  -Cos[θ′[λ]] Sin[θ′[λ]] φ′[λ]^2 + 2 Coth[χ[λ]] θ′[λ] χ′[λ] + θ′′[λ] == 0,
  2 Cot[θ′[λ]] θ′[λ] φ′[λ] + 2 Coth[χ[λ]] φ′[λ] χ′[λ] + φ′′[λ] == 0, χ[0] == 0.2, θ[0] == π/2,
  φ[0] == π/2, χ′[0.2] == 0, θ′[0.2] == 0, φ′[0] == -1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0.2, 10}]
```

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

General: Further output of Infinity::indet will be suppressed during this calculation.

NDSolve: Encountered non−numerical value for a derivative at λ == 0.`.

Out[ ]=

```
NDSolve[
 {-Cosh[χ[λ]] Sinh[χ[λ]] θ′[λ]^2 - Cosh[χ[λ]] Sin[θ′[λ]]^2 Sinh[χ[λ]] φ′[λ]^2 + χ′′[λ] == 0,
  -Cos[θ′[λ]] Sin[θ′[λ]] φ′[λ]^2 + 2 Coth[χ[λ]] θ′[λ] χ′[λ] + θ′′[λ] == 0,
  2 Cot[θ′[λ]] θ′[λ] φ′[λ] + 2 Coth[χ[λ]] φ′[λ] χ′[λ] + φ′′[λ] == 0, χ[0] == 0.2, θ[0] == π/2,
  φ[0] == -π/2, χ′[0.2] == 0.1, θ′[0] == 0, φ′[0] == 1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0, 10}]
```

For k = 0

In[ ]:=

```
g = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, χ^2, 0}, {0, 0, 0, χ^2 * Sin[θ]^2}}
```

Out[ ]=

```
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, χ^2, 0}, {0, 0, 0, χ^2 Sin[θ]^2}}
```

In[ ]:= **xCoord = {t, χ, θ, φ}**

Out[ ]=

{t, χ, θ, φ}

In[ ]:= **RGtensors[g, xCoord]**

$$gdd = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \chi^2 & 0 \\ 0 & 0 & 0 & \chi^2 \mathrm{Sin}[\theta]^2 \end{pmatrix}$$

$$LineElement = -d[t]^2 + \chi^2 d[\theta]^2 + d[\chi]^2 + \chi^2 d[\varphi]^2 \mathrm{Sin}[\theta]^2$$

$$gUU = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\chi^2} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{\chi^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Flat Space!

Out[ ]=

Aborted after 0.

In[ ]:= **GUdd**

Out[ ]=

$$\left\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\right.$$
$$\left\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,-\chi,0\},\left\{0,0,0,-\chi\,\mathrm{Sin}[\theta]^2\right\}\right\},$$
$$\left\{\{0,0,0,0\},\left\{0,0,\frac{1}{\chi},0\right\},\left\{0,\frac{1}{\chi},0,0\right\},\{0,0,0,-\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\}\right\},$$
$$\left.\left\{\{0,0,0,0\},\left\{0,0,0,\frac{1}{\chi}\right\},\{0,0,0,\mathrm{Cot}[\theta]\},\left\{0,\frac{1}{\chi},\mathrm{Cot}[\theta],0\right\}\right\}\right\}$$

In[ ]:=
```
GUdd = GUdd /. {χ → χ[λ], θ → θ'[λ], φ → φ[λ]};
derivs = {t'[λ], χ'[λ], θ'[λ], φ'[λ]};
geo7 =
  χ''[λ] + Sum[Part[GUdd, 2, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
geo8 = θ''[λ] + Sum[Part[GUdd, 3, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
geo9 = φ''[λ] + Sum[Part[GUdd, 4, i, j] * derivs[[i]] * derivs[[j]], {i, 2, 4}, {j, 2, 4}];
```

In[ ]:=
```
sol5 = NDSolve[{geo7 == 0, geo8 == 0, geo9 == 0, χ[0] == 0.2, θ[0] == π/2, φ[0] == π/2,
    χ'[0] == 0, θ'[0] == 0, φ'[0] == -1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0, 10}]
sol6 = NDSolve[{geo7 == 0, geo8 == 0, geo9 == 0, χ[0] == 0.2, θ[0] == π/2, φ[0] == -π/2,
    χ'[0] == 0, θ'[0] == 0, φ'[0] == 1}, {χ[λ], θ[λ], φ[λ]}, {λ, 0, 10}]
```

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

NDSolve: Encountered non–numerical value for a derivative at λ == 0.`.

Out[ ]=

$$\text{NDSolve}\Big[\Big\{-\chi[\lambda]\,\theta'[\lambda]^2-\text{Sin}[\theta'[\lambda]]^2\,\chi[\lambda]\,\varphi'[\lambda]^2+\chi''[\lambda]==0,\ -\text{Cos}[\theta'[\lambda]]\,\text{Sin}[\theta'[\lambda]]\,\varphi'[\lambda]^2+\frac{2\,\theta'[\lambda]\,\chi'[\lambda]}{\chi[\lambda]}+\theta''[\lambda]==0,\ 2\,\text{Cot}[\theta'[\lambda]]\,\theta'[\lambda]\,\varphi'[\lambda]+\frac{2\,\varphi'[\lambda]\,\chi'[\lambda]}{\chi[\lambda]}+\varphi''[\lambda]==0,\ \chi[0]==0.2,\ \theta[0]==\frac{\pi}{2},\ \varphi[0]==\frac{\pi}{2},\ \chi'[0]==0,\ \theta'[0]==0,\ \varphi'[0]==-1\Big\},\ \{\chi[\lambda],\theta[\lambda],\varphi[\lambda]\},\ \{\lambda,0,10\}\Big]$$

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

NDSolve: Encountered non–numerical value for a derivative at λ == 0.`.

Out[ ]=

$$\text{NDSolve}\Big[\Big\{-\chi[\lambda]\,\theta'[\lambda]^2-\text{Sin}[\theta'[\lambda]]^2\,\chi[\lambda]\,\varphi'[\lambda]^2+\chi''[\lambda]==0,\ -\text{Cos}[\theta'[\lambda]]\,\text{Sin}[\theta'[\lambda]]\,\varphi'[\lambda]^2+\frac{2\,\theta'[\lambda]\,\chi'[\lambda]}{\chi[\lambda]}+\theta''[\lambda]==0,\ 2\,\text{Cot}[\theta'[\lambda]]\,\theta'[\lambda]\,\varphi'[\lambda]+\frac{2\,\varphi'[\lambda]\,\chi'[\lambda]}{\chi[\lambda]}+\varphi''[\lambda]==0,\ \chi[0]==0.2,\ \theta[0]==\frac{\pi}{2},\ \varphi[0]==-\frac{\pi}{2},\ \chi'[0]==0,\ \theta'[0]==0,\ \varphi'[0]==1\Big\},\ \{\chi[\lambda],\theta[\lambda],\varphi[\lambda]\},\ \{\lambda,0,10\}\Big]$$

In[ ]:=
```
Show[
 ParametricPlot[Evaluate[{χ[λ] * Cos[φ[λ]], χ[λ] * Sin[φ[λ]]} /. sol5], {λ, 0, 10}],
 ParametricPlot[Evaluate[{χ[λ] * Cos[φ[λ]], χ[λ] * Sin[φ[λ]]} /. sol6], {λ, 0, 10}],
 PlotStyle → Orange]
```

ReplaceAll:

$$\Big\{\text{NDSolve}\Big[\Big\{-\chi[\lambda]\ll1\gg^{(\ll1\gg)}[\ll1\gg]^2-\text{Sin}[\ll1\gg]^2\,\chi[\lambda]\ll1\gg^{(\ll1\gg)}[\ll1\gg]^2+\chi''[\lambda]==0,\ -\text{Cos}[\ll1\gg^{(\ll1\gg)}[\ll1\gg]]\,\text{Sin}[\ll1\gg^{(\ll1\gg)}[\ll1\gg]]\ll1\gg^{(\ll1\gg)}[\ll1\gg]^2+\frac{2\,\theta'[\lambda]\,\chi'[\lambda]}{\chi[\ll1\gg]}+\theta''[\lambda]==0,\ \ll5\gg,\ \theta'[0]==0,\ \varphi'[0]==-1\Big\},\ \{\chi[\lambda],\theta[\lambda],\varphi[\lambda]\},\ \{\lambda,0,10\}\Big]\Big\}$$

is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolve: 0.0002040816326530612` cannot be used as a variable.

ReplaceAll:

$$\{\text{NDSolve}[\{-\chi[0.000204082]\ll1\gg^{(\ll1\gg)}[\ll1\gg]^2-\text{Sin}[\ll1\gg]^2\,\chi[0.000204082]\ll1\gg^{(\ll1\gg)}[\ll1\gg]^2+\chi''[0.000204082]==0,\ \ll7\gg,\ \varphi'[0]==-1\},\ \{\chi[0.000204082],\theta[\ll23\gg],\varphi[0.000204082]\},\ \{0.000204082,0,10\}]\}$$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolve: 0.0002040816326530612` cannot be used as a variable.

••• ReplaceAll:
{NDSolve[{−1. χ[0.000204082] ≪1≫^(≪1≫)[≪1≫]^2 − 1. Sin[≪1≫]^2 χ[0.000204082] ≪1≫^(≪1≫)[≪1≫]^2 + χ''[0.000204082] == 0., ≪7≫
, φ'[0.] == −1.}, {χ[0.000204082], θ[≪23≫], φ[≪23≫]}, {0.000204082, 0., 10.}]} is neither a list of
replacement rules nor a valid dispatch table, and so cannot be used for replacing.

••• General: Further output of ReplaceAll::reps will be suppressed during this calculation.

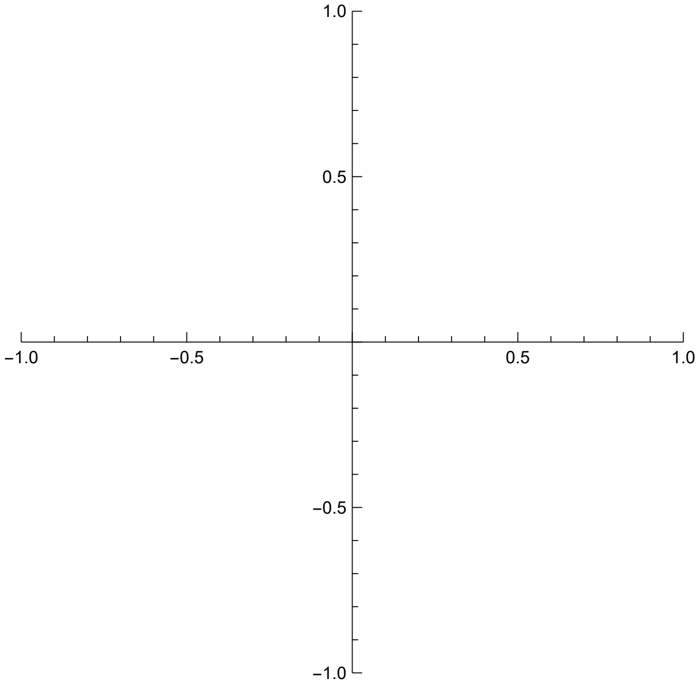
••• NDSolve: 0.2042857142857143` cannot be used as a variable.

••• General: Further output of NDSolve::dsvar will be suppressed during this calculation.

••• Infinity: Indeterminate expression 0. ComplexInfinity encountered.

••• NDSolve: Encountered non−numerical value for a derivative at λ == 0.`.

Out[•]=



In[127]:=

In[128]:=

In[129]:=

In[130]:=

4. Perfect Fluid

In[131]:=

```
xCoord = {t, r, θ, φ};
g = {{-1, 0, 0, 0},
  {0, (a[t])^2/(1 - k*r^2), 0 , 0},
  {0, 0, r^2*(a[t])^2 , 0},
  {0, 0, 0, r^2*(a[t])^2} Sin[θ]^2}
```

Out[∘]=

$$\left\{\{-1, 0, 0, 0\}, \left\{0, \frac{a[t]^2}{1-k\,r^2}, 0, 0\right\}, \left\{0, 0, r^2\,a[t]^2, 0\right\}, \left\{0, 0, 0, r^2\,a[t]^2\,\mathrm{Sin}[\theta]^2\right\}\right\}$$

In[133]:=

```
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a[t]^2}{1-k\,r^2} & 0 & 0 \\ 0 & 0 & r^2\,a[t]^2 & 0 \\ 0 & 0 & 0 & r^2\,a[t]^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = -\frac{a[t]^2\,d[r]^2}{-1+k\,r^2} - d[t]^2 + r^2\,a[t]^2\,d[\theta]^2 + r^2\,a[t]^2\,d[\varphi]^2\,\mathrm{Sin}[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -\frac{-1+k\,r^2}{a[t]^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2\,a[t]^2} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2\,a[t]^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Conformally Flat

Einstein computed in 0. sec

Out[∘]=

All tasks completed in 0.

In[134]:=

In[135]:=

```
Rdd
```

Out[•]=

$$\left\{\left\{-\frac{3\,a''[t]}{a[t]},0,0,0\right\},\left\{0,-\frac{2\,k+2\,a'[t]^2+a[t]\,a''[t]}{-1+k\,r^2},0,0\right\},\left\{0,0,r^2\left(2\,k+2\,a'[t]^2+a[t]\,a''[t]\right),0\right\},\left\{0,0,0,r^2\,\mathrm{Sin}[\theta]^2\left(2\,k+2\,a'[t]^2+a[t]\,a''[t]\right)\right\}\right\}$$

In[136]:=

```
R
```

Out[•]=

$$\frac{6\left(k+a'[t]^2+a[t]\,a''[t]\right)}{a[t]^2}$$

In[137]:=

```
FullSimplify[Rdd]
```

Out[•]=

$$\left\{\left\{-\frac{3\,a''[t]}{a[t]},0,0,0\right\},\left\{0,\frac{2\left(k+a'[t]^2\right)+a[t]\,a''[t]}{1-k\,r^2},0,0\right\},\left\{0,0,r^2\left(2\left(k+a'[t]^2\right)+a[t]\,a''[t]\right),0\right\},\left\{0,0,0,r^2\,\mathrm{Sin}[\theta]^2\left(2\left(k+a'[t]^2\right)+a[t]\,a''[t]\right)\right\}\right\}$$

In[138]:=

```
GUdd
```

Out[•]=

$$\left\{\left\{\{0,0,0,0\},\left\{0,-\frac{a[t]\,a'[t]}{-1+k\,r^2},0,0\right\},\left\{0,0,r^2\,a[t]\,a'[t],0\right\},\left\{0,0,0,r^2\,a[t]\,\mathrm{Sin}[\theta]^2\,a'[t]\right\}\right\},\left\{\left\{0,\frac{a'[t]}{a[t]},0,0\right\},\left\{\frac{a'[t]}{a[t]},-\frac{k\,r}{-1+k\,r^2},0,0\right\},\left\{0,0,r\left(-1+k\,r^2\right),0\right\},\left\{0,0,0,r\left(-1+k\,r^2\right)\mathrm{Sin}[\theta]^2\right\}\right\},\left\{\left\{0,0,\frac{a'[t]}{a[t]},0\right\},\left\{0,0,\frac{1}{r},0\right\},\left\{\frac{a'[t]}{a[t]},\frac{1}{r},0,0\right\},\{0,0,0,-\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]\}\right\},\left\{\left\{0,0,0,\frac{a'[t]}{a[t]}\right\},\left\{0,0,0,\frac{1}{r}\right\},\{0,0,0,\mathrm{Cot}[\theta]\},\left\{\frac{a'[t]}{a[t]},\frac{1}{r},\mathrm{Cot}[\theta],0\right\}\right\}\right\}$$

In[139]:=

```
EUd
```

Out[•]=

$$\left\{\left\{-\frac{3\left(k+a'[t]^2\right)}{a[t]^2},0,0,0\right\},\left\{0,-\frac{k+a'[t]^2+2\,a[t]\,a''[t]}{a[t]^2},0,0\right\},\left\{0,0,-\frac{k+a'[t]^2+2\,a[t]\,a''[t]}{a[t]^2},0\right\},\left\{0,0,0,-\frac{k+a'[t]^2+2\,a[t]\,a''[t]}{a[t]^2}\right\}\right\}$$

In[140]:=

```
R
```

Out[•]=

$$\frac{6\left(k+a'[t]^2+a[t]\,a''[t]\right)}{a[t]^2}$$

In[141]:=

```
Part[Rdd, 2, 2]
```

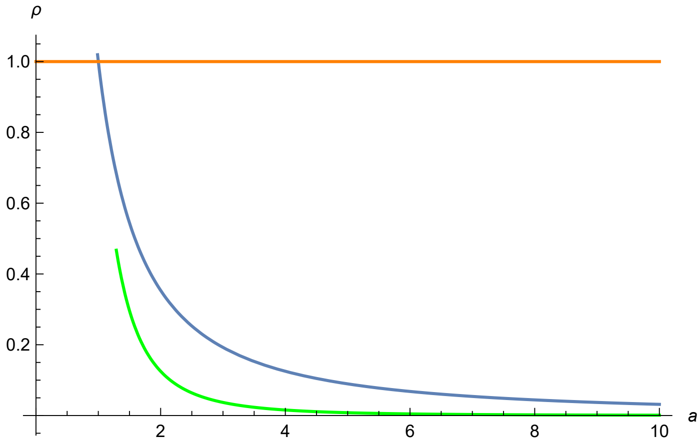
Out[•]=

$$-\frac{2\,k + 2\,a'[t]^2 + a[t]\,a''[t]}{-1 + k\,r^2}$$

In[142]:=

```
Show[Plot[y[x] = x^(-3/2), {x, 0, 10}, AxesLabel → {a, ρ}],
 Plot[y[x] = x^(-3), {x, 0, 10}, PlotStyle → Green],
 Plot[y[x] = 1, {x, 0, 10}, PlotStyle → Orange]]
```

Out[•]=



5. Friedmann Equations

In[143]:=

```
Part[Rdd, 2, 2] + Part[Rdd, 3, 3] + Part[Rdd, 4, 4]
```

Out[•]=

$$r^2\left(2\,k + 2\,a'[t]^2 + a[t]\,a''[t]\right) - \frac{2\,k + 2\,a'[t]^2 + a[t]\,a''[t]}{-1 + k\,r^2} + r^2\,\mathrm{Sin}[\theta]^2\left(2\,k + 2\,a'[t]^2 + a[t]\,a''[t]\right)$$

In[144]:=

```
lhs = FullSimplify[Part[Rdd, 2, 2] + Part[Rdd, 3, 3] + Part[Rdd, 4, 4]]
```

Out[•]=

$$\left(r^2 + \frac{1}{1 - k\,r^2} + r^2\,\mathrm{Sin}[\theta]^2\right)\left(2\left(k + a'[t]^2\right) + a[t]\,a''[t]\right)$$

In[145]:=

In[146]:=

```
rhs = FullSimplify[Part[gdd, 2, 2] + Part[gdd, 3, 3] + Part[gdd, 4, 4]]
```

Out[•]=

$$a[t]^2\left(r^2 + \frac{1}{1 - k\,r^2} + r^2\,\mathrm{Sin}[\theta]^2\right)$$

In[147]:=

In[148]:=

In[149]:=

```
lhs
```

Out[•]=

$$\left(r^2 + \frac{1}{1 - k\, r^2} + r^2\, \mathrm{Sin}[\theta]^2\right) \left(2 \left(k + a'[t]^2\right) + a[t]\, a''[t]\right)$$

In[150]:=

```
rhs
```

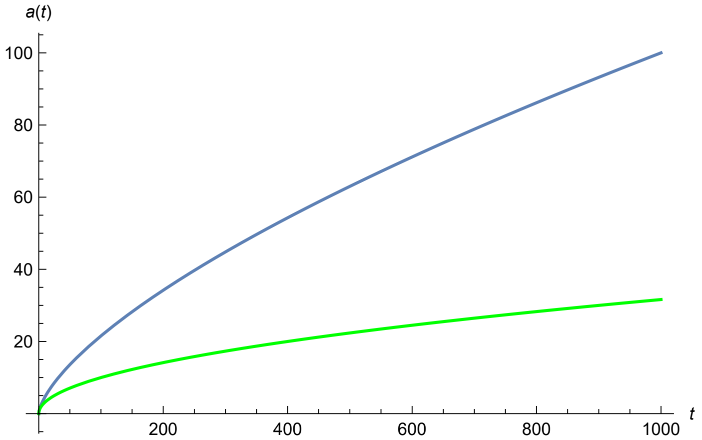
Out[•]=

$$a[t]^2 \left(r^2 + \frac{1}{1 - k\, r^2} + r^2\, \mathrm{Sin}[\theta]^2\right)$$

In[151]:=

```
Show[Plot[y[x] = x^(2/3), {x, 0, 1000}, AxesLabel → {t, a[t]}],
 Plot[y[x] = x^(1/2), {x, 0, 1000}, PlotStyle → Green]]
```

Out[•]=



In[•]:=

Out[•]=

Failure[Message: No Wolfram Language translation found.
Tag: NoParse
Query:]