Assignment 2: Question 1

Wednesday, 23 March 2022 5:48 pm

(1) A short cylinder of radius a and length L carries a 'frozen-in' uniform polarisation $\vec{P}$ parallel to its axis. Find the bound charge and sketch the electric field for $L \gg a$, $L \ll a$ and $L \sim a$

Solution:

The volume bound charge for the short cylinder is given by:

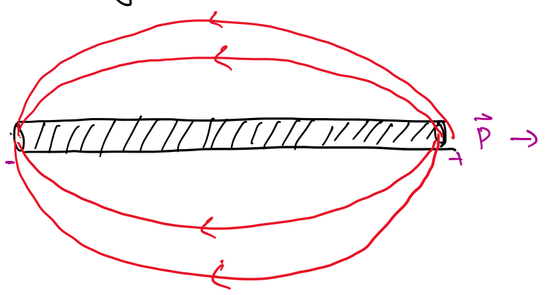
$$\rho_b = -\nabla \cdot \vec{P}$$

Since $\vec{P}$ is uniform, $\rho_b = 0$ (the volume bound charge density)

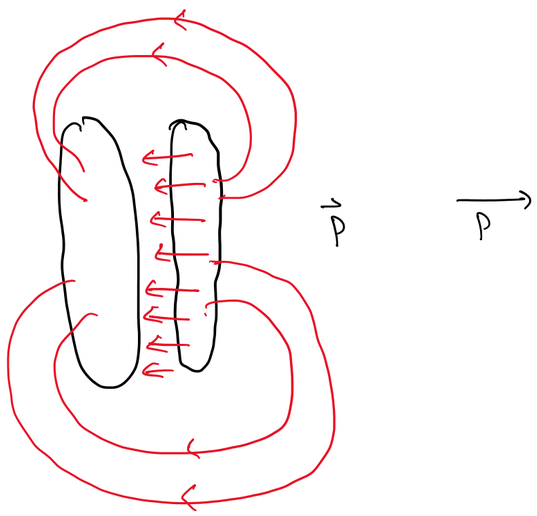
The surface bound charge for the cylinder is given by:

$$\sigma_b = \vec{P} \cdot \hat{n} \quad \hat{n} \text{ is the unit normal vector}$$
$$= \pm P \quad \text{(where P represents the magnitude of the polarisation)}$$

- When $L \gg a$ (i.e. the length of the cylinder is far greater than its radius), the two ends of the cylinder act as two point charges. Hence:



- For $L \ll a$:



The above electric field has a similar form to that of a parallel plate capacitor, where the ends of the cylinder are analogous to the plates of the capacitor.

- For $L \sim a$: the electric field can be sketched as: