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## 2 RG&TC-Code

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```
In[54]:= xCoord = {t, x, θ, φ};
```

```
g = {  
  {-x y, 0, 0, 0},  
  {0, x y t, 0, 0},  
  {0, 0, z, 0},  
  {0, 0, 0, x t}  
};
```

```
RGtensors[g, xCoord]
```

$$g_{dd} = \begin{pmatrix} -x y & 0 & 0 & 0 \\ 0 & t x y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & t x \end{pmatrix}$$

$$\text{LineElement} = -x y d[t]^2 + z d[\theta]^2 + t x d[\varphi]^2 + t x y d[x]^2$$

$$g_{UU} = \begin{pmatrix} -\frac{1}{x y} & 0 & 0 & 0 \\ 0 & \frac{1}{t x y} & 0 & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{t x} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

Einstein computed in 0. sec

```
Out[56]=
```

All tasks completed in 0.

```
In[57]:= (* Ricci Scalar *)
```

```
In[58]:= R
```

```
Out[58]=
```

$$-\frac{1}{2 t^2 x y}$$

```
In[59]:= (* Einstein Tensor *)
```

```
In[60]:= EUd
```

```
Out[60]=
```

$$\left\{ \left\{ -\frac{1}{4 t^2 x y}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{4 t^2 x y}, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{4 t^2 x y}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{4 t^2 x y} \right\} \right\}$$

```
In[61]:= (* Christoffel Symbol *)
```

In[62]:= **GUdd // MatrixForm**

Out[62]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2y} \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{2t} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2t} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2t} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2t} \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[63]:= **Part[GUdd, 1, 2, 2]**

**Part[GUdd, 2, 2, 1]**

Out[63]=

$$\frac{1}{2}$$

Out[64]=

$$\frac{1}{2t}$$

In[65]:= **(\* Riemann tensor \*)**

In[66]:= RUddd

Out[66]=

$$\begin{aligned}
& \left\{ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \right. \\
& \quad \left\{ \left\{ 0, -\frac{1}{4t}, 0, 0 \right\}, \left\{ \frac{1}{4t}, 0, 0, 0 \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left\{ \left\{ 0, 0, 0, -\frac{1}{4ty} \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{ \frac{1}{4ty}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ \left\{ \left\{ 0, -\frac{1}{4t^2}, 0, 0 \right\}, \left\{ \frac{1}{4t^2}, 0, 0, 0 \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \right. \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, 0, \frac{1}{4ty} \right\}, \{0, 0, 0, 0\}, \left\{ 0, -\frac{1}{4ty}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \right. \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\} \right\}, \\
& \left\{ \left\{ \left\{ 0, 0, 0, -\frac{1}{4t^2} \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{ \frac{1}{4t^2}, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, 0, -\frac{1}{4t} \right\}, \{0, 0, 0, 0\}, \left\{ 0, \frac{1}{4t}, 0, 0 \right\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \quad \left. \left. \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\} \right\} \right\}
\end{aligned}$$

In[67]:= (\* Ricci Tensor \*)

In[68]:= Rdd

Out[68]=

$$\left\{ \left\{ \frac{1}{2t^2}, 0, 0, 0 \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}$$

In[69]:= Part[Rdd, 1, 1]

Out[69]=

$$\frac{1}{2t^2}$$

In[70]:= xCoord = {t, r, φ};

$$\begin{aligned}
\mathbf{g} = & \left\{ \left\{ -\left( 1 + 2 \frac{G * M}{r} \right), 0, 0 \right\}, \right. \\
& \left\{ 0, \left( 1 + 2 \frac{G * M}{r} \right), 0 \right\}, \\
& \left. \{0, 0, r^2\} \right\}
\end{aligned}$$

Out[71]=

$$\left\{ \left\{ -1 - \frac{2GM}{r}, 0, 0 \right\}, \left\{ 0, 1 + \frac{2GM}{r}, 0 \right\}, \{0, 0, r^2\} \right\}$$

In[72]:= RGtensors[g, xCoord]

$$g_{dd} = \begin{pmatrix} -1 - \frac{2GM}{r} & 0 & 0 \\ 0 & 1 + \frac{2GM}{r} & 0 \\ 0 & 0 & r^2 \end{pmatrix}$$

$$\text{LineElement} = \frac{(2GM + r) d[r]^2}{r} - \frac{(2GM + r) d[t]^2}{r} + r^2 d[\varphi]^2$$

$$g_{UU} = \begin{pmatrix} -\frac{r}{2GM+r} & 0 & 0 \\ 0 & \frac{r}{2GM+r} & 0 \\ 0 & 0 & \frac{1}{r^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

*Testing for 3-dim conformal flatness...*

 **Outer:** Heads Times and List at positions 3 and 2 are expected to be the same. 

Einstein computed in 0. sec

Out[72]=

All tasks completed in 0.

In[73]:= **GUdd**

Out[73]=

$$\left\{ \left\{ \left\{ 0, -\frac{GM}{r(2GM+r)}, 0 \right\}, \left\{ -\frac{GM}{r(2GM+r)}, 0, 0 \right\}, \{0, 0, 0\} \right\}, \right. \\ \left. \left\{ \left\{ -\frac{GM}{r(2GM+r)}, 0, 0 \right\}, \left\{ 0, -\frac{GM}{r(2GM+r)}, 0 \right\}, \left\{ 0, 0, -\frac{r^2}{2GM+r} \right\} \right\}, \right. \\ \left. \left\{ \{0, 0, 0\}, \left\{ 0, 0, \frac{1}{r} \right\}, \left\{ 0, \frac{1}{r}, 0 \right\} \right\} \right\}$$

In[74]:= **xCoord = {ψ, θ, φ};**

**g = {{1, 0, 0},**  
**{0, Sin[ψ]^2, 0},**  
**{0, 0, Sin[ψ]^2 \* Sin[φ]^2}}**

Out[75]=

$$\left\{ \{1, 0, 0\}, \{0, \text{Sin}[\psi]^2, 0\}, \{0, 0, \text{Sin}[\varphi]^2 \text{Sin}[\psi]^2\} \right\}$$

In[76]:= **RGtensors[g, xCoord]**

$$g_{dd} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin[\psi]^2 & 0 \\ 0 & 0 & \sin[\varphi]^2 \sin[\psi]^2 \end{pmatrix}$$

$$\text{LineElement} = d[\psi]^2 + d[\theta]^2 \sin[\psi]^2 + d[\varphi]^2 \sin[\varphi]^2 \sin[\psi]^2$$

$$g_{UU} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \csc[\psi]^2 & 0 \\ 0 & 0 & \csc[\varphi]^2 \csc[\psi]^2 \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

*Testing for 3-dim conformal flatness...*

 **Outer:** Heads Times and List at positions 3 and 2 are expected to be the same. 

Einstein computed in 0. sec

Out[76]=

All tasks completed in 0.

In[77]:= **RUddd**

Out[77]=

```
{ {{ {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}, {{0, Sin[ψ]^2, 0}, {-Sin[ψ]^2, 0, 0}, {0, 0, 0}},
  {{0, 0, Sin[φ]^2 Sin[ψ]^2}, {0, 0, 0}, {-Sin[φ]^2 Sin[ψ]^2, 0, 0}} },
  {{ {0, -1, 0}, {1, 0, 0}, {0, 0, 0}}, {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}},
  {{0, 0, 0}, {0, 0, -Cos[ψ]^2 Sin[φ]^2}, {0, Cos[ψ]^2 Sin[φ]^2, 0}} },
  {{ {0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 0, 0}, {0, 0, Cos[ψ]^2}, {0, -Cos[ψ]^2, 0}},
  {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}} }
```

In[78]:= **GUdd**

Out[78]=

```
{ {{ {0, 0, 0}, {0, -Cos[ψ] Sin[ψ], 0}, {0, 0, -Cos[ψ] Sin[φ]^2 Sin[ψ]} },
  {{0, Cot[ψ], 0}, {Cot[ψ], 0, 0}, {0, 0, 0}},
  {{0, 0, Cot[ψ]}, {0, 0, 0}, {Cot[ψ], 0, Cot[φ]}} }
```

In[79]:=

In[80]:=

In[81]:=

In[97]:= **xCoord = {x, y};**

**g = {{x\*y, 0}, {0, x^2}}**

Out[98]=

```
{{x y, 0}, {0, x^2}}
```

In[99]:= **RGtensors[g, xCoord]**

$$g_{dd} = \begin{pmatrix} xy & 0 \\ 0 & x^2 \end{pmatrix}$$

$$\text{LineElement} = xy d[x]^2 + x^2 d[y]^2$$

$$g_{UU} = \begin{pmatrix} \frac{1}{xy} & 0 \\ 0 & \frac{1}{x^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0.015 sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

## Conformally Flat

Einstein computed in 0. sec

## Einstein Space

Out[99]=

All tasks completed in 0.015625

In[85]:= **GUdd**

Out[85]=

$$\left\{ \left\{ \left\{ \frac{1}{2x}, \frac{1}{2y} \right\}, \left\{ \frac{1}{2y}, -\frac{1}{y} \right\} \right\}, \left\{ \left\{ -\frac{1}{2x}, \frac{1}{x} \right\}, \left\{ \frac{1}{x}, 0 \right\} \right\} \right\}$$

In[86]:= **RUdd**

Out[86]=

RUdd

In[100]:=

**RUddd // MatrixForm**

Out[100]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & \frac{x+2y}{4xy^2} \\ -\frac{x+2y}{4xy^2} & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -\frac{x+2y}{4x^2y} \\ \frac{x+2y}{4x^2y} & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

In[88]:= **Rdd**

Out[88]=

$$\left\{ \left\{ \frac{x+2y}{4x^2y}, 0 \right\}, \left\{ 0, \frac{x+2y}{4xy^2} \right\} \right\}$$

In[89]:= **R**

Out[89]=

$$\frac{x+2y}{2x^3y^2}$$

In[90]:= **xCoord** = {t, r,  $\theta$ ,  $\varphi$ }

Out[90]=

{t, r,  $\theta$ ,  $\varphi$ }

In[91]:= **g** = {{-(1 - 2 \* G \* M / r), 0, 0, 0},  
 {0, (1 - 2 \* G \* M / r) ^ (-1), 0, 0},  
 {0, 0, r^2, 0},  
 {0, 0, 0, r^2 \* Sin[ $\theta$ ]^2}}

Out[91]=

$\left\{ \left\{ -1 + \frac{2GM}{r}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{1 - \frac{2GM}{r}}, 0, 0 \right\}, \left\{ 0, 0, r^2, 0 \right\}, \left\{ 0, 0, 0, r^2 \sin^2[\theta] \right\} \right\}$

In[92]:= **RGtensors**[g, xCoord]

$$g_{dd} = \begin{pmatrix} -1 + \frac{2GM}{r} & 0 & 0 & 0 \\ 0 & \frac{1}{1 - \frac{2GM}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2[\theta] \end{pmatrix}$$

$$\text{LineElement} = -\frac{r d[r]^2}{2GM - r} + \frac{(2GM - r) d[t]^2}{r} + r^2 d[\theta]^2 + r^2 d[\varphi]^2 \sin^2[\theta]$$

$$g_{UU} = \begin{pmatrix} \frac{r}{2GM - r} & 0 & 0 & 0 \\ 0 & -\frac{2GM - r}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\csc^2[\theta]}{r^2} \end{pmatrix}$$

gUU computed in 0. sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0. sec

Riemann(Uddd) computed in 0. sec

Ricci computed in 0. sec

Weyl computed in 0. sec

## Ricci Flat

Out[92]=

All tasks completed in 0.

In[93]:= **GUdd**

Out[93]=

$\left\{ \left\{ \left\{ 0, -\frac{GM}{(2GM - r)r}, 0, 0 \right\}, \left\{ -\frac{GM}{(2GM - r)r}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\}, \right.$   
 $\left\{ \left\{ -\frac{GM(2GM - r)}{r^3}, 0, 0, 0 \right\}, \left\{ 0, \frac{GM}{(2GM - r)r}, 0, 0 \right\}, \right.$   
 $\left\{ 0, 0, 2GM - r, 0 \right\}, \left\{ 0, 0, 0, (2GM - r) \sin^2[\theta] \right\} \right\},$   
 $\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{r}, 0 \right\}, \left\{ 0, \frac{1}{r}, 0, 0 \right\}, \left\{ 0, 0, 0, -\cos[\theta] \sin[\theta] \right\} \right\},$   
 $\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{r} \right\}, \left\{ 0, 0, 0, \cot[\theta] \right\}, \left\{ 0, \frac{1}{r}, \cot[\theta], 0 \right\} \right\}$