$$H(p,x;\mathbf{P},\mathbf{X}) = H_S + H_B + H_{SB} \tag{1}$$

(S)

m

$$H_S = \frac{p^2}{2m} - a\frac{x^2}{2} + b\frac{x^4}{4}, \tag{2}$$

p

q

a, b

(B)

$$H_B(\mathbf{P},\mathbf{X}) = \sum_{n=1}^{N}\left(\frac{P_n^2}{2M_n} + M_n\omega_n^2\frac{X_n^2}{2}\right), \tag{3}$$

n

X_n

P_N

ω_n

M_n

$$H_{SB}(p,x;\mathbf{P},\mathbf{X}) = -x\sum_{n=1}^{N} g_n X_n + x^2\sum_{n=1}^{N}\frac{g_n^2}{2M_n\omega_n} \tag{4}$$

$$\dot{p} = \frac{-\partial H}{\partial x} = ax - bx^3 + \sum_{n=1}^{N} g_n X_n - x\sum_{n=1}^{N}\frac{g_n^2}{M_n\omega_n^2}, \tag{5}$$

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}. \tag{6}$$

n

$$\dot{P}_n = \frac{-\partial H}{\partial X_n} = -xg_n - M_n\omega_n^2 X_n \tag{7}$$

$$\dot{X}_n = \frac{\partial H}{\partial P_n} = \frac{P_n}{M_n} \tag{8}$$

$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4, \tag{9}$$

V

$$\frac{d}{dx}V = 0 \to x(bx^2 - a) = 0. \tag{10}$$

$x = 0, \pm\sqrt{\frac{a}{b}}$

$x = 0$

$x = \pm\sqrt{\frac{a}{b}}$

$x_0 = \sqrt{\frac{a}{b}}$

$$V(x_0+\epsilon) = V(x_0) + U'(x_0)\epsilon + \frac{1}{2}V''(x_0)\epsilon^2 + ... \tag{11}$$

$V'(x_0) = 0$

$V''(x_0) = \frac{1}{2}(-a + 3bx^2)|_{x=x_0}$

$$V(x_0+\epsilon) \approx \frac{1}{2}(2a)\epsilon^2. \tag{12}$$

$k = 2a$

$V_{HO} = \frac{1}{2}kx^2$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2a}{m}} \tag{13}$$