Thm. 1.10 (Law of Total Probability)

• From thm 1.9 & conditional prob.

- For any event A & partition $\{B_1, B_2, \dots B_n\}$ with $P[B_i] > 0$, then $P[A] = \sum_{i=1}^{n} P[A|B_i]P[B_i]$

Ex: ADM company consists of 3 programmers: Alex (A), Drew (D), Morgan (M)

- Percentage of program each write: 60, 25, 15
- Prob. program compiles first try: .8, .9, .7

• Find prob that program compile 1st try.

Partition: $\{A, D, M\}$

F - program compile first try

Then: $P[F|A] = .8$ $P[F|D] = .25$ $P[F|M] = .15$

$P[A] = .6$ $P[D] = .25$ $P[M] = .15$

So $P[F] = (.8)(.6) + (.9)(.25) + (.7)(.15) = .81$

Thm 1.11 (Baye's Rule)

- Allow calculate $P[B|A]$ if we know $P[A|B]$ & $P[A]$ & $P[B]$

- $$P[B|A] = \frac{P[A|B] \cdot P[B]}{P[A]}$$

Ex: Prob that a prog that compiles 1st time was written by Drew?

$$P[D|F] = \frac{P[F|D] \cdot P[D]}{P[F]}$$

$$= \frac{(.9)(.25)}{.81} = .278$$

Partion Form of Bayes'

for partition: $\{B_1 \dots B_n\}$

s'pose we know $P[B_i]$ & $P[A|B_i]$ for some event A.

- Combine Bayes' & Total Prob.

$$P[B_i|A] = \frac{P[A|B_i] \cdot P[B_i]}{\sum_{i=1}^{M} P[A|B_i] \cdot P[B_i]}$$

Ex: Disease Screening Problem

Consider screening for disease.

Have disease (D) or not (D^c)

Test positive (T_p) or negative (T_n)

Characteristics

Sensitivity - Prob of detection: $P[T_p | D]$

Specificity - $P[T_n | D^c]$

False Positive - $P[T_p | D_c]$

Characteristic of Population

Prevalence - $P[D]$ → Prob. someone has disease

$\underline{Q}$: with $P[T_p | D] = P[T_N | D^c] = .99$

& $P[D] = .01$ (rare)

What is prob. that person that tests positive has the disease?

Bayes': $P[D | T_p] = \dfrac{P[T_p | D] \cdot P[D]}{P[T_p]}$

Total Prob.:
$$\frac{P[T_p | D] \cdot P[D]}{P[T_p | D] \cdot P[D] + \underbrace{P[T_p | D^c]}_{1 - P[T_n | D^c] = .01} \cdot P[D^c]}$$

$$\frac{(.99)(.01)}{(.99)(.01) + (.01)(.99)} = .5$$

- but if $P[D] = .1$, $P[D | T_p] = .917$

Independence

- 2 events independant if observing one doesn't affect prob. of other.

- A & B independent if & only if $P[AB] = P[A] \cdot P[B]$

Relationships if indep.

- $P[A|B] = P[A]$

Prev Ex:

Is being a junior & having a Mac indendent?

$P[J \cdot M] = .8$ $\quad$ $P[J] \cdot P[M] = (.23)(.3)$

$= .69$

Not independent!

Longer defn for ≥ 3 events.

<u>Defn</u>: events $A_1, \ldots, A_n$

are independent iff:

a) all collections of $n-1$ events are mutually independent.

&

b) $P[A_1 \cap A_2 \cap \ldots A_n] = P[A_1] \cdot P[A_2] \ldots$

Ex. 3 - write program that generates 0 or 1 w/ prob. of .5 for each

- Run 1000 times & get 1 1000 times
  what prob. of getting 1 again?:
  A1 - If truly indep. → .5
  A2 - Likely indep. is not there

Ex. 4 Monty Hall Problem

Let C_i : event that car is behind door i

H_i : empty door

Y_i : choose door i

Assume: $P[C_i] = 1/3$

&

where Car is & choice independent

$\Rightarrow P[C_i Y_j] = P[C_i] \cdot P[Y_j] \; \forall i,j$

<u>Consider:</u> case Y_1 & H_3

If C_2 occured, win by switching

Find $P[C_2 \mid (H_3, Y_1)]$

$$P[C_2 \mid (H_3, Y_1)] = \frac{P[C_2, (H_3, Y_1)]}{P[H_3, Y_1]}$$

$$\Rightarrow \frac{P[H_3, (C_2, Y_1)]}{P[H_3, Y_1]} \Rightarrow P[H_3$$

$$\Rightarrow \frac{\overbrace{P[H_3 \mid (C_2, Y_1)]}^{=1.0} \cdot P[C_2, Y_1]}{P[H_3, Y_1]}$$

$$P[C_2 \mid (H_3, Y_1)] = \frac{P[C_2, Y_1]}{P[H_3, Y_1]}$$

$$= \frac{P[C_2] \cdot P[\cancel{Y_1}]}{\underbrace{P[H_3]}_{=1/2} \cdot P[\cancel{Y_1}]} \Rightarrow \frac{1/3}{1/2} = 2/3$$