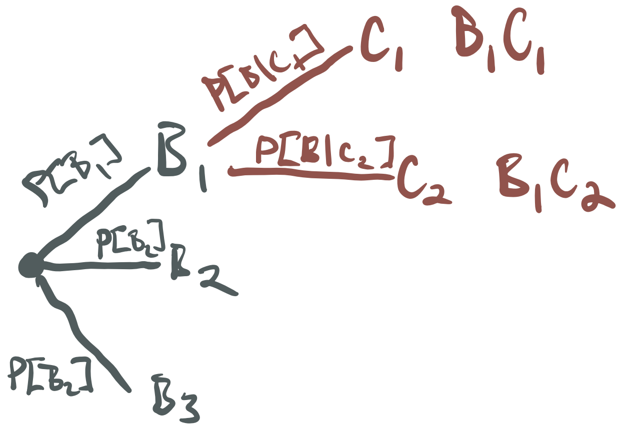
Tree Diagrams

Read 2.1–2.4

Start @ root of tree

1st set of branches

- From roots to a partition. (Sub experiment)
- Labels are prob. of events in partition



* Labels are conditional probs.

2nd Set Branches

- From each event of 1st sub exp.

- End w/ joint prob. of the seq. exp.

* Prob. of these are product of branch probs. back to root

B) Monty Hall Problem

C_i – Car behind door i

H_i – Host shows door i

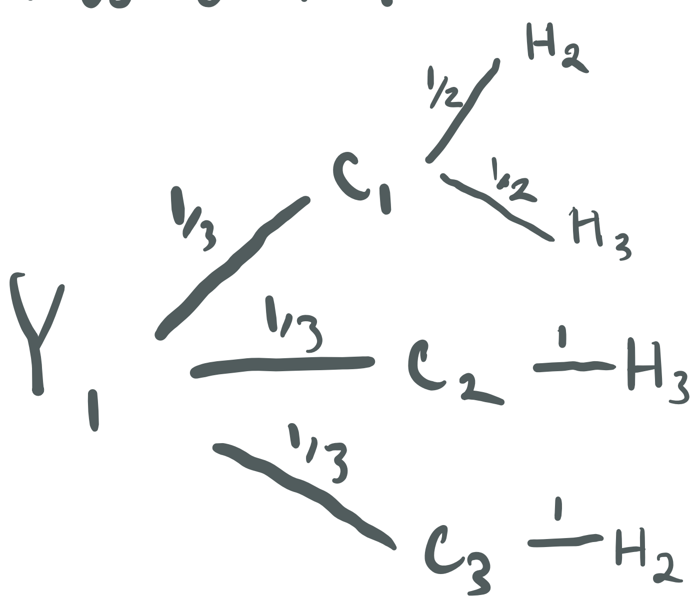
Y_i – You choose door i

W – win

L – lose

Start

Choose door 1



Switch	Stay	Prob
L	W	1/6
L	W	1/6
W	L	1/3
W	L	1/3

$P[W] = 2/3$ $\quad$ $P[W] = 1/3$

Counting Methods

A) Prelims:

- For experiment w/ countable outcomes, need to be able determine # of outcome in event.

Thm 2.1: Fund. theorem of counting

- For exp. w/ 2 sub exp.: If one sub-exp has i outcomes, and other has J outcomes. The total exp. has is $I \cdot J$ outcomes

Ex: $\{1,2,3\}$ & $\{A,B\}$ 6 outcomes: A1 A2 A3
B1 B2 B3

B) Sampling, Permutations, & Combinations

<u>Sampling</u>: Choosing items randomly from collection, an act of Sampling is a trial of exp.

Sampling w/ Replacement: After choosing item, return back to bag

Sampling w/o: Don't replace item

<u>Permutation</u>: Result of a series of Samplings, w/ order of selections preserved

<u>Combination</u>: Results again, but w/o preserving

Notation:

m - number of items in selection (size of alphabet)

k - number of selections made

n - number of repititions of particular experiment

$(m)_k$ - # of k-permutations of m items w/o replacement

$\binom{m}{k}$: "m choose k": # of k combinations w/o replacement

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Example Permutation W/ replacement

Collection: letters A, B, C, D $\Rightarrow m = 4$

# of selections: $k = 2$

- List & count & generalize all outcomes

A) Permutations of sampling w/ replacement

AA	AB	AC	AD
BA	BB	BC	BD
CA	CB	CC	CD
DA	DB	DC	DD

Generalization: selections here $m=4$. w/ $k=2 \Rightarrow \boxed{m \cdot m = m^k}$

w/ replacement

Permutations W/O Replacement

A) $\cancel{AA}$ AB AC AD Removed bc no replacement

BA $\cancel{BB}$ BC BD

CA CB $\cancel{CC}$ CD

DA DB DC $\cancel{DD}$

Generalization:

1st choice: $m=4$ possibilities

2nd: $3=m-1$

Total num: $(m)(m-1)\dots(m-k+1)$

$$\Rightarrow \frac{m!}{(m-k)!} = (m)_k$$

Combinations w/o replacement

~~AA~~	AB	AC	AD
~~BA~~	~~BB~~	BC	BD
~~CA~~	~~CB~~	~~CC~~	CD
~~DA~~	~~DB~~	~~DC~~	~~DD~~

Excluded perms. for combination & for no replacement

Generalization: 2 sub-exp. to produce a k-permutation

Step 1: Choose k-combination of n-items

Step 2: Choose k-permutation of k-items in k-combo

1) Has $\binom{n}{k}$ outcomes

2) has $(k)_k = k!$ outcomes

Entire exp. has $(n)_k$ outcomes:

So, $(n)_k = \binom{n}{k} k!$

$$\binom{n}{k} = \frac{(n)_k}{k!} = \frac{n!}{(n-k)! \cdot k!}$$

3) Example

A-Z + 1-10

License Plate: 6 characters, letter or #

# of possible allowing repeat symbols? w/ replacement

$(36)^6 = \ldots$

W/o Repeating

$(36)_6 = \frac{36!}{(36-6)!} = \ldots$

W/ ordering (letters in order, then #'s)

$\binom{36}{6} \Rightarrow \frac{36!}{30!\,6!} = \ldots$

How many possible ~~sq~~ quences of 1000 bits have exactly 997 1's

choosing $k = 997$ positions from

$n = 1000$ positions

$$\binom{1000}{997} = \frac{(1000)(999)(998)}{3 \cdot 2}$$

$$= 166{,}167{,}000$$