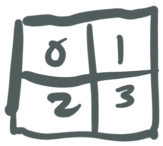
4 sided dice

$P[\{0\}] = P[\{1\}] = P[\{2\}] = P[\{3\}] = .25$

n trials $= 1000$

$N_{\{1\}} \approx 250$

Venn Diagram:

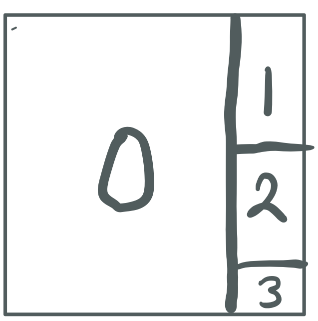


Ex 2: Skeet shooting

- 3 targets launched, 1 shot

<u>Observe</u>: # targets hit

Sample Space, events identical as 1 but more factors



Other probability Theorems:

- A3 holds for finite collections, too. (prob 1.3.13)
- $P[\emptyset] = 0$
- $P[A^c] = 1 - P[A]$
- $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
- M.E. $P[A \cup B] = P[A] + P[B]$
  - if $A \subset B$, then $P[A] \leq P[B]$

Notation: $P[A \cap B]$ same as $P[A, B]$ same as $P[AB]$

- if s_i is an outcome, $\{s_i\}$ is an event, so should write like $P[\{s_i\}]$

Conditional Properties

1) Defn & Properties

- Occurance of 1 event can give info on another event

Conditional Prob of A given B has occured is:

$$P[A|B] = \frac{P[AB]}{P[B]}$$

Note:

- Occurance of B eliminates outcomes not in B, so $P[A|B]$ is based on a new prob. measure for reduced sample space
- So for that prob. space:
  1) $P[A|B] \geq 0$
  2) $P[B|B] = 1$
  3) for $A = A_1 \cup A_2 \cup A \ldots$ with all A_i disjoint
     $P[A|B] = P[A_1|B] + P[A_2|B] \ldots$

Note: $P[A]$ is unconditioned a priori prob

Ex: Power surges can damage PC. Due to CPU, memory, etc...

For each component, let w indicate working & b is broken

Outcomes are all possible states. of 3 components for computer that has failed.

Probs: Mem most susceptible, and disk is least

Outcome

CPU	Mem	Disk	Prob
W	W	W	.24
W	W	B	.02
W	B	W	.22
W	B	B	.12

∴ Let C be event CPU is broken.

Let D be disk broken

$P[D] = .02 + .12 + .10 + .15 = .39$

$P[C] = .10 + .10 + .05 + .15 = .40$

$P[CD] = .10 + .15 = .25$

$P[C|D] = P[CD]/P[D] = .25/.39 = .641$

To see this another way: Give that disk is broken, only 4 possible outcomes in reduced sample space

Outcome	Prob.	
W W B	.021/.39 →	.051
W B B	.12/.39 →	.308
B W B	.10/.39 →	.256
B B B	+ .15/.39 →	.385
	.39	

Divide each by .39

Now they all add to 1

D) Partition in Prob.

1) Def. partion divides sample space into M.E. events

2) Partition set of events that are both M.E. & exhaustive

Note 1: Partition is set of events

Note 2: Portion not necessarily a sample space (Partition is events) & S S are outcomes)

a) Partition $\not\Rightarrow$ S.S

b) Union of events in partition is S.S.

2) Theorems related to partitions

a) For partition $B = \{B_1, B_2, \ldots\}$ and any event in S.S., let $C_i = A \cap B$

Then for $i \neq j \Rightarrow C_i$ & C_j are M.E. & $A = C_1 \cup C_2$

Power Surge Ex.

Partition: $B_1 = \{$CPU working$\}$

$B_2 = \{$CPU broken, mem working$\}$

$B_3 = \{$CPU broken, mem broken$\}$

$A = \{$Mem working$\} = \{$www, wwb, bww, bwb$\}$

$C_1 = \{$www, wwb$\}$ $C_2 = \{$bww, bwb$\}$ $(= B_2)$ $C_3 = \emptyset$

a & b theorem are true here

Thm 1.9

From thm 1.8 & 1.3 for any event A & partion $\{B_1, B_2, \ldots B_m\}$

$$P[A] = \sum_{i=1}^{m} P[A \cap B_i]$$

Thm 1.9 works well when SS probs can be written as table w/ 2 orthogonal partitions

Ex: All KU EECS students are either freshman, soph, ... (F S J R)
& everyone has laptop with different OS. (M, W, L)

$\{F, S, J, R\}$ & $\{M, W, L\}$

Put joint probs in table:

	F	S	J	R	{M,W,L}
M	.06	.07	.08	.09	.30
W	.06	.03	.05	.04	.20
L	.18	.14	.10	.08	.50
{F,S,J,R}	.30	.26	.23	.21	1.0

Find probability using steps 1-6 or 6-1