**Huarongdao Rule:** A piece of the puzzle can move from position A to B if:
1. A and B are adjacent
2. B is empty.

**Relaxed version:** A piece of the puzzle can move from position A to B if:
1. A and B are adjacent
2. ~~B is empty.~~

**Description of the advanced Heuristic function:**
The advanced heuristic function is based on the Manhattan Heuristic. The puzzle is a 5x4 squared space. For Caocao (2x2) to be at the goal state, it has to occupy the middle position of the last two rows.
1. Any horizontal (1x2) pieces cannot be presented in the last two rows for the goal state, since each piece cannot overlap with others. It requires at least one additional step to move one horizontal piece away from the last two rows.
2. Any vertical (1x2) pieces or single (1x1) pieces cannot occupying the middle position of the last two rows (Caocao goal state). It requires at least one additional step to move one peice away from the Caocal goal state.

**Implementation of the advanced Heuristic function:**
- For a given state, if the top left corner of Caocao is at position (x,y), the heuristic of Caocao is **H_manhatten(n) = H_caocao(n) = |3-x| + |1-y|.**
- Then we check the horizontal pieces, whose left block is at position (a,b). **If a in [3,4], H_horizontal(n) += 1.**
- Then we check the vertical pieces, whose top block is at position (a,b) and single pieces, whose position is at (a,b). **If (a,b) in [(3,1)(3,2)(4,1)(4,2)], H_other(n) += 1.**
- So, the final total **H(n) = H_manhatten(n) + H_horizontal(n) + H_other(n)**.

**Why it is Admissible:**
*Definition:* For every state n, 0<=h(n)<=h*(n).
*Proof:*
Since the Manhatten heuristic function is admissible, it satisfied that for every state n, 0<=**H_manhatten(n)**<=h*(n). Then next step is to prove that after adding the additional values **H_horizontal(n) & H_others(n)**, the H(n) is still underestimated compared to the minimum movements.

For the states that **H_horizontal > 0 and/or H_other(n) > 0**, from the relaxed version of the rule, we are not considering if position B is empty for moving each piece and calculate the minimum steps for that piece. In the real huarongdao rule, there will be at least equal or more steps involved for the movement to clear up position B and then move the desired piece to position B. So, the inequality satisfied that for every state n, **H_manhatten(n)+H_horizontal(n)+H_other(n)<=h*(n)**. Since H_horizontal(n) >= 0 and H_other(n) >= 0, so, definition satisfied for every state n, **0<=H_manhatten(n)+H_horizontal(n)<=h*(n).**

**Why it dominates the Manhattan distance heuristic:**
*Definition:* If h1(n) dominates h2(n), if and only if:
1. h1(n) >= h2(n). for every state n
2. h1(n) > h2(n), for at least one state n.

*Proof:*
- To prove requirement 2, we can easily create a state that a single piece is at Caocao goal state. In this case, H_other(n) = 1, so, H(n) > H_manhatten(n).
- To prove requirement 1, since H_horizontal(n) >= 0 & H_other(n) >=0, H(n) = H_manhatten(n) + H_horizontal >= H_manhatten(n). So, for every state of n, h1(n) >= h2(n).