APS106 Lab 8

This lab will test your ability to define classes and methods. Place appropriate comments in your program. **Due: 11:59pm, March 29, 2019**

Question

A **matrix** of dimensions $m \times n$ (an m-by-n matrix) is an ordered collection of $m \times n$ elements, which are called elements (or components). The elements of an $(m \times n)$-dimensional matrix A are denoted as $a_{i,j}$, where $1 \leq i \leq m$ and $1 \leq j \leq m$, symbolically, written as, $A = a_{i,j}, (1,1) \leq (i,j) \leq (m,n)$. Written in the familiar notation:

$$\begin{bmatrix} a_{1,1} & \dots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{m,1} & \dots & a_{m,n} \end{bmatrix}$$

A 3×3 matrix

The horizontal and vertical lines of entries in a matrix are called rows and columns, respectively. A matrix with the same number of rows and columns is called a square matrix.

Several operations are defined for matrices, you are asked to implement the following:

- **Addition**: the matrix addition is defined as $c_{i,j} = a_{i,j} + b_{i,j}$. ***Matrices A and B must have same dimensions***. For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+4 & 3+7 \\ 4+2 & 5+5 & 6+8 \\ 7+3 & 8+6 & 9+9 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 10 \\ 6 & 10 & 14 \\ 10 & 14 & 18 \end{bmatrix}$$

- **Scaler multiplication** Matrix A multiplied by scaler k is defined as $c_{i,j} = k \times a_{i,j}$. For example:

$$2 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

- **Trace** The trace of a ***square matrix*** is the sum of its diagonal elements, namely $a_{i,j}, where i = j$. For instance,

$$\text{tr}\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\right) = 1 + 5 + 9 = 15$$

- **Transpose** The transpose of a matrix is simply inverting the column and rows, namely $a_{i,j} = a_{j,i}$. For instance,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

- **Matrix multiply** Assume A is a $m \times k$ matrix and B is a $k \times n$ matrix, the product of AB is defined as

$$C_{i,j} = \sum_k A_{i,k} B_{k,j}$$

You are asked to complete the implementation of a Python class that can be used to represent matrices and perform various operations on them. The elements of a matrix are passed as a nested list to the constructor of the class, e.g., `Matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]])` creates an object representing the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

In the code you implement, make sure to check whether the matrices' dimensions match so that the computation can be carried out. Pay attention to the ***bold italic font***.

In any case the operation is invalid (dimension mismatch, for instance), return an empty `Matrix`, namely with zero row and zero column. If the matrix passed to operator `trace` is not a square matrix, return `float('inf')`.

Note: you should write your own code for all the matrix operations. Do not import non-default python library like numpy. Follow the docstring provided to you, closely.

TODO: Download the file `lab8.py`, complete the functions inside according to their descriptions and upload your version of lab8.py to MarkUs. Feel free to add new helper functions/methods, but all the functions defined for you should be implemented.