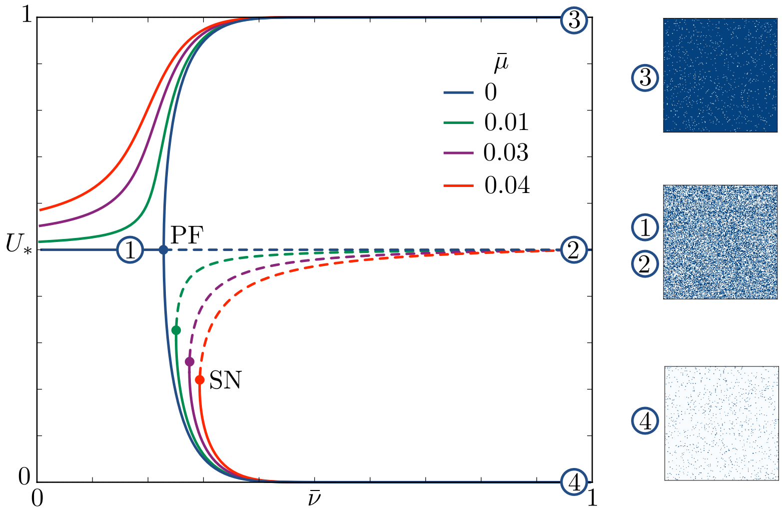

Fig. 3.1: Fixed points of the approximate coarse map Φ_a as a function of the coupling parameter $\bar{\nu}$ for $\bar{\xi} = 0.236$ and various values of $\bar{\mu}$. Solid (dashed) lines represent stable (unstable) coarse equilibria. Representative microscopic solutions are plotted on the right. If agents are unbiased on average ($\bar{\mu} = 0$), the mixed state becomes unstable at a pitchfork bifurcation (PF), attained at the critical value given by (3.7), and two locked-in states emerge. If agents have an average bias towards product 1 ($\bar{\mu} > 0$), the pitchfork breaks down, giving rise to a saddle node bifurcation (SN). A similar scenario occurs if $\bar{\mu} < 0$ (not shown).

approximate this distribution using a Dirac-delta distribution. However, over long time scales, both metastable states are equally likely to occur, as Figure 2.3 shows.

4. Equation-free Newton-Krylov method.

In this section, we aim to obtain a numerical closure relation for the evolution of the *spatially distributed* macroscopic state $\boldsymbol{U} = (U_n)_{n=1}^N$. In this case, an analytical closure approximation is no longer valid. We thus propose an equation-free method. We first outline the general principle of the equation-free methodology (Section 4.1). Next, we describe the concrete lifting and restriction operators that will be used (Section 4.2). The main algorithmic contribution of the present paper is the introduction of a weighted lifting and restriction operator that allows the accurate computation of Jacobian-vector products, as will be discussed in Section 4.3.

4.1. Principle.

As was shown in the previous sections, the lock-in model (2.5) consists, at the microscopic level, of individual agents whose state keeps evolving, owing to the probabilistic nature of their choices. Nevertheless, at the macroscopic level, the ensemble average (2.8) is seen to evolve to a metastable equilibrium. In this paper, we are interested in performing a bifurcation analysis at the macroscopic level, at which an exact, closed model is not available. The equation-free framework was developed for such tasks [37, 38].

The main building block in an equation-free method is the *coarse time-stepper*, which allow the performance of time-steps at the macroscopic level (defined by (2.9)), using only the simulation of M realizations of the microscopic model (2.5). To achieve this, the procedure relies on the definition of two operators (*lifting* and *restriction*)