Equation-free Analysis of Agent-Based Models

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Abstract

Agent based models (ABM)s are used across a large range of disciplines, including social science, economics, mathematics, biology and computer science. These models are particularly useful where a closed form description is difficult, and consist of a set of individual agents and a specification for how each agent moves and interacts with other agents. In spite of their increasing ubiquity, few tools are currently available for the systematic analysis of ABM behaviour.

Numerical continuation and bifurcation analysis is a well-established tool for the study of deterministic systems and has been widely used to investigate the behaviour of nonlinear dynamical systems. There are two barriers to the application of standard numerical continuation techniques to ABMs: firstly, ABMs are in general stochastic; secondly, standard continuation methods require an analytic description of the model, which is unknown for ABMs in the vast majority of cases.

Recent work has shown how to use equation-free (EF) methods to perform parameter continuation of a system where the dynamics are described at a microscopic scale and continuation of a macroscopic measure is considered. Essentially these work by replacing equations for macroscopic behaviour with an ensemble of microscopic simulations. This provides a platform for performing macroscopic analysis of a system without the need for an explicit equation describing macroscopic behaviour.

To date, this formulation has been applied to a few specific examples. For each case, choices have to be made for algorithmic parameters, such as: the number of micro-simulations carried out at each parameter point; the time window for each micro-simulation and the size of steps in parameter space. These parameters have been chosen largely by trial-and-error and large numbers of micro-simulations are often needed. As a result the use of EF analysis is currently limited to users with knowledge of EF continuation and there is a significant over-head in configuring the EF method to the specific problem.

Here we develop a generic framework for the EF continuation of ABMs which does not require any knowledge of the mathematics of programming implementation involved, enabling numerical continuation to be performed by any ABM user. Our implementation is coded in Java, compatible with the all major operating systems, as it can interface with the NetLogo programming language which is a popular tool for coding ABMs. Our generic framework includes a systematic method for automatic configuration of EF methods to a specific ABM, indicating algorithmic parameters essential for EF continuation. The object orientated design provides a flexible, 'modular' structure that additionally increases computational efficiently. An adaptive approximate convergence bound is used in the continuation corrector to enhance robustness and algorithmic speed.

We demonstrate our method with application to several ABM models revealing parameter dependence, bifurcation and stability analysis of these complex systems that are not otherwise easily obtainable.

Problem: analysis of ABMs

Agent-based models (ABMs) consist of individual entities (agents) which move and interact with each other according to a defined set of rules. They are increasingly used to model complex systems where it is difficult to write down an explicit system-level description of macroscopic behaviour and have been used in a wide range of disciplines such as social science and psychology, mathematics and network theory, policy making and economics, computer science and biology.

ABMs often contain non-linear interactions which can result in macroscopic (emergent) behaviour that is based on the initial conditions. Many ABMs have stochastic components arising from random initial configurations (agent location) and random interaction order between agents leading to multiple outcomes from the same initial (parameter) states (see Fig. 1 and 2).

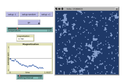

Figure 1: Ising ABM dynamics.

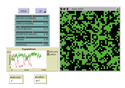

Figure 2: Altruism ABM dynamics.

Currently analysis tools for ABMs perform parameter sweeps and some statistical analysis, though most rely on external software for the latter. None of the available methods are able to perform analysis of the system-level (emergent) behaviour of models, which is an area of particular interest for social and complexity scientists.

Continuation

In a numerical continuation approach, the focus is on steady-state (or periodic) solutions. Rather than time evolving to find $x(t)$, in the case of steady-states, solutions for $\tilde{x}$ such that $f(\tilde{x}, \lambda) = 0$, or an extended version of f for periodic solutions.

Having found a solution for one particular set of parameter values, the solution is 'followed' by stepping in λ and solving f such that $x_{n+1} = f(x_n; \lambda)$. This is both faster than finding steady solutions using simulations and has the advantage that both stable and unstable solutions can be found. These techniques are well-established in their value in understanding deterministic systems have been applied to problems in a number of different areas [1].

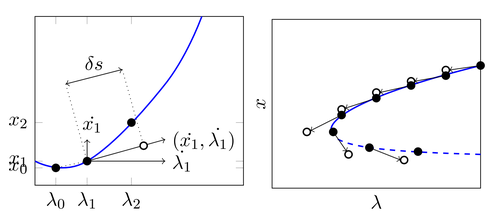

Figure 3: Pseudo-arclength predictor-corrector continuation method

Equation-free Methods

Equation-free methods [2] provide a enable numerical continuation by replacing the macroscopic behaviour with an ensemble simulations of the model at the microscopic level. N independent microscopic states (x) are initialised from the macroscopic state (X) at time $t = 0$. After a time evolution window of τ, the $X(t = \tau)$ is estimated from the distribution of microscopic final state, i.e. $x(t = \tau)$.

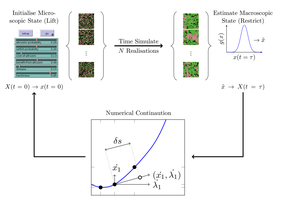

Figure 4: Equation-free method for numerical continuation of agent-based models.

Parameter Determination

For an ensemble of stochastic microscopic models, all initialised at $x = x_0$, the variance (σ^2) of the distribution of states after a simulation time τ will depend on the level of noise and dynamics in the model, and therefore on τ. In the general case the underlying distribution is unknown, and may vary for stochastic systems. However, according to the central limit therm (CLT), the distribution of means from independent ensembles is normal. This provides a platform to calculate properties of a generic system regardless of the underlying distribution or dynamics. Increasing τ from an arbitrarily small value will lead to an increase in (σ^2) as the dynamics and noise in the system evolve. Over sufficiently large time windows, (σ^2) will; 1) increase indefinitely, 2) increase to a constant, or 3) increase to a maximum then decrease (see Fig. 5).

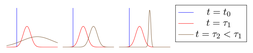

Figure 5: Dependence of σ^2 on τ. For increasing τ, σ^2 increases; indefinitely (left), to a constant (mid), or to a maximum before reducing (right).

Figure 6: Variance of the distribution of means, ω with γ limit. For small τ (left) $\omega < \gamma$, whereas for larger τ (right) $\omega \approx \gamma$.

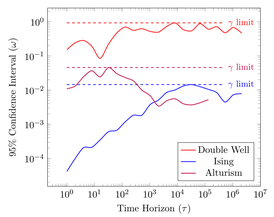

Figure 7: Confidence interval of the distribution of ensemble means for the 3 examples in this work showing the maximum value of γ.

Defining τ for equation-free methods is non-trivial as it must be sufficiently large to enable some evolution in the system but will depend on the specific problem. By using the CLT and monitoring the relationship between τ and the confidence interval (ω) on the mean of the means, we can monitor the variance of the underlying distribution of x. This enables us to determine a threshold (γ) of ω for a specific problem, providing a systematic method for determining τ directly from simulation of the problem (see Fig. 6). Note this process can be used with an arbitrarily large γ to observe the maximum ω (γ limit) and an indication of γ for the problem (see Fig. 7). The whole process can be repeated in order to systematically determine τ for an unknown problem. Requiring $\omega > \gamma$ ensures τ is large enough to observe some level of dynamics in the model.

Similarly, we can use ω to determine the minimum N such that $\omega < \gamma$, i.e. we are confident in the mean of the means. Using our obtained values of τ and N we determine the step size in parameter space (δs) in terms of the standard error (Δ) of the distribution of N realisations of the microscopic state time evolved to $t = \tau$ (see Fig. 8).

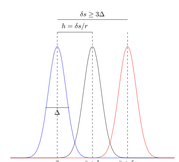

Figure 8: Illustration of the δs predictor.

This method enables the automatic configuration of τ, N and δs (all problem specific) to any arbitrary model. Previously the application of equation-free has been restricted to a small number of well studied or low noise systems, thus our contribution is a platform for a wide range of applications across many domains of science.

Examples: Stochastic Double Well Potential

The stochastic double well is modelled with a potential;

$$V_x = -x(x^2 - \mu) - \nu \ , \tag{1}$$

where V_x is the derivative of V wrt x, μ is the barrier hight and ν is the asymmetric term in the well. The state of a particle in the well is given by;

$$\dot{x} = V_x + \eta \dot{W} \ , \tag{2}$$

where η is the noise level and $\dot{W}$ is Gaussian noise between 0 and 1. Figure 9 details the deterministic case of Eq. (2), where $\eta = 0$, and Fig. 10 demonstrates the equation-free continuation of the stochastic model in Eq. (2).

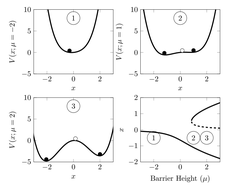

Figure 9: Deterministic double well potential for varying μ and bifurcation digram of the model. Number indicate the where the illustrates wells correspond to the bifurcation curve. Stable (unstable) points are illustrated with black (white) circles in the potential wells and solid (dashed) lines in the bifurcation diagram.

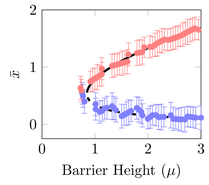

Figure 10: Equation-free continuation of a stochastic double well potential. Error bars are 2 s.e.m and red (blue) points are stable (unstable). Continuation here is performed at a very low number of realisations ($N = 100$) and for a large time horizon ($\tau = 1$).

Examples: Ising

This models [3] the spins states of atoms in a magnet, where the spin state is a function of the state of itself, the surrounding atoms ($\tilde{\mathbf{x}}_n$) and temperature (T);

$$x_{n+1} = f(x_n, \tilde{\mathbf{x}}_n, T) \ . \tag{3}$$

The effect of the surrounding atoms dominates in the low temperature range (region 1), but has a negotiable effect in the high temperature limit (region 2), see Fig 11. In low temperature limit (ferromagnetic phase) the magnet will undergo spontaneous magnetisation, where all the atoms will align. In the high temperature limit (paramagnetic phase) there is no spontaneous magnetisation as spins states for all atoms are random.

The total magnetisation of the magnet, M, is the average of magnetisations across all the atoms. In the ferromagnetic phase $|M| \to 1$ and in the paramagnetic phase $M \to 0$.

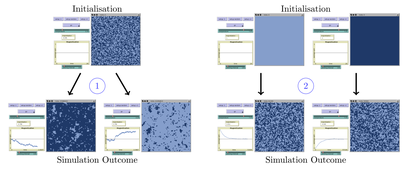

Figure 11: Time simulations in the Ising model showing transition in the ferromagnetic (1) and paramagnetic (2) regions.

Figure 12 shows how the fixed point of M varies with temperature, agreeing with observations from simulations (Fig 11). Using our method for parameter configuration to obtain τ=16000, N=10 and δs = 0.2 and perform equation-free continuation on the Ising model (Fig 12).

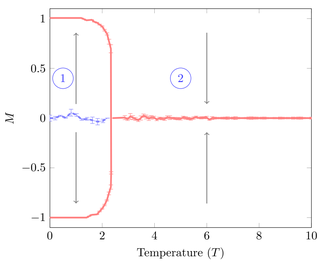

Figure 12: Equation-free continuation of the Ising ABM model [3] showing a pitchfork bifurcation. Stable (unstable) states are illustrate by red (blue) lines, arrows indicate transient behaviour of the model, and errorbars are 2 s.e.m. Numbered regions correspond to those in Fig. 11.

Examples: Altruism

The altruism model [3] simulates the competition between selfish and altruistic agents for survival in a 2D world. Interestingly, similar models are used in a number of studies regarding cooperation, competition, game theory and social dynamics [4, 5, 6]. All agents have a fitness value associated with the current state of the agent and its neighbours. At each time step an agent may change its type based on a weighted combination of fitness values of an agent and its neighbours.

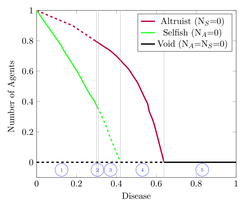

Figure 13: Bifurcation curves for the Altruism model

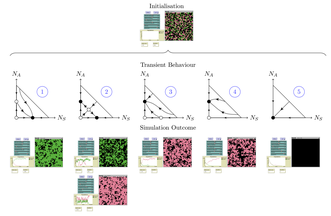

Figure 14: Behaviour of Altruism Model

A Future Direction

Future developments to enable the analysis of periodic orbits, the continuation in multiple parameters, and parallel execution are to be included in future version of this tool. Additional functionalities to analyses escape rates of basins of attractions and the computation of special points are also possible. A user friendly interface is also planned to enable use of this tool without knowledge of the implementation or mathematics involved.

Beyond algorithmic developments, we can apply this tool to other ABMs and models to examine the macroscopic behaviour of systems for a wide range of disciplines. In particular we will apply our algorithm to a number of ABMs used in social science and policy making in order to understand the underlying behaviour of the model.

References

[1] E. L. Allgower and K. Georg Numerical Continuation Methods, An Introduction, Springer-Verlag Berlin Heidelberg 1990

[2] I. G. Kevrekidis and G. Samaey, Equation-Free Multiscale Computation: Algorithms and Applications, Annu. Rev. Phys. Chem., 2009, 60(1), pp. 321–344

[3] U. Wilensky, NetLogo, http://ccl.northwestern.edu/netlogo/ 1999

[4] R. Boyd *et al.*, The evolution of altruistic punishment, PNAS, 2003, 100(6):3531-3535.

[5] C. F. Camerer, Behavioral game theory: Experiments in strategic interaction, Russel Sage Fondation, New York, NY, 2003

[6] M. A. Nowak and K. Sigmund, Evolution of indirect reciprocity by image scoring, Nature, 1998, 393:573-577.

Acknowledgements

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