Predicting a sine wave

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Scripot to predict a sine wave and get practice using LSTM layers in a recurrent neural network. This notebook shows how to construct a three layer recurrent neural network using PyTorch in which the first two layers use LSTM units and the last layer uses a nn.linear output with a single node which applies a linear transformation to the output of the previous layer to produce a single prediction at some time (t). The results of training and testing can be seen towards the bottom of the notebook. Using 21 LSTM units in the hidden layer, it takes about six epochs for the network to learn the sine function well enough to achieve an extremely low error or loss. (The loss function in this case is simply mean squared error)

Sources:

The main source for this notebook was [Soumith's Githib - Time Sequence Prediction] in which he goes through the problem of using a recurrent neural network to predict a sine wave.

Other sources include:

PyTorch Containers

Python Tips

Understanding Class Inheritance in Python

PyTorch AutoGrad

Source Code for TORCH.NN.MODELS.LINEAR

Generating Sine Wave Data

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[85]

```
import numpy as np
import torch
import matplotlib.pyplot as plt
import cycler
import time
```

Our goal here will be to generate 100 sine waves with equal wavelength, but each with a different phase shift. In order to achieve this goal, we will generate a vector of 100 random numbers using a random uniform distrobution, then create a 100x1000 matrix of 100 sine waves each starting from a different place between our upper and lower bounds of -4T and 4T.

Below we define parameters for our 100 sine waves:

```
[86] np.random.seed(2)

T = 20
L = 1000
N = 100

#Create an empty np matrix (of zeros) with shape 100x1000
x = np.zeros((N, L), 'int64')
print("Shape of matrix x:", x.shape)
print(" ")
print("Top left 5x5 matrix of matrix (x):")
print(x[:5, :5])
```

```
Shape of matrix x: (100, 1000)

Top left 5x5 matrix of matrix (x):
[[0 0 0 0 0]
 [0 0 0 0 0]
 [0 0 0 0 0]
 [0 0 0 0 0]
 [0 0 0 0 0]]
```

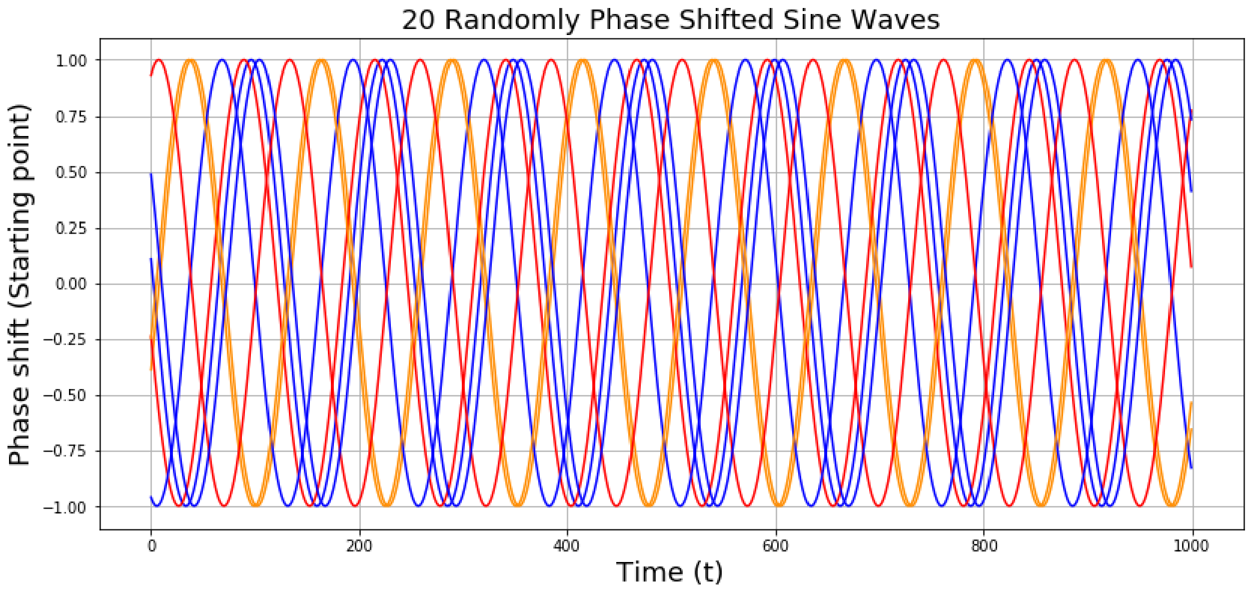
```
[87] linear_vector = np.array(range(L))
x[:] = linear_vector + np.random.randint(-4 * T, 4 * T, N).reshape(N, 1)
```

We first create a numpy vector with length L, then we can populate it with random numbers generated from a unifrom distrobution between bounds of -4T and 4T.

These will be the starting points of our 100 phase shifted sine waves.

```
[88] data = np.sin(x / 1.0 / T).astype('float64')
```

```
[89] fig = plt.figure(figsize=(30, 6))
ax1 = fig.add_subplot(121)
plt.gca().set_prop_cycle('color', ['blue', 'darkorange', 'red'])

plt.plot(data[0, :])
plt.plot(data[1, :])
plt.plot(data[2, :])
plt.plot(data[3, :])
plt.plot(data[4, :])
plt.plot(data[5, :])
plt.plot(data[6, :])
plt.xlabel('Time (t)', fontsize = 18)
plt.ylabel('Phase shift (Starting point)', fontsize = 18)
plt.title('20 Randomly Phase Shifted Sine Waves', fontsize = 18)
plt.grid()
```



Define Network Parameters

Now that we have several randomly phase shifted sine waves stored in 'data' we can begin to build out the network we will use for prediction

[98]
```
from __future__ import print_function
import torch
import torch.nn as nn
import torch.optim as optim
import numpy as np
import matplotlib
#matplotlib.use('Agg')
import matplotlib.pyplot as plt
```

Define the Sequence Class

I'm still learning all of the benifits that are derrived from using the super() function here. It seems like it's a very common practice when creating a sequence class in pytorch. According to Digital Ocean, "With the super() function, you can gain access to inherited methods that have been overwritten in a class object. When we use the super() function, we are calling a parent method into a child method to make use of it. For example, we may want to override one aspect of the parent method with certain functionality, but then call the rest of the original parent method to finish the method."

[91]
```
"""
We can now create a Sequence class which we will later use to instantiate
This will be the network which we use to train on our time series data an
trained to approximate our function, we will use the 'future' method to p
into the future.
```

```
    This is all made possible by the use of the LSTM nodes in the network
    """
    units = 21 #<-- Define the number of hidden LSTM units in the middle laye

    class Sequence(nn.Module):
        def __init__(self):

            """
            ---------- SELF ----------
            The 'self' function will construct a three layer neural network u
            This output will represent the resulting position of the next poi
            """

            super(Sequence, self).__init__()
            self.lstm1 = nn.LSTMCell(1, units)#<-- LSTM Cell 1 to 51 units
            self.lstm2 = nn.LSTMCell(units, units)#<-- LSTM Cell 51 units
            self.linear = nn.Linear(units, 1)#<-- Linear cell for final predi

            """
            ---------- FORWARD ----------
            The forward function takes three inputs including 'input' and 'fu

            'input' represents the training data: a 100x1000 matrix of sine w
            each new row of inputs represents a different phase shifted sine 
            network will subsequently learn. If future=0, then no predictions
            and the network will simply generate outputs

            'output' represents the network's approximation of the data, or t
            output of the function after training. If future=/0 then the Sequ
            will simply run the below "Future" loop and continue to append ea
            vector which resides in the input tensor with the next time serie
            according to the function which the network has approximated. In 
            the true function f*(x)=sin(x)+b, where b is some random number b

            """

        def forward(self, input, future = 0):
            outputs = [] #<-- initialize an empty array which will be populat

            h_t = torch.zeros(input.size(0), units, dtype=torch.double)#<-- d
            c_t = torch.zeros(input.size(0), units, dtype=torch.double)#<-- d
            h_t2 = torch.zeros(input.size(0), units, dtype=torch.double)#<-- 
            c_t2 = torch.zeros(input.size(0), units, dtype=torch.double)#<-- 

            """
            ---------- FOR TRAINING ----------
            *Note that this loop takes in 'input_t' and not simply 'input' be
            to grab one row at a time from our input vector and use that one0
            for training.
            """
```

```
        #remember that 'enumerate' allows us to loop over a certain value
        for i, input_t in enumerate(input.chunk(input.size(1), dim=1)):

            # We now use the input layer and the hidden layer values from
            # previous time step to predict the new h_t2, c_t2 values
            h_t, c_t = self.lstm1(input_t, (h_t, c_t))

            # We now use the result from the previous time step to predic
            h_t2, c_t2 = self.lstm2(h_t, (h_t2, c_t2))

            output = self.linear(h_t2)#<-- use the final, linear layer to

            outputs += [output]#<-- append the resulting output values on

        """
        ---------- FOR TESTING ----------
        This loop iterates through a 'future' vector to use a trained net
        'future' examples forward in time.

        We will later create a Sequence object and initialize future to z
        it will only run the above loop and populate the output array wit
        training.

        """

        for i in range(future):# if we should predict the future
            # We now use the out layer and the hidden layer values from t
            # previous time step to predict the new h_t2, c_t2 values

            """
            And now we can use the output array which we have predicted a
            will be used to continue the time series prediction.

            This is the loop which allows the sequence object to continue
            the future by looking back and using the output, h_t, and c_t
            next value which will be appended onto the 'output' array.
            """

            h_t, c_t = self.lstm1(output, (h_t, c_t))#<-- Calculate h_t a

            h_t2, c_t2 = self.lstm2(h_t, (h_t2, c_t2))#<-- Calculate h_t2

            output = self.linear(h_t2)#<-- Use the final linear layer to

            outputs += [output]#<-- ONLY when future/=0 we append our out

            """
            We always generate 'outputs' here, but only when we want to p
            do we use the lstm1 and lstm2 layers to predict into the futu
            """

        outputs = torch.stack(outputs, 1).squeeze(2)
```

```
        #print("Inputs:", input) #<-- UNCOMMENT to see an overview of the
        #print("Outputs:", outputs) #<-- UNCOMMENT to see the network's a
        return outputs #<-- Finally we return our outputs
```

We set np random seed to zero so that the random numbers generated will be the same on each run. We do the same with the random number generator in torch

```
[92]  # set random seed to 0
      np.random.seed(0)
      torch.manual_seed(0)
```

```
<torch._C.Generator at 0x113bf6230>
```

Define X and y data

We can now use the 'data' dataframe defined above to create our training and testing dataset

We define our training dataset as 97% of the data, we can simply take our [97, 999] matrix of data and use each of the 97 phase shifted sine waves as teraining data

```
[93]  input = torch.from_numpy(data[3:, :-1])
      target = torch.from_numpy(data[3:, 1:])
```

We can now take the remining three sine waves and use them as the testing data. This will result in three sine waves with different phase shifts as our target functions that the network will need to learn

```
[94]  test_input = torch.from_numpy(data[:3, :-1])
      test_target = torch.from_numpy(data[:3, 1:])
```

Build the Model

Here we use the Sequence model in PyTorch and apply sequnce.double() which casts all floating point parameters and buffers to double datatype source.

Becasue we're measuring the linear deviation in the y-direction between our prediction and the true sine wave, we can simply use MSELoss as the loss function. We define this function as *Criterion* meaning that this is the criterion we will use to measure the relative success, or error, of the model.

```
[95]  # We can use the above defined Sequence class to instantiate a new sequen
      seq = Sequence()

      #Set all floating point parameters to doubles (change dataypes to double)
      seq.double()

      # Use mean squared error loss as the loss function
      criterion = nn.MSELoss()

      # use LBFGS as optimizer since we can load the whole data to train
      optimizer = optim.LBFGS(seq.parameters(), lr=0.8)

      #define number of epochs for training
      epochs = 15
```

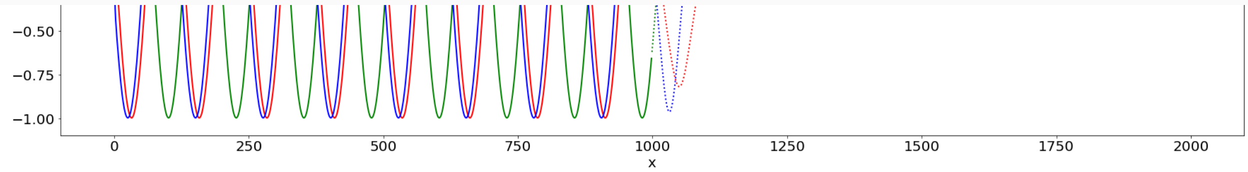
Why zero_grad()?

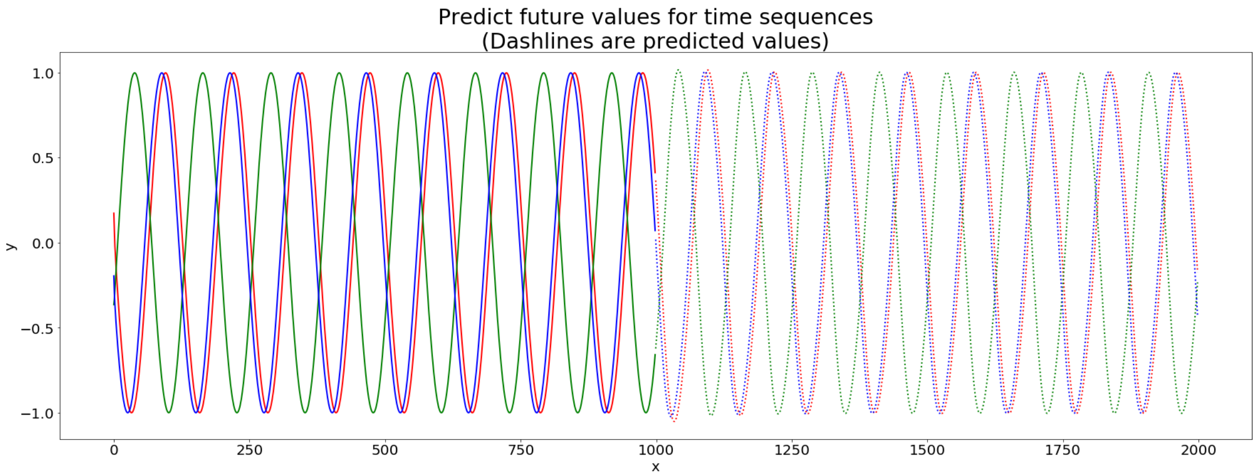
A fundamental difference here between recurrent neural networks and fully connected or convolutional networks is the need for gradient accumulation instead of replacement during backpropogation, hence the *optimizer.zero_grad()*. The *zero_grad()* function simply clears the gradients of all optimized torch tensors (source.) Because we're using an RNN, we want to accumulate the gradients during backpropogation instead of simply replacing the gradient each time a variable is backpropogated. Unlike other neural networks, this is important for RNNs so that we can accumulate the gradient through several time steps. It's important to remember, however, that at the start of each minibatch we need to zero out the gradient so that we can start again from scratch, which is why we use the *zero.grad()* function to zero the gradients at the start of each new minibatch. There is a good discussion of why we need accumulate and zero gradients in RNNs here.

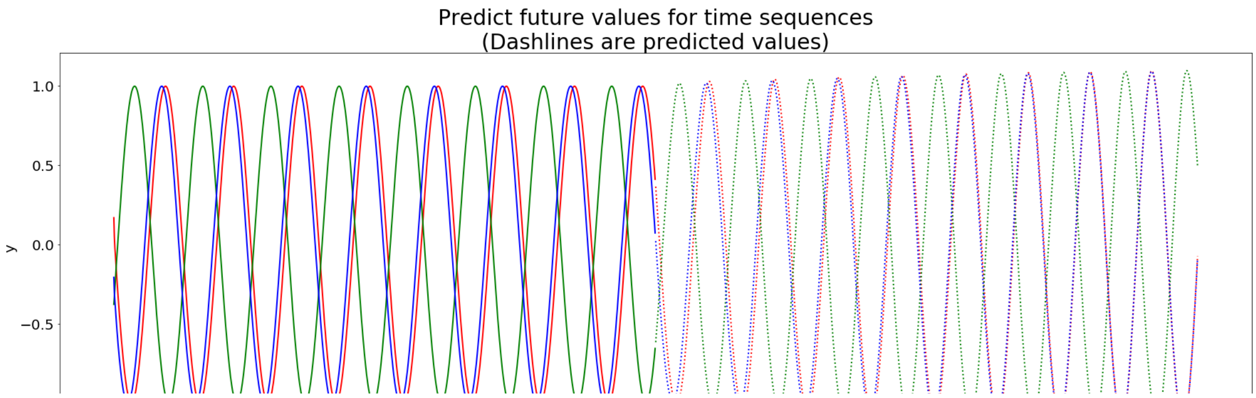
```
[96]  #begin to train
      tic = time.time()
      history = []#<-- We could store loss values here for later

      for i in range(epochs):
          print('STEP: ', i)


          """
          *** THIS CODE RUNS TWENTY TIMES PER EPOCH ***

          This section runs 20 minibatches over the data, allowing for the accu
          backpropogated gradients through each minibatch. At the start of the 
          the gradients are reset to zero and the training process continues.
          """

          def closure():
              #print("Closure")
              # Use zero_grad() with the LBFGS Optimizer so we accumulate gradi
              optimizer.zero_grad()

              # use the 'seq' object and provide it with the training set 'inpu
```

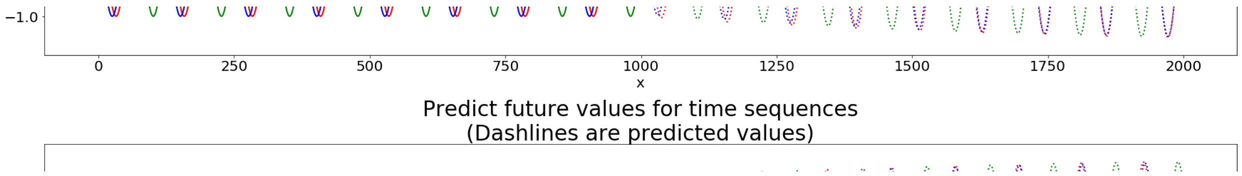
```
        # the sine function through time from the data in 'input'
        out = seq(input)

        # remember our loss function is simply MSELoss, so our loss funct
        # in the previous line and finding the sum of the squared error i
        # actual sine waves and the predicted next step in the wave
        loss = criterion(out, target)

        # we can now print the loss, or the mean squared error between th
        print('loss:', loss.item())


        loss.backward()

        return loss

    optimizer.step(closure)

    # begin to predict, no need to track gradient here

    """
    *** THIS CODE RUNS ONLY ONCE PER EPOCH ***

    This section sets future to 1000, changing the role of the seq object
    to predicting the next 1000 elemnts of the approximated function f*(x
    """
    with torch.no_grad():
        print("TEST")
        future = 1000#<-- Set future=1000 which will allow the sequence o
        """
        Remember that when we created our Sequence class we built a funct
        we initialized 'future' to zero. Here, however, we set future=100
        know that we want to predict values into the future! (1000 values

        And now future=1000 now gets passed into the sequence object alon
        which is simply the first three of our randomly phase shifted sin

        The design of the Sequence class is very clean so subsequently ou
        is easy to use, we simply train it above and use it to predict ne
        """

        # The seq object has now been trained for i epochs and we are rea
        # of the network by passing a future value of 1000 to the object
        # We give it 'test_input' which is simply three of our sine waves
        # our Sequence object to generate 1000 predictions into the futur
        pred = seq(test_input, future=future)#<-- future = 1000

        # We can now find the MSELoss between our N=3 sine waves and the
        loss = criterion(pred[:, :-future], test_target)

        # Using loss.item() simply returns the scalar value of the item s
        print('test loss:', loss.item())
```

```
        # Lastly, we want to plot our predictions so we use the .detatch(
        # torch dataframe and convert it into numpy vectors which can be
        y = pred.detach().numpy()

        """
        Finally we can draw the result to show that our network is either
        or show that we still have error in our test predictions
        """

    # draw the result
    plt.figure(figsize=(30,10))
    plt.title('Predict future values for time sequences\n(Dashlines are p
    plt.xlabel('x', fontsize=20)
    plt.ylabel('y', fontsize=20)
    plt.xticks(fontsize=20)
    plt.yticks(fontsize=20)
    def draw(yi, color):
        plt.plot(np.arange(input.size(1)), yi[:input.size(1)], color, lin
        plt.plot(np.arange(input.size(1), input.size(1) + future), yi[inp
    draw(y[0], 'r')
    draw(y[1], 'g')
    draw(y[2], 'b')


toc = time.time()
```










```
[97] #We can use the 'time' package to measure how long it takes to train the
train_time = toc - tic
secs = train_time%60
mins = (train_time - secs)/60

print('Time taken to train on', epochs, 'epochs was:', mins, 'minutes and
```

```
Time taken to train on 15 epochs was: 5.0 minutes and 39.49153470993042
seconds.
```

We can see that the final iteration produced a result with a test loss of 6*10^-6 and a qualitatively accurate visual continuation of our testing sine wave.

Final Remarks

This project was a good example of why LSTM units are needed and the advantages that RNNs have over traditional dense, fully connected networks. It also shows how to create a sequence class using LSTM units in pytorch which allows us to instantiate a sequence object which is able to learn from time series data and make accurate predictions into the future. One key takeaway from this project was connecting the conceptual idea that RNNs work by learning from both the input features and the previous state of the network, and we were able to code that directly using h_t and h_t2 in the Sequence class. It was also interesting to discover the correct way to generate output predictions for test (future) values is simply by feeding the previously predicted value back into the function to subsequently get propagated through the network. This way the network is either training and generating output in an attempt to approximate the training data, or testing and attempting to predict some f(t) for some t value in the future.