In Depth Analysis of the 2008 Financial Crisis Using Nonlinear Optimization, Simulation Theory and Game Theory

Spencer Bertsch, Amritansh Varshney, Ross Warner

Abstract – **The global financial crisis of 2008 caused by the collapse of the American housing market is often considered to be the worst economic downturn since the great depression in 1929. There were several players that had roles in this crisis including investment banks, commercial banks, the securities exchange commission, the public, investors such as hedge funds, and rating agencies. In this report, we consider the interaction of several of these players and, through use of nonlinear optimization, simulation theory, and game theory, study their interaction before and during the 2008 financial crisis.**

I. INTRODUCTION

This report outlines the three modules used to analyze the 2008 financial crisis from the perspective of investors, commercial banks, investment banks, rating agencies, the public, and the SEC. Each of the three modules is aimed at answering specific questions regarding the reasons behind the crisis in 2008 and the factors that contributed to the collapse.

Firstly, the optimization section uses historical data and a nonlinear solver in Matlab to maximize the annual return of an initial investment of 1 Billion USD. We collected monthly data for 20 top stocks in the mid 2000s, four popular mutual funds, one T-Bill, and one Goldman Sachs mortgage backed security (MBS) for a total of 26 investment options. The optimization was subject to several constraints including keeping the portfolio below a risk ceiling, and always keeping 100 percent of the portfolio's value invested.

The data used for the optimization was acquired from Yahoo Finance. The solver examined both the monthly returns for each investment option as well as the risk associated with those investments. In order to calculate risk, we created a matrix of covariance values between each investment for each month between Jan. 1, 2003 and Dec. 31, 2008. We calculated risk by multiplying the proportion of amount invested in option A, times the amount invested in option B, times the covariance of option A and B. We did this for all the investment combinations and then took the square root of the summation of these values to get the final risk value.

In the simulation module, mortgage default rates were randomized based on a normal distribution fitted to historical data. The objective was to simulate the morally hazardous behavior leading up to the crash of the credit rating agencies who were financially incentivized to rate these mortgage backed securities as low risk despite rising default rates and delinquencies. Credit Ratings of low risk were assigned with higher probabilities based on these randomized default rates. This credit rating would inform the optimization model by determining what percentage of the portfolio is allowed to be invested in the Goldman Sachs' MBS. The results showed that this MBS was not invested in at all in 2003 while being highly invested in during years 2007 and 2008.

Game Theory was used to model the behavior of the major market players in the years leading up to 2008. The purpose of this was to pit a player indulging in moral hazard (i.e. doing ethically wrong things because one does not have to pay for its consequences) against a seemingly pro-public player, to understand the impact on the larger society and the incentives of both the players.

Data used in this project was sourced from Statista and Bloomberg, with other data coming from Yahoo Finance and Google Finance.

II. OPTIMIZATION MODULE

A. Objective Function, Constraints, and Decision Variables

The first of the three modules in this project studies the role of an investor between the years of 2003 and 2008. We categorize investor as a party that owns lots of capital enough to invest 1 Billion openly into the market across 26 different investment options. Starting in 2003, our optimization systems goal was to maximize the annual return with the given initial investment. The decision variables in the optimization function were simply the amount of the portfolio (0 to 100%) invested in each of the 26 options. If the solver found that the return was relatively high and the risk for that year was relatively low, it would invest accordingly into that investment option. If the solver found that the risk to return ratio was too high, however, it would simply invest 0 percent of the portfolio in that option and move on. The only constraints in this system were that the total risk must be kept below a risk ceiling of 0, and the total value of the portfolio must be invested in each subsequent year.

B. Risk Calculations

The data used for the optimization was acquired from Yahoo Finance. The solver examined both the monthly returns for each investment option as well as the risk associated with those investments. In order to calculate risk, we created a matrix of covariance values between each investment for each month between Jan. 1, 2003 and Dec. 31, 2008. We calculated risk by multiplying the proportion of amount invested in option A, times the amount invested in option B, times the covariance of option A and B. We did this for all the investment combinations and then took the square root of the summation of these values to get the final risk value.

$$\sqrt{\sum_i \sum_j (\sigma_i * \sigma_j * COV_{i,j})} = RISK$$

The covariance matrix used was calculated using Excel, and the resulting tables were then exported to CSVs and imported into Matlab for use in the optimization. Resulting values of risk for different simulations were then found and stored for use in the constraints of the Matlab solver.

III. OPTIMIZATION RESULTS AND CONCLUSIONS

The results of the optimization show that the solver initially diversified investments across most of the 26 investment options, but as time grew closer to the 2008 crisis, the solver invested heavily in the Goldman Sachs mortgage backed security. The reasons for this result are that the perceived risk (calculate using the method discussed above) was small compared to the payout of the MBS. Similar investments such as stocks may have had larger returns, but the risk associated with these investments was too high, so the solver had to invest in very risk averse options such as the MBS.

Once the optimization was working smoothly, we began varying the risk and investment constraint to observe the changes in the ideal investment strategy. As expected, we can see in Figure 4 that, as the risk ceiling is shifted up by 0.05, the solver chooses to invest more heavily in stocks which yield greater returns, but carry more risk. We can also see in Figure 3 that when the overall portfolio cap is shifted down from 100%, the overall portfolio becomes much more diversified as we would expect. The "portfolio cap" or "high cap" is simply the maximum percentage of the overall portfolio that can be invested in a single investment option. Lastly, we can see from Figure 5 that when the investment limit is not constrained and the risk is capped at 0, the solver chooses to invest heavily in the Goldman Sachs MBS leading into the 2008 crisis.

A main takeaway from this module is that the way that portfolio managers or investors of any type are calculating risk cannot simply be naïve. Our crude calculation of risk caused the solver to suggest remarkably heavy investments into a mortgage backed security in 2007 and 2008; if this action had been carried out by a real portfolio manager, that investment would have likely been lost during the economic crisis. It is important to understand how investment are related, and that investment options can be strongly correlated in times of crisis, even if the covariance is relatively small. Developing a sophisticated approach for calculating risk is an essential part of keeping an investment safe.

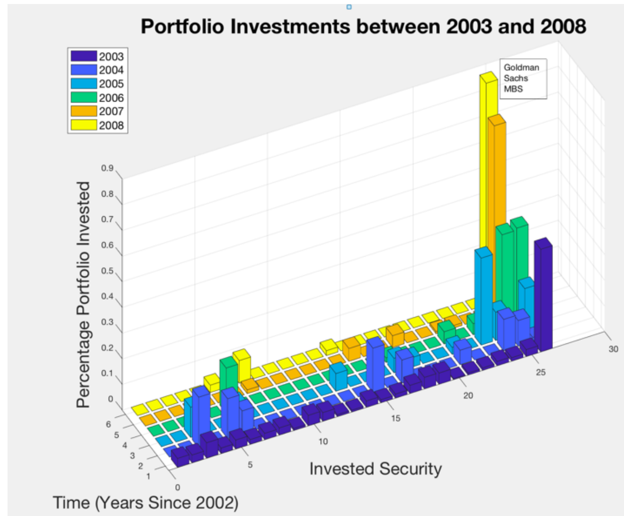

Fig. 1 Solver Results: Optimized investment strategy from 2003 to 2008 over 26 investment options. Option 25 is the Goldman Sachs MBS

IV. SIMULATION MODULE

A. Motivation for Simulation

The simulation portion of this project was approached from the perspective of a Credit Rating Agency such as Moody's. At the time leading up to the 2008 crisis, these agencies would perform analysis of an MBS by looking at Mortgage Default Rates and then provide credit rating to this type of security. These ratings ranged from AAA (least risky) to B (most risky). Unfortunately, these credit agencies were financially incentivized to rate these MBSs highly irrespective of the default rates and even as these mortgage default rates increased across the country in the years leading up to the crisis, as seen in Table I. Thus, subprime mortgages, those with delinquencies and much risk that should have been rated BB or B, were actually securitized and rated much higher such as AA or AAA in CDOs. The increased frequency of these subprime MBS can be seen from Figure 6 and increase in delinquencies associated with these mortgages close to the time of the crisis can be seen in Figure 7 (Ref. 1 & 2).

B. The Model

Our objective was to replicate this morally hazardous behavior in credit rating and to see how our optimization model might change its investing behavior towards MBSs in the time leading up to the crisis. To replicate this behavior, we aimed to assign lower risk credit values with a higher probability while assigning higher risk values with a lower probability. To do so the historical data (Ref. 3) provided in Table I was utilized, and Mortgage Default Rates were assumed to lie on a normal distribution given by the means and standard deviations of 2003 to 2008. A random variable was generated from this distribution for each round of the simulation, and, depending on what range it was found within the distribution, the MBS of our potential portfolio was given a particular credit rating. Therefore, the credit rating assigned to the Goldman Sachs MBS was randomized each simulation and each year. Figure 8 shows the range on the normal distribution for each credit rating assignment and Table II shows the likelihood of a particular credit rating being assigned. It was ignored that there was a probability that these default rates were negative since the objective was to assign a likelihood to each credit rating.

TABLE I
HISTORICAL MORTGAGE DEFAULT RATES (Ref. 3)

Year	Mean	Standard Deviation
2003	1.83	0.0901
2004	1.55	0.0897
2005	1.55	0.0788
2006	1.72	0.1249
2007	2.55	0.3858
2008	4.98	1.095

An optimization was performed for each iteration of the simulation and for each year. The flow is as follows: the randomized mortgage default rates of each year would inform a credit rating for our portfolios GS MBS and this credit rating would determine an optimization constraint. It was this constraint that tied together the simulation and optimization portions of this project. The optimization constraint determined what percentage of the portfolio could be placed into the GS MBS based on how risky this MBS seemed. Credit Ratings and the corresponding allowable percentages or risk ceilings are provided in Table III.

TABLE II
LIKELIHOOD OF CREDIT RATING ASSIGNMENT

Credit Rating	Probability of Assignment
AAA	50%
AA	19.1%
A	15%
BBB	9.2%
BB	4.4%
B	1.7%

C. Simulation Results

For 1000 simulations, a new investment portfolio was made for each year. Looking at Figure 2 the behavior of investing in the MBS is revealed. It can be seen that for year 2003 close to none of the MBS was invested in for all 1000 simulations. This value drastically increased for years 2007 and 2008 across simulations.

TABLE III
CREDIT RATING AND MBS RISK CAPACITY

Credit Rating	Portfolio Capacity in MBS
AAA	100%
AA	90%
A	70%
BBB	50%
BB	30%
B	10%

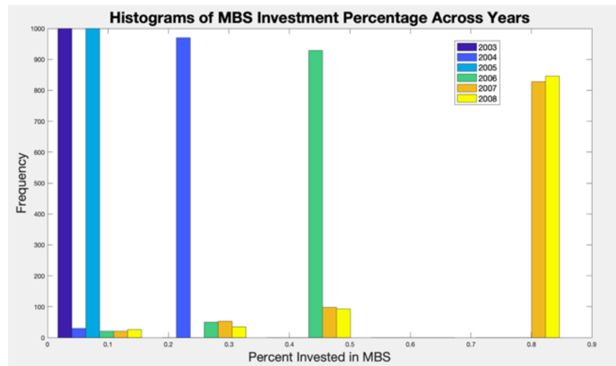

Fig. 2 The Percentage of the Portfolio Invested in Goldman Sachs MBS is shown across each of the years leading up to the crisis along the x axis. The y axis shows the frequency of this percentage across the 1000 simulations run. It can be seen that nearly no investments were made in the MBS in 2003 for all 1000 simulations while a high percentage was invested for 20007-2008 for 80-85% of the simulations.

V. GAME THEORY MODULE

A. Introduction to Game Theory

To understand the 2008 financial crisis better, Game Theory was used to mimic the potential behavior of 2 players. These are Securities and Exchange Commission (SEC) and Investment Bankers (IB). Over here, clear distinction needs to be drawn between Investment Bankers and the Investment Banks because both of these do not necessarily have the same intentions and payoffs. Lastly, to make the problem more relatable for all the readers, the impact on the larger society was also studied through the decisions made by the two players. The table below gives the different actions of each of the player in decreasing order of preference.

TABLE IV
ACTION LIST IN DECREASING ORDER OF PREFERENCE

Securities & Exchange Commission	Investment Bankers
Organically reduce the economic bubble	Indulge in Moral Hazard
Negotiate with Investment Bankers	Self-Regulate
Impose Fines	
Do Nothing	
Halt Trading	

B. Game Setup

This is a Multi-Stage game and IB get to decide first as to what they want to do. If they Self-Regulate, then their payoffs are minimal and the SEC and the public benefit the most. Please see Figure 9 in the Appendix for a graphical depiction of the game. If the IB indulge in Moral Hazard, then the SEC has 5 options available; (1) Negotiate (2) Impose Fines (3) Threaten to halt trading (4) Halt Trading (5) Do Nothing. If the SEC chooses to negotiate, IB can either comply or not. For both of these decisions, the SEC can choose to comply or not too. In both the situations, SECs payoffs are maximized when it chooses not to comply. Given that information, the IBs payoffs are maximized when it too chooses not to comply. This already tells us that negotiations would have been tricky to navigate through in those times. If the SEC imposes fines, IB can either file a lawsuit, pay the fine and regulate themselves or pay the fines and continue indulging in moral hazard. The payoffs for the IB are maximized when they decide to file a lawsuit because proving such a huge allegation in a court of law will take a few years of time, until which the moral hazard will not stop and the IB will make huge gains. Over here the distinction between the Bankers and the Banks becomes clearer because the Banks will eventually be proven guilty and will have to pay off large amounts of fines, but most Bankers would walk away with huge pay checks. If the SEC tries to threaten, the IB can either succumb to the threat and self-regulate or they can call SECs bluff. If this happens, then SEC can make good on its threat or do nothing. In both the cases, the payoffs are the same for the SEC, but those of the IB are different and depend on what the SEC does. This subgame is also critical because it significantly changes the Nash Equilibria. If the SEC decides to halt trading, both the players are equally worse off, and if it decides to do nothing then the IB receive large payoffs. Do nothing is an important decision because a lot of the employees in the SEC come from similar Investment Banks and might have an ulterior motive(s) for doing nothing. Solving the above game, we arrive at the following Nash Equilibria:

TABLE V
Potential Nash Equilibria

SEC's decision when IB calls their bluff	Investment Bankers	SEC	Maximum payoff for the Public
Do Nothing	Indulge in Moral Hazard	Do nothing, impose fines or negotiate	If SEC negotiates or imposes fines
Halt Trading	- Self-Regulate - Moral Hazard	- Work peacefully - Threaten	If SEC and IB work peacefully

VI. CONCLUSIONS & RECOMMENDATIONS

A. Optimization Module Conclusion

We saw from the results of the optimization that the solver initially chose to diversify investments across the 26 investment options, but in 2006, 2007, and 2008 it chose to invest heavily in the Goldman Sachs mortgage backed security. This result shows that naïve risk calculations and limited investment constraints can lead to heavy investments in single sources. If it turns out that the perceived risk is lower than the actual risk especially in times of economic crisis investments could be put in jeopardy. Moving forward, investors and portfolio managers need to be cognizant of the fact that risk can change drastically in times of crisis because of the correlation between investments such as mortgage backed securities and other stocks, or any two investment options. Sophisticated risk calculations are already a very integral part of any investment firms' operations, and maintaining a high standard of risk mitigation will continue to be crucial for large scale investors in the future.

A more in-depth analysis could be conducted by providing the solver with additional investment options, perhaps an MBS from other investment banks such as Morgan Stanley, Lehman Brothers, UBS, J.P. Morgan, etc. More investment options would allow the solver to choose better investment options and would provide a more realistic scenario in terms of the investment options that were available during the mid 2000s.

B. Simulation Module Conclusion

The intention of the simulation was to mimic the lowering of standards that the credit agencies were indulging in during the time leading up to the crisis. The results of our simulation and optimization show that this behavior would lead investors at the time to invest heavily in mortgage backed securities through the propagation of misinformation (faulty ratings) in the market. The reason the agencies participated in this morally hazardous behavior was due to the fact that they were financially incentivized to do so. Thus, based on our results, our recommendation is that the structure of payment and incentivization of the rating system be realigned to reflect accuracy rather than quantity of highly rated mortgages.

C. Game Theory Module Conclusion

Game Theory results suggest that because of the intricate relationships amongst the multiple players in the market, including government regulators like the Securities and Exchange Commission, public intentions were not kept in the best of interests. This could be a reason as to why the bubble grew so large when it did and resulted in the worst US economic crisis after the Great Depression. Some possible steps which the US government can take to mitigate these issues are: 1) Have a very strict vetting process for the SEC Chairman to ensure that he/she doesn't have any personal interests in the market 2) Ensure that insurance companies cannot insure a security if they don't have a minimum amount of assets available 3) Put regulations which make all parties liable for indulging in Moral Hazard.

VII. APPENDIX

The stocks, securities, government bills, and mutual funds that were part of the portfolio and from which historical data was found on Yahoo Finance are provided below.

• JAGLX	• EQT
• FBIOX	• MKC
• PHSZX	• MCO
• FBSOX	• PPL
• MSFT	• RTN
• GE	• REGN
• ABMD	• SBAC
• LNT	• SHW
• CCI	• TXN
• CVS	• TMK
• DHI	• YUM
• DWDP	• GS – MBS
• EA	• T-Bills

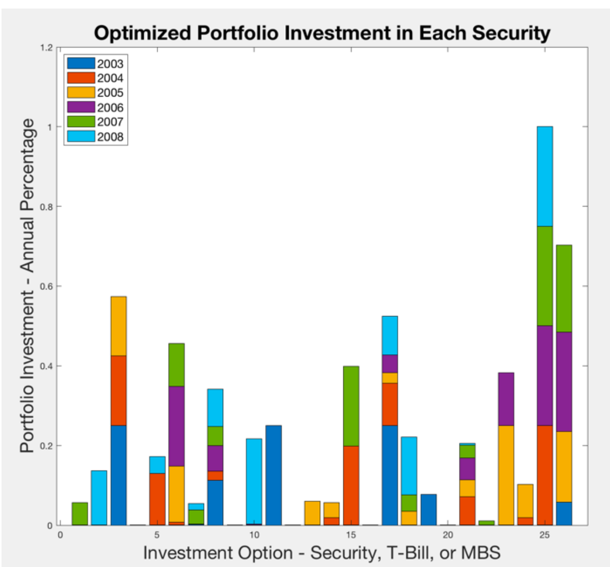

Fig. 3 Solver Results: Optimized investment strategy when a constraint is applied which limits the maximum amount of the overall portfolio which can be invested in a single investment option. Constraint – Max Investment = 20% portfolio value.

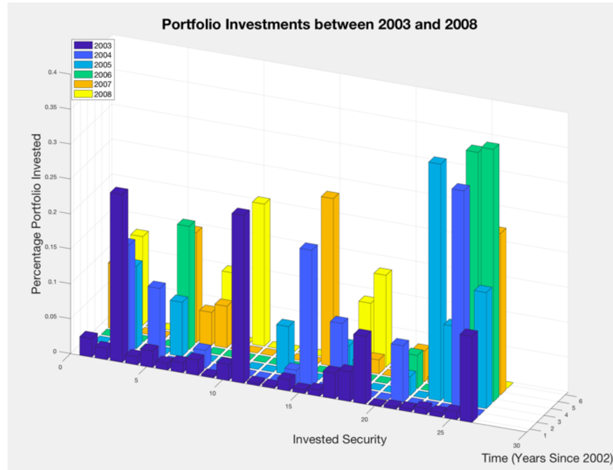

Fig. 4 Solver Results: Optimized investment strategy when a constraint is applied which increases the maximum amount of allowable risk in the system by 0.05.

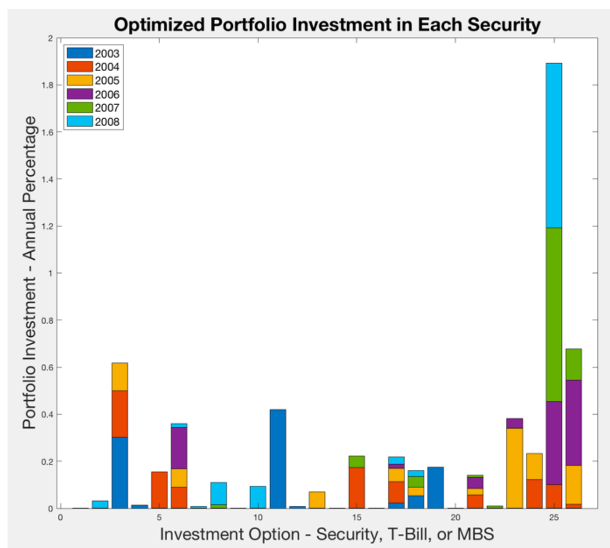

Fig. 5 Solver Results: Optimized investment strategy when minimum constraints are applied. No maximum investment constraints were applied and a maximum allowable risk of 0.

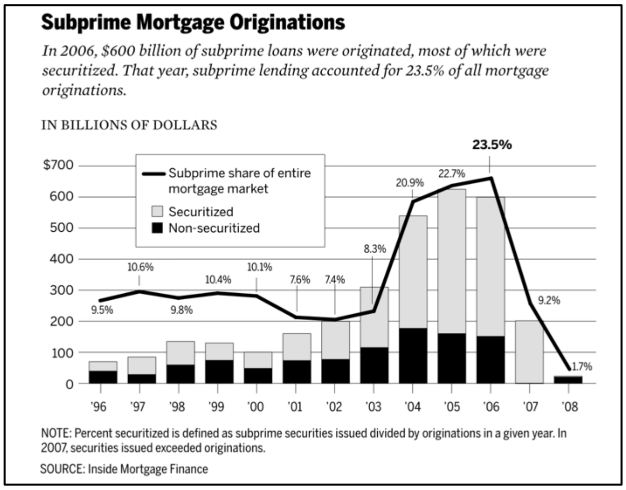

Fig. 6 This Figure is from the Federal Investigation into the Financial Crisis and shows the growth in subprime mortgages that were securitized leading up to the 2008 crash (Ref. 1).

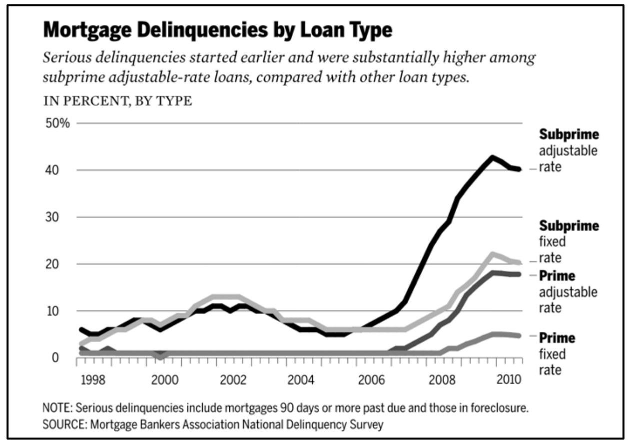

Fig. 7 This Figure is from the Federal Investigation into the Financial Crisis and shows the drastic increase in delinquencies on mortgages and how this varied by the type of loan these mortgages were (Ref. 1).

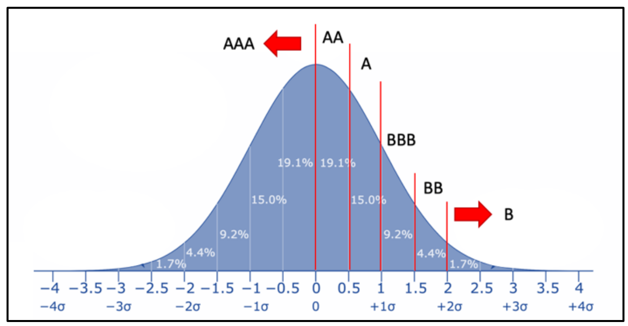

Fig. 8 A random number was generated from a normal distribution for each year using the historical data provided in Table I. Based on where this value lied on the distribution, a credit rating was assigned to the MBS in the portfolio. The ranges along the normal distribution and their corresponding credit ratings are shown above.

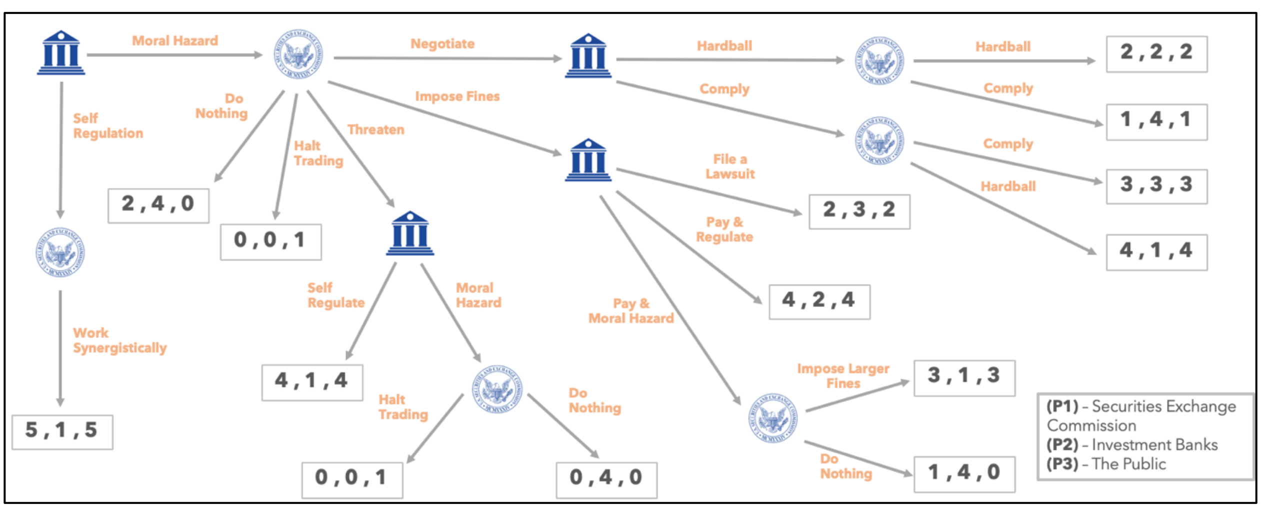

Fig. 9 Complete Game Tree for an Extended form multi-stage game between the SEC and Investment Bankers

IX. REFERENCES

[1] *Financial Crisis Inquiry Report: Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States*; The Financial Crisis Inquiry Commission, United States Government; January 2011; https://www.gpo.gov/fdsys/pkg/GPO- FCIC/pdf/GPO-FCIC.pdf

[2] *Securitization*; Investopedia; https://www.investopedia.com/terms/s/securitization.asp

[3] https://fred.stlouisfed.org/series/DRSFRMACBS

[4] https://finance.yahoo.com

[5] https://www.bls.gov/opub/mlr/2010/12/art1full.pdf