

Top-Down Problem Solving

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Work with your team to write a report showing your knowledge of the Recursion. Submit the team's report on Canvas. Include a task matrix indicating who did what.

Recursion

There is a cute definition of recursion in the Hacker's Dictionary: Recursion: See Recursion

There is a good description of recursion on Wikipedia, read it. Top-down problem solving requires solving a recurrence relation. There are similarities between recurrence equations and ordinary differential equations should you desire to explore.

After successful completion of this quest you will understand how to model the time complexity of a recursive algorithm by a recurrence of the form

$$T(n) = aT(n/b) + f(n)$$

together with some initial conditions to get things started. Interpret the recurrence above as saying: To solve a problem with input size n , solve a problem of size n/b (you may need to do this a times) and apply a forcing function $f(n)$ at each step.

There are many ways to solve a recurrence:

1. Guess or Given and Prove
2. Unrolling also called substitution
3. The Master Theorem
4. Generating Functions

Guess or Given and Prove

I like this approach to my third grade teacher Mrs. Beavis asking me to prove $x = 2$ is a solution to the polynomial equation

1. Show that $\log(n)$ (the log base 2 of n) solves the recurrence:

$$T(n) = T(n/2) + 1, T(0) = 1$$

Mention an algorithm that is described by this recurrence. (Note: you may assume n is a power of 2 so that dividing by 2 never introduces a fraction)

Solution:

Assume $n = 2^k$

$$T(2^k) = T(2^k/2) + 1$$

$$T(2^k) = T(2^{k-1}) + 1$$

$$T(2^k) = (T(2^{k-2}) + 1) + 1$$

$$T(2^k) = T(2^{k-2}) + 2$$

$$T(2^k) = [T(2^{k-3}) + 1] + 2$$

$$T(2^k) = T(2^{k-3}) + 3$$

$$T(2^k) = T(2^{k-k}) + k$$

$$T(2^k) = T(2^0) + k$$

$$T(2^k) = T(1) + k$$

$$T(2^k) = 1 + k$$

$$T(2^k) = \log(n) + 1$$

Therefore,

$$T(n) = \log(n)$$

Binary search is a recursive algorithm that has a time complexity of $(\log(n))$.

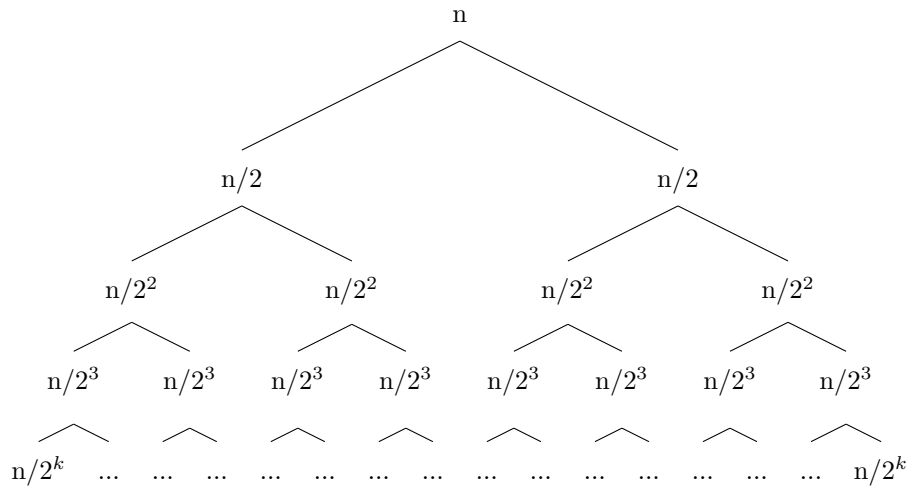
2. Show that $n \log(n)$ solves the recurrence:

$$T(n) = 2T(n/2) + n, T(0) = 1$$

Mention a algorithm that is described by this recurrence.

Solution:

I started by using a recursion tree, because it is easy to visualize.



Sum of each row is n . There are k columns.

$$T(n) = nk$$

Assume

$$n/2^k = 1$$

$$n = 2^k$$

$$k = \log(n)$$

$$nk = n \log(n)$$

Additionally,

$$T(n) = 2T(n/2) + n$$

$$T(n/2) = 2T(n/2^2) + n/2$$

$$T(n) = 2[T(n/2) + n]$$

$$T(n) = 2[2T(n/2^2) + n/2] + n$$

$$T(n) = 2^k T(n/2^k) + kn$$

$$T(n/2^k) = T(1)$$

$$n/2^k = 1$$

$$n = 2^k$$

$$k = \log(n)$$

$$kn = n\log(n)$$

Therefore,

$$T(n) = n\log(n)$$

Mergesort is a recursive algorithm that has a time complexity of $O(n\log(n))$.

Name	Section
Remington Greko	Reviewed and corrected solutions
Tyler Gutowski	$O(n \log(n))$ Recurrence
Spencer Hirsch	$O(\log(n))$ Recurrence and Example Algorithms