Final Undergraduate Psuedo-Sudoku Submission

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Summary of Project

Following our original submission we have implemented another algorithm. With our original submission we implemented a backtracking algorithm, which worked very well for our first milestone. For this final milestone we have implemented a backtracking with a heuristic algorithm. Both algorithms solve the given test cases, however after doing an analysis of the two comparing the input size and the number of removed cells there is a clear difference in the complexity of the two different algorithms.

In order to generate reasonable test cases to demonstrate the time complexity of our our two algorithms. We have implemented a generator to generate test cases to give to our algorithms. Each algorithm tests a total of 2700 different test cases with varying difficulty. The goal of implementing a generator was to make the tests as random as possible and to easily generate a larger sum of test cases. For each n (the size of the matrix, n x n) there are 15 additional test cases that have varying m (number of missing cells). Each of these n x m combinations is tested a total of 30 times. This is how we have determined that there are a total of 2700 different test cases run on each of the algorithms.

All aspects of the board aside from the size of the board is randomly determined. The values for each squares are randomly assigned for the size of the board. Once all of the values are determined the number of removed cells is randomly selected and unique to each row. The purpose of creating this random algorithm was to ensure that bias was reduced.

In order to demonstrate the time complexity of each of the algorithms the cpu time is recorded for each test and plotted against the number of removed squares from the sudoku board. To ensure the accuracy of our results the same test cases are run on both of the algorithms. Two scatter plots are constructed for the two algorithms, one displaying every point gathered from the test sets and the other plot demonstrating the averages of the 15 ponints for each n x m pair.

Backtracking Algorithm

```
# Function next empty cell in the matrix


def find_cell(board, size):
       iterate through the board to find the next instance of a 0 value in
       a cell


       If no value is found, return -1, -1


       otherwise return the coordinate of the cell that contains a 0 value




# Function checks for a valid move given the value of the current
# Cell


def valid_move(coordinate_1, coordinate_2, board, size, number):


       Iterate through the size of the board
              Check to see if there is a cell in the column has the current val
                     if so, return false


       Iterate through the size of the board
              Check to see if there is a cell in the row that has the current val
                     If so, return false


       return true # It is a valid move for the algorithm to make




def solve(board, size):
       Create a list of all possible numbers based on size of the board
       Find the coordinate to the next empty cell
```

```
        Check to see if cell is valid


        iterate through all possible numbers to fill a cell
               Check to see if the move is a vlid move


        Continue until all cells are filled or no solution is found
```

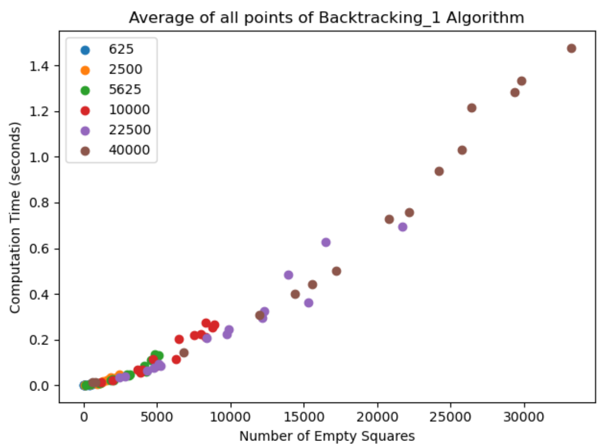
How the backtracking algorithm works:

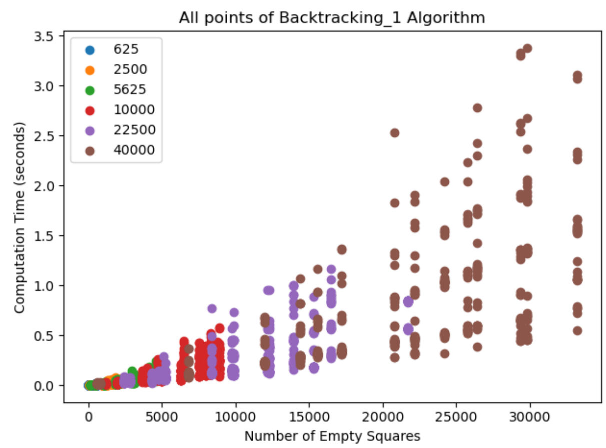
Backtracking with Heuristic

How the backtracking with heuristic algorithm works:

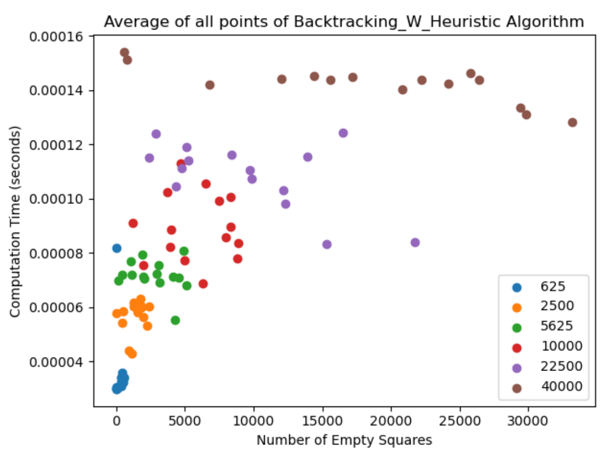
Analysis of Algorithms

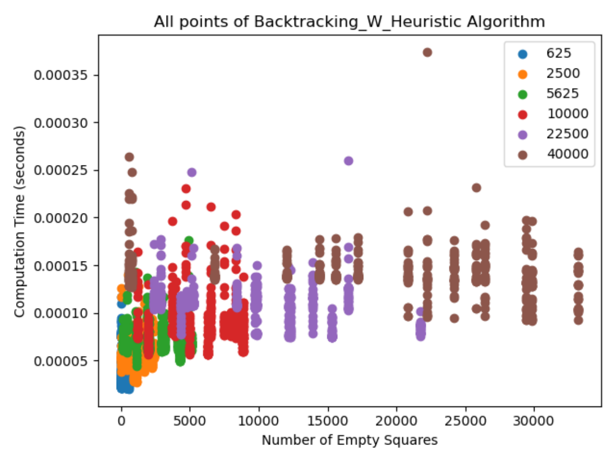
Backtracking





Backtracking with Heuristic

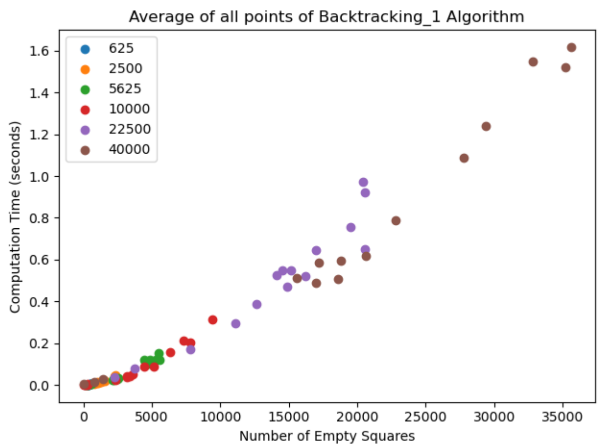


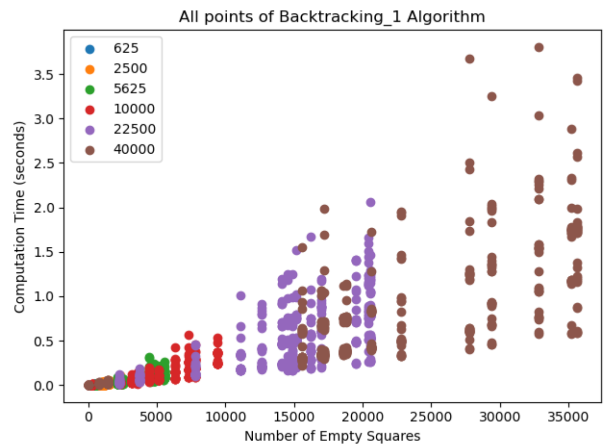


From the images it is clear to see that there is a significant difference in time complexity between the backtracking algorithm and the forward-checking with heuristic algorithm. The graph with the average of the points of the backtracking shows what we believe to be O(n^2) for time complexity. As the number of missing cells increases there appears to be an exponential growth with respect to time. However, in the case of the backtracking algorithm with a heuristic, it is much more difficult to determine Big-Oh complexity. In this specific test case it appears to loosely conform to O(logn) complexity. However, strictly looking at the y-axis on both of the graphs we can see that the backtracking with heuristic algorithm performs much quicker than the previous backtracking algorithm.

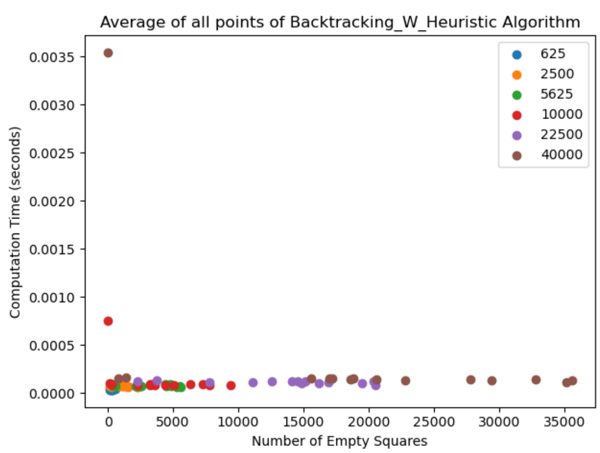
Here are some additional examples with some other test cases.

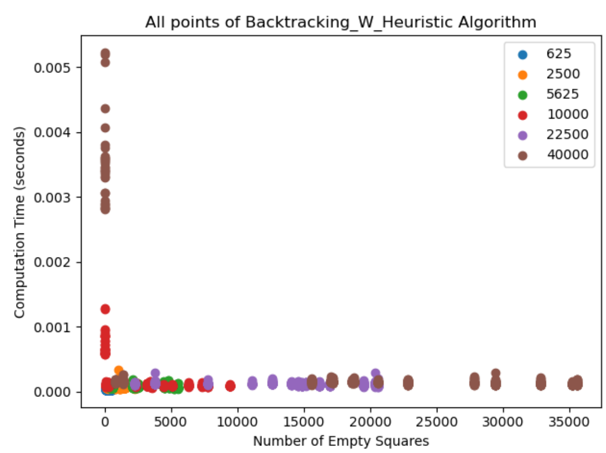
Backtracking





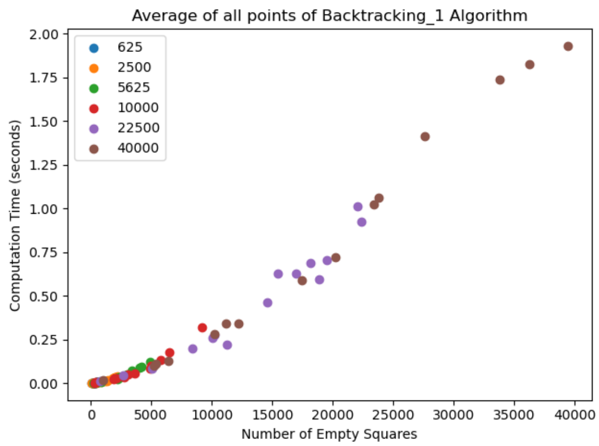
Backtracking with Heuristic

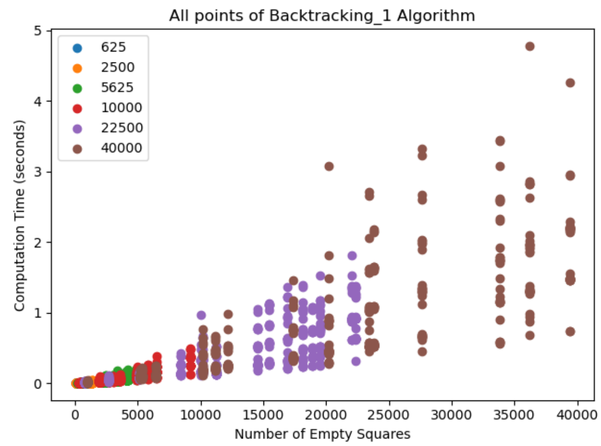




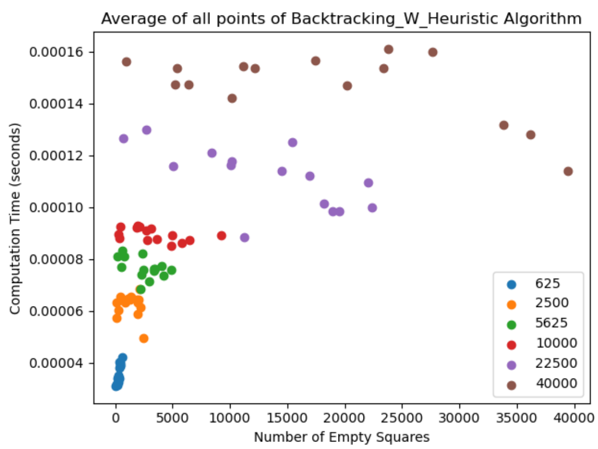
Again, with this test case the backtracking algorithm appears to be in the shape of O(n^2). However, the backtracking algorithm with heuristic appears to be more in the shape of constant time, besides the single point which is clearly an outliar. Despite the drastic change in the shape of the graph for the backtracking with heuristic algorithm, the backtracking with heuristic algorithm performs much quicker than the backtracking algorithm that was originally implemented.

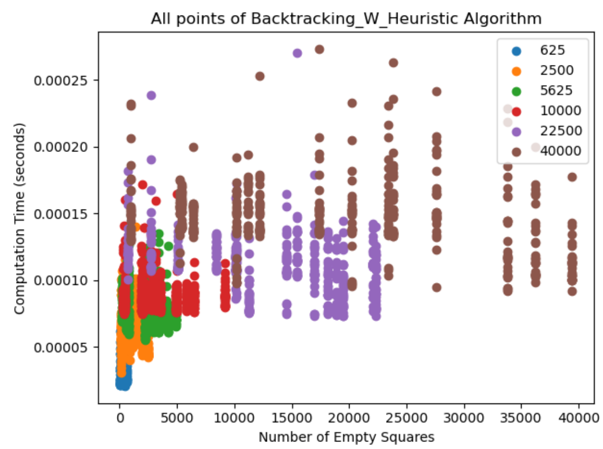
Backtracking





Backtracking with Heuristic





These test cases aligned more with the first test case listed above. The backtracking algorithm again having more of a O(n^2) complexity while the backtracking with heuristic algorithm had more of a O(logn) shape. Again, the averages of the tests showed that the backtracking algorithm with heuristic signigicantly outperformed the backtracking algorihtm that was originally used.

Overall, throughout all of the test cases that we ran on both of the algorithms. The newly implemented, backtracking with heuristic, significantly outperformed our algorithm from the first milestone submission. Solving the problem in the fraction of the time. The backtracking algorithm would usually take more than a second to solve the larger n x m problems while the backtracking with heuristic was capable of solving it in less than 0.0002 seconds. We are very pleased and surprised with the outcome of these results. It was very interesting to see such a drastic increase in performance when comparing the two algorithms and their ability to solve such large test cases. The largest test case is a matrix of 40000 cells and our backtracking with heuristic algorithm was capable of solving it in less than 0.0002 seconds.