Final Undergraduate Psuedo-Sudoku Submission

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Summary of Project

In our first submission we implemented a backtracking algorithm, this algorithm worked incredibly well. However, we have now implemented a backtracking algorithm with a heuristic. Both of the algorithms work in solving the test cases given, however, there is a clear difference in the time complexities of the algorithms.

In our intermediate submission all of our test cases were written by hand, however, we believe that this did not challenge the algorithm enough. Therefore, for this final submission, we have implemented a a function that generates the test boards using randomness. Both algorithms are tested with 2700 different test cases each. Both of our algoriths are fed the exact same test cases to better demonstrate the time complexities of the algorithms.

In order to generate reasonable test cases to demonstrate the time complexity of our our two algorithms. We have implemented a generator to generate test cases to feed to our algorithms. Each algorithm tests a total of 2700 different test cases with varying difficulty. The goal of implementing a generator was to make the tests as random as possible and to quickly generate a more considerable sum of test cases. For each n (the matrix size, n x n), 15 additional test cases have varying m (number of missing cells). Each of these n x m combinations are tested 30 times. This is how we have determined that 2700 different test cases are run on each of the algorithms.

All aspects of the board, aside from the size of the board, are randomly determined. The values for each square are randomly assigned depending on board size. Once all of the values are determined, the number of removed cells is randomly selected and unique to each row. This random algorithm was created to ensure that bias was reduced.

In order to demonstrate the time complexity of each of the algorithms, the CPU time is recorded for each test and plotted against the number of removed squares from the sudoku board. To ensure the accuracy of our results, the same test cases are run on both of the algorithms. Two scatter plots are constructed for the two algorithms, one displaying every point gathered from the test sets and the other plot demonstrating the averages of the 30 points for each n x m pair. The legend of the plots shows the total number of cells for the test, being n^2. We felt that this would better visualize the size of the test as opposed to n, the number of rows or columns.

Backtracking Algorithm

```
# Find next empty cell in the matrix
def find_cell(board, size): -> int, int
    for i in range size:
        for j in range size:
            if board[i][j] is empty
                return i,j
    return -1, -1 # Full board, return sentinal values -1, -1

# Function checks for a valid move given the value of the current cell
def valid_move(cord1, cord2, board, size, number): -> boolean
    # Given current cordinatess iterate through the row, and column
    # to make sure psuedo sudoku rules are not violated
    for i in range size:
        if board[i][cord2] = number
        return false
    for j in range size:
        if board[cord1][j] = number
        return false
    return true

# Implements the full back tracking algorithm
solve(board, size): -> boolean
    nums = list(1, ... N + 1) # stores all possible numbers
    cord1, cord2 = find_cell (board, size) # finds the next position
    if cord1 and cord2 == -1: # board is filled with legal moves
        return true
```

```
        for num cell in nums # iterates through every possible move
            if valid_move(cord1, cord2, board, size, num):
                board[cord1][cord2] = num
                if solve(board, size): # continue down this branch
                    return true
                board[cord1][cord2] = 0 # failure, begin back tracking
        return false # could not be solved
```

How the backtracking algorithm works:

The original algorithm is built on the idea of back tracking. To begin we first need to have an array storing all possible number selections for an NxN board. This means that if the board is 3x3 we will store the numbers [1, 2, 3] in the array `nums`. The next step is to find which cell we should make the next valid move in, using `cord1, cord2 = find_cell(board, size)`. This function works simple enough; it merely iterates through the rows, and columns of the inputted board to find the next blank space. If no empty space is found we return the values -1, -1. This signifies the board is filled up fully, and that it is solved.

Moving into the main bulk of the algorithm, we enumerate through each `num in nums`, and use the code `if valid_move(cord1, cord2, board, size, num)`. `valid_move(...)` works on taking in the required information, and seeing if the current `num` is a valid move at that cordinate given the ruleset of psuedo sudoko. If it is valid we return true else we return false, and move on to another number. If its a valid move however we set that boards cell to the current number `board[cord1][cord2] = num`.

Now that a valid move has been found we begin to recursively call the function using the code `if solve(board, size)`. If this evaluates to true then the board will recursively be filled up with valid moves. However if `solve(...)` evaluates to false at any point in time, the current cell is reflagged as empty, and the back tracking begins.

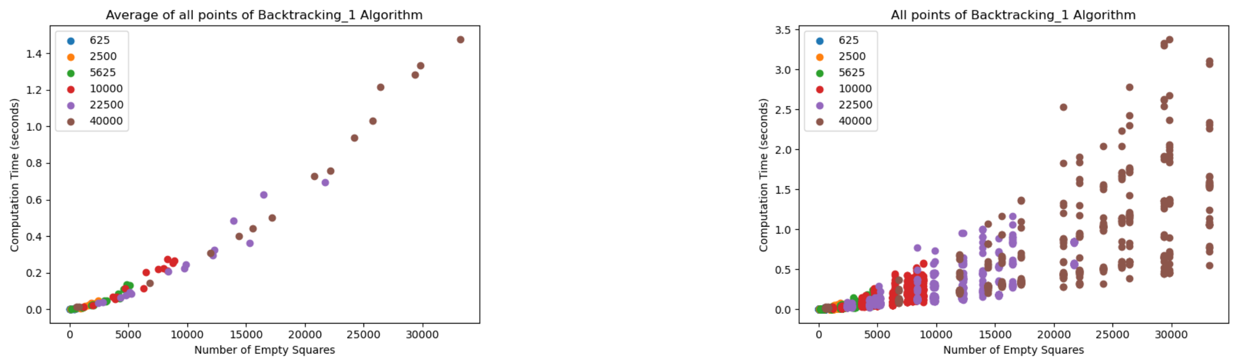
Backtracking with Heuristic

```
# Functions used prior that are unchanged:
def find_cell(board, size) -> int, int
def valid_move (cord1, cord2, board, size, number) -> boolean

# New function to implement basic forward checking
def get_unused_numbers(board, cord1, cord2, size) -> list
       nums = set of the range(1, size + 1) to hold all possible values.

       # Iterate through the row, and column of current cords to remove duplicates
       for i in range of size:
              current number = board[i][cord2]
              if current number is in nums:
                     remove current number from nums
       for j in range of size:
              current number = board[cord1][j]
              if current number is in nums:
                     remove current number from nums
       return list(nums)

# This will slightly the code for solve2
def solve2 (board, size):
       cord1, cord2 = find_cell (board, size) -> int, int
       nums = get_unused_numbers(board, cord1, cord2, size) -> list
       if cord1 and cord2 == -1:
              return true
       for num cell in nums
              if valid_move(cord1, cord2, board, size, num):
                     board[cord1][cord2] = num
                     if solve(board, size):
                            return true
                     board[cord1][cord2] = 0
```

How the backtracking with heuristic algorithm works:

The only real change to the algorithm is the inclusion of the forward checking heuristic. This is implemented by the function `get_unused_numbers(...)`. This works by checking the prior moves, and using this information to adjust which values we will attempt to use in our current move. We do this by simply taking our current list of numbers, iterating through the row, and column of our current cords, and then removing any numbers found during the iterations from our current number list.

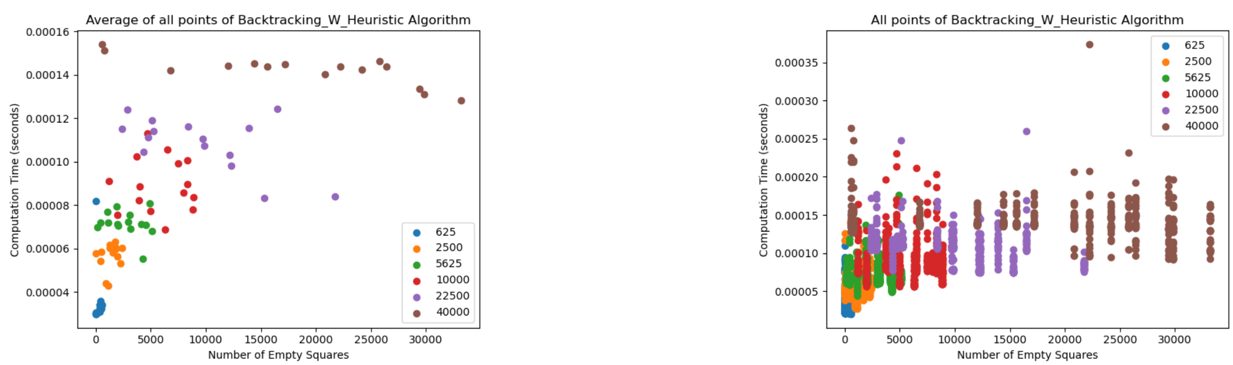
Analysis of Algorithms



(a) Items in the legend demonstrate the total number of cells, n^2

(b) Items in the legend demonstrate the total number of cells, n^2

Figure 1: Data Collected from Backtracking algorithm, Test 1.



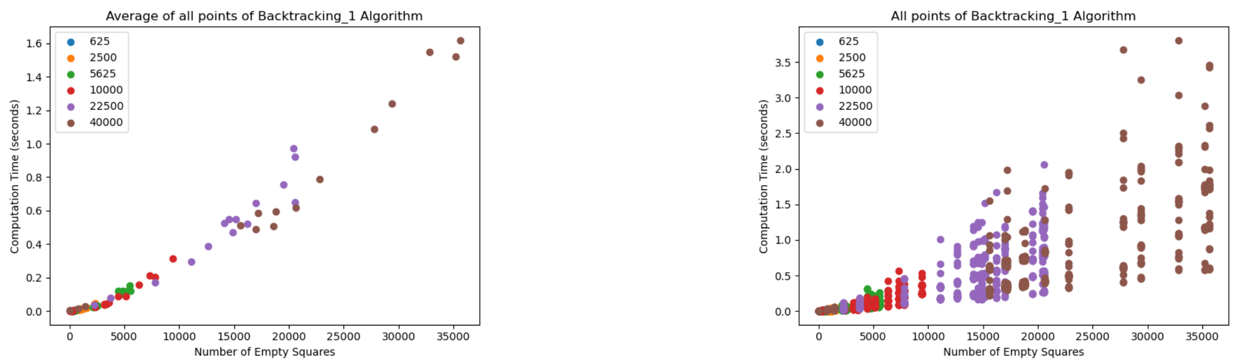
(a) Items in the legend demonstrate the total number of cells, n^2

(b) Items in the legend demonstrate the total number of cells, n^2

Figure 2: Data Collected from Backtracking algorithm with a Heuristic, Test 1.

From the graphs it is clear to see that there is a significant difference in time complexity between the backtracking algorithm and the backtracking with heuristic algorithm. *Figure 1* shows what we believe to be O(n^2) time complexity. As the number of missing cells increase there appears to be an exponential growth with respect to time. However, in the case of the backtracking algorithm with a heuristic (*Figure 2*), it is much more difficult to determine Big-Oh complexity. In this specific test case, Figure 2 appears to loosely conform to O(logn) complexity. However, strictly looking at the y-axis(CPU time in seconds) on both *Figure 1* and *Figure 2* we can see that the backtracking with heuristic algorithm performs much quicker than our previous backtracking algorithm.

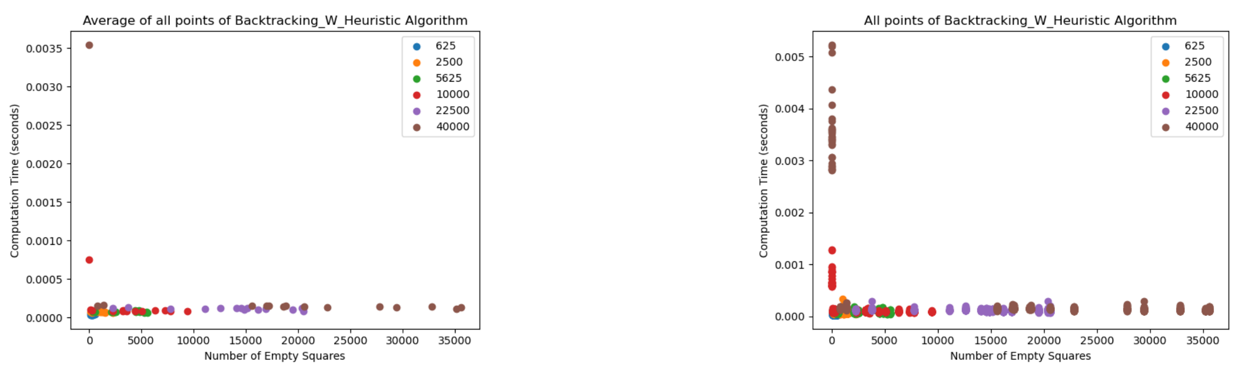
Here are some additional examples with some other test cases.



(a) Items in the legend demonstrate the total number of cells, n^2

(b) Items in the legend demonstrate the total number of cells, n^2

Figure 3: Data Collected from Backtracking algorithm, Test 2.

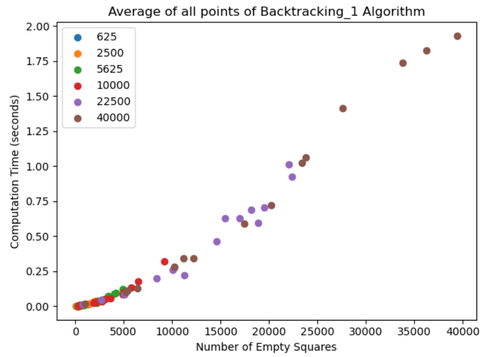


(a) Items in the legend demonstrate the total number of cells, n^2

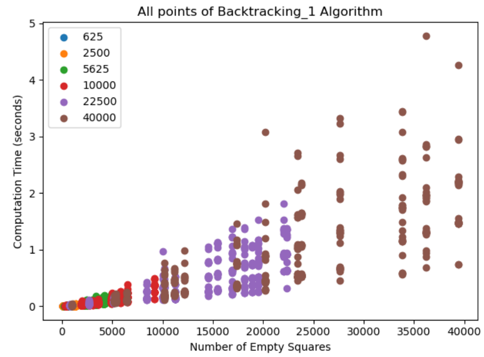
(b) Items in the legend demonstrate the total number of cells, n^2

Figure 4: Data Collected from Backtracking algorithm with a Heuristic, Test 2.

Again, with this test case the backtracking algorithm(*Figure 3*) appears to be in the shape of O(n^2). However, the backtracking algorithm with heuristic(*Figure 4*) appears to be more in the shape of constant time, besides the single point which is clearly an outliar. Despite the drastic change in the shape of the graph for the backtracking with heuristic algorithm, the backtracking with heuristic algorithm performs much quicker than the backtracking algorithm that was originally implemented.

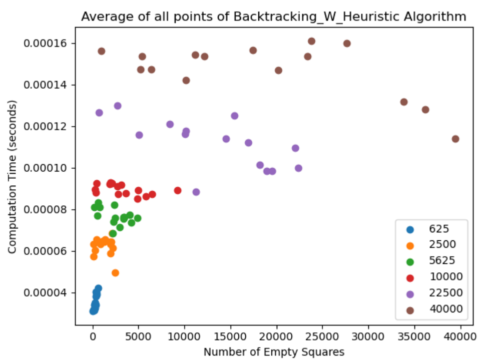


(a) Items in the legend demonstrate the total number of cells, n^2

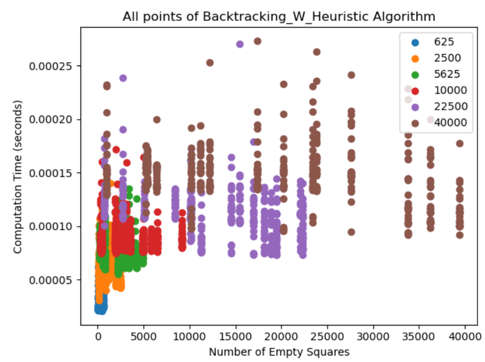


(b) Items in the legend demonstrate the total number of cells, n^2

Figure 5: Data Collected from Backtracking algorithm, Test 3.



(a) Items in the legend demonstrate the total number of cells, n^2



(b) Items in the legend demonstrate the total number of cells, n^2

Figure 6: Data Collected from Backtracking algorithm with a Heuristic, Test 3.

These test cases aligned more with the first test case depicted in both *Figure 1* and *Figure 2*. The backtracking algorithm again having more of a O(n^2) complexity while the backtracking with heuristic algorithm had more of a O(logn) shape. Again, the averages of the tests showed that the backtracking algorithm with heuristic signigicantly outperformed the backtracking algorihtm that was originally used.

Overall, throughout all of the test cases that we ran on both of the algorithms. The newly implemented, backtracking with heuristic, significantly outperformed our algorithm from the first milestone submission, backtracking. Solving the problem in the fraction of the time. The backtracking algorithm would typically take more than a second to solve the larger n x m problems while the backtracking with heuristic was capable of solving it in less than 0.0002 seconds. We are very pleased and surprised with the outcome of these results. It was very interesting to see such a drastic increase in performance when comparing the two algorithms and their ability to solve such large test cases. The largest test case is a matrix of 40000 cells, a matrix 200 x 200, our backtracking with heuristic algorithm was capable of solving it in less than 0.0002 seconds.