Uniform FBGs

Grating (power) reflectivity (simplified from [Erdogan97] Eq. 22):

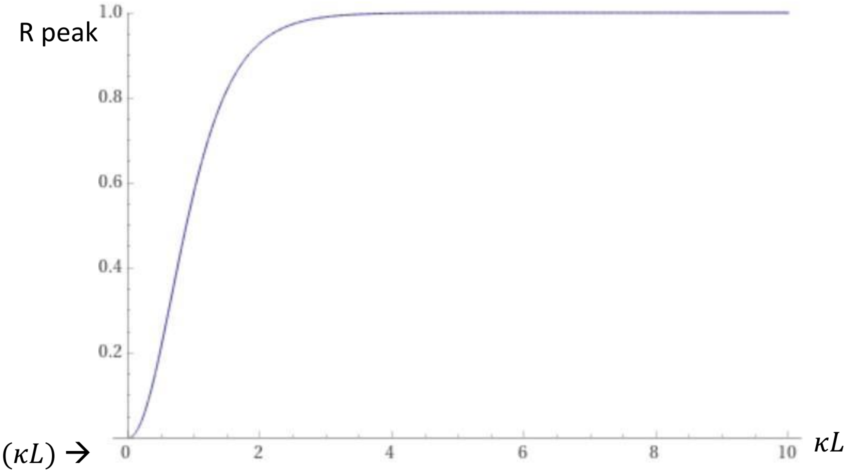
$$R = \frac{\sinh^2\left(\sqrt{1-\frac{\delta^2}{\kappa^2}}\kappa L\right)}{\cosh^2\left(\sqrt{1-\frac{\delta^2}{\kappa^2}}\kappa L\right)-\frac{\delta^2}{\kappa^2}}$$

Where κ is a number that encapsulates the refractive index difference in the grating – greater index difference means larger κ – and δ represents the frequency detuning from the Bragg frequency:

$$\delta = \left(\frac{2\pi n_{eff}}{c}\right)\delta f$$

L is just the total length of the grating.

Important scaling rules:



$R_{max} = \tanh^2(\kappa L)$ →

$\kappa L \gg 1$ →the grating will become more "saturated" in shape
$\kappa L \gg \pi$ →the grating becomes a "strong grating", the bandwidth no longer depends on the length (these scalings are basically the same, i.e. a strong grating is always saturated, but they mean slightly different things)

The implications of the above scalings:

- if the reflectivity is significantly less than one and the reflectivity curve is not "saturated", then it must not be a "strong grating" → the bandwidth will still scale inversely with length
- When increasing the length, the bandwidth will decrease, but the peak reflectivity will also increase and the reflectivity curve will also become more "saturated"
- The only way to avoid changing reflectivity and curve shape when changing the length is to alter the index change when making the grating
- But, the bandwidth will still have to change when changing the length, unless it is in the strong grating regime, where the reflectivity is already essentially fixed at 1 anyway

Chirped-FBGs

Simple calculations:

Assuming no apodization, and a purely linear chirp in the grating period:

$$\lambda(z) = \lambda_0 + \left(\frac{d\lambda_D}{dz}\right) z$$

Or

$$\omega(z) = \frac{2\pi c}{\lambda_0} - \frac{2\pi c}{\lambda_0^2}\left(\frac{d\lambda_D}{dz}\right) z$$

Invert either equation and then either the wavelength chirp or the group-delay dispersion (GDD) can be found: $t(\lambda) = 2n_{eff}z(\lambda)/c$

Wavelength chirp: $d_p = \frac{dt(\lambda)}{d\lambda} = \frac{2n_{eff}}{c}\left(\frac{d\lambda_D}{dz}\right)^{-1}$

GDD: $\phi_2 = \frac{dt(\omega)}{d\omega} = \frac{-n_{eff}\lambda_0^2}{\pi c^2}\left(\frac{d\lambda_D}{dz}\right)^{-1}$

Remember that $\lambda_D = 2n_{eff}\Lambda$, where Λ is the physically inscribed grating period.

The bandwidth is related to the span of the grating period:

$$\Delta\lambda \sim L\left(\frac{d\lambda}{dz}\right)$$

Detailed model:

This can be analyzed according to a very old model from [Matsuhara75] and compared to data presented in [Ouellette87].

The model according to [Matsuhara75], is based on coupled mode theory as described in [Yariv73]. I use notation with normalized parameters from [Ouellette87]. The model is as follows:

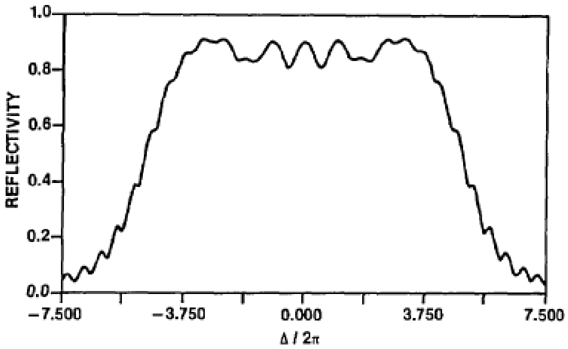
A forward going wave with amplitude A^+ is coupled via a structure in the region {-L/2, +L/2} to a backwards-going wave with amplitude A^-. The boundary conditions are $A^+(-L/2)=1$, and $A^-(+L/2)=0$. We need to solve the system of differential equations and find $A^-(-L/2)$ which is equal to the complex reflectivity out of the structure.

The differential equation and boundary conditions for A^- are:

$$\frac{d^2A^-}{dz^2} - j(2\Delta - Fz)\frac{dA^-}{dz} - (\kappa L)^2 A^- = 0$$
$$\frac{dA^-}{dz}\left(-\frac{1}{2}\right) = (\kappa L)\exp\left(-j\left(\Delta + \frac{F}{8}\right)\right)$$
$$A^-\left(\frac{1}{2}\right) = 0$$

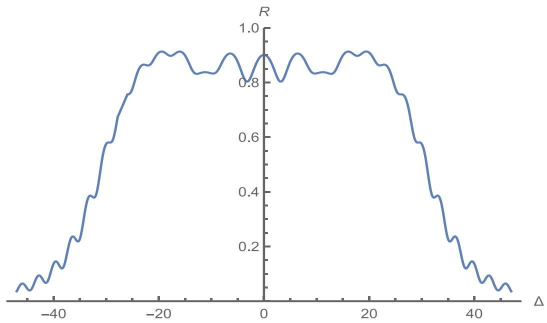
Where F is related to the chirp of the grating, Δ is related to the detuning, and z is normalized to z/L for convenience. The solution to this is actually solvable analytically (apparently…) but it is a very complicated sum of Kummer functions and Hermite functions. So, it is easier to solve it numerically. I have done this first in Mathematica, $R=|A^-(-L/2)|^2$, and it produces the correct results.

From [Ouellette87] Figure 1a:



(a)

Reflectivity from the Mathematica script “chirpedFBG_Ouellette87_full.nb”:

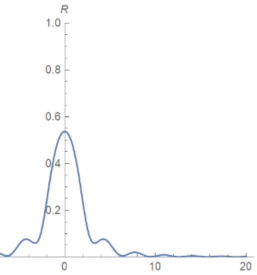


One can relate the normalized parameters in this model to physical values as follows:

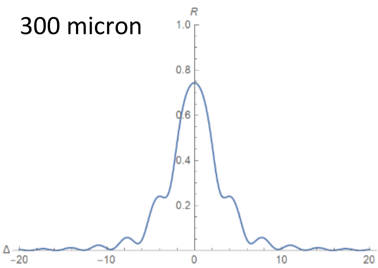
$$\Delta = \delta\beta L = \left(\frac{2\pi n_{eff} L}{c}\right)\delta f = \left(\frac{-2\pi n_{eff} L}{\lambda_0^2}\right)\delta\lambda$$
$$\frac{d\lambda}{dz} = \frac{-\lambda_0^2 F}{4\pi n_{eff} L^2}$$
$$\Delta\lambda = \lambda\left(\frac{L}{2}\right) - \lambda\left(-\frac{L}{2}\right) = L\left(\frac{d\lambda}{dz}\right) = \frac{-\lambda_0^2 F}{4\pi n_{eff} L}$$
$$\phi_2 = \frac{4 n_{eff}^2 L^2}{c^2 F} = \frac{-\lambda_0^2 n_{eff} L}{\pi c^2 \Delta\lambda}$$

We can then use these parameters to look at properties of a chirped-DBR on the InP Photonic Integrated Circuit platform from SMART Photonics (Eindhoven, NL). The parameters are: central wavelength 1550 nm, κ=50 1/cm (fixed), n_eff=3.266 (fixed), a design bandwidth of 3 nm and lengths L=200, 300, 400 micron. The Mathematica notebook “chirpedFBG_INP-SMART-PDK_full.nb” produces the following reflectivity profiles:

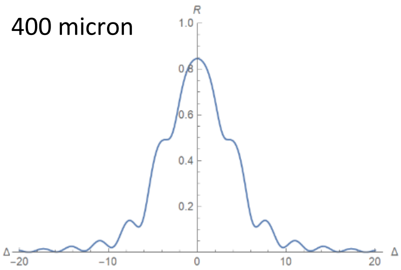
200 micron



300 micron



400 micron



Which is as expected, since δλ=(-0.6)Δ (in nm) for these parameters, so the bandwidth is indeed around 3 nm, although it increases with length. With $\kappa L = 1, 2, 3$ the peak reflectivity is higher at a larger length, also as expected. This has also been copied to Python and Matlab scripts, which produce the same results.

Lastly, we can look at chirped gratings that are apodized, which is when the profile of the index modulation is smoothed. Apodization is not possible in the SMART PDK, but generally desirable. This is because it creates much smoother reflectivity and phase profiles.

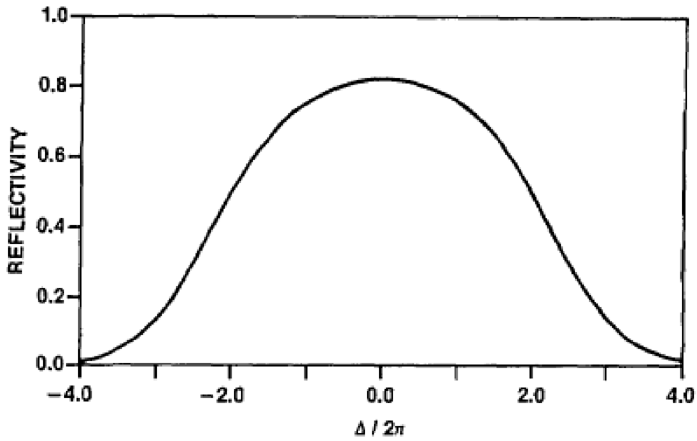
We choose the same Gaussian apodization profile as in [Ouellette87]:

$$\kappa(z) = \kappa_0 L e^{-16z^2}$$

Which produces the modified boundary value problem:

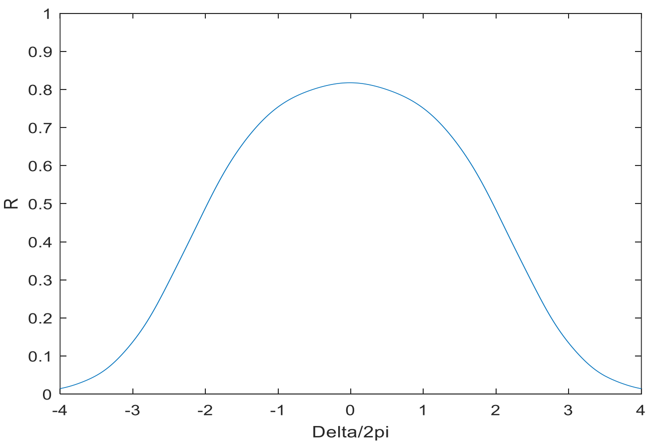
$$\frac{d^2A^-}{dz^2} - (j(2\Delta - Fz) - 32z)\frac{dA^-}{dz} - \left(\kappa_0 L e^{-16z^2}\right)^2 A^- = 0$$
$$\frac{dA^-}{dz}\left(-\frac{1}{2}\right) = (\kappa_0 L)\exp\left(-4 - j\left(\Delta + \frac{F}{8}\right)\right)$$
$$A^-\left(\frac{1}{2}\right) = 0$$

Where z is again normalized to z/L. The same procedure produces R=$|A^-(-L/2)|^2$, and the correct results is reproduced for [Ouellette87] figure 2a:



(a)

And from Matlab script “chirpedFBGApod_Ouellette87.m”:



Phase-shifted FBGs

This is based mainly on the work from [Agrawal94] and [Liu98]. The “phase-shifted grating” is a grating that is composed of two gratings with lengths and strengths resulting in their own reflectivities r_1 and r_2, and a length with no structure in-between that results in a phase-shift. This phase-shift can also be imagined as a certain fraction of a grating period is missing.

The resulting total reflectivity is:

$$r_{total} = \frac{r_1 - e^{j\phi} r_1 r_2 / r_1^*}{1 - e^{j\phi} r_1 r_2}$$

Where r is the complex reflectivity from [Erdogan97] for example. The simplest case is when the first and second grating in the composite grating are identical such that r_1=r_2=r and the phase shift is pi. This is what we will call the “pi-shifted grating”, which has a reflectivity of:

$$r_{total} = \frac{1 + r/r^*}{r + 1/r}$$

References:

[Agrawal94] G. P. Agrawal and S. Radic, “Phase-Shifted Fiber Bragg Gratings and their Application for Wavelength Demultiplexing,” *IEEE Photonics Technology Letters* **6**, 995-997 (1994).

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[Ouellette87] F. Ouellette, “Dispersion cancellation using linearly chirped Bragg grating filters in optical waveguides,” *Optics Letters* **12**, 847-849 (1987).

[Liu98] Y. Liu, S. B. Lee, and S. S. Choi, “Phase-Shifted Fiber Bragg Grating Transmission Filters Based on the Fabry-Perot Effect,” *Journal of the Optical Society of Korea* **2**, 30-33 (1998).

[Matsuhara75] M. Matsuhara, K. O. Hill, and A. Watanabe, “Optical-waveguide filters: Synthesis,” *JOSA* **65**, 804-809 (1975).

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[Yariv73] A. Yariv, “Coupled-Mode Theory for Guided-Wave Optics,” *IEEE JQE* **9**, 919-933 (1973).