

# Heat Exchanger Design Project

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# Introduction

Engine oil's thermal stability is essential to internal combustion engines' lifetime and efficiency. High operating temperatures can cause lubricating oil to thermally break down, reducing its viscosity, lubricating moving parts less effectively, and causing premature engine wear. The goal of this project was to design an efficient heat exchanger that guaranteed oil cooling under high-load circumstances.

The challenge assigned was to create an effective oil cooler that rejects air via forced convection. The system was made up of the heat exchanger and the duct that delivers the cooling air. Using computational fluid dynamics, the thermal and flow behavior of different configurations were examined to find the ideal geometry that satisfies the performance and spatial restrictions.

The goal was to assess and optimize the heat exchanger shape using CFD techniques to minimize weight, pressure losses, and overall dimensions while guaranteeing efficient heat rejection. Meshing optimization, iterative 3D modeling in ANSYS Fluent, and theoretical hand calculations for validation were all part of the design process. The following paper will describe the design concept, meshing methodologies, boundary condition definitions, hand-calculation verification, CFD simulation results, and final performance metrics, such as pressure drops, temperature differentials, and predicted efficiency.

## Problem Statement and Design Goals

The goal of the project is to design a heat exchanger to ensure the cooling oil in a gasoline IC engine does not thermally break down. The rejecting medium, air, is delivered to the heat exchanger by a duct with specified inlet dimensions. The design goal is to optimize the efficiency of the heat exchanger by finding the optimal geometry, using the following design requirements.

### Specific Design Requirements

- Minimum vehicle speed is 39.6 ft/s (27 mph)
- Air temperature is 110°F
- Duct inlet dimensions are 20 inches by 10 inches
- Heat exchanger is mounted 60 inches downstream and 36 inches to the side of the duct inlet
- Maximum width and height of heat exchanger is 36 inches
- Maximum face area of heat exchanger is 275 in<sup>2</sup>
- Heat exchanger can be as deep as required
- Oil enters at 350°F
- Oil exits at 240°F maximum
- Oil flow rate is between 4.5 gpm and 6 gpm

### Design Concept

The final design implemented a crossflow heat exchanger at the end of a duct, where the air and oil flow perpendicular to each other. The duct was defined relative to the specified requirements of the heat exchanger. To ensure that a minimal air speed is lost, the duct and heat exchanger inlet faces were designed parallel to each other.

The crossflow exchanger design was chosen due to its high surface area between the hot and cold fluids. This design was also beneficial as the series of tubes could be repeated along the depth until the desired surface area was achieved.

For the initial operating conditions, the duct should handle an initial air velocity of 39.6 ft/s and heat exchanger an oil volumetric flow rate of 6 gallons per minute.

## Geometry and Design Assumptions

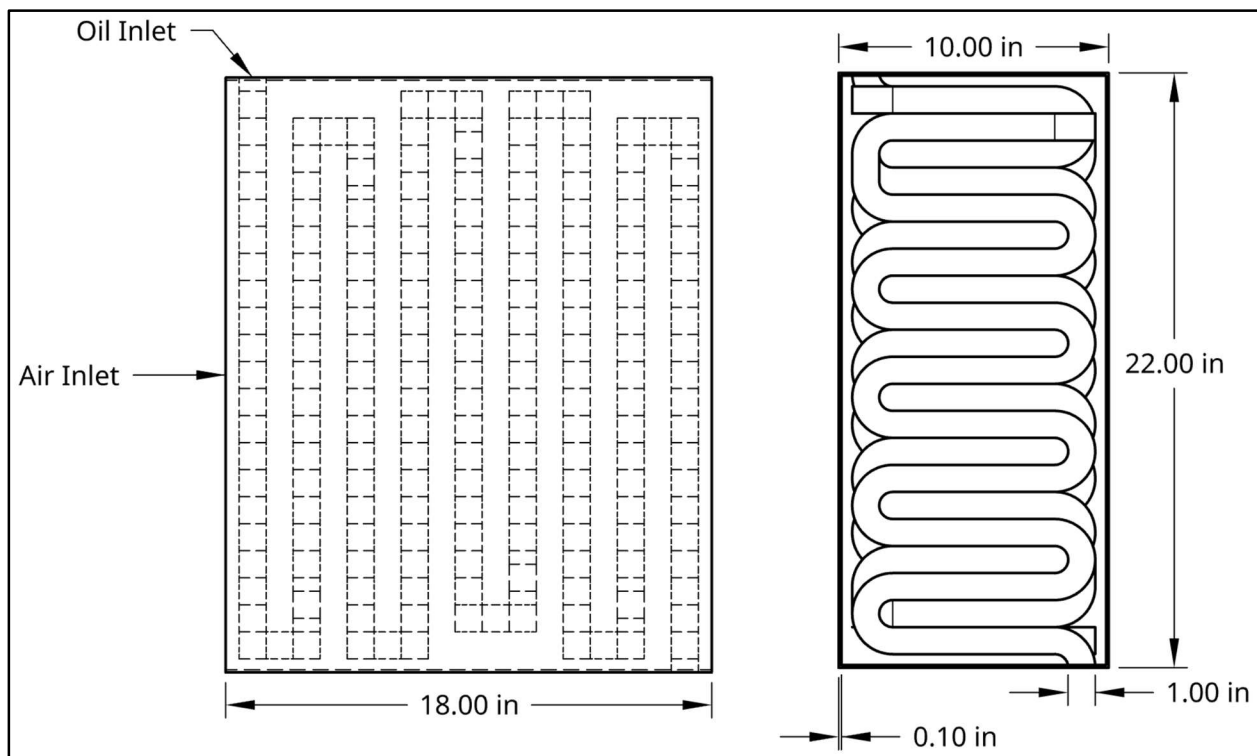
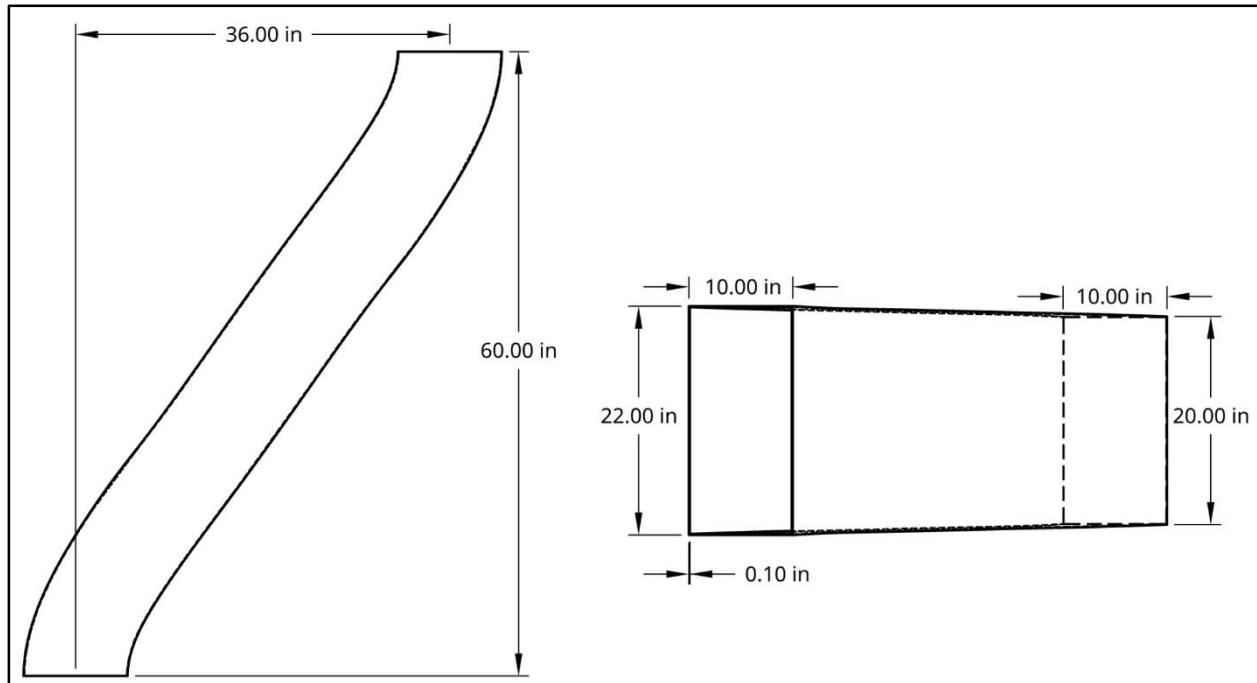
The duct and heat exchanger were modeled, meshed, and simulated separately. All the modeling was completed on OnShape. Changes made to the geometry of the heat exchanger had no impact on the duct. This assumption was also utilized for the hand-calculations.

From the design requirements, the duct is designed to be 36 inches offset to the side of the heat exchanger and allow the heat exchanger to be mounted 60 inches down from the duct inlet. The duct smoothly transitions from the 20-inch x 10-inch duct inlet to the 22-inch x 10-inch heat exchanger inlet over a length of 60 inches. The outlet air conditions of the duct, such as velocity and temperature, are assumed to be equivalent to the inlet air conditions of the heat exchanger. The design for the duct is shown in Figure 1.

Several geometries were originally considered for the heat exchanger. Geometry for a laminar model for oil flow was favored over a turbulent model due to simplification of meshing and ease of importing boundary conditions. A finer mesh near the pipe walls is required to accurately capture velocity gradients for a turbulent model, resulting in a more complex mesh and computationally expensive simulation.

The final design for the heat exchanger consists of a series of 9 staggered tubes rows that are connected, which is shown in Figure 2 below. To validate the hand calculations and efficiency of the design, a preliminary simulation was run using a heat exchanger model with one row of tubes. Detailed results can be found in Table 16 and 17.

## Drawings



## Overall Unit Dimensions

The overall dimensions for length, width, depth, and area for the duct and heat exchanger are shown below. The inlet area for the heat exchanger is 220 in<sup>2</sup>, which is less than the 275 in<sup>2</sup> design requirement.

**Table 1: Dimensions for Duct**

	Dimensions
Length	60 in
Width	10 in
Depth	22 in
Air Inlet Area	200 in <sup>2</sup> (20x10)
Air Outlet Area	220 in <sup>2</sup> (22x10)
Overall Wall Thickness	0.1 in

For the final design of the heat exchanger, the overall oil pipe surface area is 3427 in<sup>2</sup>. This is greater than the calculated required surface area of 2322 in<sup>2</sup> for a laminar model, indicating that the heat exchanger was overdesigned. This is further emphasized with the results of the preliminary simulations on one row of tubes. Calculations for the required surface area can be found in Appendix I.

**Table 2: Dimensions for Heat Exchanger**

	Dimensions
Length	18 in
Width	10 in
Depth	22 in
Pipe Row Spacing	1 in
Pipe Wall Thickness	0.125 in
Oil Inlet Area	1 in <sup>2</sup> (1x1)
Air Inlet and Outlet Area	220 in <sup>2</sup> (22x10)
Overall Pipe Surface Area	3427 in <sup>2</sup>
Overall Wall Thickness	0.1 in



**Table 3: Overall Unit Dimensions**

	Dimensions
Length	78 in
Width	10 in
Depth	22 in

## Hand Calculations

### Material Properties

To properly model the flow of air and oil at the desired conditions, the density, heat capacity, viscosity, and thermal conductivity were defined. The Engineering Toolbox was used to define the air's density and viscosity at 110°F [1, 2]. The oil density was calculated with the given specific gravity of oil and ambient air density.

**Table 4: Calculated Properties Required for Fluent Modeling**

	Density (lbm/ft <sup>3</sup> )	Dynamic Viscosity (lbm/ft-s)	Thermal Conductivity (Btu/s-in-R)	Specific Heat Capacity (Btu/lbm-R)
Oil	53.499	0.0232	$3.472 * 10^{-6}$	0.50
Air	0.0696	$1.292 * 10^{-5}$	0.150	0.24

### Temperature and Pressure Drop

The Reynolds number was then calculated to apply the appropriate equations and analyze temperature and pressure drop across the heat exchanger and duct. For this, the oil inlet velocity was calculated to be 1.925 ft/s with a volumetric flow rate of 6 gpm at maximum operating load and pipe cross sectional area of 1 in<sup>2</sup>. The Reynolds number for air in both locations indicates turbulent flow while the Reynolds number of oil indicates laminar flow, which is expected. The Reynolds number for air decreases significantly across the heat exchanger because velocity and momentum will decrease due to friction and flow separation as air flows over the tubes.

**Table 5: Calculated Reynolds Number for Air and Oil**

	Reynolds Number
Air (Duct Outlet/Heat Exchanger Inlet)	237111
Air (Heat Exchanger Outlet)	2843
Oil (Heat Exchanger Inlet)	370

From the design requirement restrictions, the entrance and exit temperatures of the oil at maximum operating load are 350°F and 240°F, resulting in a temperature drop of 110°F. Since it is assumed that there is no net heat loss to the surroundings, the magnitude of heat transfer from the hot oil stream is equivalent to the heat transfer from the cold air stream,  $-Q_{oil} = Q_{air}$ . Using this thermodynamic theory and equation for heat transfer  $\dot{Q} = \dot{m}C_p\Delta T$ , the approximate temperature difference of the air was calculated to be approximately 38°F. Since the heat exchanger should reject heat when the air is 110°F, the outlet temperature should be approximately 148°F.

**Table 6: Calculated Temperature Drops in Heat Exchanger**

	Inlet Temperature (°F)	Outlet Temperature (°F)	Temperature Difference (°F)	Rate of Heat Transfer $\dot{Q}$ (Btu/s)
Oil	350	240	110	-39.335
Air	110	148	38	39.335

The pressure drop was calculated for the air across the duct and heat exchanger and oil across the heat exchanger, by using the Darcy-Weisbach equation. Various specifications were made to account for the material and geometry of the design. Frictional factors were calculated based on whether the flow was turbulent or laminar. Specifically, the pressure drop for air across the duct is extremely turbulent and requires a relative roughness value to calculate frictional factor. According to the Engineering Toolbox, aluminum has a relative roughness of  $6.56 \times 10^{-5}$  ft [3]. The pressure drop for air across the heat exchanger is similarly calculated, accounting for a total of 91 pipe channels. The channels are defined as the horizontal parts of the pipe that the air flows past.

The overall pressure drops for oil account for major and minor pressure loss. Major loss occurs due to friction between the oil and the pipe walls while minor loss occurs from disturbance in the geometry from factors such as fittings and expansions or contractions. The geometry has a total of 90 bends in the pipes. From the Engineering Toolbox, it is assumed that the bends are return bends, flanged 180° with a minor loss coefficient  $k = 0.2$  [4]. Since the flow of oil is laminar, the friction factor was calculated with the following equation. The oil has a higher-pressure drop of 24.329 psi due to its relatively high viscosity and the number of bends that result in frictional losses, flow separation into eddies, etc. Full detailed calculations for all tabulated and mentioned results can be found in Appendix I.

**Table 7: Calculated Pressure Drops in Duct and Heat Exchanger**

	Pressure Drop (psi)
Air (Duct)	0.0296
Air (Heat Exchanger)	0.000166
Oil (Heat Exchanger)	24.329

**Table 8: Overall Temperature Change and Pressure Drop Across Device**

	Overall Temperature Change (°F)	Overall Pressure Drop (psi)
Oil	110	24.329
Air	38	0.0298

# Ansys Fluent

## Mesh Parameters

A high-quality structured mesh was generated in ANSYS Fluent to ensure accurate resolution of flow features within the heat exchanger and inlet duct. Multizone method, body sizing, and inflation layers were applied to capture near-wall gradients while maintaining numerical efficiency.

The table below shows the meshing method and body sizing used for the duct and air and oil body in the heat exchanger. The mesh for the duct and oil body of the heat exchanger was meshed using the Multizone method to produce a hexahedral-dominant topology appropriate for internal flows. This maintains a structured element alignment along the flow direction and minimizes numerical diffusion. The external air body in the heat exchanger was meshed using the automatic method, which is well suited for complex external geometries. Compared to the 0.15-inch body sizing used for the heat exchanger, a coarser 0.5-inch body sizing was applied to the duct to reduce the total element count without compromising flow development accuracy.

**Table 9: Method and Body Sizing Used**

	Method	Body Sizing (in)
Duct	Multizone	0.5
Air Body (Heat Exchanger)	Automatic	2.0
Oil Body (Heat Exchanger)	Multizone	0.2

To resolve the boundary layer behavior of the relatively viscous oil, inflation layers were added with a first layer height of 0.063 inches. A Y+ Wall Distance Estimation calculator was used to compute the value for first layer height and an estimation of Reynolds number [5]. The calculation was based on a target Y+ and flow properties, such as freestream air velocity, density, and dynamic viscosity. The Reynolds number in this region is approximately 370, agreeing with the hand-calculation. The wall spacing ensures that the first cell lies within the viscous sublayer.

Inflation layers were similarly computed for air for the duct and heat exchanger. For the heat exchanger, the first layer thickness placed the adjacent node within the

logarithmic region of the turbulent boundary layer for a target  $Y^+$  of approximately 200, making it compatible with wall function turbulence models. For the duct, the inflation layers resolved turbulent boundary layer along the walls, ensuring appropriate resolution within the log-law region. The parameters and results for computing the first layer height values are tabulated below.

**Table 10: Computational Parameters ( $Y^+$  Wall Distance Estimation)**

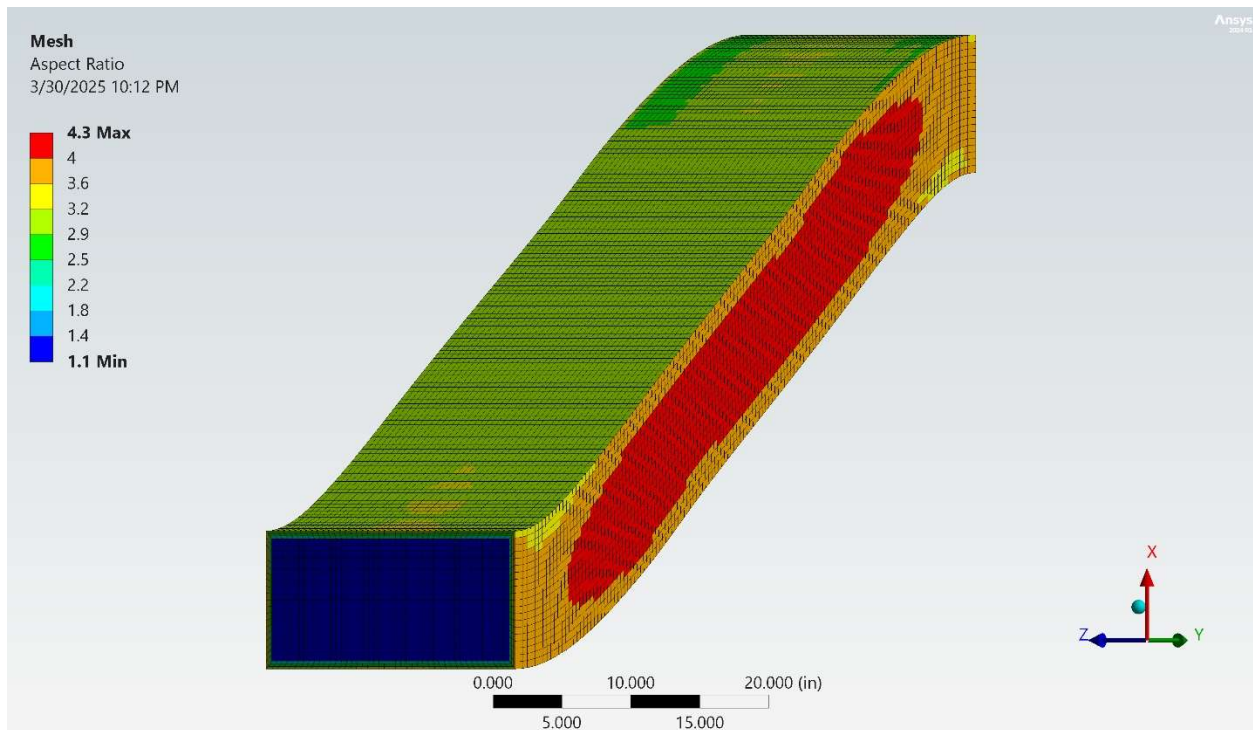
	Freestream Air Velocity (ft/s)	Density (lbm/ft <sup>3</sup> )	Dynamic Viscosity (lbm/ft-s)	Boundary Layer Length (in)	Desired $Y^+$
Air (Duct)	39.600	0.0696	$1.292 * 10^{-5}$	13.333	200
Air (Heat Exchanger)	35.523	0.0696	$1.292 * 10^{-5}$	13.750	200
Oil (Heat Exchanger)	1.925	53.499	0.0232	1.000	5

**Table 11: Results for Reynolds Number and Estimated Wall Distance**

	Reynolds Number	Estimated Wall Distance (in)
Air (Duct)	240000	0.209
Air (Heat Exchanger)	220000	0.240
Oil (Heat Exchanger)	370	0.063

## Mesh

Various views and cross-sections of the duct and heat exchanger mesh are shown below. The meshes are highlighted with aspect ratios, which measure the stretching of a cell. Lower aspect ratios closer to 1 indicate higher quality cells while higher aspect ratios indicate lower quality, resulting in numerical inaccuracy or instability. The mesh for the duct is relatively uniform and has a low aspect ratio, due to the simplicity of the geometry. Higher aspect ratios are concentrated towards the middle of the sides of the duct.



*Figure 3: Overview of Duct Mesh*

The aspect ratio at the cross-section and inlet of the duct are close to 1, indicating high quality cells.

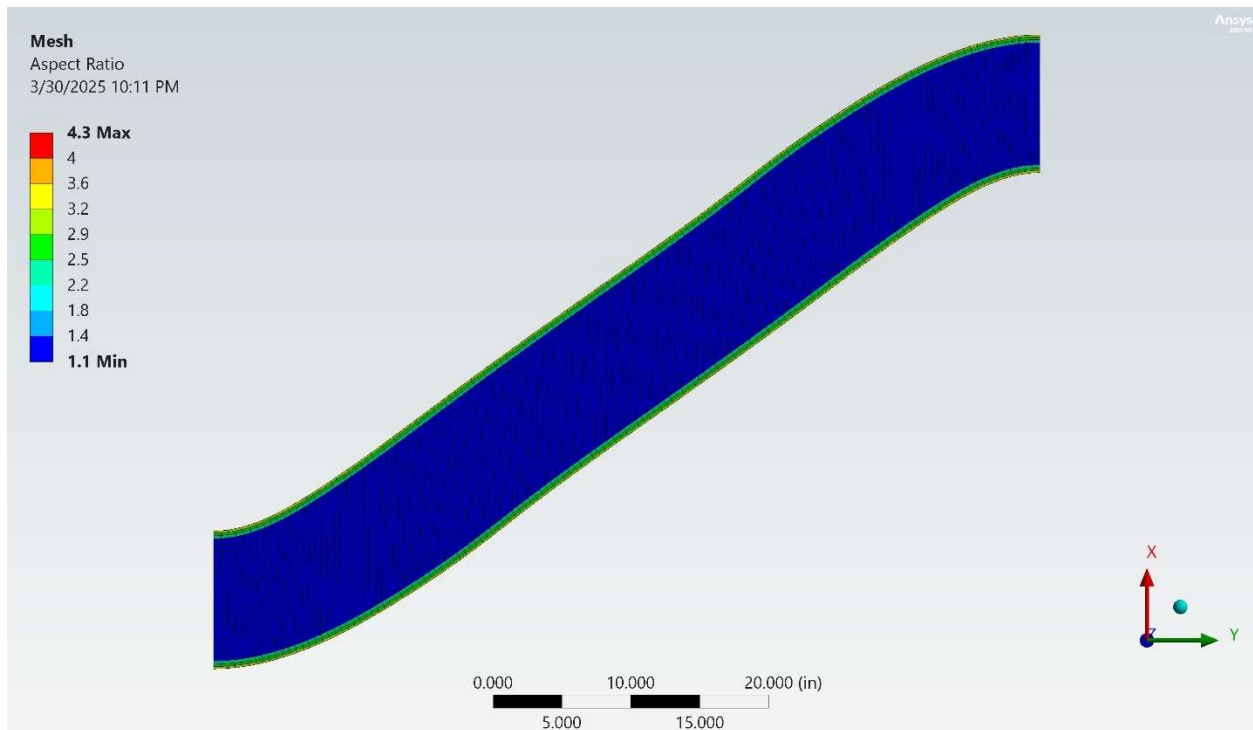


Figure 4: Cross-Section of Side View of Duct Mesh

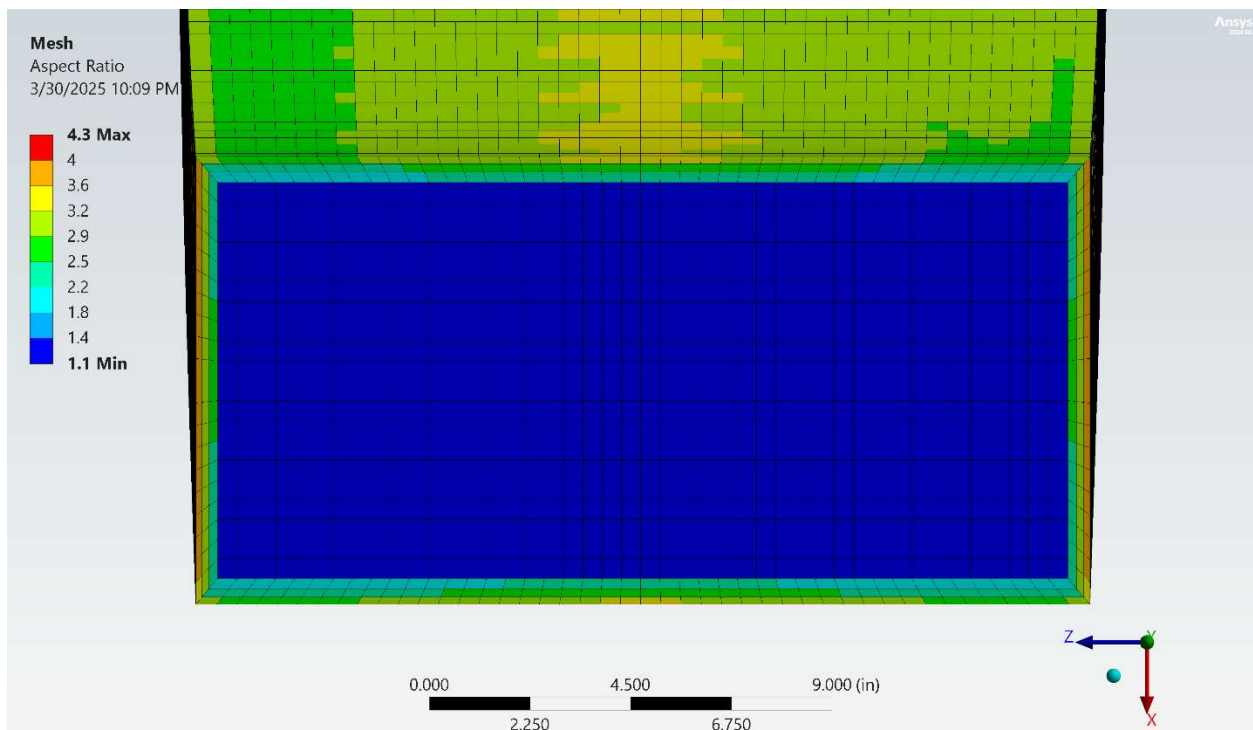


Figure 5: Close-Up of Mesh at Duct Inlet

The aspect ratio for the heat exchanger varies more and is generally higher, due to the complexity of the geometry. Lower aspect ratios are concentrated at the oil pipes and surrounding air while higher aspect ratios are concentrated down the heat exchanger, with free air flow.

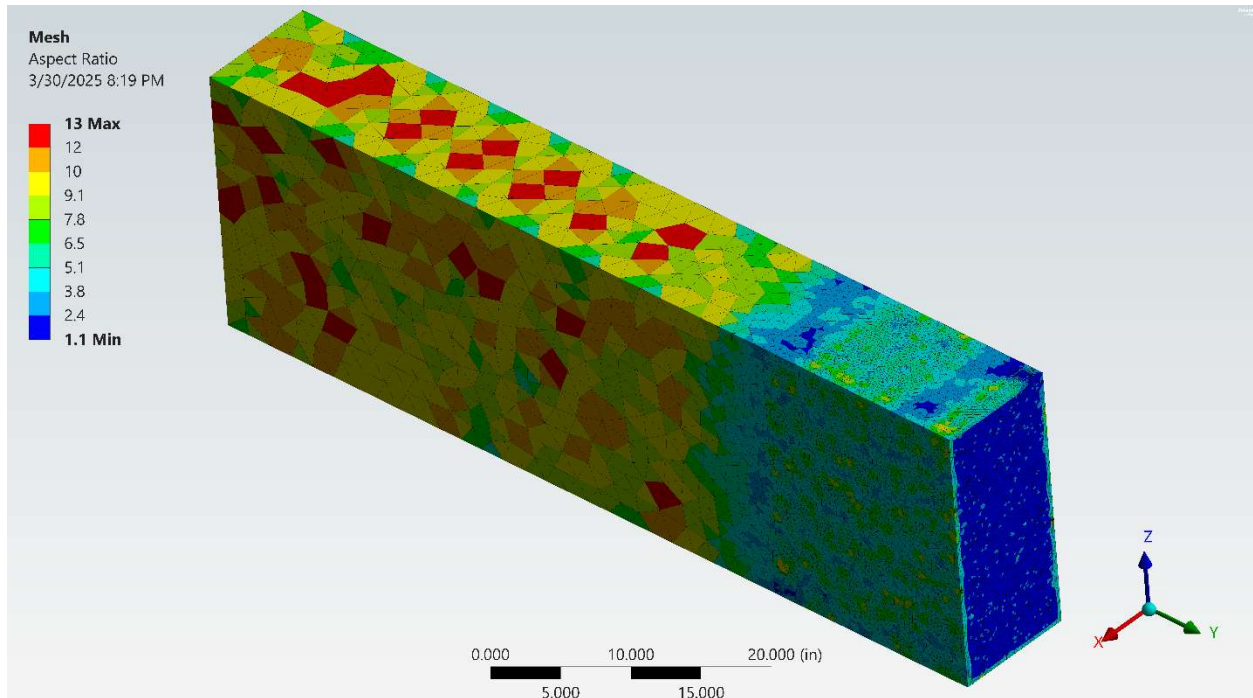


Figure 6: Overview of Heat Exchanger Mesh

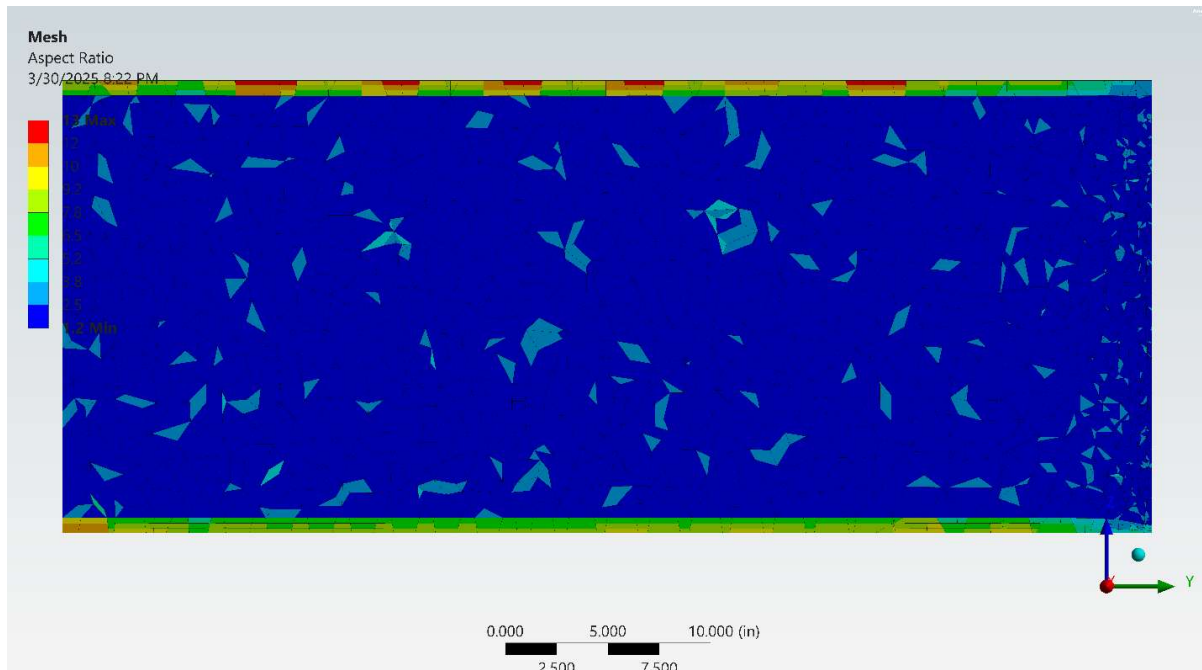
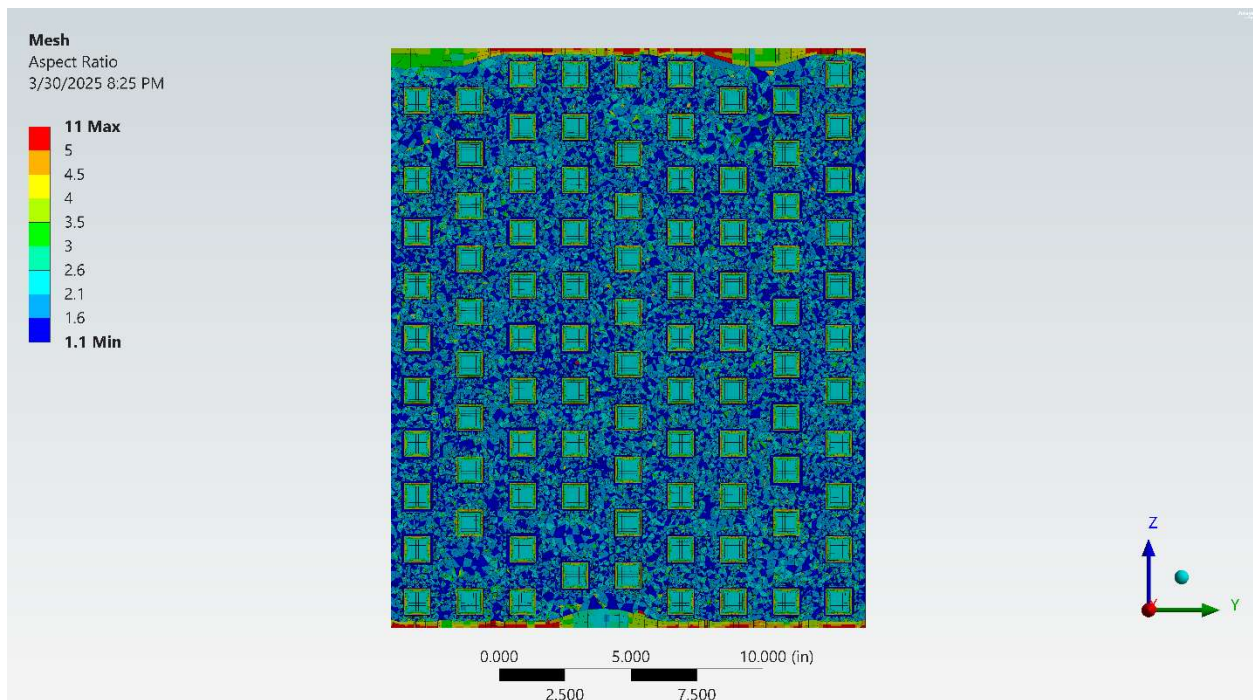
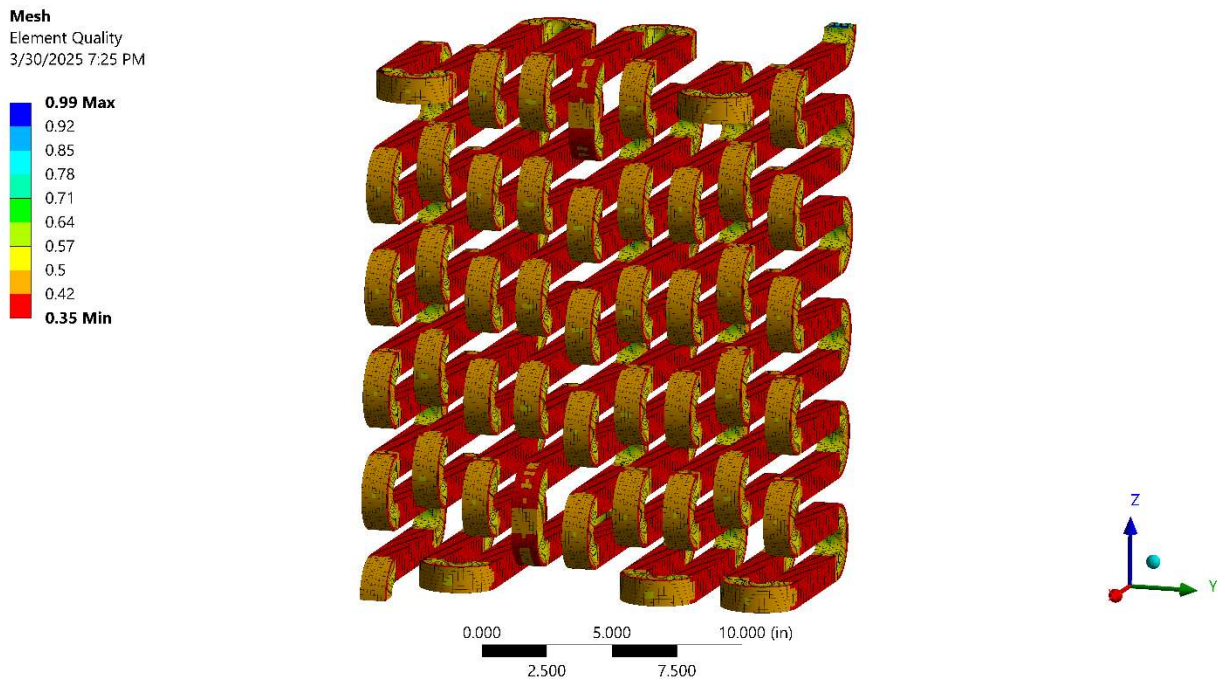


Figure 7: Full Developed Flow Area Mesh



The element quality decreases greatly at the tube rows, where the quality is slightly higher at the bends. However, the quality is above 0.1 for all elements.



Mesh  
Aspect Ratio  
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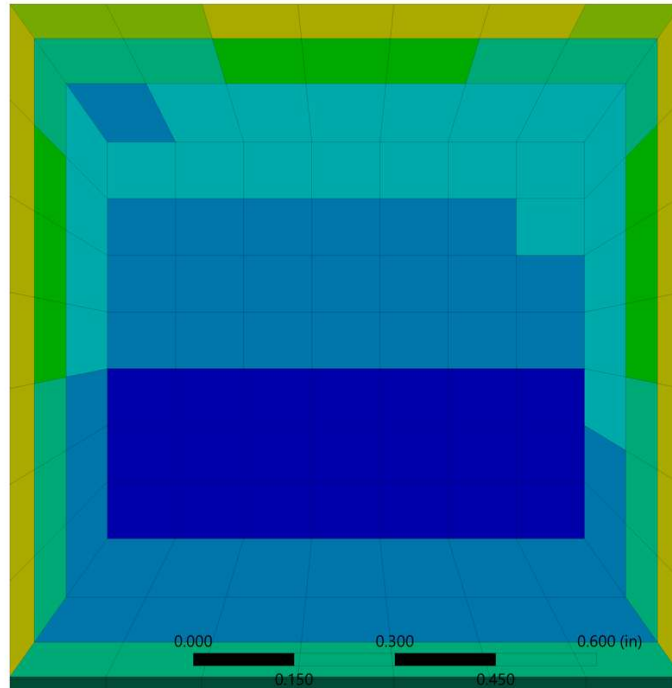
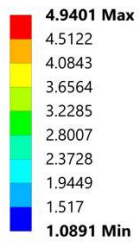


Figure 10: Oil Inlet Mesh

Mesh  
Aspect Ratio  
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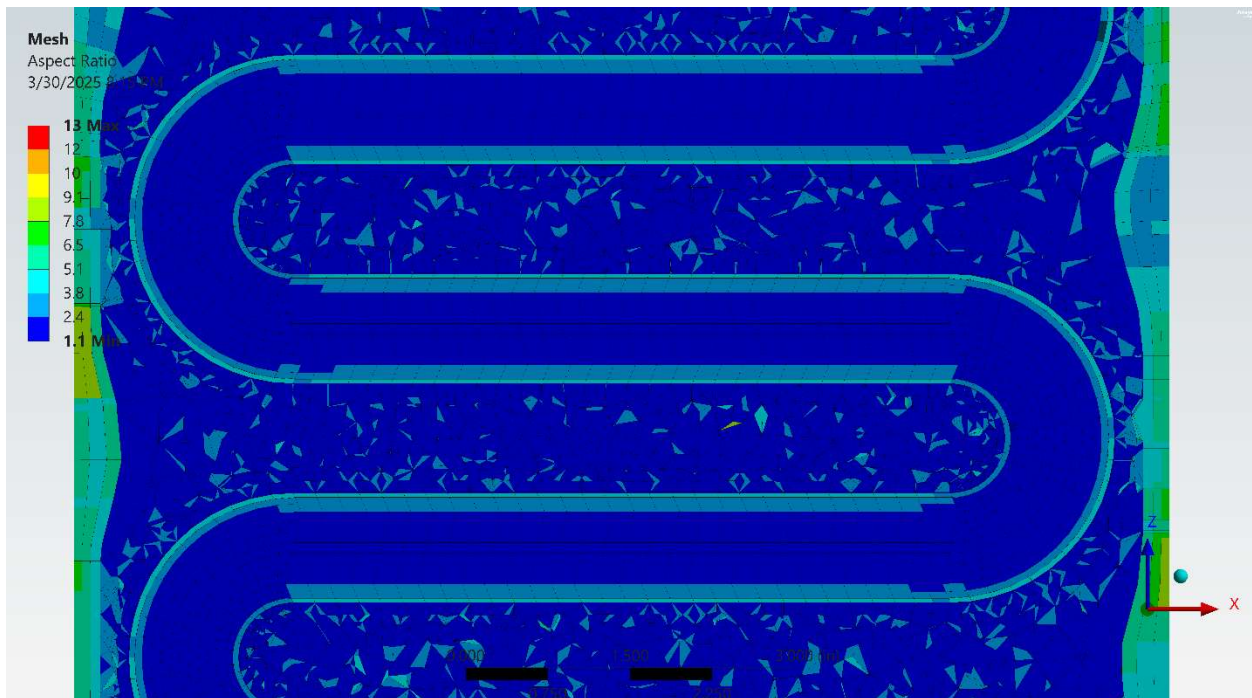
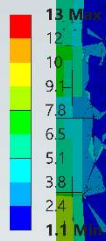


Figure 11: Close-Up of Side View of Pipe Mesh

## Material Property Assignment

The material for the duct and heat exchanger was selected to be aluminum, as a standard durable material to withstand air flow. The material for the tubes inside the heat exchanger was selected to be copper as copper has a high thermal conductivity and allows for efficient heat transfer. For the material properties listed in the table, heat capacity and thermal conductivity of aluminum are ignored as temperature change of air was not considered for the duct.

**Table 12: Material Properties for Duct and Heat Exchanger Pipe**

Material	Density (lbm/ft <sup>3</sup> )	Heat Capacity (Btu/lbm-R)	Thermal Conductivity (Btu/h-ft-R)
Aluminum	169.74		
Copper	560.48	0.091	224.01

## Dry Weight Estimate

From Onshape, the volumes of the duct walls, heat exchanger shell, and pipe walls were taken. Using the densities of the respective materials, aluminum and copper, the dry weight of each part and overall device was calculated.

**Table 13: Dry Weight Calculations**

	Volume (in <sup>3</sup> )	Weight (lbm)
Duct Wall	435.582	42.787
Heat Exchanger Shell	114.28	11.226
Pipe Wall	374.574	121.493
Total		175.506

## Boundary Conditions

The oil and air domains were subjected to suitable boundary conditions to precisely describe the temperature and flow behavior of both fluids in the system. All simulations described used the same boundary conditions.

When simulating the duct, the velocity and temperature at the air inlet were defined by the design requirements. The velocity corresponded to a vehicle moving at 27 mph or 39.6 ft/s and the temperature was set to 110°F. The outlet of the duct and heat exchanger were modeled as a pressure outlet with zero-gauge pressure. To maintain consistent mass flow through the system, the outlet conditions were set to allow backflow of ambient air at 110°F, although negligible backflow occurred in simulations. The turbulent intensity was calculated using the formula  $I = 0.16 \cdot Re^{-1/8}$  with a Reynolds number of 237112, resulting in a value of 3.41%. This is consistent with external forced convection.

When simulating the heat exchanger, the boundary conditions were directly imported into the air inlet. Specifically, the resultant velocity at the air outlet of the duct was imported into the air inlet of the heat exchanger. At the oil inlet, the velocity boundary condition corresponded to a volumetric flow rate of 6 gallons per minute at maximum operating load and temperature was set to 350°F. Similarly, the outlet was modeled as a pressure outlet with zero-gauge pressure to allow for full development of the thermal and velocity fields. The turbulent intensity was calculated to be 7.64% with a Reynolds number of 370, appropriate for internal laminar flow. The oil and air backflow were set to 350°F and 110°F, respectively.

Thermal boundary conditions were also applied to all relevant solid-fluid interfaces. The tube walls were assumed to be made of copper and set to participate in conjugate heat transfer, with the air and oil exchanging heat across the wall without any external heat loss to the surroundings. The outer walls of the duct were modeled as adiabatic, assuming negligible heat transfer to ambient air outside the system. The table below summarizes the boundary conditions applied.

**Table 14: Boundary Conditions for Duct Simulation**

Boundary/Domain	Type	Value
Air Inlet	Velocity	39.6 ft/s
Air Inlet	Temperature	110°F
Air Inlet	Turbulence Intensity	3.41%
Air Outlet	Pressure Outlet	0 psi (gauge)
Duct Wall	External Surface	Adiabatic

**Table 15: Boundary Conditions for Heat Exchanger Simulation**

Boundary/Domain	Type	Value
Air Inlet	Velocity, Temperature, etc.	Imported Values
Oil Inlet	Velocity	1.925 ft/s
Oil Inlet	Temperature	350°F
Oil Inlet	Turbulence Intensity	7.64%
Air & Oil Outlet	Pressure Outlet	0 psi (gauge)
Tube Wall (Fluid-Solid Interface)	Conjugate Heat Transfer	-

## Results

### Single Tube Row Test (CFD Simulation)

The final geometry of the crossflow heat exchanger includes 9 staggered rows of tubes for oil flow. Before finalizing the design, a simulation was run on a single tube row with the same boundary conditions. This allowed for preliminary analysis of the efficiency of the design without having to run simulation on a much larger and more complex geometry, saving computational time. From this, the final design was able to be adjusted.

After running a preliminary simulation, the area-weighted average was taken for the total pressure and static temperature and the inlets and outlets for oil and air.

**Table 16: Temperature Difference for Single Tube Heat Exchanger**

	Inlet Temperature (°F)	Outlet Temperature (°F)	Temperature Difference (°F)
Air	110.00	113.75	3.75
Oil	350.00	334.00	16.03

**Table 17: Pressure Drop for Single Tube Heat Exchanger**

	Inlet Pressure (psi)	Outlet Pressure (psi)	Pressure Drop (psi)
Air	0.0509	0.0103	0.0407
Oil	0.0990	0.0381	0.0608

From the results for a single tube row, the temperature of the air increased by approximately 3.75°F and the temperature of the oil cooled by 16.03°F. The results indicate that in order to achieve a desired temperature drop of 110°F for oil, the heat exchanger must have at least 7-8 tube rows to fully cool the oil. The final heat exchanger was designed to have 9 tube rows to create a safe overestimate of the required tube length. This overdesign is also shown by the excess pipe surface area, as the current design has a pipe surface area of 3427 in<sup>2</sup> versus the required surface area of 2322 in<sup>2</sup> for a laminar model.

## Temperature Contour

The outlet temperature for air was 139°F, rising from 110°F. The outlet temperature for oil was 271°F, cooling from 350°F. This shows that the heat exchanger was not able to fully cool the oil down to 240°F. The graphs for convergence of temperature, velocity, and residuals can be viewed in Appendix II.

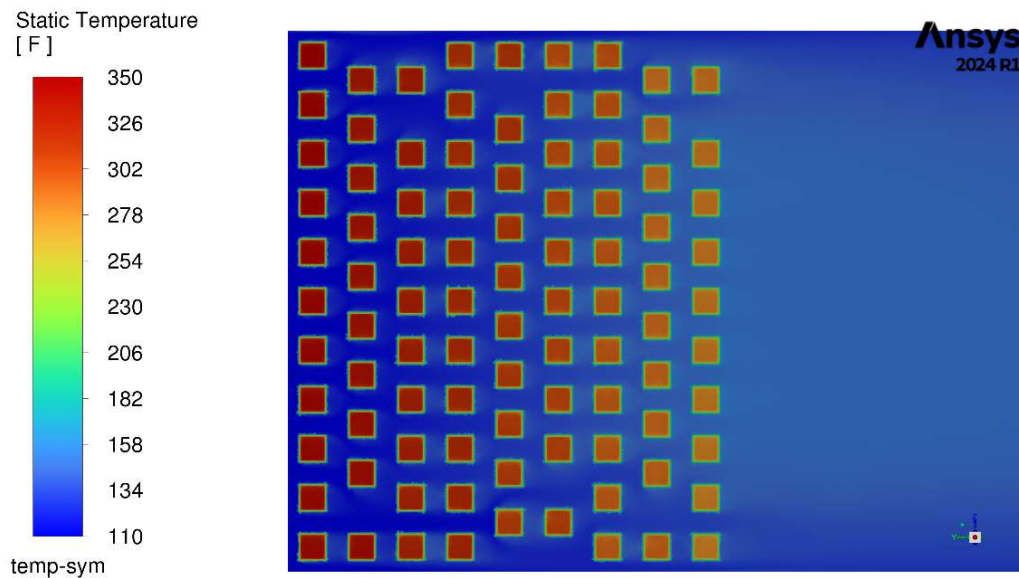


Figure 12: Heat Exchanger Temperature Contour

## Pressure Contour

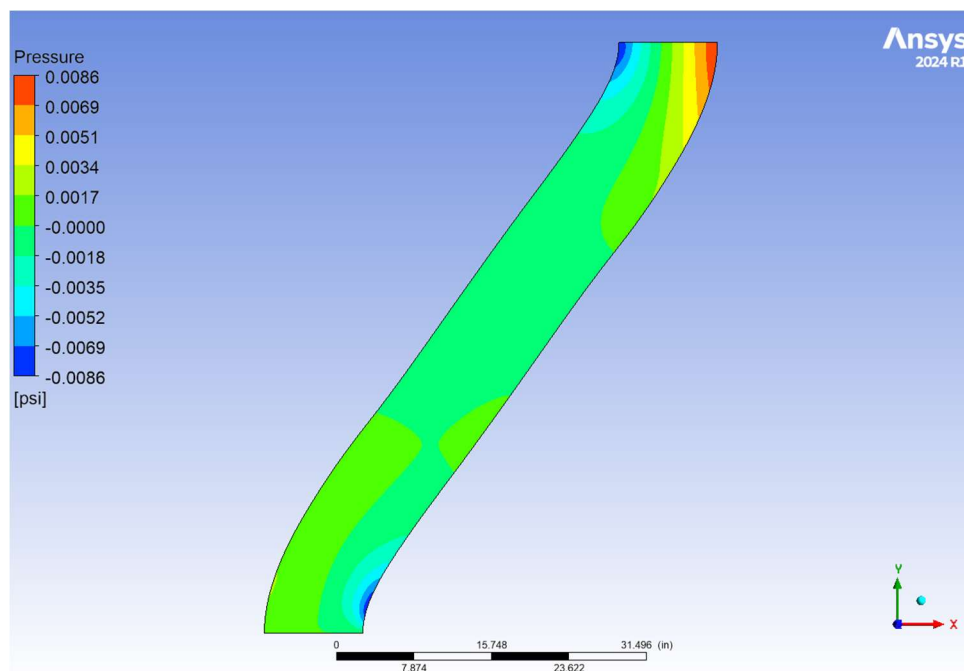
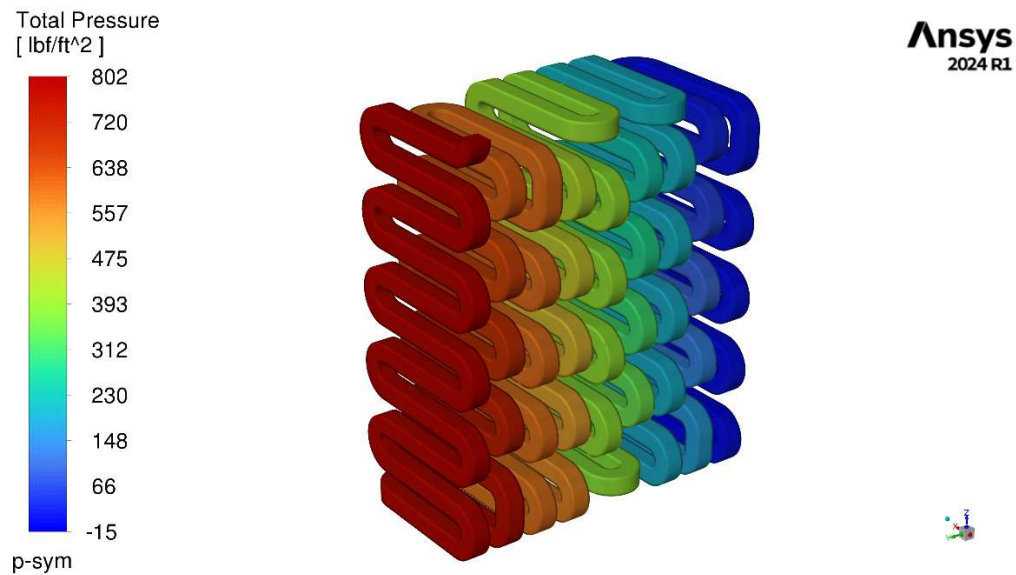
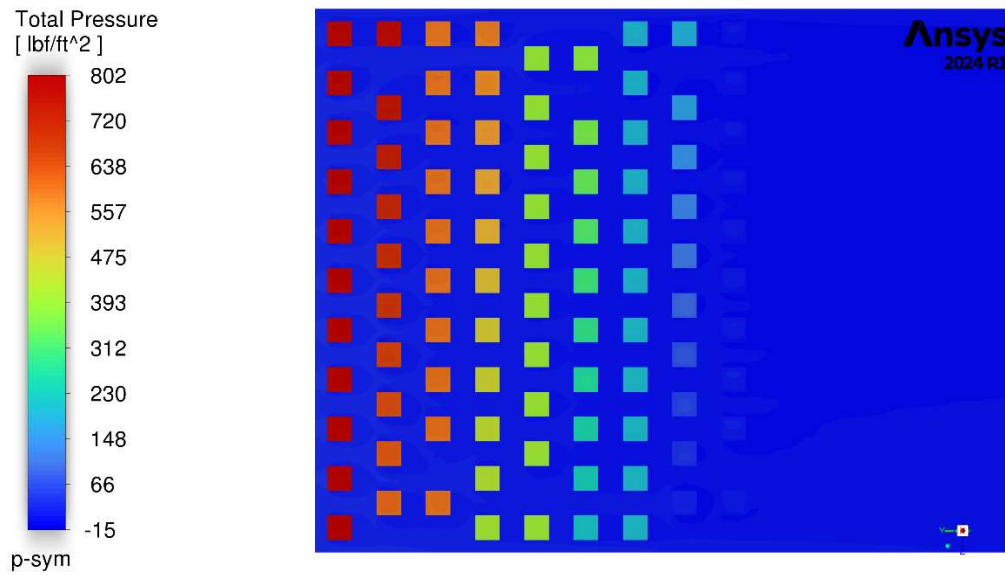


Figure 13: Duct Pressure Contour







## Velocity Contour

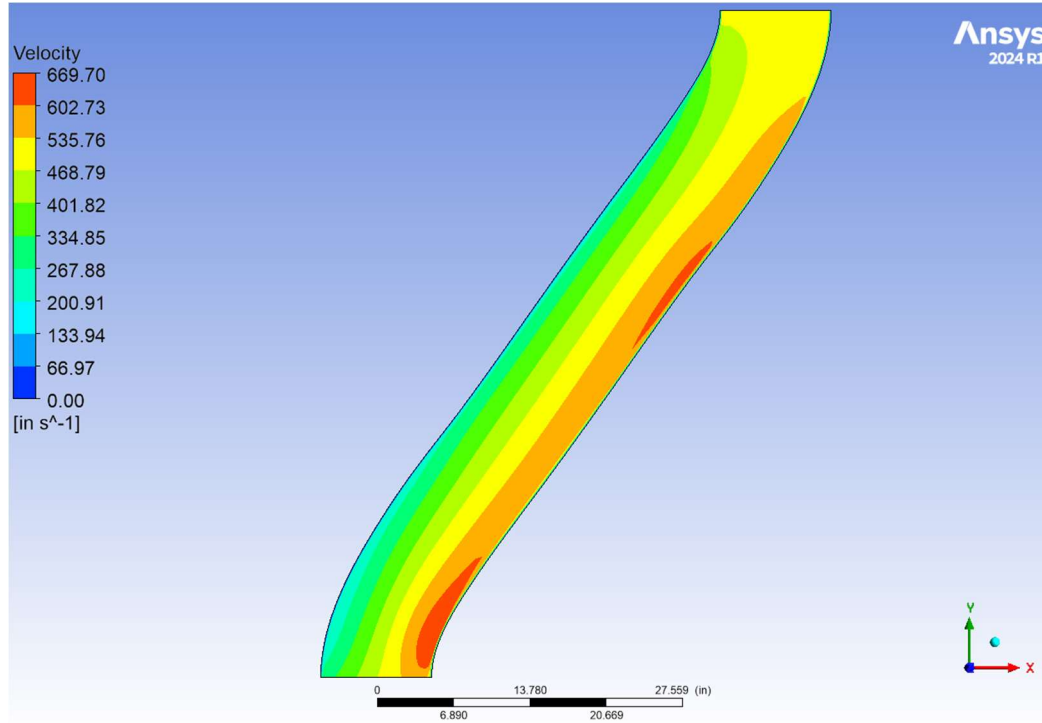


Figure 16: Duct Velocity Contour

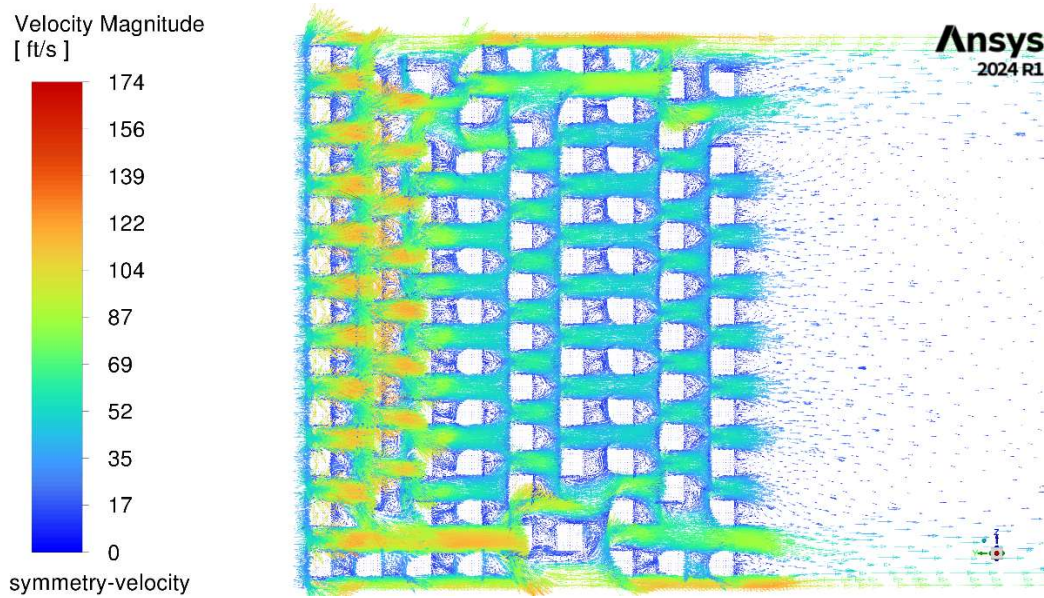


Figure 17: Heat Exchanger Velocity Vector about Symmetry

## Heat Exchanger Efficiency

From the simulation results, the heat exchanger efficiency was calculated from the actual heat transfer against the maximum heat transfer. With an oil inlet temperature of 350°F and outlet temperature of 271°F,  $\dot{Q} = \dot{m}C_p\Delta T$  was used to calculate an actual heat transfer rate of 27.936 Btu/s. With the oil inlet and air inlet temperature, the maximum heat transfer was calculated to be 85.821 Btu/s. From this, the heat efficiency is 32.55%. This is a reasonable value as the heat exchanger was not fully able to cool the oil down to 240°F.

## Result Comparison

In comparison to the hand-calculated results, the oil outlet temperature did not reach the desired 240°F and converged to 271°F instead, resulting in a 29°F difference. The air outlet temperature correspondingly did not reach the expected 148°F and converged to 139°F, resulting in a 79°F change. This is reasonable due to the heat transfer between streams.

The simulated air pressure drop across the duct is lower than the hand-calculated value while the air in heat exchanger is higher. The simulated overall air pressure drop is greater than the hand-calculated result. The simulated oil pressure drop is significantly lower than the hand-calculated value. These discrepancies may be a result of an overestimation or assumption made about the geometry in the hand-calculations.

**Table 18: Hand-Calculation vs Simulation Results**

	Hand-Calculation Result	Simulation Result
Air Outlet Temperature	148°F	139°F
Oil Outlet Temperature	240°F	271°F
Air Pressure Drop Across Duct	0.03 psi	0.0023 psi
Air Pressure Drop Across Heat Exchanger	0.0002 psi	0.18 psi
Overall Air Pressure Drop	0.03 psi	0.1823 psi
Oil Pressure Drop Across Heat Exchanger	24.3 psi	5.5 psi

## Discussion

The heat exchanger did not meet the required temperature change to 240°F; it was 31°F higher, at 271°F. The current design has 9 rows of tubing. As this configuration stands, there is an average temperature drop of 8.8°F per tube. At this rate, an additional 4 tubes, totaling 13 tubes, are needed to reach below the 240°F design requirement. Besides this, the heat exchanger remains relatively compact at 10" x 18" x 20". An addition of 4 tubes would increase the 18" width to 26", which is still relatively compact. CFD simulations helped the design meet compact design requirements whilst providing effective heat transfer. CFD allows the final design to be scaled to meet heat exchange requirements easily within relatively small-time constraints.

## Design Time

The cumulative design time was approximately 100 hours. Hand calculation took 10 hours, designing the heat exchanger, meshing, setup, and simulation took approximately 90 hours.

## Appendix I: Hand-Calculation Verification

For further calculations, the density of oil was calculated using the given specific gravity of oil 0.86 and ambient air density 0.036 lbm/in<sup>3</sup>. The velocity of oil at the heat exchanger inlet was calculated using the given flow rate of 6 gpm and cross-sectional area.

$$\begin{aligned}\rho_{oil} &= SG_{oil} \cdot \rho_{air, ambient} = 0.86 \cdot 0.036 \frac{lb}{in^3} = 0.0310 \frac{lb}{in^3} \\ &= 53.499 \frac{lb}{ft^3}\end{aligned}$$

$$A_{pipe} = 1.0 \text{ in} * 1.0 \text{ in} = 1.00 \text{ in}^2$$

$$\begin{aligned}v_{oil, inlet} &= \frac{Q_{pipe}}{A_{pipe}} = 6 \text{ gpm} \cdot 3.85 \frac{in^3}{s} \cdot \frac{1}{gpm} * 1.00 \text{ in}^2 = 23.100 \frac{in}{s} \\ &= 1.925 \frac{ft}{s}\end{aligned}$$

The hydraulic diameter for the duct inlet and outlet and pipes were calculated. The hydraulic diameter for the duct outlet and heat exchanger air inlet are the same.

$$\begin{aligned}D_{duct, inlet} &= 4 \cdot \frac{A_{duct, inlet}}{P_{duct, inlet}} = 4 \cdot \frac{200 \text{ in}^2}{2 \cdot (20 \text{ in} + 10 \text{ in})} = 13.333 \text{ in} \\ &= 1.111 \text{ ft}\end{aligned}$$

$$\begin{aligned}D_{HE, inlet} &= D_{duct, outlet} = 4 \cdot \frac{A_{HE, duct}}{P_{HE, duct}} = 4 \cdot \frac{220 \text{ in}^2}{2 \cdot (22 \text{ in} + 10 \text{ in})} \\ &= 13.750 \text{ in} = 1.146 \text{ ft}\end{aligned}$$

$$\begin{aligned}D_{pipe} &= 4 \cdot \frac{A_{tube, inlet}}{P_{tube, inlet}} = 4 \cdot \frac{1.00 \text{ in}^2}{2 \cdot (1.00 \text{ in} + 1.00 \text{ in})} = 1.000 \text{ in} \\ &= 0.0833 \text{ ft}\end{aligned}$$

The Reynolds number for the oil in the heat exchanger and air in both the heat exchanger and duct were calculated.

$$Re = \frac{\rho v D}{\mu}$$

$$Re_{air,duct} = \frac{\left(0.03096 \frac{lb}{in^3} \cdot 23.100 \frac{in}{s} \cdot 1 in\right)}{0.001932 \frac{lbm}{in s}} = 370$$

$$Re_{oil,HE} = \frac{\left(4.03 \cdot 10^{-5} \frac{lb}{in^3} \cdot 475.2 \frac{in}{s} \cdot 13.333 in\right)}{1.077 \cdot 10^{-6} \frac{lbm}{in s}} = 237112$$

To specifically solve for the Reynolds number of air in the heat exchanger, the volumetric flow rate at the inlet, free area, and velocity were calculated to account for rows of tubes in the air body. The volumetric flow rate at the inlet of the heat exchanger was calculated using conservation of mass of air in the duct.

$$Q = A \cdot v$$

$$Q_{duct,inlet} = 39.6 \frac{ft}{s} \cdot 1.388 ft^2 = 54.996 \frac{ft^3}{s}$$

$$A_{duct,inlet} \cdot v_{duct,inlet} = A_{duct,outlet} \cdot v_{duct,outlet}$$

$$Q_{duct,outlet} = Q_{HE,inlet} = \frac{1.389 ft}{1.528 ft} \cdot 39.6 \frac{ft}{s} \cdot 1.528 ft = 50.00 \frac{ft^3}{s}$$

$$A_{free} = A_{air} - A_{tube} = 220 ft^2 - 100.55 ft^2 = 119.45 ft^2$$

$$v_{free} = \frac{Q_{air,inlet}}{A_{free}}$$

$$v_{free} = \frac{Q_{air,inlet}}{A_{free}} = \frac{50.00 \frac{ft^3}{s}}{119.45 ft^2} = 0.460 \frac{ft}{s}$$

$$Re_{air,HE} = \frac{\left(0.0696 \frac{lb}{ft^3} \cdot 0.46 \frac{ft}{s} \cdot 1.146 ft\right)}{0.001932 \frac{lb}{ft s}} = 2843$$

For the boundary conditions at the oil and air inlet of the heat exchanger, the turbulent intensity was calculated.

$$I = 0.16 \cdot Re^{-\frac{1}{8}}$$

$$I_{oil} = 0.16 \cdot 370^{-\frac{1}{8}} \cdot 100\% = 7.640\%$$

$$I_{air} = 0.16 \cdot 237112^{-\frac{1}{8}} \cdot 100\% = 3.406\%$$

From the given inlet and outlet temperature conditions of oil, the rate of heat transfer from the hot oil stream was calculated. The rate of heat transfer out of the hot oil stream is equivalent to the rate of heat transfer into the cold air stream.

$$\dot{Q} = \dot{m}C_p\Delta T$$

$$\Delta T_{oil} = 350^\circ F - 240^\circ F = 110^\circ F$$

$$\dot{Q}_{oil} = \left(0.715 \frac{lb}{s}\right) \left(0.5 \frac{Btu}{lbm R}\right) (350 - 240) R = 39.335 \frac{Btu}{s}$$

$$-\dot{Q}_{oil} = \dot{Q}_{air}$$

$$\begin{aligned} \dot{m}_{air} &= v_{air} \cdot A_{HE,inlet} \cdot \rho_{air} = 475.2 \frac{in}{s} \cdot 220 in^2 \cdot 4.03 \cdot 10^{-5} \frac{lb}{in^3} \\ &= 4.213 \frac{lb}{s} \end{aligned}$$

$$\Delta T_{air} = \frac{\dot{Q}_{air}}{\dot{m}_{air}C_{p,air}} = \frac{39.335 \frac{Btu}{s}}{4.213 \frac{lb}{s} \cdot 0.24 \frac{Btu}{lbm R}} = 38.90^\circ F$$

Using a Nusselt's number of 0.46, the required surface area for a laminar model was calculated. The required surface area was then compared to the actual surface area to analyze the overdesign of the model.

$$Nu = \frac{hD}{k}$$

$$h_{oil} = \frac{Nu \cdot k}{D} = \frac{\left(4.6 \cdot 3.472 \cdot 10^{-6} \frac{Btu}{s \cdot in \cdot R}\right)}{0.104 \text{ in}} = 0.000154 \frac{Btu}{s \cdot in^2 \cdot R}$$

$$\dot{Q} = h \cdot A \cdot \Delta T$$

$$A = \frac{\dot{Q}}{h \cdot \Delta T} = \frac{39.335 \frac{Btu}{s}}{0.000154 \frac{Btu}{s \cdot in^2 \cdot R} \cdot (350 - 240) R} = 2321.731 \text{ in}^2$$

The pressure drop for air across the duct was calculated using the Darcy-Weisbach equation. Since the flow of air is assumed to be turbulent from its Reynolds number, the friction factor was calculated with the following equation. A relative roughness of  $6.56 \cdot 10^{-5}$  ft was used.

$$\frac{1}{\sqrt{f}} = -2 \log \left( \frac{\frac{k}{D}}{3.7} + \frac{2.52}{Re \sqrt{f}} \right)$$

$$\frac{1}{\sqrt{f}} = -2 \log \left( \frac{\frac{6.56 \cdot 10^{-6} \text{ ft}}{1.111 \text{ ft}}}{3.7} + \frac{2.52}{237111 \sqrt{f}} \right)$$

$$f \approx 0.0151$$

$$\Delta P = f \cdot \frac{L}{D} \cdot \frac{\rho v^2}{2} D_{h, duct \text{ avg}} = \frac{(1.111 \text{ ft} + 1.456 \text{ ft})}{2} = 1.28 \text{ ft}$$

$$\begin{aligned}\Delta P_{duct,air} &= 0.0151 \cdot \frac{5.831 \text{ ft}}{1.128 \text{ ft}} \cdot \frac{\left(0.0696 \frac{\text{lb}}{\text{ft}^3} \cdot \left(39.6 \frac{\text{ft}}{\text{s}}\right)^2\right)}{2} = 4.268 \text{ psf} \\ &= 0.0296 \text{ psi}\end{aligned}$$

The major, minor, and overall pressure drops were calculated for oil. The geometry has a total of 90 return bends, flanged 180° with a minor loss coefficient  $k = 0.2$ . Since the flow of oil is laminar, the friction factor was calculated with the following equation.

$$\begin{aligned}f &= \frac{64}{Re} = \frac{64}{370} = 0.173 \\ \Delta P_{major,oil} &= 0.173 \cdot \frac{(8.379 \text{ ft})}{0.0833 \text{ ft}} \cdot \frac{\left(53.437 \frac{\text{lb}}{\text{ft}^3} \cdot \left(1.925 \frac{\text{ft}}{\text{s}}\right)^2\right)}{2} \\ &= 1721.193 \text{ psf} = 11.953 \text{ psi}\end{aligned}$$

$$\begin{aligned}\Delta P_{minor} &= k \cdot \frac{\rho v^2}{2} \\ \Delta P_{minor,oil} &= 90 \cdot 0.2 \cdot \frac{\left(53.437 \frac{\text{lb}}{\text{ft}^3} \cdot \left(1.925 \frac{\text{ft}}{\text{s}}\right)^2\right)}{2} = 1782.156 \text{ psf} \\ &= 12.376 \text{ psi}\end{aligned}$$

$$\Delta P_{total,oil} = 11.953 \text{ psi} + 12.376 \text{ psi} = 24.329 \text{ psi}$$

Similarly, the pressure drop for air across the heat exchanger was calculated using the Darcy-Weisbach equation. The air is flowing past a total of 91 channels of pipes.

$$f = 0.24 \cdot Re^{-0.24} = 0.24 \cdot 2843^{-0.24} = 0.0356$$

$$k = f \cdot N = 0.0356 \cdot 91 = 3.238$$



$$\begin{aligned}\Delta P_{HE,air} &= 3.238 \cdot \frac{\left(0.0696 \frac{lb}{ft^3} \cdot \left(0.460 \frac{ft}{s}\right)^2\right)}{2} = 0.0239 \text{ psf} \\ &= 0.000166 \text{ psi}\end{aligned}$$

The overall pressure drop for air could then be calculated.

$$\begin{aligned}\Delta P_{total,air} &= \Delta P_{duct,air} + \Delta P_{HE,air} = 0.0296 \text{ psi} + 0.000166 \text{ psi} \\ &= 0.0298 \text{ psi}\end{aligned}$$

Using the volumes of the individual parts and densities of the respective materials, the dry weight of the device was calculated.

$$W_{pipe,wall} = 374.574 \text{ in}^3 \cdot \frac{1 \text{ ft}^3}{1728 \text{ in}^3} \cdot 169.742 \frac{lb}{ft^3} = 42.787 \text{ lb}$$

$$W_{HE,shell} = 114.280 \text{ in}^3 \cdot \frac{1 \text{ ft}^3}{1728 \text{ in}^3} \cdot 169.742 \frac{lb}{ft^3} = 11.226 \text{ lb}$$

$$W_{pipe,wal} = 374.574 \text{ in}^3 \cdot \frac{1 \text{ ft}^3}{1728 \text{ in}^3} \cdot 560.478 \frac{lb}{ft^3} = 121.493 \text{ lb}$$

$$W_{total} = W_{pipe,wall} + W_{HE,shell} + W_{pipe,shell}$$

$$W_{total} = 42.787 \text{ lb} + 11.226 \text{ lb} + 121.493 \text{ lb} = 175.506 \text{ lb}$$

From the simulation results, the heat exchanger efficiency could then be calculated.

$$\dot{Q}_{actual} = \left(0.715 \frac{lb}{s}\right) \left(0.5 \frac{Btu}{lbm R}\right) (350 - 271) R = 27.936 \frac{Btu}{s}$$

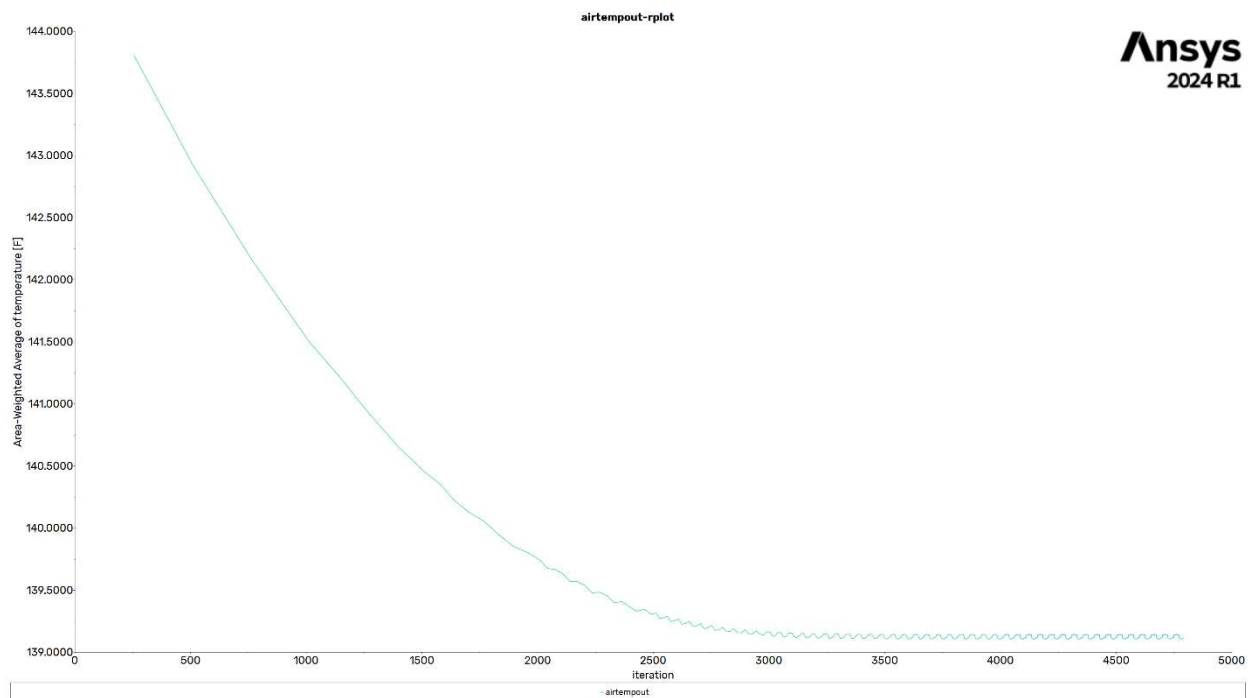
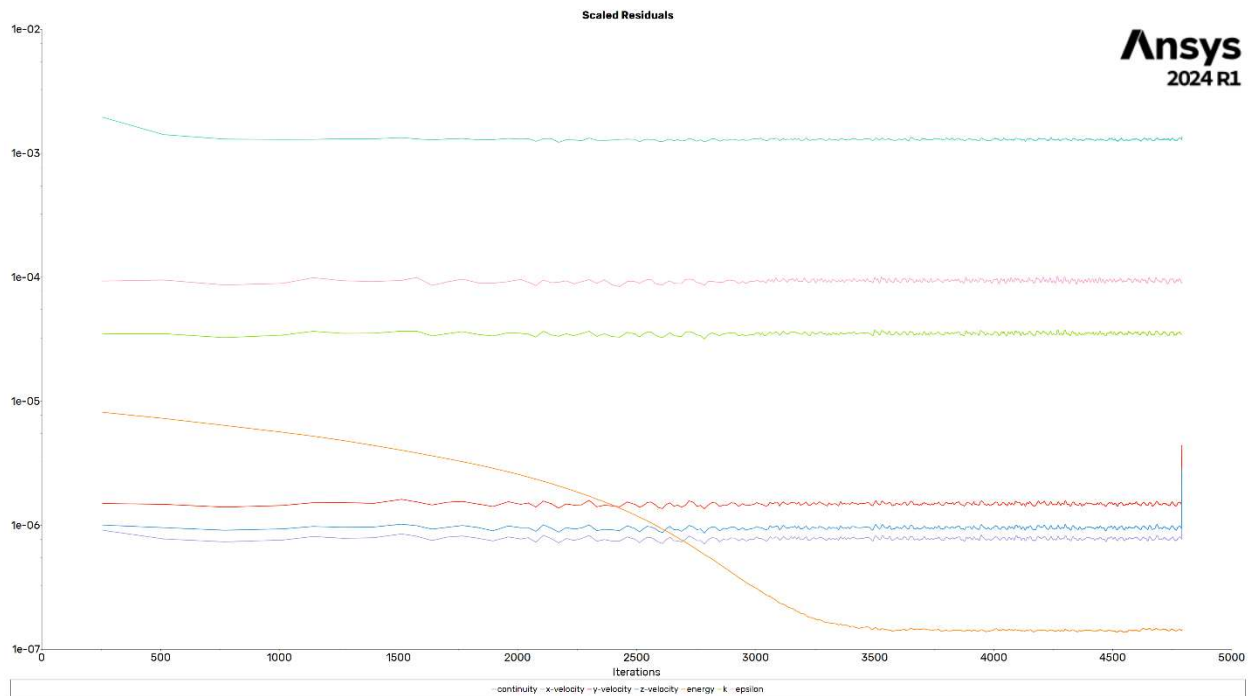
$$\dot{Q}_{max} = C_{min} \cdot (T_{oil, inlet} - T_{air, inlet})$$

$$\begin{aligned} C_{min} &= \min(\dot{m}_{oil} C_{p, oil}, \dot{m}_{air} C_{p, air}) = \min\left(0.358 \frac{Btu}{s R}, 1.011 \frac{Btu}{s R}\right) \\ &= 0.358 \frac{Btu}{s R} \end{aligned}$$

$$\dot{Q}_{max} = 0.358 \frac{Btu}{s R} \cdot (350 - 110) R = 85.821$$

$$\varepsilon = \frac{\dot{Q}_{actual}}{\dot{Q}_{max}} \cdot 100\% = \left(\frac{27.936}{85.821}\right) \cdot 100\% = 32.55\%$$

## Appendix II: Additional Fluent Plots



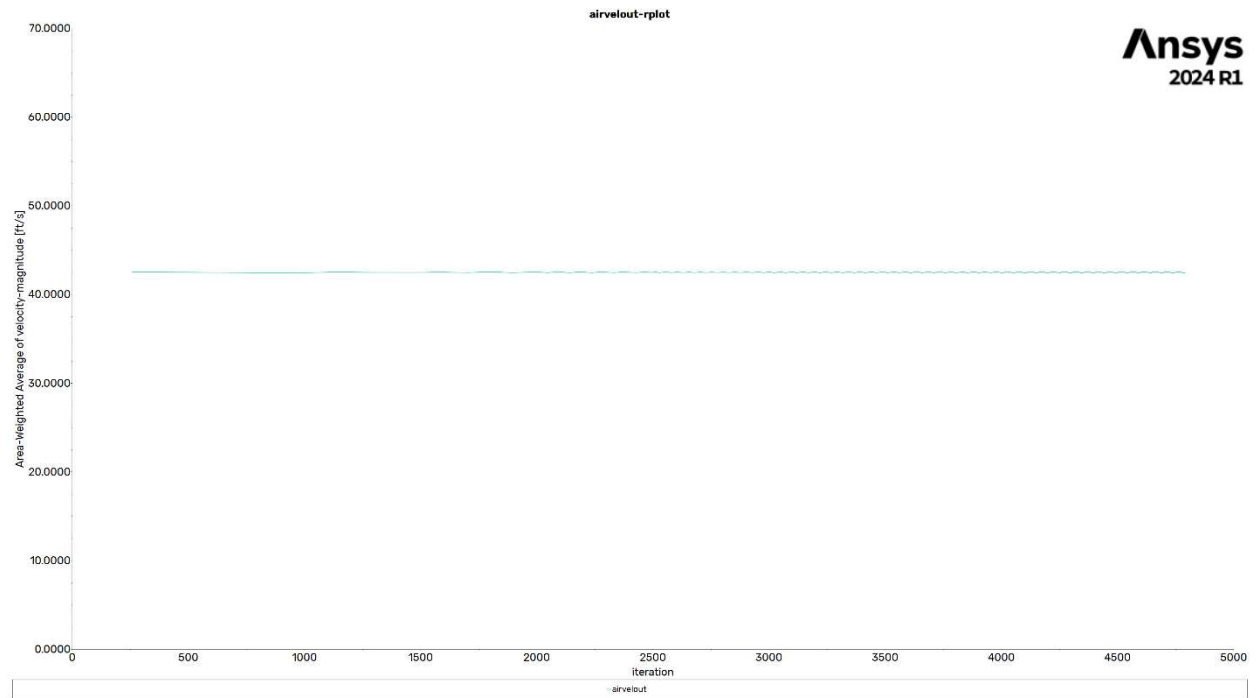


Figure 20: Air Outlet Velocity

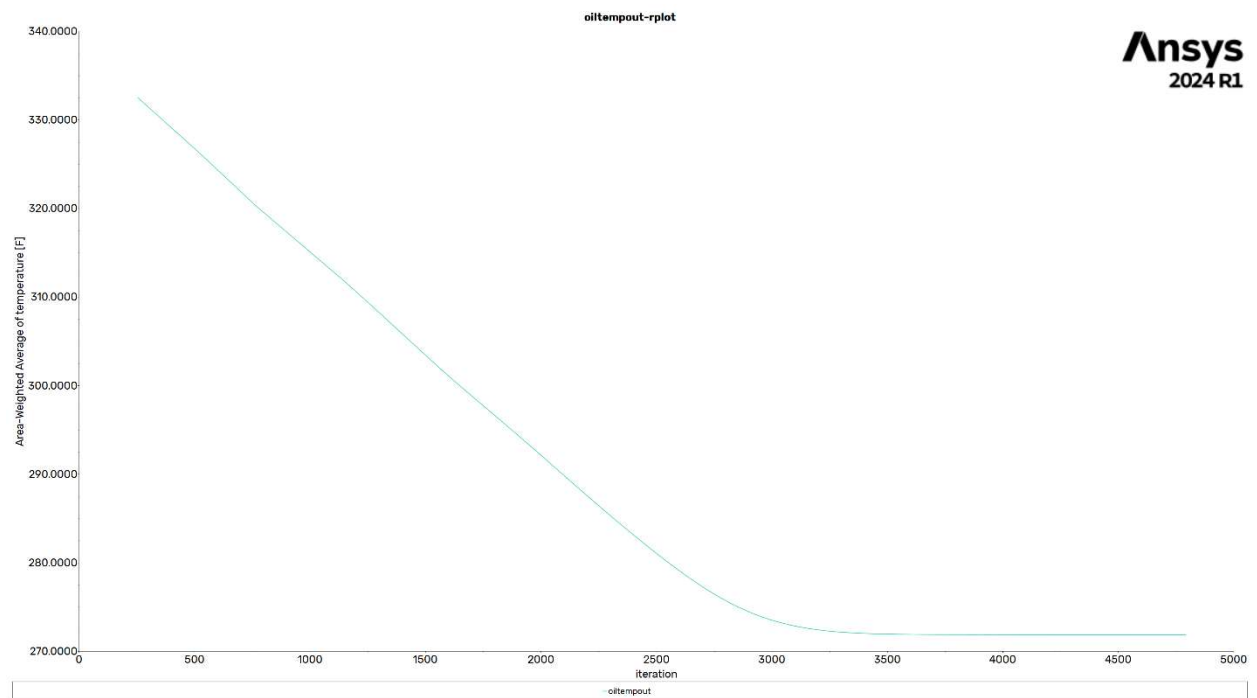


Figure 21: Oil Outlet Temperature

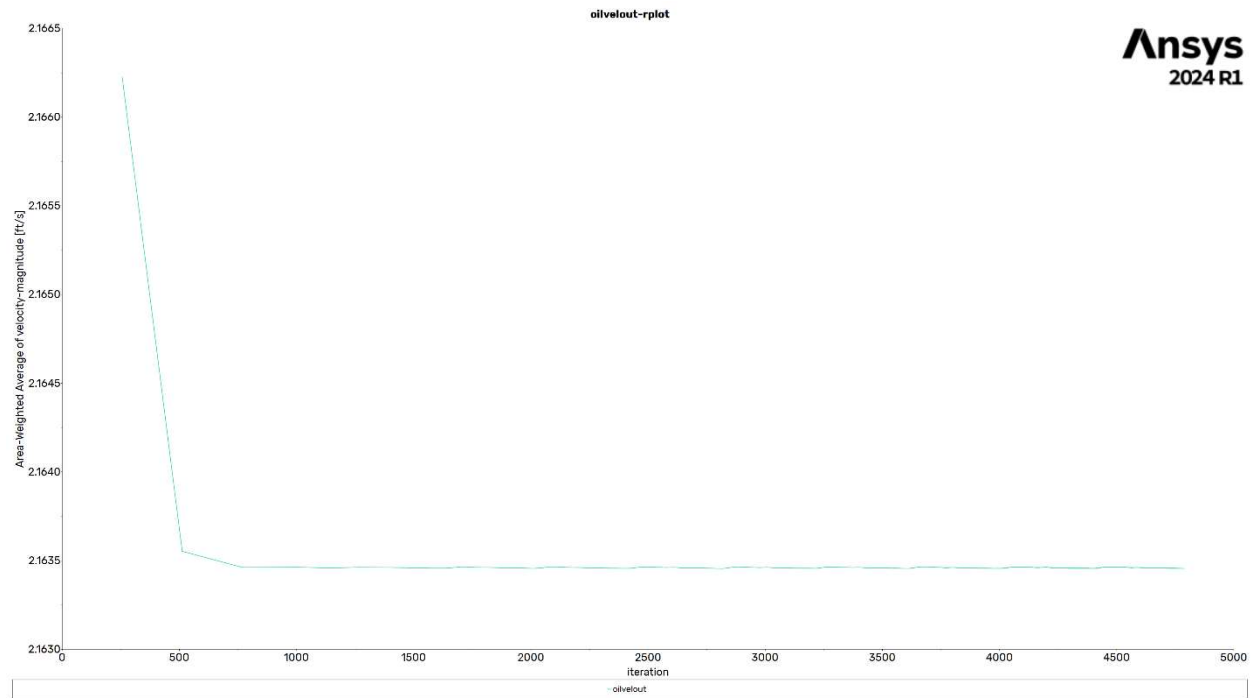


Figure 22: Oil Outlet Velocity

## Appendix III: Citations

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