Dynamic Vibration Absorber

By

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Technical Memo for ME-301

Professor Baglione

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Abstract

This project aims to analyze a rig created as part of our ME-360 Engineering Experimentation course that demonstrates the behavior of a tuned, undamped, two-degree-of-freedom (2DOF) dynamic vibration absorber (DVA) for future Mechanical Vibrations courses. The demonstration rig is intended to provide a clear physical example of how a DVA can mitigate resonance in a primary system through the appropriate tuning of an absorber mass and spring stiffness. Experimental measurements of the amplitude response of both the primary and absorber masses across a range of driving frequencies are compared to theoretical predictions to validate the system's performance. The rig will not only demonstrate a 2DOF system with a DVA but also highlight the design process for effective tuning, prompting future students to consider the following key questions:

1. How does a DVA reduce the amplitude of vibration in the primary mass during resonance?
2. What design parameters ensure that the DVA is tuned to mitigate resonance in the primary mass system?
3. How does the addition of a DVA influence the natural frequencies of the coupled system?

To answer these questions and provide an in depth analysis of the effectiveness of our tuning parameters two sets of data were taken: one for a mass ratio (μ) of 0.31 and frequency ratio (β) of 1 and the other for a mass ratio of 0.44 and frequency ratio of 0.84. Experimental results were compared to theoretical models for an undamped, tuned vibration absorber as well as a damped, tuned vibration absorber. The parameters for the damped model were determined through measurements of the natural frequencies and damping ratios of the coupled system using accelerometers, a DAQ device, and LabView VI.

Motivation

Vibrations are a significant concern in the design and operation of engineering systems. When left uncontrolled, vibratory motion can lead to structural fatigue or failure, increased noise, and reduced performance. This is particularly true when systems operate near or at resonance frequencies, in which small periodic inputs can result in large oscillatory responses. To mitigate such effects, engineers often incorporate dynamic vibration absorbers (DVAs), which are secondary systems composed of a mass, spring, and sometimes damper to be optimally tuned to absorb vibrational energy from the primary oscillatory system. DVAs are used in high-rise buildings to reduce motion due to wind or seismic activity, to control noise and vibrations in rotating machinery, and to stabilize automotive suspension systems among other applications. When tuned correctly, the DVA can shift or split the resonance peaks of the primary system, minimizing the amplitude of vibration (i.e. the displacement of the primary mass) at critical frequencies. The plots, created by Dr. Daniel A. Russell, a professor at Pennsylvania State University, shown in Figure 1, plot the displacement of the primary mass as a function of normalized frequency for a system with a DVA and without a DVA [1].

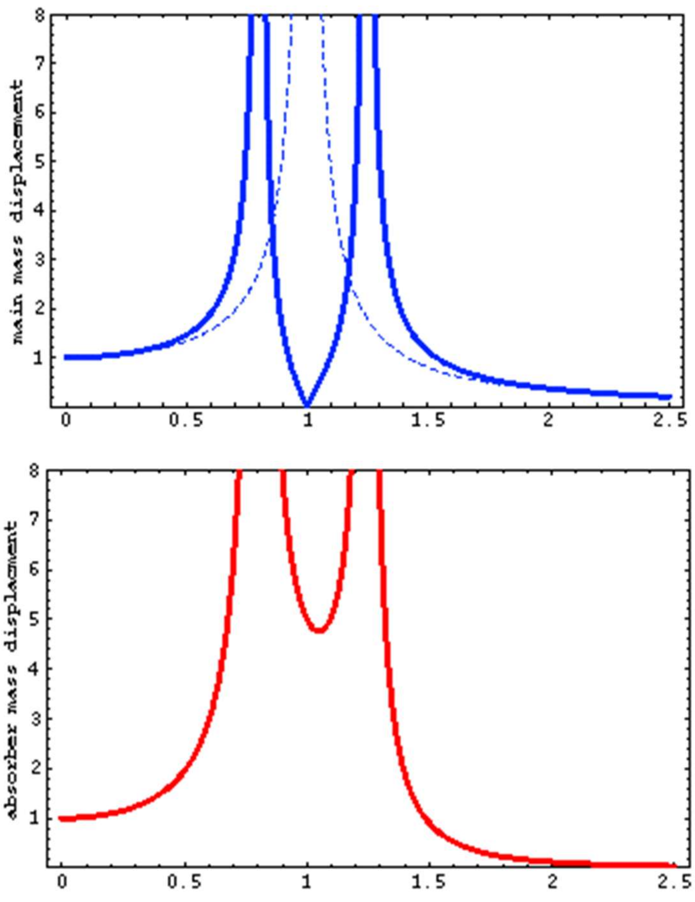

Figure 1: Displacement as a Function of Normalized Frequency for an Undamped Primary Mass without a DVA (red) and with a DVA (blue) [1]

The primary objective of our project is to analyze the behavior of the two-degree-of-freedom (2DOF) DVA demonstration rig we designed and constructed as part of our ME-360 Engineering Experimentation course. Our rig consists of a primary mass-spring system on which a rotating unbalanced force acts as a source of harmonic excitation and an undamped tuned DVA. Figures 2 and 3 below illustrate our rig design and setup. As the rotational speed of the eccentric mass approaches the natural frequency of the primary system, resonance occurs, leading to larger oscillations. Based off our calculations the DVA is tuned such that the frequency ratio, β, is one, therefore reducing the amplitude of oscillation of the primary mass at the mass' resonant frequency (before the DVA is attached). In this project, we will compare theoretical calculations to experimental data measured using accelerometers and a data acquisition (DAQ) system to understand the effectiveness of the DVA in mitigating resonance, assess how our tuning parameters perform, and identify any discrepancies between theoretical and measured responses.

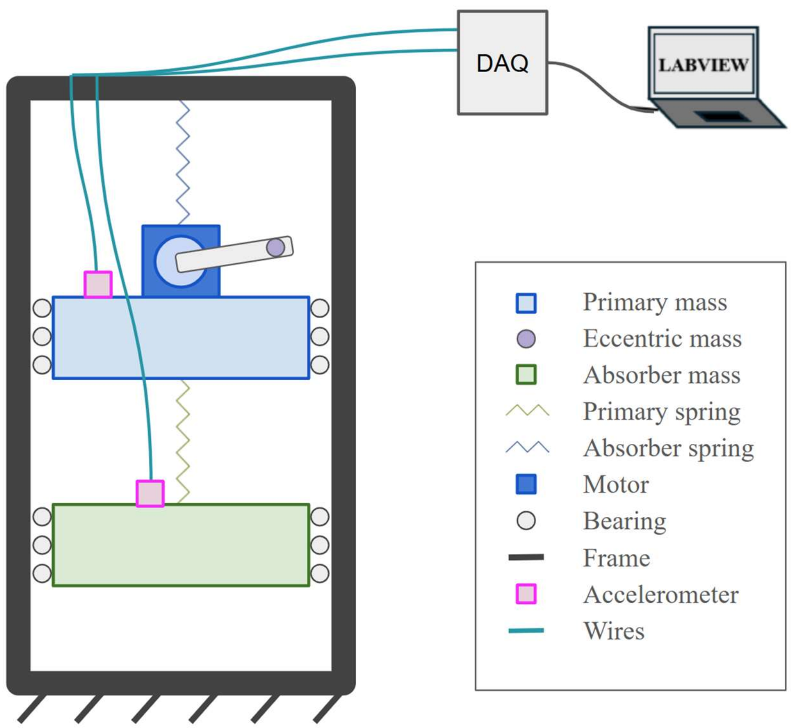

Figure 2: System Diagram

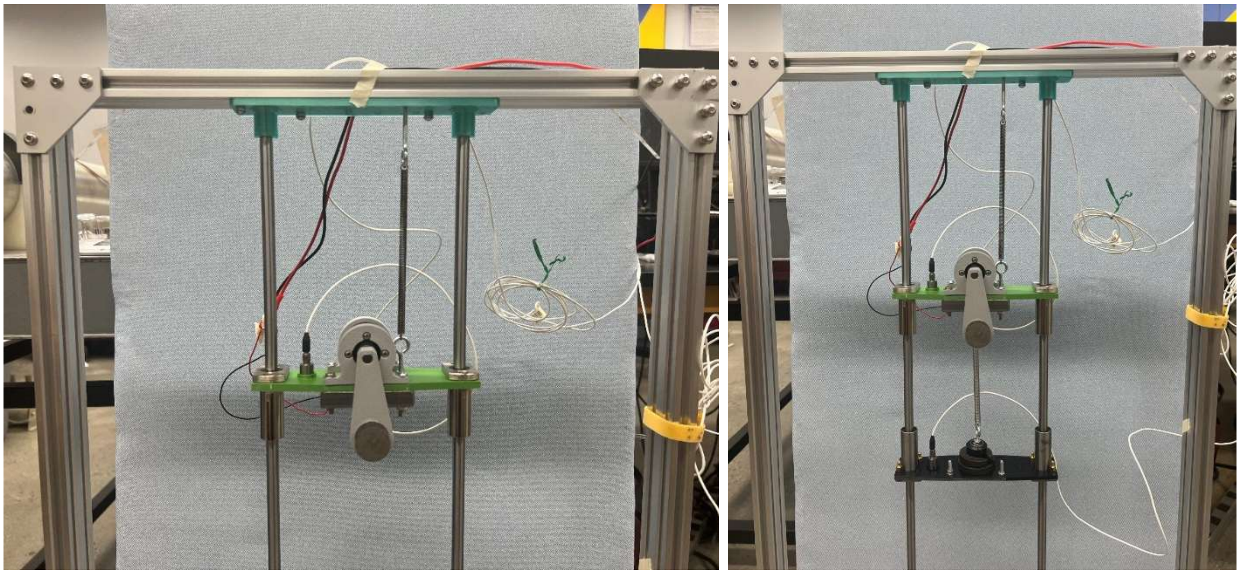

Figure 3: Rig Setup with Primary Mass Alone (left) and Primary Mass with DVA (right)

Theoretical Calculations

All relevant variables can be found in Appendix A. The demonstration rig was designed with a primary spring constant of 228.45 N/m and an absorber spring constant of 71 N/m. These spring constants were not taken from specs but rather calculated from static deflection measurements. The primary mass was 0.9869 kg, and the absorber mass was 0.3067 kg, resulting in a mass ratio of about 0.31. The natural frequencies of each mass independently can be calculated as follows:

$$\omega_p = \sqrt{\frac{k}{m_p}} \qquad \omega_a = \sqrt{\frac{k_a}{m_a}}$$

Undamped Tuned Absorber

If we assume there is no damping in the system, the equation of motion for the primary system with harmonic excitation due to a rotating imbalance without an absorber is:

$$m\ddot{x} + kx = m_o e\omega^2 \sin(\omega t)$$

Thus, the maximum amplitude of the primary system without an absorber can be calculated as follows:

$$X_{without\ DVA} = \frac{m_o e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

where $r = \frac{\omega}{\omega_p}$ and $\zeta = 0$ since damping is negligible.

With the addition of a DVA, the amplitude of the primary mass would decrease significantly assuming that the DVA is tuned properly. The amplitudes of the primary mass with a DVA and the absorber mass can be determined using the following equations for an undamped, tuned absorber, where $r_p = \frac{\omega}{\omega_p}$ and $r_a = \frac{\omega}{\omega_a}$.

$$X_{wit\quad DVA} = \frac{m_o e\omega^2}{k}\frac{1 - r_a^2}{\left(1 + \mu\beta^2 - r_p^2\right)(1 - r_a^2) - \mu\beta^2}$$

$$X_a = \frac{k_a F_0}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2}$$

The stiffness ratio is $\mu\beta^2 = \mu\frac{\omega_a^2}{\omega_p^2} = \frac{k_a}{k}$, and the frequency ratio is $\beta = \frac{\omega_a}{\omega_p}$. The magnitude of the maximum amplitude values for the primary mass without a DVA, the primary mass with a DVA, and the absorber mass at driving frequencies near and at the resonance frequency are tabulated in Table 1. Resonance occurs when the driving frequency is equal to the natural frequency of the primary system (i.e. $\omega = \omega_p$).

Table 1: Theoretical Maximum Amplitudes of Primary Mass With and Without DVA and of Absorber Mass

Voltage (V)	Driving Frequency (rad/s)	Amplitude of Primary Mass Without DVA (mm)	Amplitude of Primary Mass With DVA (mm)	Amplitude of Absorber Mass (mm)
11.0	14.70	85.82	1.314	19.86
11.32	15.21	∞	0	19.56
11.5	15.50	167.4	0.7403	19.65

Damped Tuned Absorber

Taking the damping of the system calculated, which was determined from experimental data using the log decrement method, into consideration, the equation of motion for the primary system with harmonic excitation due to a rotating imbalance without an absorber becomes:

$$m\ddot{x} + c\dot{x} + kx = m_o e\omega_r^2 \sin(\omega_r t)$$

$$\begin{bmatrix} m & o \\ o & m_a \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c + c_a & -c_a \\ -c_a & c_a \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k + k_a & -k_a \\ -k_a & k_a \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The maximum amplitudes of the primary system without an absorber, the primary system with an absorber, and the absorber system are as follows:

$$X_{without\ DVA} = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1 - r^2)^2 + \left(2\zeta_p r\right)^2}}$$

The calculations for damping are further explained in the damping section.

where $\zeta_p = 0.034275$ for the primary system and $\zeta_a = 0.034692$ for the absorber system. These were found using the log decrement method as mentioned in later sections. For ease of calculation, the amplitude of the primary mass with the absorber was determined using the MATLAB script in Appendix B.

Note that for the tests with $\mu = 0.44$ and $\beta = 0.84$ the mass was added to the absorber platform so no new data for the primary mass without the absorber was required. The frequency however was normalized against the new ω_a. This can be seen in the normalized amplitude to normalized frequency plots found in the processed data section.

Bandwidth

Since the useful range of an absorber is defined such that $\left|\frac{Xk}{F_0}\right| < 1$, the bandwidth of the coupled system can be computed by solving for $\frac{\omega}{\omega_a}$ when $\left|\frac{Xk}{F_0}\right| = 1$ and for when it equals -1 and choosing the two most restricting roots about the natural frequency. Although the amplitude of the primary mass never exceeds a normalized magnitude of 1, systems in which the amplitude does exceed this value could drift into resonance and the amplitude of vibration becomes an amplification of the driving force amplitude [2]. The calculation for the bandwidth of the coupled system with a mass ratio of 0.31 and is included below.

$$\frac{Xk}{F_0} = \frac{1 - \omega^2/\omega_n^2}{\left[1 + \mu\left(\omega_a/\omega_p\right)^2 - \left(\omega/\omega_p\right)^2\right]\left[1 - (\omega/\omega_a)^2 - \mu\left(\omega_a/\omega_p\right)^2\right]}$$

The following equation can be derived from the equation above with $\left|\frac{Xk}{F_0}\right| = 1.$

$$1 - \left(\frac{\omega}{\omega_a}\right)^2 = \left[1 + \mu\left(\frac{\omega}{\omega_a}\right)^2 - \left(\frac{\omega}{\omega_p}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_a}\right)^2\right] - \mu\left(\frac{\omega_a}{\omega_p}\right)^2$$

Solving the equation above yields:

$$\frac{\omega_a}{\omega_p} = \pm\sqrt{1 + \mu}$$

$$\frac{\omega_a}{\omega_p} = 1.144897$$

The equation below can be derived the first bandwidth equation, which is quadratic in $\left(\frac{\omega}{\omega_a}\right)^2$.

$$\left(\frac{\omega_a}{\omega_p}\right)^2\left(\frac{\omega}{\omega_a}\right)^4 - \left[2 + (\mu + 1)\left(\frac{\omega_a}{\omega_p}\right)^2\right]\left(\frac{\omega}{\omega_a}\right)^2 + 2 = 0$$

Using the values of $\omega_a^2 = \omega_p^2 = 231.4828\, rad^2/s^2$ *and solving for* $\frac{\omega}{\omega_a}$ *yields:*

$$\left(\frac{\omega}{\omega_a}\right)^2 = 2.516, 0.795$$

$$\left(\frac{\omega}{\omega_a}\right) = 1.586, 0.892$$

For $\frac{Xk}{F_0} = 1$, $\left(\frac{\omega}{\omega_a}\right)^2 = \mu + 1$ *yields the other root.*

$$\frac{\omega}{\omega_a} = 1.145$$

Solving for ω, the bandwidth the system with a mass ratio of 0.31 and frequency ratio of 1 is:

$$13.57\, rad/s < \omega < 17.42\, rad/s$$

$$0.9 < \frac{\omega}{\omega_a} < 1.14$$

Therefore, the driving frequency may vary between 13.5655 rad/s and 17.4191 rad/s , (10.23V-12.92V) before the response of the primary mass is amplified.

For a mass ratio of 0.43, $\omega_p = 12.78$ rad/s and $\omega_a = 15.21$ rad/s (as it was before). Following the same procedure for calculating bandwidth with a mass ratio of 0.31, the bandwidth of the system becomes:

$$12.70\, rad/s < \omega < 15.34\, rad/s$$

$$0.99 < \frac{\omega}{\omega_a} < 1.2$$

Note that for the larger mass ratio one might expect the bandwidth to increase rather than staying the same. However, since the mass ratio and frequency ratio both changed for the second test these counteract one another leading to a similar bandwidth. These values give us a better understanding of the voltage range we expect the absorber to be effective in when converting these frequencies to voltages based off the voltage-natural frequency curve of the loaded motor (measured via tachometer). The recorded values for this curve can be found in Appendix C.

Experimental Results

Experimental results were measured using accelerometers mounted to both the absorbing and primary masses and connected to a DAQ system. Data was collected from LabVIEW's Acceleration vs Time and FFT plots. Examples of these plots can be found in Appendix B.

Natural Frequency

The natural frequencies of the primary and absorber systems were measured by isolating each system and plucking the respective mass, using the LabView VI to measure the time between peaks on the Acceleration vs Time plot.

The measured time between peaks corresponds to the period of oscillation (T), and natural frequency (in rad/s) was determined using the following relation:

$$\omega_n = \frac{2\pi}{T}$$

The Amplitude vs Time plots for the free response of the primary system and absorber system as well as the experimental natural frequencies are included below.

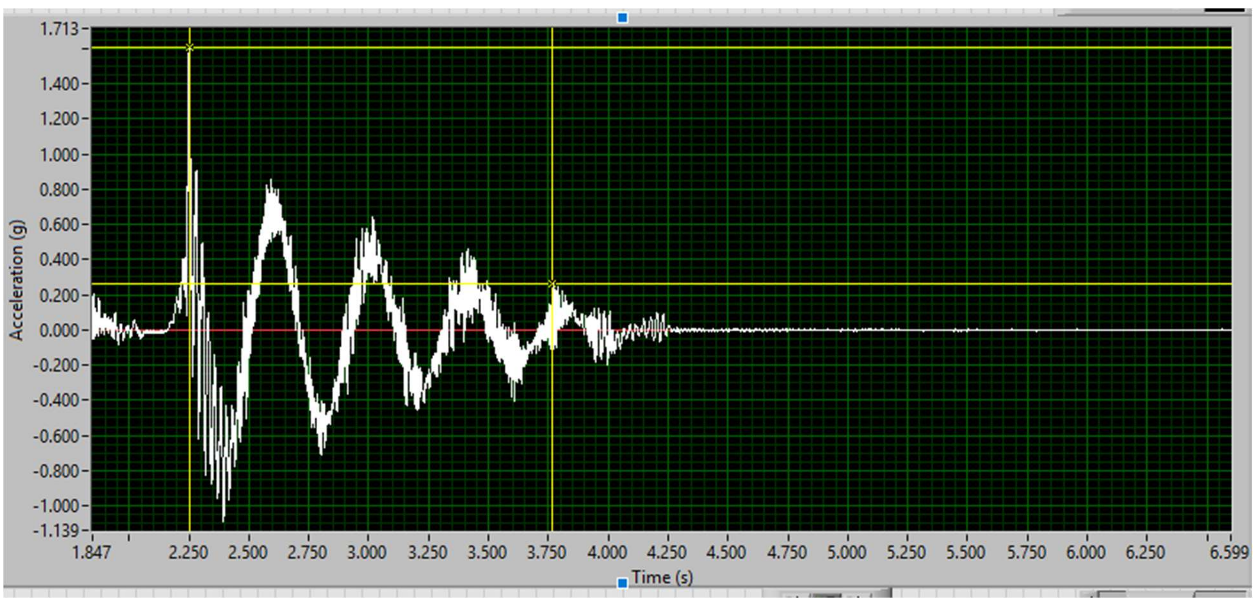

Figure 4: Acceleration vs. Time plot of Primary Mass Free Response

*This plot allowed us to find the damping ratio of the primary mass and the real natural frequency (as opposed to the theoretical one)

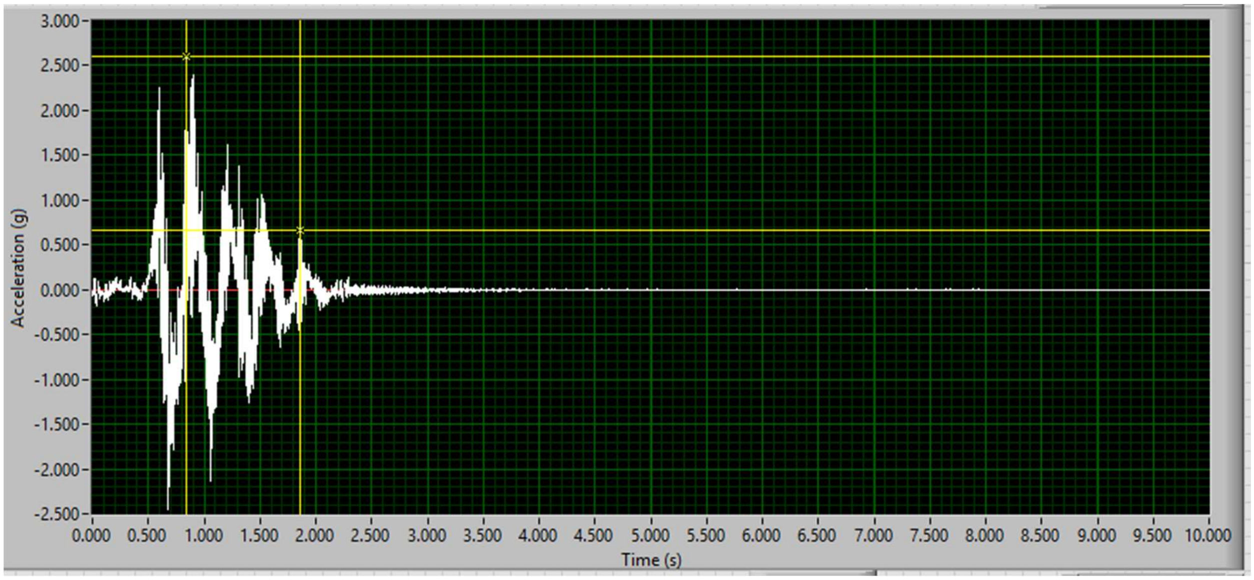

Figure 5: FFT Plot of Primary Mass Free Response

*This plot allowed us to find the damping ratio of the primary mass and the real natural frequency (as opposed to the theoretical one)

Table 2: Experimental Natural Frequencies of Primary and Absorber Systems

System	Natural Frequency (rad/s)
Primary	16.578
Absorber	18.498

To measure the amplitudes of the primary system with and without a DVA, primary system with a a power supply was used to sweep through a voltage range of about 8V to about 20V by increments of about 0.5V.

Without the DVA, the peak amplitude near the system's natural frequency on the FFT plot was recorded at each voltage. The amplitude measurements were converted from dB to mm, for input voltages that fall between the values in Appendix C, the driving frequency was interpolated. The raw data for both

Damping

The damping of the system can be calculated using the log decrement method. Two peaks from the Acceleration vs Time plot were measured to determine the damping ratio for the primary and absorber systems (Note that the axes are mislabeled in LabView).

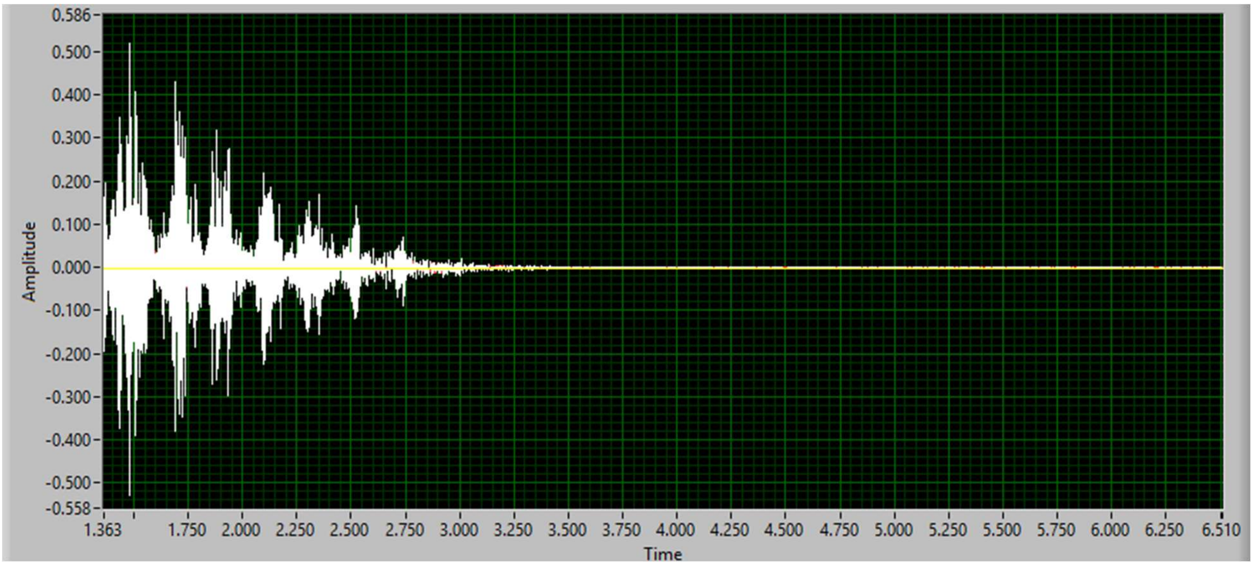

Figure 6: Acceleration vs Time Plot of the Primary System

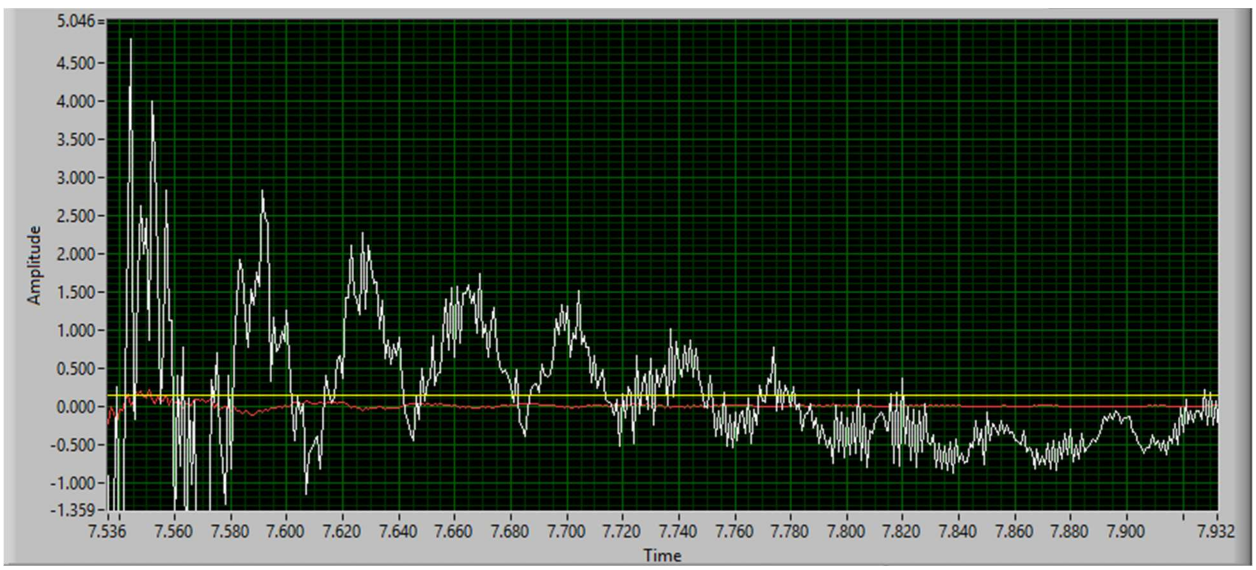

Figure 7: Accleration vs Time Plot of the Absorber System

The damping ratio for the primary system and absorber system are included in Table 2 below.

Table 3: Damping Ratio for Primary System and Absorber System

	Amplitude at Peak 1	Amplitude at Peak 6	Damping Ratio
Primary System	0.521	0.143	0.0343
Absorber System	2.824	0.763	0.0347

The damping ratio was then calculated using the log-decrement method.

$$\delta = \frac{1}{n}\ln\left(\frac{B_1}{B_{n+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

It should be noted here that these are the damping ratios of each single degree of freedom system and is not a mixed damping ratio. This damping ratio was then used to calculate each system's respecting damping coefficient using the following formula.

$$c = 2\zeta\omega_n m$$

Amplitude

To measure the amplitudes of the primary system without a DVA, primary system with a DVA, and absorber system, a power supply was used to sweep through a voltage range of 10V to 18V by increments of 0.5V. With only the primary mass attached to the apparatus (no DVA), the peak amplitude near the system's natural frequency on the FFT plot was recorded at each voltage, and corresponding RPM values were measured with a tachometer. The amplitude measurements were converted from dB to mm, and the relationships between voltage input and driving frequency are included in Table 6 in Appendix B. For input voltages that fall between the values in Table 6, the driving frequency was interpolated. An example of the FFT plot at a voltage input of 11.5 V (driving frequency of 15.50 rad/s) is included in Appendix B.

The absorber system and a second accelerometer were then added to the system, and the process was repeated to measure the amplitudes of the primary and absorber masses. An example of the FFT plot at a voltage input of 11.5 V (driving frequency of 15.50 rad/s) is included in Appendix B, where the primary system measurements are white and the absorber system measurements are red. The experimental results for amplitudes (in mm) at driving frequencies ranging from 13.22 rad/s to 24.79 rad/s are included in Table 4 below. The experimental results for the amplitudes in dB are included in Appendix B as well.

Table 4: Amplitude of Primary and Absorber Masses at Varying Input Voltages in mm μ=0.31 and β=1

Trial	Voltage (V)	Driving Frequency (rad/s)	Amplitude of Primary Mass without DVA (mm)	Amplitude of Primary Mass with DVA (mm)	Amplitude of Absorber Mass (mm)
1	10	13.22	13.16	3.55	13.90
2	10.5	13.97	18.32	2.75	13.40
3	11	14.70	24.22	1.51	11.40
4	11.5	15.50	28.08	1.53	11.19
5	12	16.23	26.62	2.43	13.64
6	12.5	16.92	31.32	3.09	13.00
7	13	17.51	38.64	4.01	16.82
8	13.5	18.32	37.96	4.76	18.59
9	14	19.10	18.33	5.32	19.72
10	14.5	19.72	16.43	5.08	19.56
11	15	20.47	11.09	4.04	15.30
12	15.5	21.15	9.53	3.97	15.36
13	16	21.93	10.48	4.19	15.79
14	16.5	22.57	11.69	4.29	16.66
15	17	23.33	10.57	5.03	19.10
16	17.5	24.06	8.26	5.00	16.15
17	18	24.79	7.77	6.35	17.65

The experimental amplitudes are compared to the undamped and damped theoretical models in the following figures:

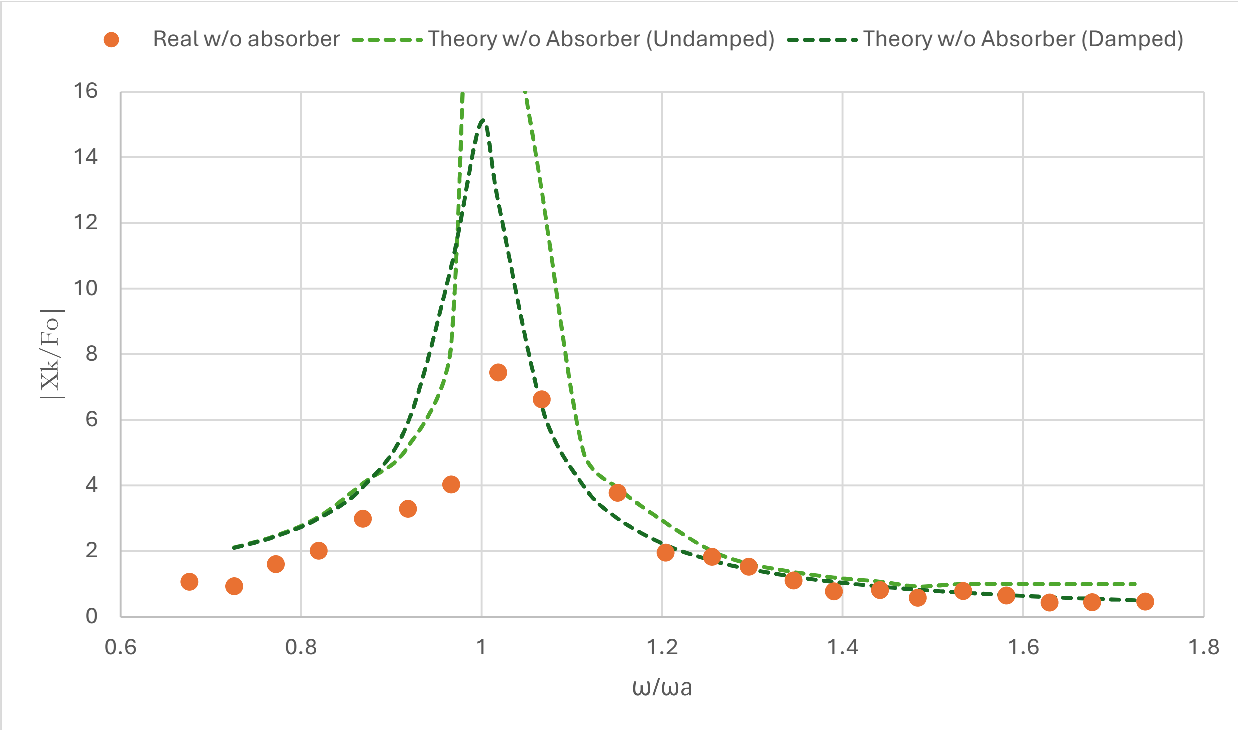

Figure 8: Normalized Amplitude of Primary System Without DVA over Normalized Driving Frequency Compared to Theoretical Models for μ=0.31 and β=1

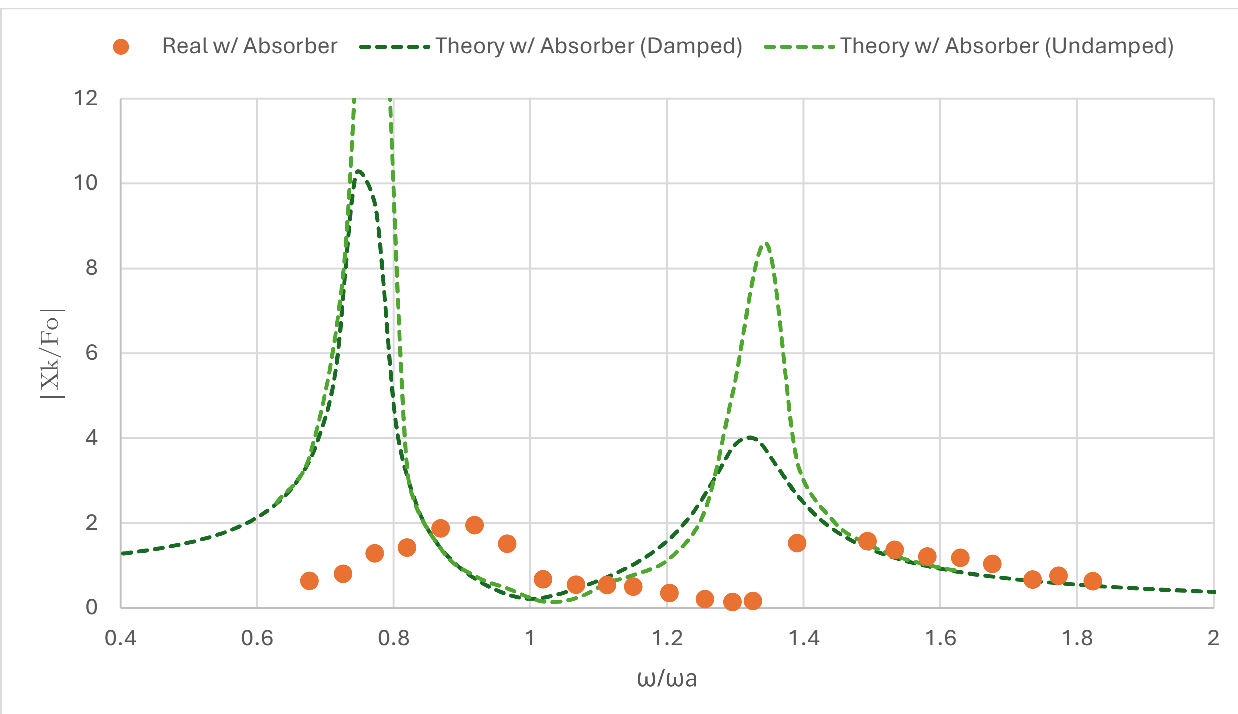

Figure 9: Normalized Amplitude of Primary System With DVA over Normalized Driving Frequency Compared to Theoretical Models for μ=0.31 and β=1

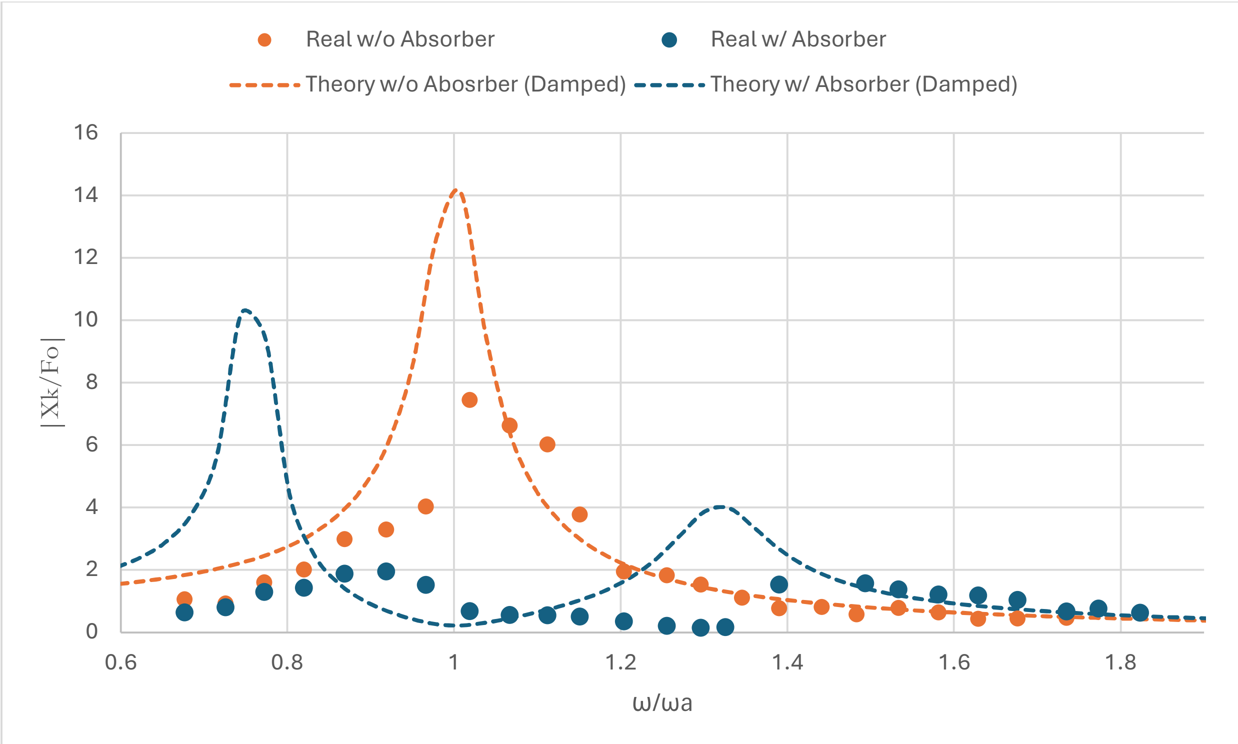

Figure 10: Comparison of Experimental Data and Theoretical Models for Normalized Primary System Amplitude With DVA and Without DVA for μ=0.31 and β=1

The amplitudes of the primary system without a DVA and the primary system with a DVA for a mass ratio of 0.43 and frequency ratio of 0.84 are included in Figures 13-15 below. The FFT plots and amplitude measurements for the system with a mass ratio of 0.43 are included in Appendix B.

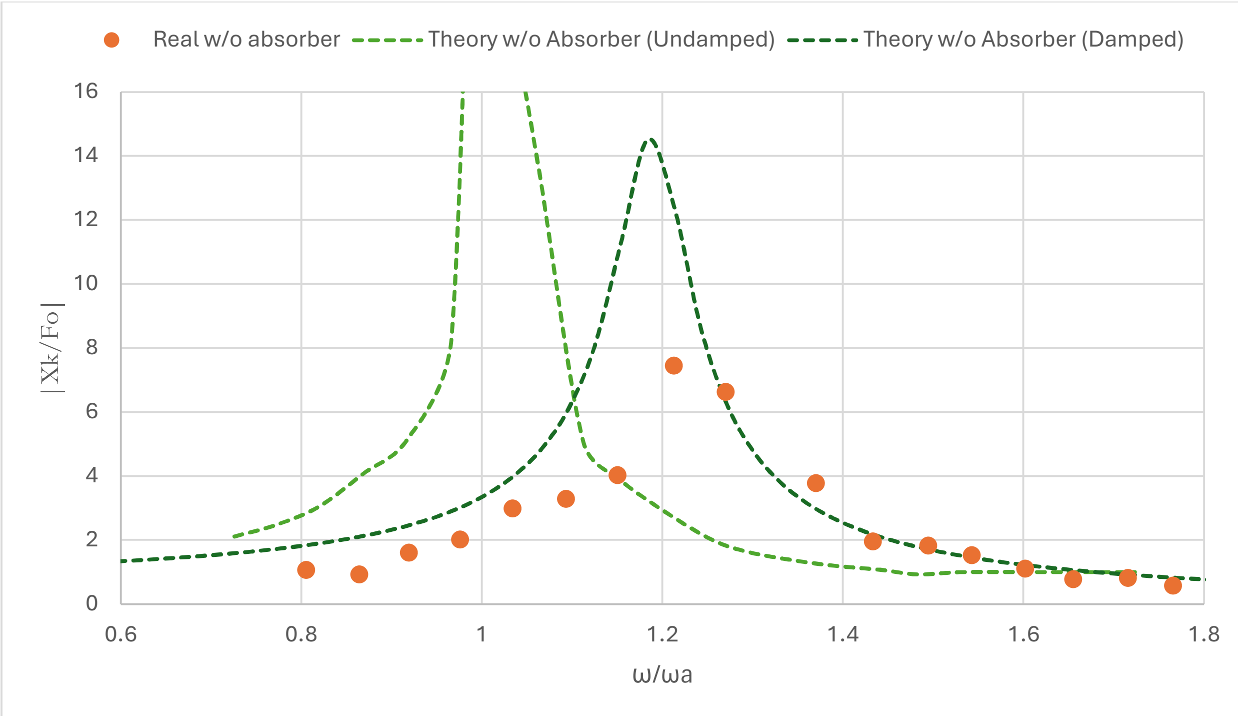

Figure 11: Normalized Amplitude of Primary System Without DVA over Normalized Driving Frequency Compared to Theoretical Models for μ=0.44 and β=0.84

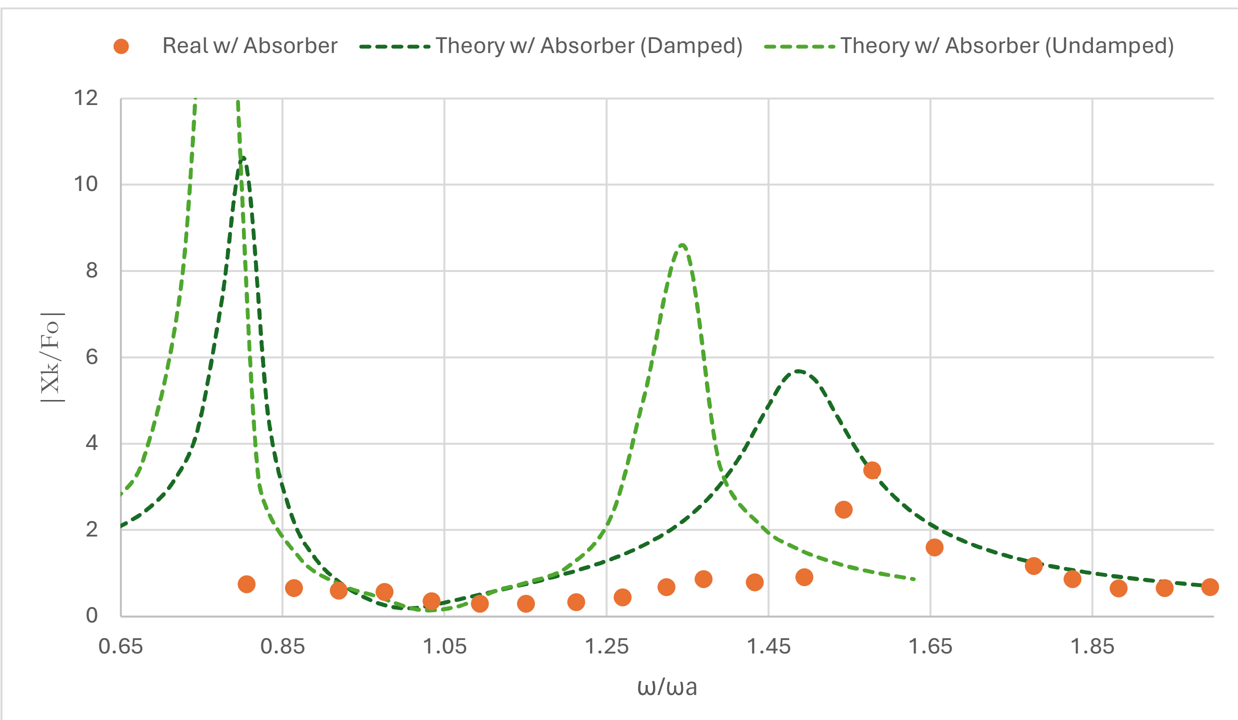

Figure 12: Normalized Amplitude of Primary System With DVA over Normalized Driving Frequency Compared to Theoretical Models for μ=0.44 and β=0.84

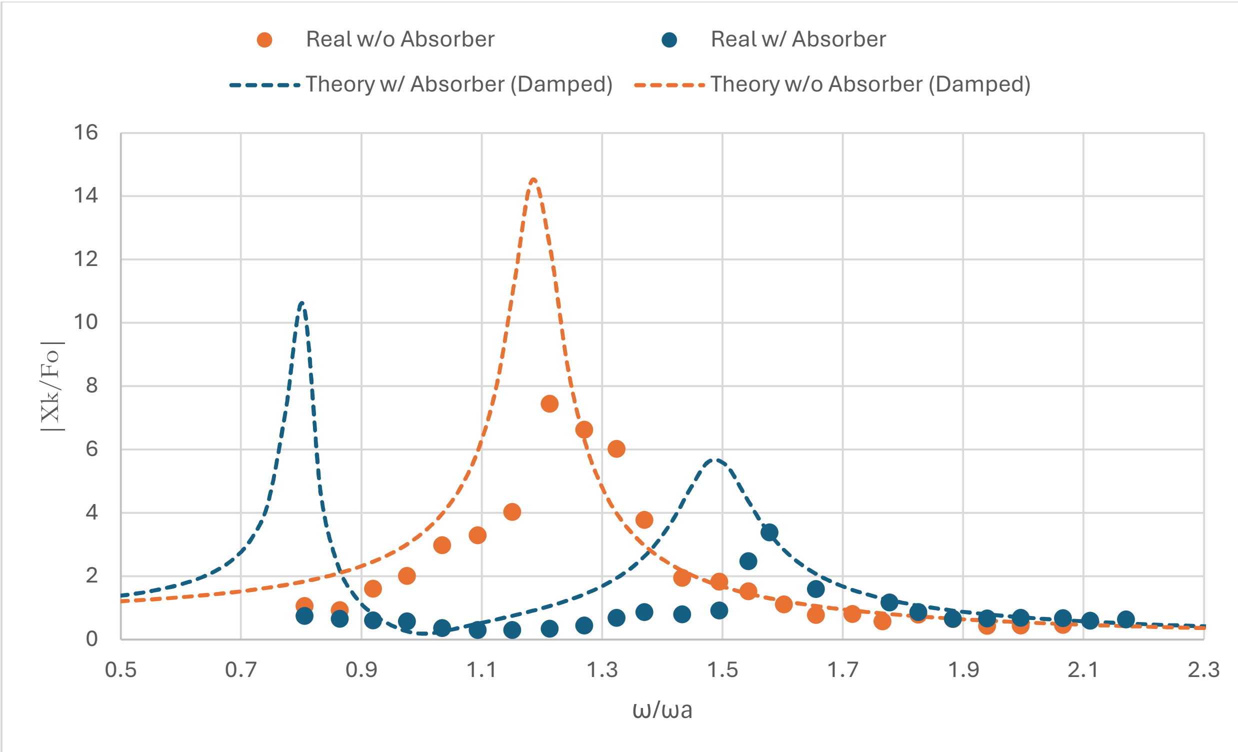

Figure 13: Comparison of Experimental Data and Theoretical Models for Normalized Primary System Amplitude With DVA and Without DVA for $\mu=0.44$ and $\beta=0.84$

Discussion & Conclusions

Our initial theoretical calculations are based on an undamped, tuned DVA, in reality, friction around the bearings in the rig and air drag introduce damping to the system. The effects of an undamped, tuned DVA on the oscillatory behavior of a primary spring-mass system are visualized in Figure 14, where resonance occurs in the animation at the center.

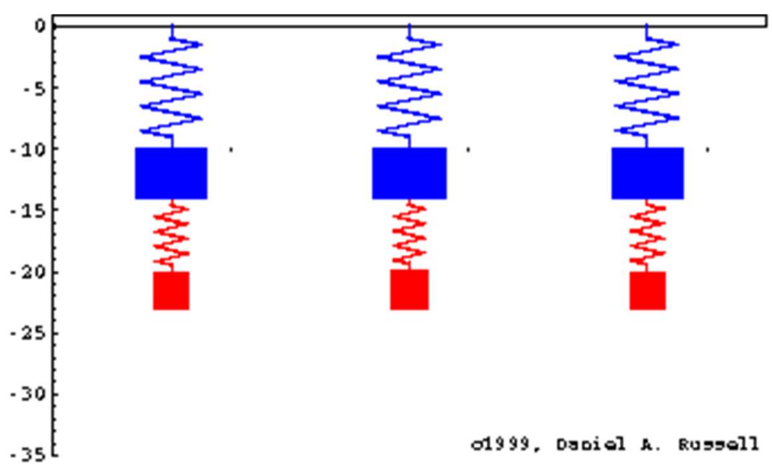

Figure 14: Motion of the Coupled System at Varying Normalized Forcing Frequencies [1]

Our absorber system does not perfectly suppress the oscillatory behavior of the primary system, so comparing our theoretical calculations to our experimental data allows us to analyze the effects of unintended damping on the effectiveness of the DVA on the primary system at resonance. The damping on the primary system and absorber system was determined from our experimental results using the log decrement method as follows:

$$\delta = \frac{1}{n}\ln\left(\frac{B_1}{B_{n+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

To further understand the effects of a damping on the amplitude of the primary system with a DVA, we compared our experimental results to a theoretical model for a damped, tuned absorber using the damping coefficients determined by the calculated damping ratios. It must be noted that damping effects due to friction was modeled as viscous damping to follow the 2DOF matrix method of solution.

Several issues were observed during testing that affected measurement accuracy. Damping was present in the system as friction along the linear rails. Off axis torques produced by the eccentric mass were also observed and thus motion was not entirely linear. The motor stalled at lower voltages, yet the system's bandwidth often extended into this low voltage range, causing issues with varying the driving frequency. This stalling also affected the sinusoidal input of the motor. Spring deflection was also a concern, as the absorbing mass spring was observed as bottoming out due to operation in the lower end of the loading range. The precision of the DAQ was limited to 0.1 Hz, which was much too large compared to the system dynamics, which is on the order of 2 Hz. The rig also bottomed out when hitting resonance, where the linear bearings of the primary and absorbing mass begin to hit each other due to high displacement amplitudes. Data points were taken every 0.5V, however figures 6-11 show large amplitude changes around where the peaks of amplitude should be, meaning there were not enough data points taken around the natural frequency of the primary mass.

Future work would involve upgrading to a DAQ system with finer resolution in the low-frequency range to enable higher-quality data collection from accelerometers. This is more convenient than the other option of operating the rig at a far greater frequency to fit the DAQ's range. More accurately determining the experimental amplitudes, natural frequencies, and damping ratios would improve our theoretical model for a damped, tuned vibration absorber, improving the correlation between theoretical model and experimental results. Another critical area of improvement lies in the springs. Acquiring or fabricating springs that are longer to avoid bottoming out and operate near the middle of their loading range would greatly improve system responsiveness and accuracy in demonstrating desired phenomena. Improving the spring length would also mitigate the masses from bottoming out, allowing for more data points to be taken around the resonance peaks of both the 1DOF and 2DOF systems. Furthermore, making the rig modular for students to change masses, springs, and eccentricities will allow for analysis on the effect of stiffness ratios and frequency ratios on vibration suppression. Having more precise voltage input or a motor encoder would allow for better analysis of driving frequency and for more data points. To meet the goal of the rig as a tool for student demonstration and analysis, the issues mentioned must be addressed.

Personal Contributions and Skills Gained

Erika Gregory:

- Experimentally measured the amplitudes, damping ratios, and natural frequencies of our system using accelerometers and DAQ systems with LabView, strengthening my understanding of FFT and Acceleration vs Time plots
- Strengthened by understanding of real world applications for DVA systems through background research
- Created a theoretical model for an undamped, tuned vibration absorber with excitation from an unbalanced force, and learned how to create a theoretical model informed by the experimentally determined damping ratio and natural frequencies of the system
- Strengthened my understanding of 1DOF and 2DOF as well as vibration absorbers, having to learn concepts before they were taught in class
- Combined concepts taught in class like rotating unbalance, 2DOF, and vibration suppression
- Helped iteratively design, construct, and test the rig and determine the best tuning parameters (i.e. springs, masses)
- Learned how to determine the actual spring constant by conducting static deflection tests on our purchased springs

Santiago Helbig:

- Conducted and recorded data for both test cases used with LabView
- Compiled and processed all raw data in excel to create all normalized magnitude to normalized frequency plots and overlaid with theoretical plots for both test cases.
- Calculated natural frequencies needed for a tuned absorber dictating our design
- Took tachometer readings of the motor from about 7V to 25V for both eccentricities we tested against
- Learned 2DOF before being taught in class which strengthened my understanding of the topic once it was covered
- Learned the limitations of reality relative to theory and the limits this imposes on rigs and their accuracy
- Solidified my understanding of acceleration-time plots vs. FFT plots and which peaks are natural frequencies vs. driving frequencies

Spencer Kirsch:

- Learned how to use accelerometers and DAQs with LabView
- Learned 1DOF and 2DOF damped and undamped theory as well as eccentric mass theory, helping my understanding greatly for the homework and exams
- Learned about real world use cases for dynamic vibration absorbers targeting specific frequencies
- Planned, designed, manufactured, and iterated the rig and testing setup
- Helped spec out and test key parts such as the motor and springs through tachometer and static deflection tests.
- Gathered data on motor Speed vs. Voltage
- Gathered data on primary and absorbing mass amplitude across varying driving frequencies
- Helped create a theoretical model of viscously damped 2DOF system in MATLAB to use as a comparison to the experimental data

References

[1] D. Russell, “The Dynamic Vibration Absorber,” *www.acs.psu.edu*. https://www.acs.psu.edu/drussell/Demos/absorber/DynamicAbsorber.html

[2] D. J. Inman, *Engineering Vibrations*. Fourth Edition. Upper Saddle River, N.J.: Prentice-Hall, 2014.

Appendix A: Relevant Variables

Table 5: Relevant Variables

Variable	Symbol	Value
Primary Mass (kg)	m	0.9869
Absorber Mass (kg)	m_a	0.3067
Primary Spring Constant (N/m)	k	228.45
Absorber Spring Constant (N/m)	k_a	71.0
Primary Natural Frequency (rad/s)	ω_p	15.215
Absorber Natural Frequency (rad/s)	ω_a	15.215
Eccentricity (m)	e	0.06
Driving Frequency (rad/s)	ω	-
Frequency Ratio	β	-
Mass Ratio	μ	-
Amplitude (m)	X	-
Force due to Eccentric Mass	F_0	-
Damping Ratio	ζ	-
Damping Coefficient	c	-

Appendix B: MATLAB Code

```matlab
%% ME360/ME301 2DOF Theory W/ Damping
% Erika Gregory, Santiago Helbig, Spencer Kirsch
% Spring 2025
close all
clear all
clc
```

```matlab
%% System Params
% Primary system
m  = 0.9869;       % Primary mass (kg)
k  = 228.45;       % Primary spring stiffness (N/m)
c  = 1.029301;     % Primary damping (kg/s)

% Absorber (DVA)
ma = 0.31079;     % Absorber mass (kg)
ka = 71;           % Absorber stiffness (N/m)
ca = 0.323788;     % Absorber damping (kg/s)

% Excitation (rotating unbalanced mass)
m0 = 0.098;         % Unbalanced rotating mass (kg)
e  = 0.06;         % Eccentricity (m)
```

```matlab
%% Derived Parameters
wa = sqrt(ka / ma);     % Absorber natural frequency
fprintf('Absorber Natural Frequency (wa): %.4f rad/s\n', wa);
```

```matlab
%% Matrices
% Mass matrix
M = [m,  0;
     0, ma];

% Damping matrix
C = [c + ca, -ca;
     -ca,    ca];

% Stiffness matrix
K = [k + ka, -ka;
     -ka,    ka];
```

```
%% Natural Frequencies
[eVecs, D] = eig(K, M);
wn = sqrt(diag(D));         % Natural frequencies (rad/s)

%% Frequency Sweep
w_ratio = linspace(0.1, 3, 100);   % omega / omega_a
omega = w_ratio * wa;            % Actual excitation frequencies (rad/s)

%% Initialize Arrays
X1_mag_2DOF = zeros(size(omega));
X1_mag_1DOF = zeros(size(omega));

%% Frequency Response Loop (to iterate normalization properly)
for i = 1:length(omega)
    w = omega(i);
    F0 = m0 * e * w^2;

    % 2DOF response
    H = -w^2 * M + 1i * w * C + K;
    F_dynamic = [F0; 0];   % Force on primary mass
    X = H \ F_dynamic;
    X1_mag_2DOF(i) = abs(X(1)) * k / F0;

    % 1DOF Response
    H1 = -m * w^2 + 1i * c * w + k;
    X1 = (F0 / H1);
    X1_mag_1DOF(i) = abs(X1) * k / F0;
end

%% Export to excel for plotting against true values
% output = [w_ratio.', X1_mag_2DOF.', X1_mag_1DOF.'];
% header = {'omega/omega_a', 'X1_k_over_F0_2DOF', 'X1_k_over_F0_1DOF'};
%
% filename = 'amplitude_normalized_by_omega_a_0.31.csv';
% fid = fopen(filename, 'w');
% fprintf(fid, '%s,%s,%s\n', header{:});
% fclose(fid);
% dlmwrite(filename, output, '-append');

%% Plot
figure;
plot(w_ratio, X1_mag_2DOF, 'LineWidth', 1.5, 'DisplayName', '2-DOF w/ DVA');
hold on;
plot(w_ratio, X1_mag_1DOF, 'LineWidth', 1.5, 'DisplayName', '1-DOF baseline');
xlabel('\omega / \omega_a');
ylabel('|X_1 \cdot k / F_0|');
title('Normalized Amplification: X_1 k / F_0 vs \omega / \omega_a');
legend('Location', 'northeast');
grid on;
```

Appendix C: Experimental Data

Figure 15: Angular Velocities at Various Voltages for Motor Loaded with Eccentric Mass

Voltage	Rpm	Rad/s
7	85.0	8.90
7.6	92.5	9.69
8	98.3	10.29
8.6	106.9	11.19
9.1	113.5	11.89
9.8	123.3	12.91
10.6	134.9	14.13
11	140.4	14.70
11.32	145.3	15.21
11.5	148.0	15.50
12	155.0	16.23
12.5	161.6	16.92
13	167.2	17.51
13.5	174.9	18.32
14	182.4	19.10
14.5	188.3	19.72
15	195.5	20.47
15.5	202.0	21.15
16	209.4	21.93
16.5	215.5	22.57
17	222.8	23.33
18	236.7	24.79
19	250.4	26.22
20	264.9	27.74
21	277.7	29.08
22	291.2	30.49
23	306.4	32.09
24	319.4	33.45
25	332.7	34.84

*Note that the light green rows are those at the angular frequencies we calculated to be within the bandwidth (for $\mu = 0.31, \beta = 1$). The mint colored row is interpolated as the voltage corresponding to the natural frequency of the primary mass without the absorber.

The FFT plots for the primary system without a DVA and coupled system with a DVA for a voltage input of 11.5 V (driving frequency of 15.50 rad/s) are included below. The measured ampltiudes in dB are included below as well. The primary system is in white line and the absorber system is in red.

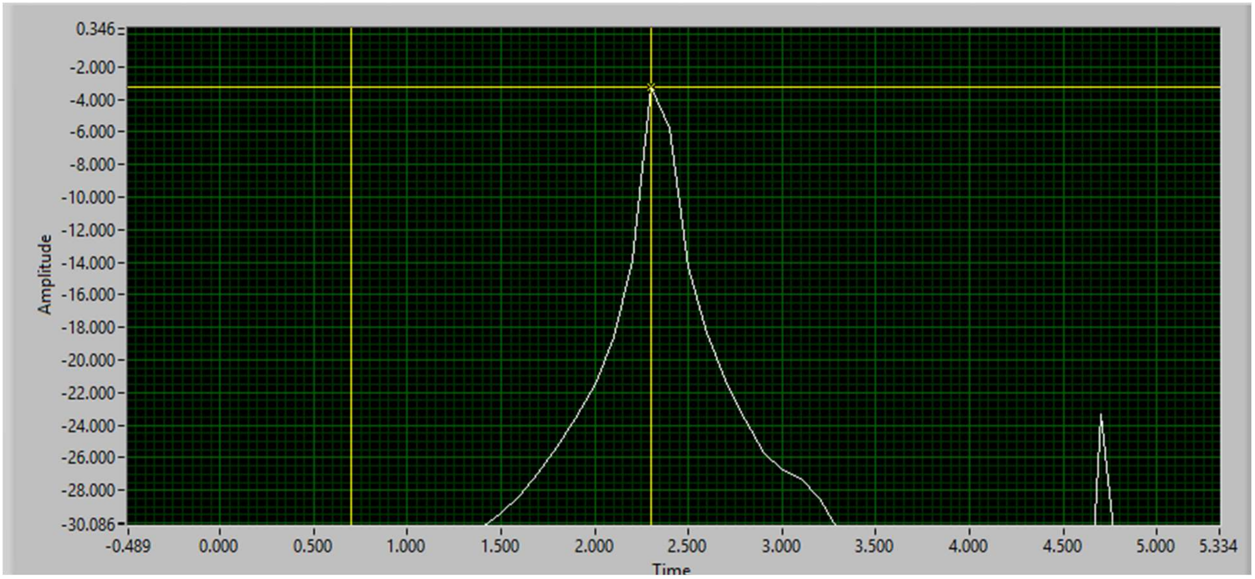

Figure 16: FFT Plot for Primary System Without Absorber at 11.5V

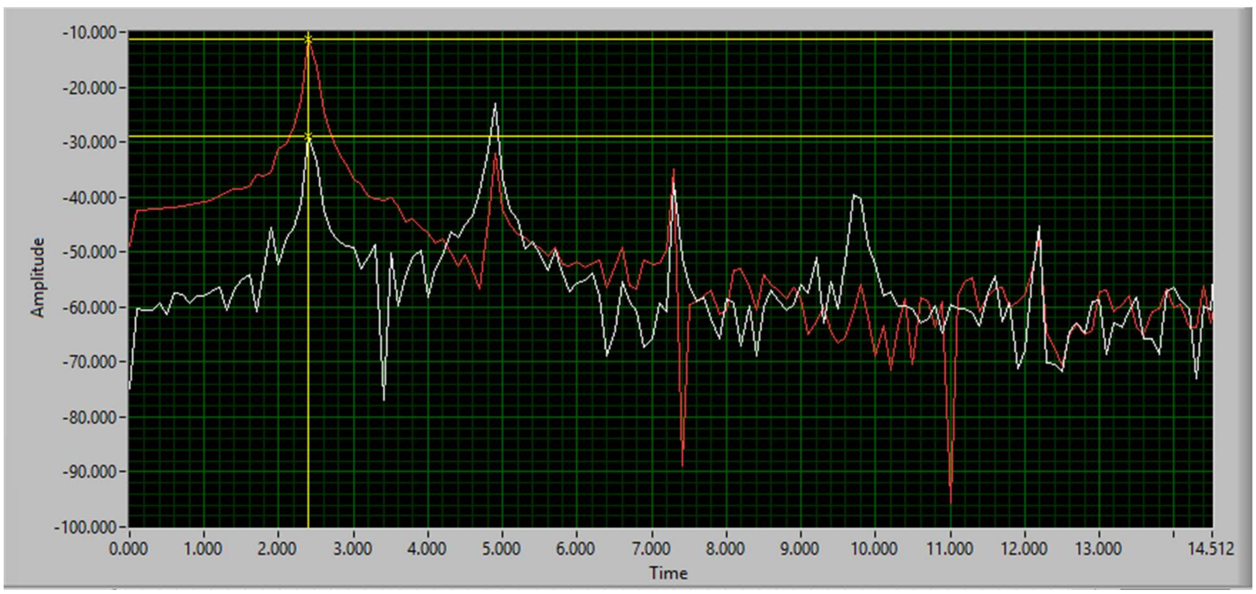

Figure 17: FFT Plot for Primary System (white) and Absorber System (red) at 11.5V

Table 6: Amplitude of Primary Mass without DVA in dB at Varying Voltage Inputs in dB for μ=0.31 and β=1

Voltage (V)	Driving Frequency (rad/s)	Amplitude of Primary Mass (dB)
8	10.29	-30.312

8.5	11.04	-28.254
9	11.75	-21.474
9.5	12.47	-17.558
10	13.22	-12.292
10.5	13.97	-9.666
11	14.70	-6.247
12	16.23	1.234
12.5	16.92	1.130
13	17.51	-1.640
13.5	18.32	-5.948
14	19.10	-3.946
14.5	19.72	-4.391
15	20.47	-5.968
15.5	21.15	-7.432
16	21.93	-5.949
16.5	22.57	-7.877
17	23.33	-4.156
17.5	24.06	-4.907
18	24.79	-6.900
18.5	25.50	-5.771
19	26.40	-4.374

Table 7: Amplitude of Primary and Absorber Masses in dB at Varying Voltage Inputs in dB μ=0.31 and β=1

Voltage (V)	Driving Frequency (rad/s)	Amplitude of Absorber Mass (dB)	Amplitude of Primary Mass (dB)
10	13.22	-12.131	-24.020
10.5	13.97	-11.478	-25.418
11	14.70	-11.998	-29.878
11.5	15.50	-11.246	-29.000
12	16.23	-8.723	-23.585
12.5	16.92	-8.416	-20.857
13	17.51	-5.587	-17.959
13.5	18.32	-3.937	-15.861
14	19.10	-2.691	-14.309
14.5	19.72	-2.211	-14.136
15	20.47	-3.690	-15.575
15.5	21.15	-3.089	-15.181
16	21.93	-2.225	-14.208
16.5	22.57	-1.261	-13.012

17	23.33	0.507	-11.154
17.5	24.06	-0.417	-10.732
18	24.79	0.870	-8.662

The Acceleration vs Time plot, FFT plot, and measured ampltiudes for the system with a mass ratio of 0.43 and frequency ratio of 0.84 at a voltage input of 11.5 V (driving frequency of 15.50 rad/s) are included below.

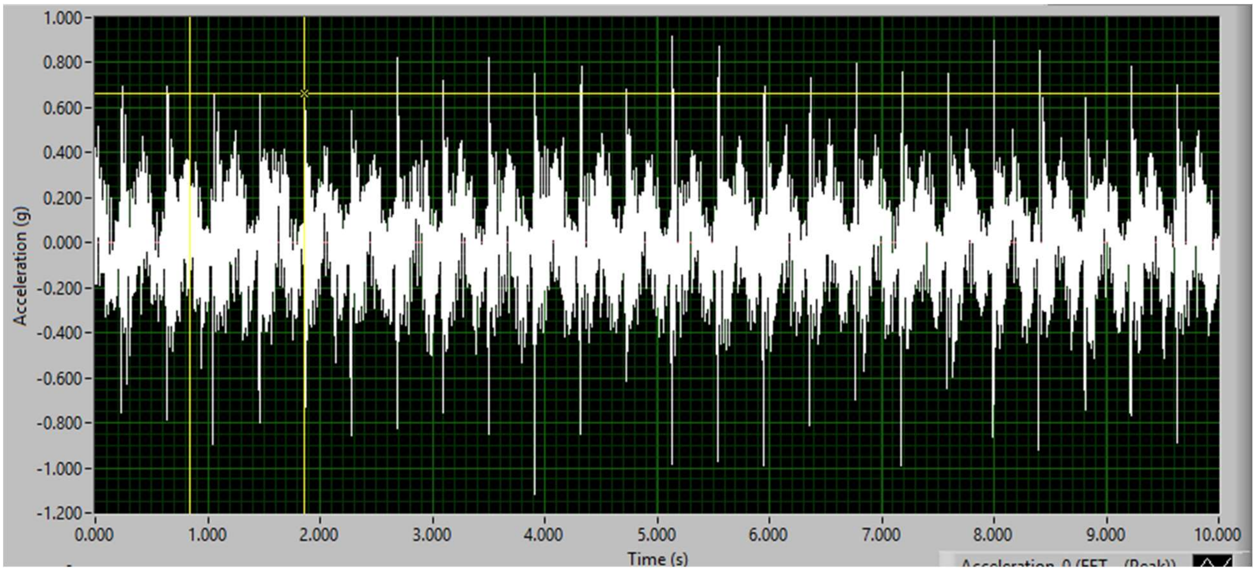

Figure 18: Acceleration vs Time Plot for Mass Ratio of 0.43 and Frequency Ratio of 0.84 at 11.5V

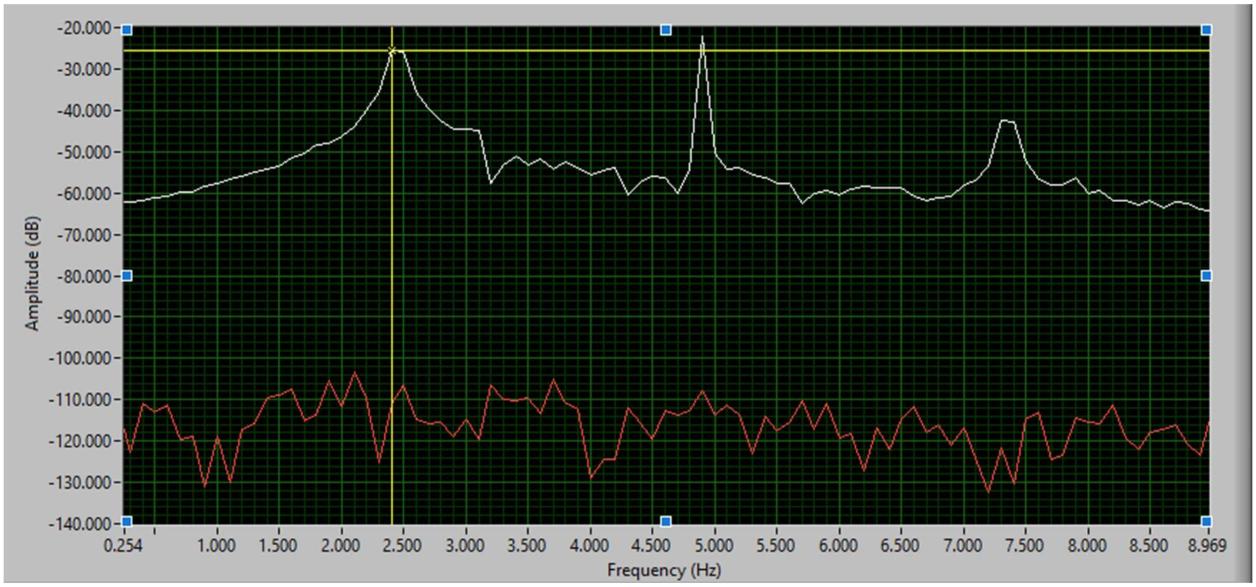

Figure 19: FFT Plot for Mass Ratio of 0.43 and Frequency Ratio of 0.84 at 11.5V

Table 8: Amplitude of Primary Mass without DVA in dB at Varying Voltage Inputs in dB for μ=0.43 and β=0.84

Voltage (V)	Driving Frequency (rad/s)	Amplitude of Primary Mass (dB)	Amplitude of Primary Mass (mm)
8	10.29	-30.312	2.96
8.5	11.04	-28.254	2.97
9	11.75	-21.474	5.81
9.5	12.47	-17.558	8.23
10	13.22	-12.292	13.69
10.5	13.97	-9.666	16.87
11	14.70	-6.247	22.88
12.5	16.92	1.234	45.83
13	17.51	1.130	45.28
13.5	18.32	-1.640	30.43
14	19.10	-5.948	17.19
14.5	19.72	-3.946	17.53
15	20.47	-4.391	15.60
15.5	21.15	-5.968	12.21
16	21.93	-7.432	9.14
16.5	22.57	-5.949	10.23
17	23.33	-7.877	7.74
17.5	24.06	-4.156	11.25
18	24.79	-4.907	9.78
18.5	25.50	-6.900	7.02
19	26.40	-5.771	7.606617132

Table 9:Amplitude of Primary and Absorber Masses in dB at Varying Voltage Inputs in dB μ=0.43 and β=0.84

Voltage (V)	Driving Frequency (rad/s)	Amplitude of Absorber Mass (dB)	Amplitude of Absorber Mass (mm)	Amplitude of Primary Mass (dB)	Amplitude of Primary Mass (mm)
10	13.22	-24.020	3.5471	-32.965	2.0809
10.5	13.97	-25.418	2.7515	-31.636	2.1066
11	14.70	-29.878	1.5064	-30.237	2.1874
11.5	15.50	-29.000	1.5307	-28.655	2.3283
12	16.23	-23.585	2.4328	-30.667	1.6449
12.5	16.92	-20.857	3.0884	-30.369	1.5224
13	17.51	-17.959	4.0091	-28.556	1.6946
13.5	18.32	-15.861	4.7584	-25.670	2.1261
14	19.10	-14.309	5.3164	-21.661	3.0754
14.5	19.72	-14.136	5.0791	-16.493	5.1295

15	20.47	-15.575	4.0389	-13.190	7.0087
15.5	21.15	-15.181	3.9740	-12.384	7.0280
16	21.93	-14.208	4.1875	-9.725	8.7763
16.5	22.57	-13.012	4.2865	0.030	25.3169
17	23.33	-11.154	5.0258	3.526	36.1834
17.5	24.06	-10.732	5.0020	-1.338	18.7936
18	24.79	-8.662	6.3481	-1.496	15.9368

Appendix D: Procedure for Data Collection

Amplitude Without DVA

To collect data on the amplitudes of the primary and absorber masses, the amplitude of the primary mass without the absorber attached was measured first. The primary mass and spring (bronze colored) were attached first according to Figure 3. The motor on the primary mass was connected to the power supply using red and black cables. An accelerometer was attached to the primary platform and connected to terminal 0 on the DAQ device using a white accelerometer cable. The DAQ device was then connected to a device with LabView installed. The DVA.vi file was created in LabView VI to generate an Acceleration vs Time plot and Amplitude vs Driving Frequency (FFT) plot from the measured accelerometer data. The power supply was turned on, the Output button was selected, and the input voltage value was adjusted to the desired value (8V). Voltage input corresponds to the driving frequency of the system. Once the motor and eccentric mass were rotating steadily, the Run button was pressed in LabView to collect data. After 10 seconds, the measured accelerometer values appeared on the Acceleration vs Time and FFT plots. The peak amplitude closest to the natural frequency of the system on the FFT plot, or the first peak, was measured using cursers, recorded on the data sheet, and later converted from dB to m. Data collection was repeated at varying input voltages spaced 0.5V apart for a total of about 20 amplitude measurements. Once measurements were completed, the power supply was turned off.

Amplitude With DVA

To measure the amplitude of the absorber mass and the suppressed amplitude of the primary mass (i.e. the primary mass with DVA attached). The absorber mass and spring (silver colored) were attached according to Figure 3. An accelerometer was attached to the absorber platform and connected to terminal 1 on the DAQ using a white accelerometer cable. The DAQ device and power supply remain attached from the previous measurements. The power supply was turned on, the Output button was selected, and the input voltage value was adjusted to the desired value (10V). Like in previous measurements, the Run button was selected in LabView to collect data and the FFT plot was referred to for the amplitude. The measurements from both the primary and absorber masses appear on each plot. The peak amplitudes of the first peak on the FFT plot of both the primary and absorber masses were measured using cursers, recorded on the data sheet, and later converted from dB to m. Data collection for both masses were repeated at varying input voltages spaced 0.5V apart for a total of about 20 amplitude measurements. Once measurements were completed, the power supply was turned off. A more detailed version of the procedure is included in Appendix III for M1 students to reference while performing their own experiments.

Appendix E: LabView VI

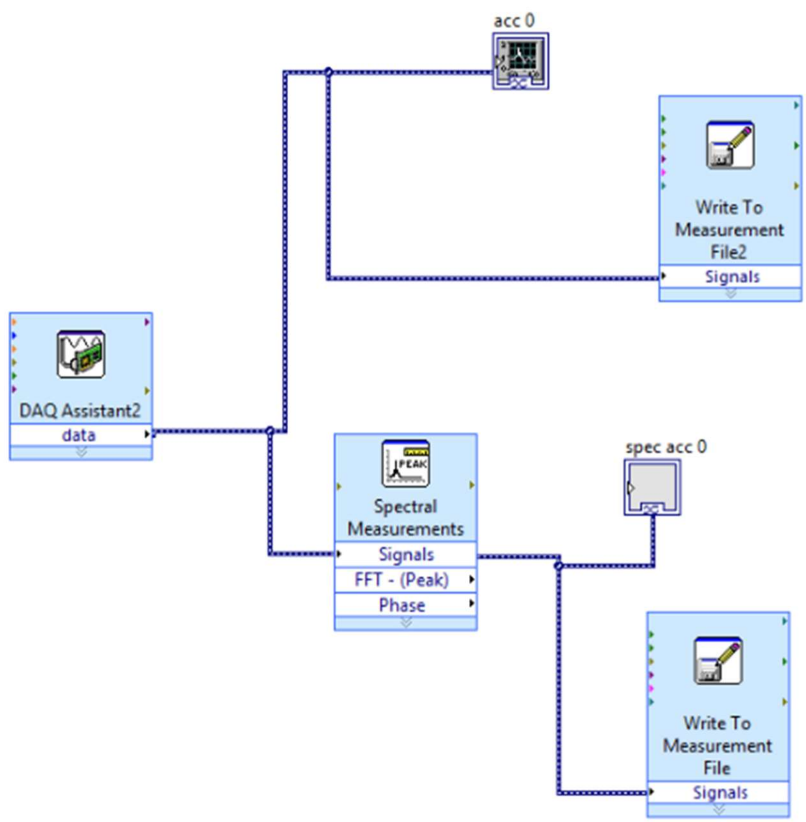

Figure 20: Block Diagram Setup in LabView VI