Final Design Project

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ME 408: Introduction to Computer Aided Engineering

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Problem Statement and Design Goals

The goal of this project is to design the fastest possible method to fully cook a genetically modified turducken using both a conventional oven and electrical resistance heaters with minimal burning. The design goals are to find the optimal oven temperature and cooking time as well as optimal skewer geometry, placement, material, and cooking time.

Given Parameters and Conditions

- The outer layer is homogeneous turkey meat
- The middle layer is a homogeneous combination of chicken and duck meat
- The inner layer is a homogenous piece of stuffing
- At the beginning of the cooking process, the Turducken is fully thawed out with all components are at 42°F
- Skewers reject a constant rate of heat
- Skewers can only be turned off or on once
- Maximum skewer cross-section length of 5/8 inches
- Maximum skewer cross section area of 1/4 square inches
- Oven Temperature Range between 350°F and 575 °F
- Up to 2 different temperature settings within this range during the cooking process

Design Concept

For the final skewer design, two skewers were used. Each skewer is a cylinder 16 inches long with a diameter of 0.56 inches. The end of the skewer is tapered and comes to a sharp point with an angle of 40° to assist with insertion into the turducken. The circular shape was chosen for ease of design, manufacturing, and handling, based on the assumption that heat transfer dynamics of the skewer would not be excessively impacted by the exact cross-sectional shape of the skewer given the restrictive conditions on its geometry. The skewer diameter of 0.56 inches is below the maximum allowed length of 0.625 inches and the cross-sectional area is 0.246 in^2, less than the maximum of 0.25 in^2.

The skewers are designed to be placed horizontally in the turkey on the vertical midline of the turducken approximately 3.5 inches apart. One skewer is inserted directly through the cavity into the stuffing and passes from there through the stuffing and ducken and slightly into the turkey on the other side. The other skewer is placed above the tailbone, and thus goes through successive layers of turkey, ducken, stuffing, and ducken before entering but not exiting a final layer of turkey. Air is to be allowed to freely reach the bottom of the turkey by placing it on a surface which allows free airflow. This placement was chosen in order to cause the central cavity of the turkey, which is farthest from the outside and therefore least likely to cook from the oven, to cook evenly with the skewers.

The skewers are to be made with a food grade aluminum shell, which has desirable heat transfer properties compared to materials such as stainless steel such as a thermal conductivity of 237 $\frac{W}{m*K}$ and is often used for cookware. The core of the skewer is made of silicon carbide, which has high electrical resistance and is often used as a resistive heating element. The Skewer emits 125 Watts at 19.5 Volts and 6.4, the derivation of which is detailed in the skewer heating calculation. A schematic of the skewer design is provided in Figure 1.

Drawings

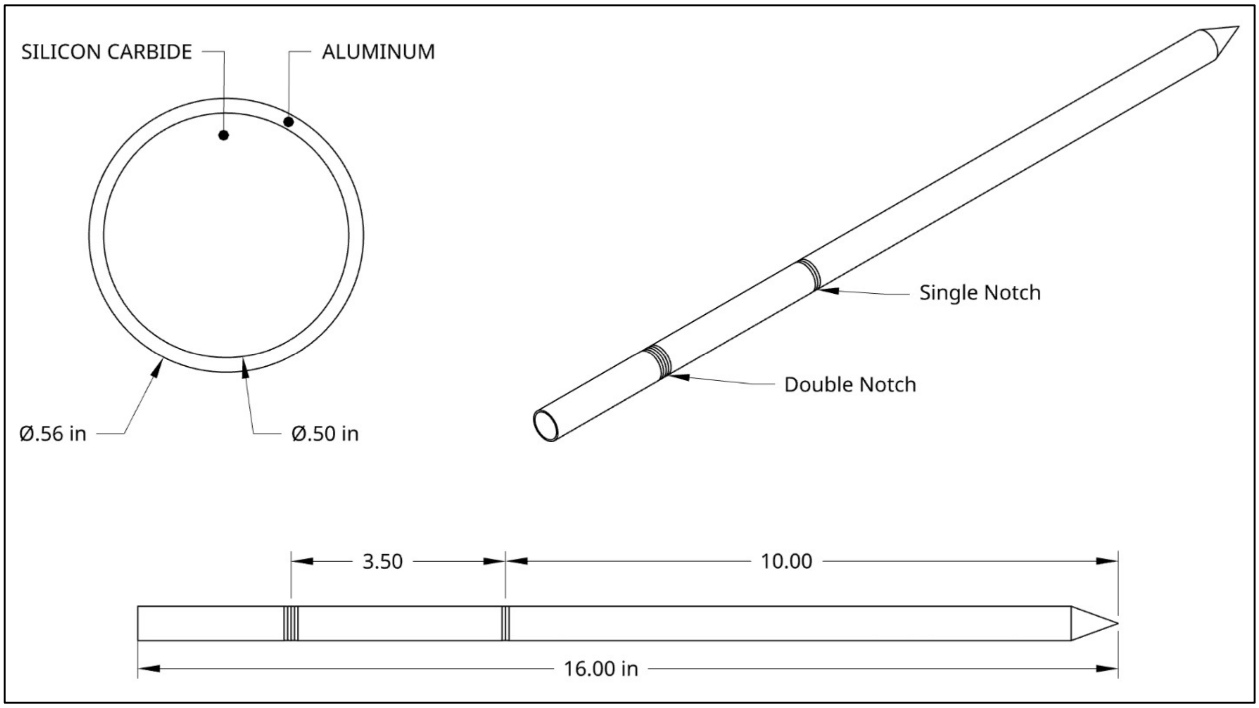


Figure 1: Skewer sketch

Hand Calculations

Ingredient Properties

To accurately model the cooking process the density, thermal conductivity, and specific heat must be found for the turkey, stuffing, and ducken at all possible temperatures. These values can be found by considering the mass composition of the nutrients that make up each ingredient; water, protein, fat, and carbohydrates. As fibers are a subset of carbohydrates they were not needed for calculations. Using the 2006 ASHRAE handbook, each nutrient has a formula for density, thermal conductivity, and specific heat (see Appendix B), which are weighted with respect to their mass percentage to determine the overall values for each ingredient. To then determine the overall values for the turducken components, a weighted average of the ingredients making up the turkey, ducken, and stuffing must be taken respectively. The general approach for these calculations is as follows.

Table 1: Variables for Calculating the Density of an Ingredient

	Water	Protein	Fat	Carbs	Ash
Density	$\rho_{H_2O}(T)$	$\rho_{protein}(T)$	$\rho_{fat}(T)$	$\rho_{carbs}(T)$	$\rho_{ash}(T)$
Mass %	$x_{H_2O}(T)$	$x_{protein}(T)$	$x_{fat}(T)$	$x_{carbs}(T)$	$x_{ash}(T)$

The variables in Table 1 can be used to find the density of any ingredient at any given temperature.

$$\rho_{ingredient}(T) = \sum \rho_i(T) x_i(T) \tag{1.1}$$

Note that the density of each ingredient is temperature dependent as both the densities and mass percentages of each nutrient are as well. How the nutrient densities vary with temperature can be found in Appendix A, Table 4. Although the formulas in these tables are only specified for 42ºF to 300ºF it's assumed they are a good enough approximation until 500ºF as property values above 300ºF are needed for simulations. How the nutrient mass percentages vary with temperature can be seen more clearly in the following table which applies to butter.

Table 2: Composition of Butter from 42°F Onwards

	$\mathbf{42}$°F $-$ $\mathbf{212}$°F	$\mathbf{212}$°F $-$ $\mathbf{290}$°F	$\mathbf{290}$°F +
$\boldsymbol{x_{H_2O,b}}$(%)	17.94	$17.94 - 0.23(T - 212)$	0
$\boldsymbol{x_{protein,b}}$ (%)	0.85	$0.85 - 0.011(T - 212)$	0
$\boldsymbol{x_{fat,b}}$ (%)	81.11	$81.11 - 1.04(\mathrm{T} - 212)$	0
$\boldsymbol{x_{carbs,b}}$ (%)	0.06	$0.06 - (7.7 * 10^{-4})(T - 212)$	0
$\boldsymbol{x_{ash,b}}$ (%)	0.04	$0.04 + 1.28(T - 212)$	100

Table 2 shows sample formulas for the composition of butter from 42°F onwards. Until 212°F the composition was assumed to be constant as no moisture is lost. From 212°F onwards the boiling temperature of water has been surpassed, consequently decreasing the water composition which will affect the composition of the other nutrients as well. It's assumed that once 290°F is reached the entire composition becomes ash. This means that the water has fully evaporated, and all other nutrients have turned to ash which is a reasonable assumption considering this temperature represents the turducken being burnt. To meet each of these conditions it was assumed that as moisture evaporates it decreases the mass composition of water linearly until reaching zero at 290°F. As the entire composition must become ash, all other components were also assumed to decrease linearly in composition until reaching zero at 290°F.

The approach for finding the thermal conductivity and specific heat of an ingredient follows the approach for finding the density, see equations A.21 and A.22. Rather than using the density of each nutrient as a function of temperature the functions for thermal conductivity and specific heat as a function of temperature were used instead, as found in Appendix A Tables 6 and 7 respectively. The approach for determining an ingredient's composition throughout the temperature range follows that of butter above, except that poultry has an additional stage, shown in the example for turkey below in Table 3.

Table 3: Composition of Turkey from 42°F Onwards

	$\mathbf{42}$°F $-$ $\mathbf{212}$°F	$\mathbf{212}$°F $-$ $\mathbf{228}$°F	$\mathbf{228}$°F $-$ $\mathbf{290}$°F	$\mathbf{290}$°F +
$\boldsymbol{x_{H_2O,t}}$(%)	70.4	$70.4 - 0.47(T - 212)$	$62.9 - 1.01(T - 228)$	0
$\boldsymbol{x_{protein,t}}$ (%)	20.42	20.42	$20.42 - 0.33(T - 228)$	0
$\boldsymbol{x_{fat,t}}$ (%)	8.02	8.02	$8.02 - 0.13(T - 228)$	0
$\boldsymbol{x_{carbs,t}}$ (%)	0	0	0	0
$\boldsymbol{x_{ash,t}}$ (%)	0.88	$0.88 + 0.47(T - 212)$	$8.38 + 1.48(T - 228)$	100

Based on USDA Food and Inspection Service: Water in Meat and Poultry, the water percentage of white and dark poultry meat drops 8% and 7% from raw to cooked respectively. Assuming the poultry meat is 50/50 white to dark meat, it was assumed that 7.5% of moisture is lost in poultry meat once cooked. Since our poultry is considered cooked between 165°F and 290°F, the temperature at which poultry is cooked was chosen as the midpoint of these two bounds, which rounds to 228°F. This means that from 212°F to 228°F the water composition drops 7.5% and for simplicity the ash was assumed to increase by 7.5%, leaving the other nutrients constant until 228°F. From 228°F to 290°F the compositions of all nutrients were assumed to drop to zero linearly other than ash which increases to 100% and remains that way from 290°F onwards. This approach was used for the chicken, duck, and turkey alike.

The density, thermal conductivity, and specific heat of each ingredient were determined using the method of weighted averages as shown above for a temperature range of 42°F to 500°F, see Appendix A for other ingredient compositions over this range. Anything over 500°F is well overburnt and if reached is deemed an unsuccessful cooking method. The values for each ingredient throughout this range were found using Excel as the formulas above could be easily applied throughout this range. Once these are found a weighted average of each ingredient relative to mass percentage must be taken to determine the values of each component, namely the turkey, ducken, and stuffing. The composition of each component with respect to its ingredients are as follows:

Table 4: Ingredient Composition (Volume %) of Components Making up Turducken at 42°F

Food	Ingredient	Composition (% Volume)
Turkey	Turkey	100%
Ducken	Duck	50%
	Chicken	50%
Stuffing	Butter	6.1%
	Celery	16.3%
	Onion	4.1%
	Potato	73.5%

The volume compositions for the stuffing were found based off the recipe which calls for ¾ cups of butter, two large celery sticks, ½ cups of onion, and nine cups of potatoes. The ducken was assumed to be even amounts of duck and chicken. To increase accuracy, the weighted averages were taken based off mass composition, which requires multiplying the density of each ingredient by volume at every temperature. Assuming the ingredient volumes are constant throughout cooking sample results at 42°F are as follows:

Table 5: Ingredient Composition (Mass %) of Components Making up Turducken at 42°F

Food	Ingredient	Composition (% Mass)
Turkey	Turkey	100%
Ducken	Duck	49%
	Chicken	51%
Stuffing	Butter	5.3%
	Celery	15.4%
	Onion	3.9%
	Potato	75.4%

The mass composition of each ingredient was taken throughout the temperature range and weighted with the density, thermal conductivity, and specific heat of each ingredient to determine these values for the turkey, ducken, and stuffing from 42°F to 500°F. These ranges of values were imported under the properties created in Ansys Mechanical. The plots for each component's imported properties over the temperature range can be found in Appendix A.

Fourier Number

A Fourier number of 1 is known as the hard stability limit for explicit methods, i.e. the maximum theoretical time limit associated with a stable solution. This number will be used to guide the time step calculation. To determine a conservative estimate of the Fourier number, the maximum

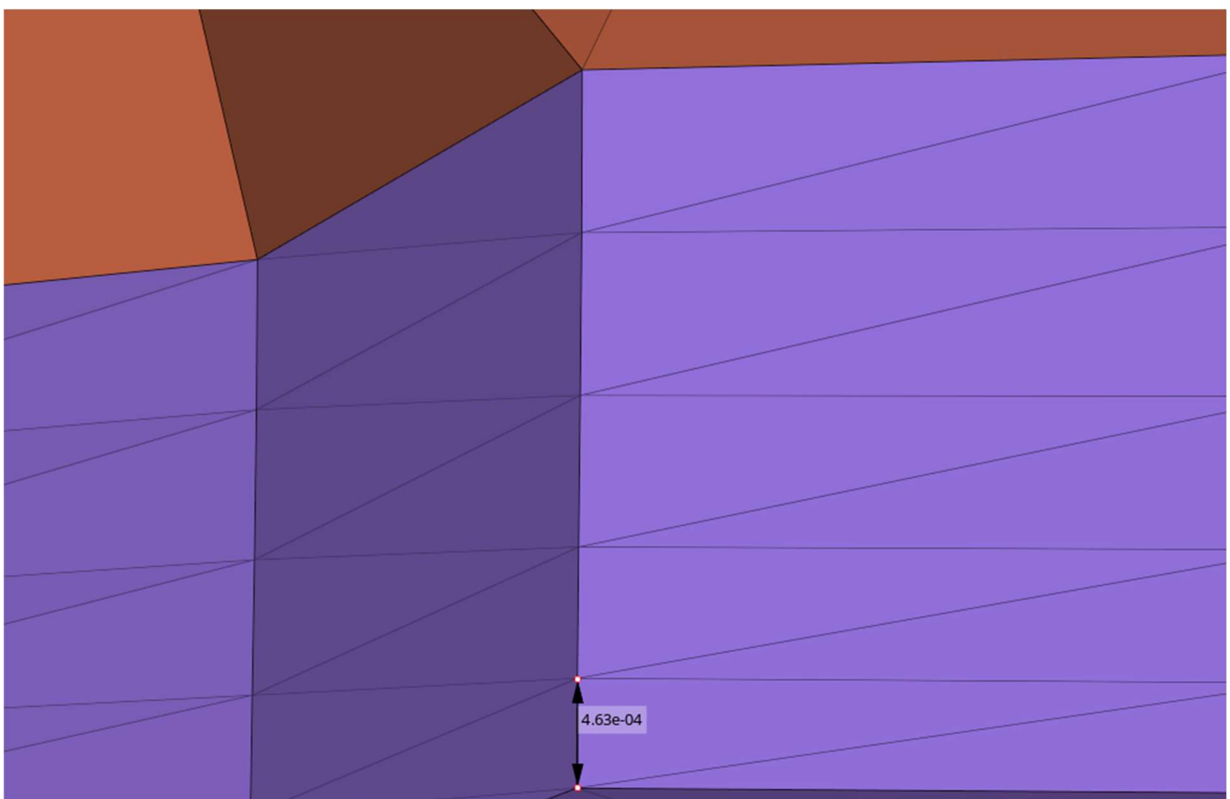


Figure 2: Minimum element size

thermal diffusivity and minimum length must be used. As seen in Table 10 of Appendix A thermal diffusivity is also temperature dependent therefore the maximum thermal diffusivity values from 42°F to 500°F for turkey, ducken, and stuffing were compared. Of these results the maximum thermal diffusivity pertained to the stuffing at 212°F with a value of 0.006087 ft^2/hr. To determine the minimum characteristic length, it's worth noting that each element has a unique value in the x, y, and z directions. To determine the minimum length the Hypermesh file was searched, and the smallest length was determined to be the width of a first boundary layer element about a skewer surrounded by stuffing (irrespective of which one).

Figure 2 shows the minimum element to be 0.463 mm, and plugging these values into Equation A.21 as seen in Appendix 1 and isolating t yields the following:

$$\frac{F_o L^2}{\alpha} = \frac{(1)(0.000463\ m)^2}{\left(\frac{0.006087 ft^2}{hr} * \frac{0.093\ m^2}{ft^2} * \frac{1\ hr}{3600\ s}\right)} = \boldsymbol{t = 1.36\ s} \tag{1.2}$$

Unfortunately, this time step would take far too long to run, for a final simulation with a time step of 10 minutes the Fourier number would be as follows for the most conservative element.

$$\frac{\alpha t}{L^2} = \frac{\left(\frac{0.006087 ft^2}{hr} * \frac{0.093\ m^2}{ft^2} * \frac{1\ hr}{3600\ s}\right)\left(10\ \text{min} * \frac{60\ sec}{1\ min}\right)}{(0.000463\ m)^2} = F_o = 440.122 \tag{1.3}$$

Theoretically this is too coarse for our simulation since it is a relatively long simulation, however in essence of time these simulations were run with a Fourier number of 440 for the most conservative element. When pulling the turducken out of the oven and resting, the time step was decreased to $F_o \approx 200$ as more precision was desired in this range to know when the turducken was fully cooked.

As the time steps did not see sharp increases in temperature, it's assumed these time steps are fine enough to get a general idea of the temperature within the turducken during cooking. Since there are no sharp spikes, it can be safely assumed that the burnt percentage estimate is reasonable, although it's understood that it's likely a slight undershot.

Skewer Heating Calculations

The skewers are modeled as resistance heaters with a silicon carbide core, which possesses an electric resistivity, ρ of 1 Ω-mm, or approximately 0.04 Ω-in. The resistance of the silicon carbide is calculated using the following equation.

$$R = \rho \frac{L}{A} = 0.04\ \Omega in * \frac{15.23\ in}{0.20\ in^2} = \mathbf{3.05}\ \Omega \tag{1.4}$$

Where L is the length of the carbide tube and A is the cross-sectional area of the carbide tube. The formula to calculate the power running through the silicon carbide is and thus the heat flux coming from it is:

$$P = I^2R = (6.4\ A)^2(3.05\ \Omega) = 125\ W \tag{1.5}$$

Based on the wattage per skewer that satisfies the cooking conditions, a current was decided on using this equation.

Based on this current, the voltage is determined via Ohms law:

$$V = IR = 6.4\ A * 3.05\Omega = 19.5V \tag{1.6}$$

The surface area in which the skewers conduct heat across is 0.38 ft^2, or 0.035 m^2. Thus, the skewers were modeled as having a heat flux of:

$$\frac{125W}{0.035\ m^2} = 3541 \frac{W}{m^2} = 0.31 \frac{Btu}{s * ft^2} \tag{1.7}$$

Turducken Heat Transfer

To solve for the heat transfer when cooking the turducken by hand, simplifications need to be made. Firstly, the geometry of the turducken is simplified as layers of spheres within one another. Next, a circuit analysis is conducted in which boundaries between materials are modeled as resistances, volumes of each layer are modeled as capacitances, the voltages are representative of the heat emitted, and heat flow is modeled by current. Two skewers are used during the cooking process; however, they do not impact the heating of each layer equally. As seen in Figure 3, one skewer passes through the turkey portion of the turducken significantly, however the other does not.

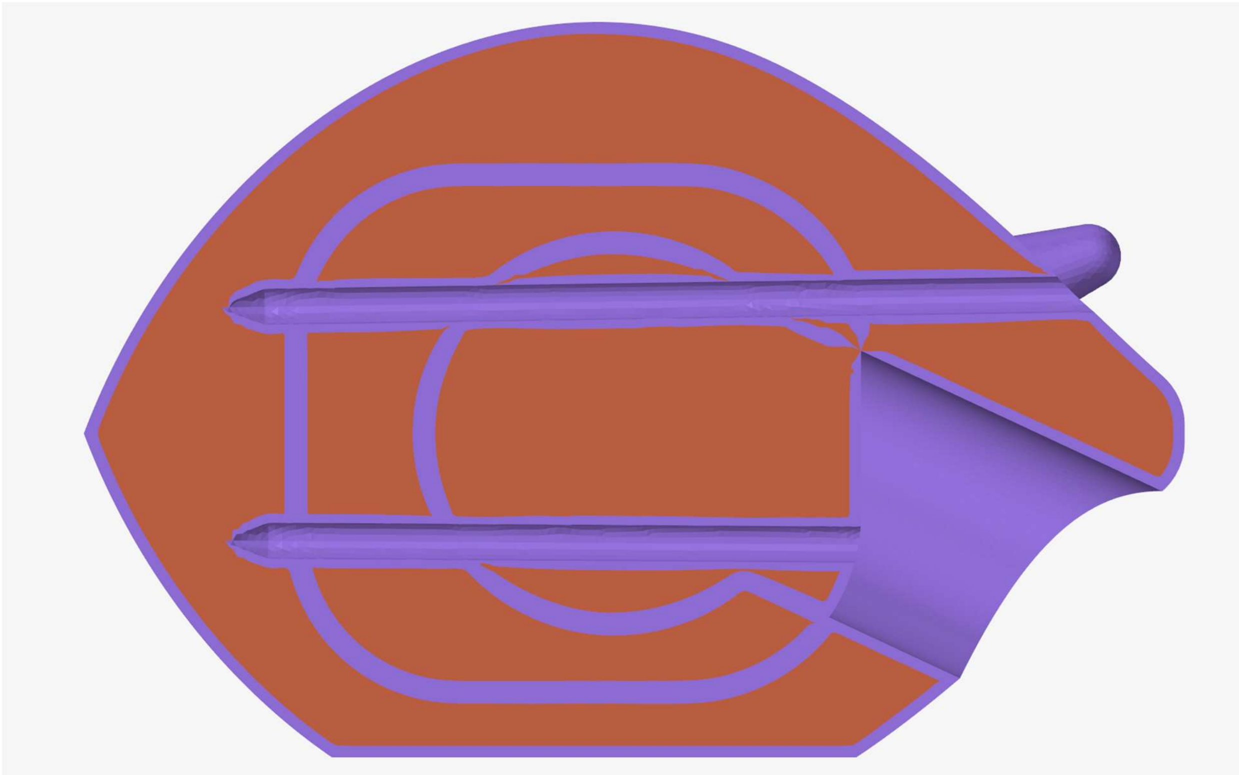


Figure 3: Turducken Geometry Cross-section

This cross-section is then simplified into a spherical model with six nodes, one for each boundary layer as well as one for the mass of each layer as follows in Figure 4:

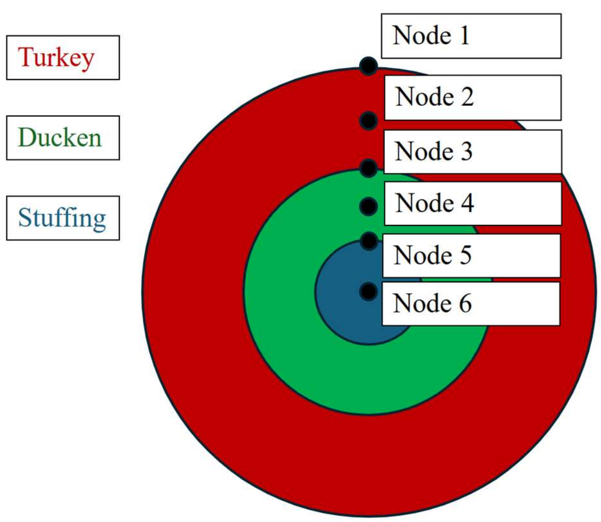


Figure 4: Thermal Model

As the impact of the skewers is complex, the total input heat is split up into the ratio of heat per layer mass. Although one of the skewers only passes through the turkey once, it is simplified to acting through the skin fully for computational simplicity. As two skewers impact the ducken and stuffing layers, the input heat, Q_{in}, is doubled. Resulting in the following equivalences:

$$Q_{in} = Total \tag{1.8}$$

$$Turkey\ Layer = \frac{Q_{in}}{5} \tag{1.9}$$

$$Ducken\ Layer = \frac{2Q_{in}}{5} \tag{1.10}$$

$$Stuffing\ Layer = \frac{2Q_{in}}{5} \tag{1.11}$$

Using the spherical schematic and ratios of heat impact above, a circuit diagram is formed to model the heat transfer. Radiation and convection of the oven, conduction between layers, and conduction from the skewers are all considered.
Conduction is expressed as:

$$Q = \frac{kA\Delta T}{\Delta x} \tag{1.12}$$

In which A is the area, Δx is the distance, and k is the thermal conductivity of the material. Thermal resistance is then found as:

$$R_{cond} = \frac{\Delta T}{Q} = \frac{\Delta x}{kA} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{1.13}$$

R_{cond} is applied between layers of the turducken. Nodes 2-6 in the upcoming circuit diagram. Convection is expressed as:

$$Q = hA\Delta T \tag{1.14}$$

In which h is the heat transfer coefficient. The thermal resistance is then found as:

$$R_{conv} = \frac{\Delta T}{Q} = \frac{1}{hA} \tag{1.15}$$

Convection is applied to the outside of the turducken, as thermal resistance for the oven and room. Radiation is expressed as:

$$Q = \varepsilon\sigma A(T_{surface}^4 - T_{surroundings}^4) \tag{1.16}$$

In which ε is the emissivity of the surface and σ is the Stefan-Boltzmann constant. From this, the thermal resistance is then found as:

$$R_{rad} = \frac{1}{\varepsilon\sigma A\left(T_{surface}^2 + T_{surroundings}^2\right)\left(T_{surface} + T_{surroundings}\right)} \tag{1.17}$$

Radiation is applied in conjunction with convection as the thermal resistance of the oven and room. From this, equations representing a change in temperature at each node are created.

Circuit Diagram

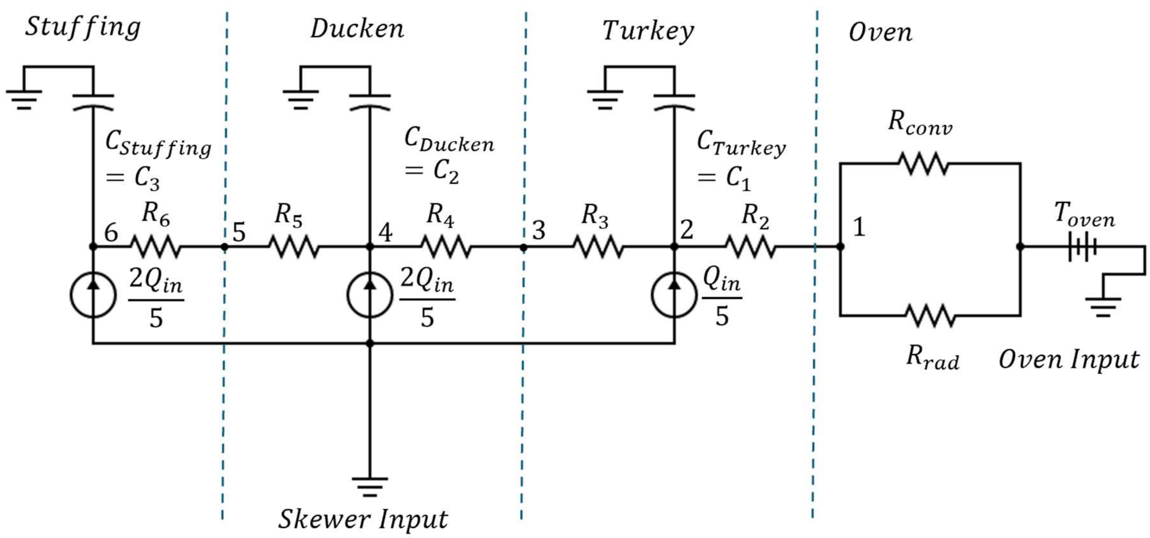


Figure 5: Circuit Diagram

Using this diagram, a nodal analysis is conducted for the turkey, ducken, and stuffing nodes. At each node, Kirchhoff's Current Law is applied. The resistances, radiative and convective, from the oven are simplified into one resistance. This creates the following simplified circuit model.

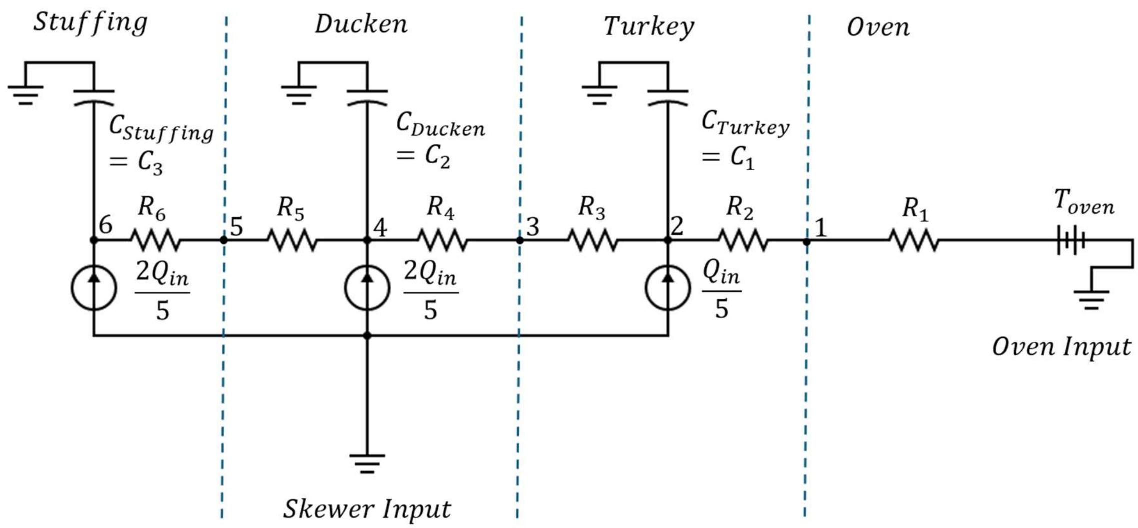


Figure 6: Simplified Circuit Diagram

This circuit model states that the sum of currents entering and leaving each node is zero and is then applied to thermal terms in which the current is representative of heat entering and exiting the node. In equations 1.18, 1.20, and 1.22, there is no capacitance at the boundary layers as these nodes do not store heat and instead represent steady-state flow. The following equations, written in differential form, represent the heat transfer within the turducken during the cooking process.

$$\frac{dT_1}{dt} = \left(\frac{T_{inf} - T_1}{R_1} - \frac{(T_1 - T_2)}{R_2}\right) \tag{1.18}$$

$$\frac{dT_2}{dt} = \frac{1}{C_2}\left(\frac{T_1 - T_2}{R_2} - \frac{T_3 - T_2}{R_3} + \frac{Q_{in}}{5}\right) \tag{1.19}$$

$$\frac{dT_3}{dt} = \left(\frac{T_3 - T_2}{R_3} - \frac{T_4 - T_3}{R_4}\right) \tag{1.20}$$

$$\frac{dT_4}{dt} = \frac{1}{C_4}\left(\frac{T_4 - T_3}{R_4} - \frac{T_5 - T_4}{R_5} + \frac{2Q_{in}}{5}\right) \tag{1.21}$$

$$\frac{dT_5}{dt} = \left(\frac{T_5 - T_4}{R_5} - \frac{T_6 - T_5}{R_6}\right) \tag{1.22}$$

$$\frac{dT_6}{dt} = \frac{1}{C_5}\left(\frac{T_6 - T_5}{R_5} + \frac{2Q_{in}}{5}\right) \tag{1.23}$$

It is important to note that the cooking method for the turducken consists of three stages. In the first stage, only the skewers are activated, in the second the turducken with activated skewers is placed into the oven, lastly, the cooked turducken is taken out of the oven and the skewers are turned off. This last stage is called the resting stage and allows for some layers to cool down, while others complete their cooking process. Due to this, it must be kept in mind that the above equations are the general form, as T_{inf} during the skewer and resting stages is equivalent to room temperature, which is 70 degrees Fahrenheit. However, while the turducken is cooking inside the oven, it is equivalent to 350 degrees Fahrenheit which is the temperature inside the oven, also expressed as T_{oven}. Similarly, R_1 depends on the stage of the cooking process as well, as the temperature of both the surface and the temperature of the surroundings changes. This is crucial as seen in eq 1.17, since R_{rad} is dependent on both of these temperatures. This is important in our model as the temperatures change from phase to phase of the cooking process. In the first stage, the turkey skin is at its frozen temperature, 42 degrees Fahrenheit. While cooking inside the oven it is assumed to be defrosted and at room temperature (70 degrees Fahrenheit). However, during the resting stage, the turkey skin stays at the temperature of the oven, 350 degrees Fahrenheit, while the room temperature returns to 70 degrees Fahrenheit.

Hand Calculation Plots and Results

The plots below represent the cooking process of the turducken at each stage.

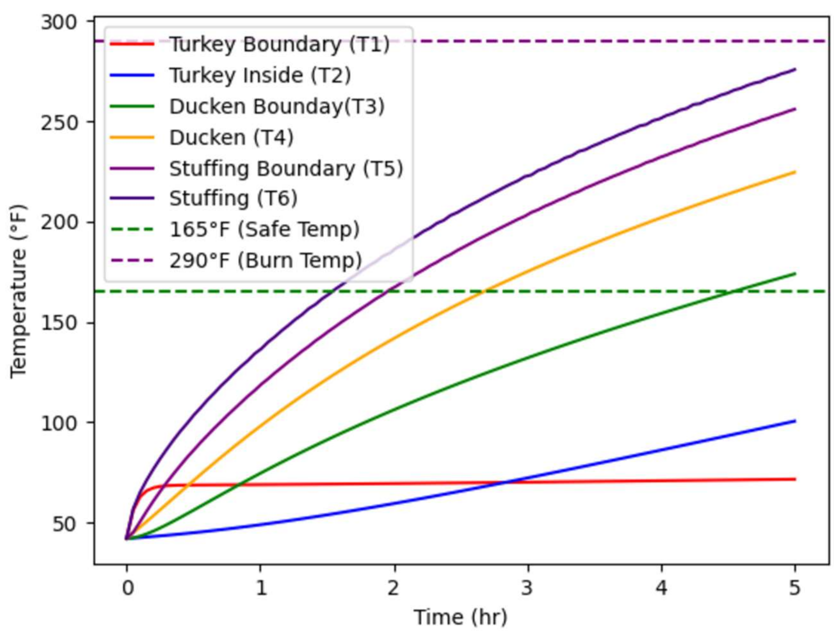


Figure 7: Skewer Cooking Stage Plot

Figure 7 represents the skewer stage, in which only heated skewers are used to cook the turducken for 4.25 hours. This technique is used to provide heat to the stuffing and ducken layers prior to the oven stage and ensures that they will be cooked at the end of the complete cooking process.

In this plot, time is shown to extend to 5 hours, however the skewers are only activated for 4.25 hours. This results in the following temperatures of each later at the conclusion of this stage (Table 6):

Table 6: Node temperatures after skewer cooking

Node	**Temperature**
T1	70.96 °F
T2	39.54 °F
T3	159.11 °F
T4	207.69 °F
T5	238.04 °F
T6	258.28 °F

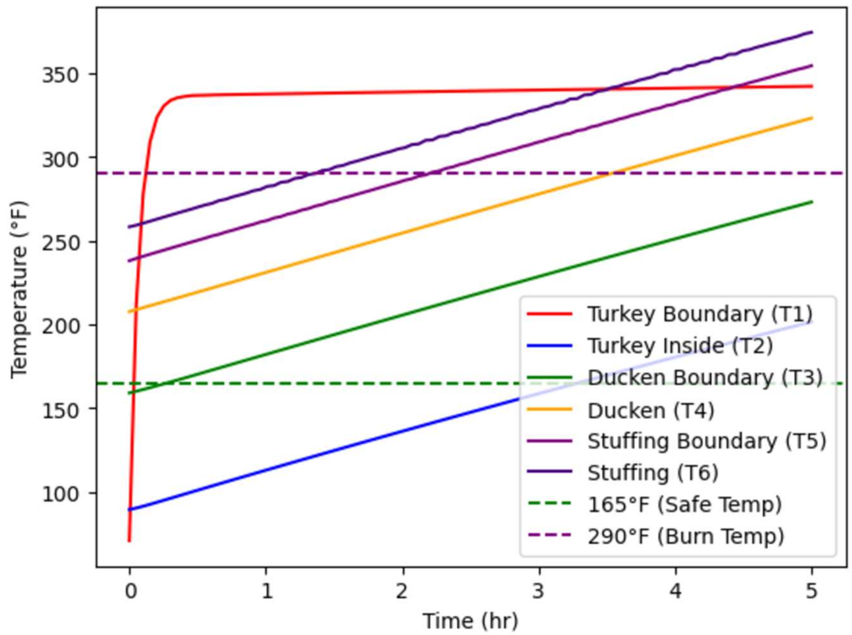


Figure 8: Oven Cooking Stage Plot

In the second stage, the skewers are still turned on and the turducken is placed into the oven. This allows for the entirety of the turducken to begin to cook and primarily focuses on cooking the outer layers through convection.

In Figure 8: Oven Cooking Stage Plot, the oven stage is represented. Similarly, while the plot extends to 5 hours, the cooking time completes at 2.5 hours, resulting in the following final values:

Table 7: Skewer Stage Hand Calculation Results

Node	Temperature
T1	339.33 °F
T2	146.99 °F
T3	216.54 °F
T4	265.66 °F
T5	296.52 °F
T6	316.76 °F

The last portion of the cooking process is the resting stage, in which the turducken is allowed to cool for two hours.

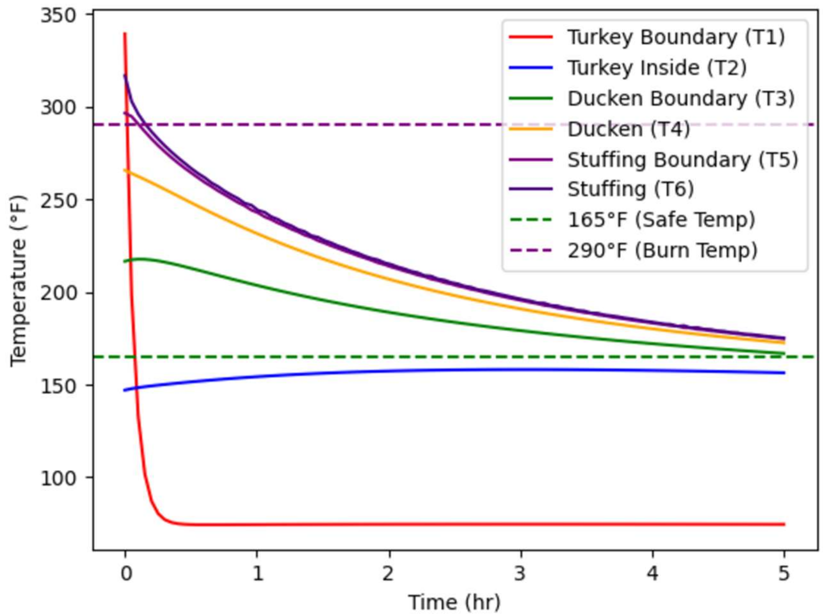


Figure 9: Resting Stage Plot

As in the previous graphs, in Figure 9: Resting Stage Plot time extends visually to 5 hours, however, at 2 hours the resting time is complete. This results in the following final values in Table 8:

Table 8: Hand Calculation Resting Stage Results

Node	Temperature
T1	74.55 °F
T2	157.29 °F
T3	188.79 °F
T4	206.53 °F
T5	213.48 °F
T6	214.81 °F

This resting process allows for the turkey layer to complete the cooking process while the rest of the turducken cools down. Heat flows from the other layers to the turkey layer. As such, the final values for the other nodes are taken from the previous plot rather than this one, to represent the maximum temperatures reached during the complete cooking process.

The maximum temperatures are compiled below in Table 9:

Table 9: Maximum Temperature Hand Calculation Results

Node	**Temperature**
T1	339.33 °F
T2	157.29 °F
T3	216.54 °F
T4	265.66 °F
T5	296.52 °F
T6	316.76 °F

As simplifications are made to the turducken heat transfer model, plots provide final temperature values slightly smaller than our simulation. This is due to the geometry simplification of the turducken as concentric spheres, the simplification of the resistances from the oven, as well as the simplification of the skewer impact on each layer of the turducken. Namely, both skewers slightly impact the cooking of the turkey layer, however, their impact is significantly smaller than equating them to fully piercing the turducken. As such, this is considered to be negligible, and only one skewer impacts the turkey, as aside from the slight contact of the skewer to the turkey after exiting the ducken, there is also a skewer impact on the turkey while entering the turducken. This is still representative of our simulation, as according to it, the turkey layer very closely reached the safe temperature. Due to our simplifications, it can be concluded that the lower final value of the turkey layer in these hand calculations can be considered negligible.

Finite Element Analysis

Mesh Creation

The turducken was meshed in HyperMesh utilizing the mesh controls. First, a surface mesh was created with Triangle elements and an element size of 3 mm upon all surfaces. This was done by right clicking on the surface mesh tab followed by; Create, Model, Size and Bias, Surfaces. The skewers were not included in the mesh but the volume they make up was considered fixed. After the Surface mesh, a volume mesh was selected as the following: Create, Model, BL + Tetra. After meshing the total element number came out to 4.97 million elements Table 10 lists the settings which were applied to the Boundary Layer and Core Mesh:

Table 10: Volume Mesh Settings

Boundary Layer	**Setting**	**Tetra Mesh (Core)**	**Setting**
First Layer Thickness	0.0006	Element Size Limit	None
Growth Rate	1.2	Quality	Optimize Quality
Final Layer Height	0.8	Tetra Mesh Method	Delaunay
Number of Layers	5	Growth Rate	1.2
Hexa Transition	All Tetras	Tetramesh Height	1.0

The smallest element side length in the mesh was 0.463 mm, as seen in Figure 10.

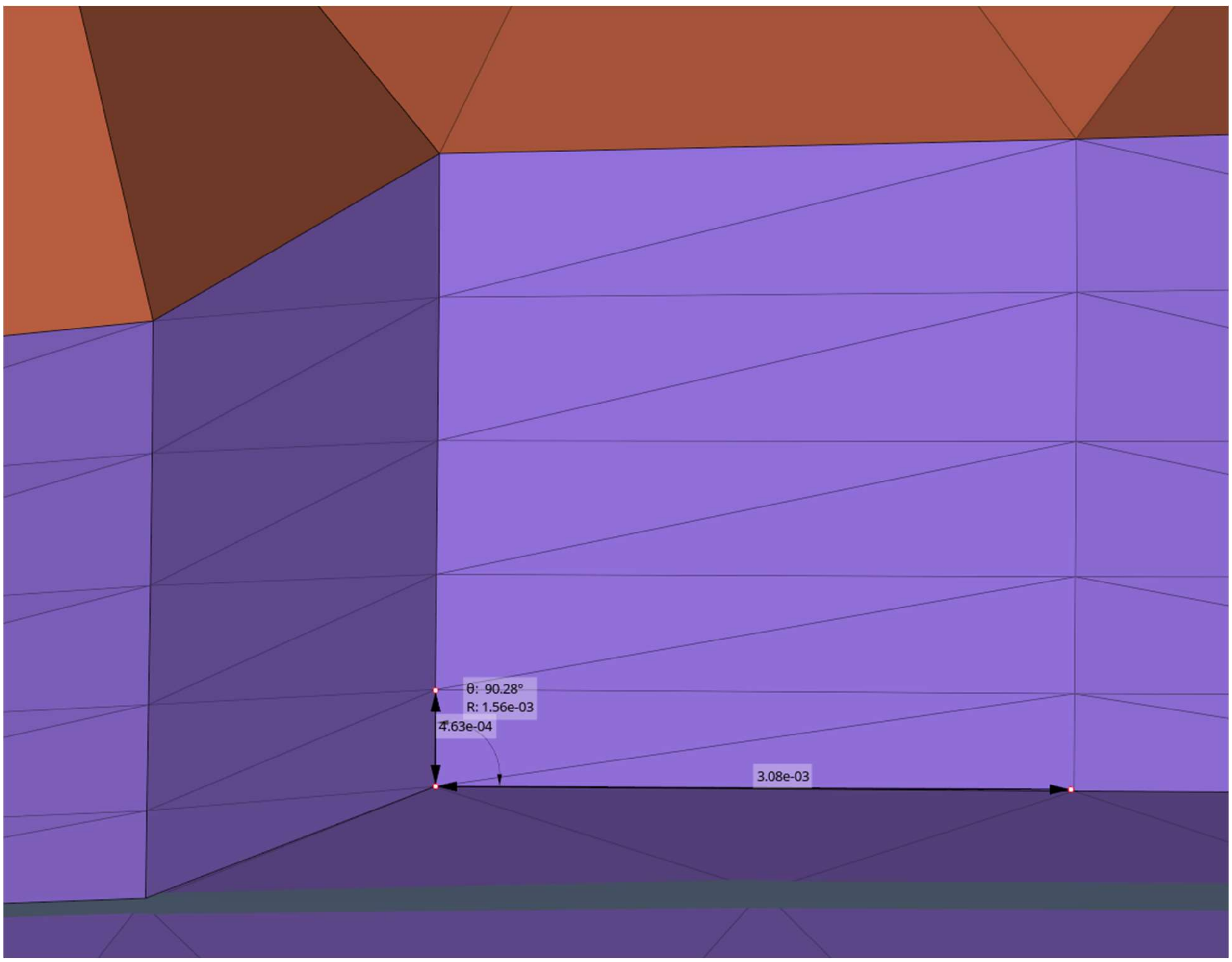


Figure 10: Smallest element size

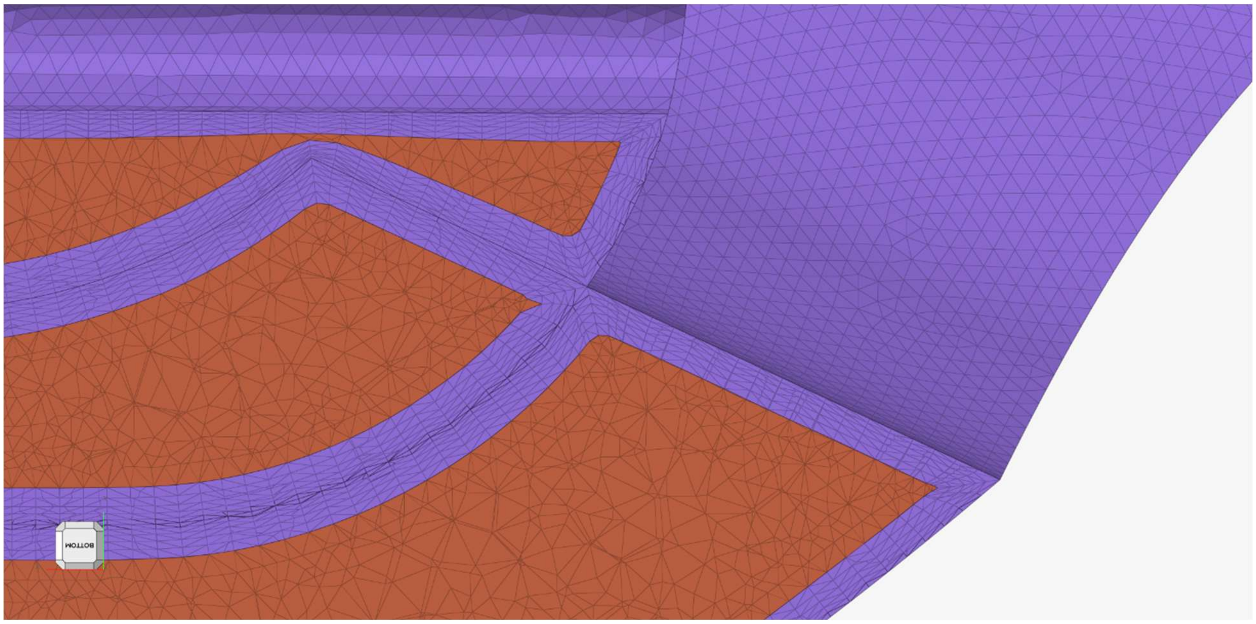


Figure 11: Intersection of stuffing, ducken, turkey, and skewer with boundary layers

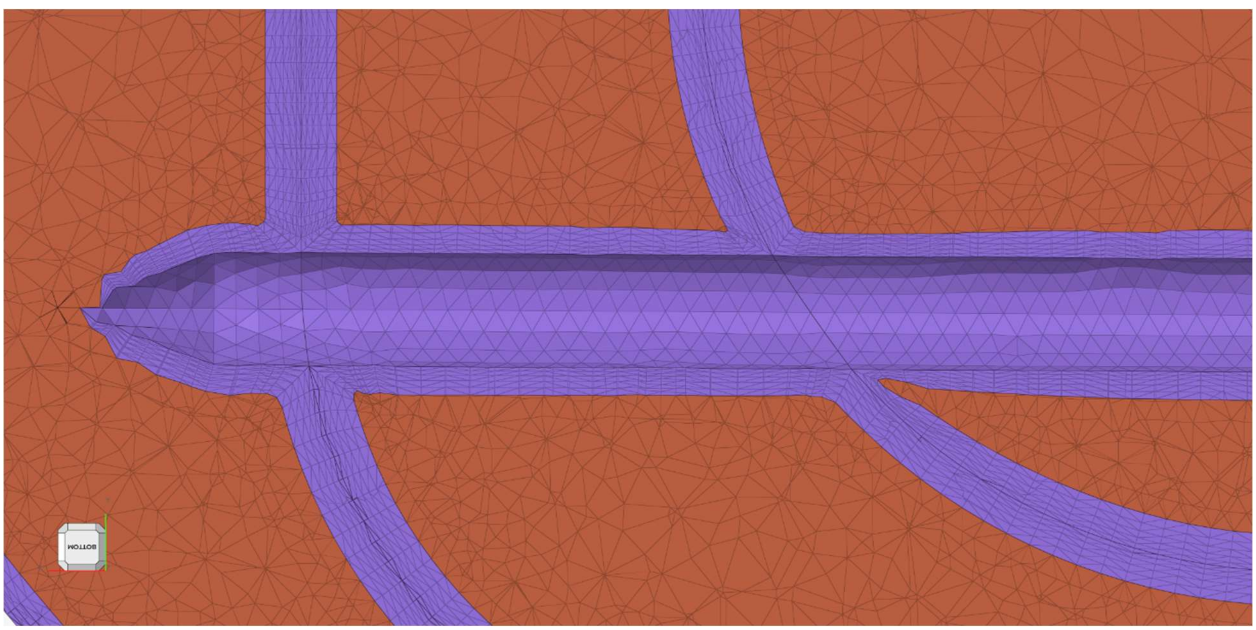


Figure 12: Boundary layers at skewer entrance to stuffing, ducken, turkey

Figure 11 and Figure 12 show that the boundary layers around the skewers and between materials is robust enough to model the turducken adequately. The junction of Ducken, Stuffing, and Turkey, as seen in Figure 11, is a proper boundary layer. It is extremely important for boundary layers to be well meshed so that nodal data can be transferred properly.

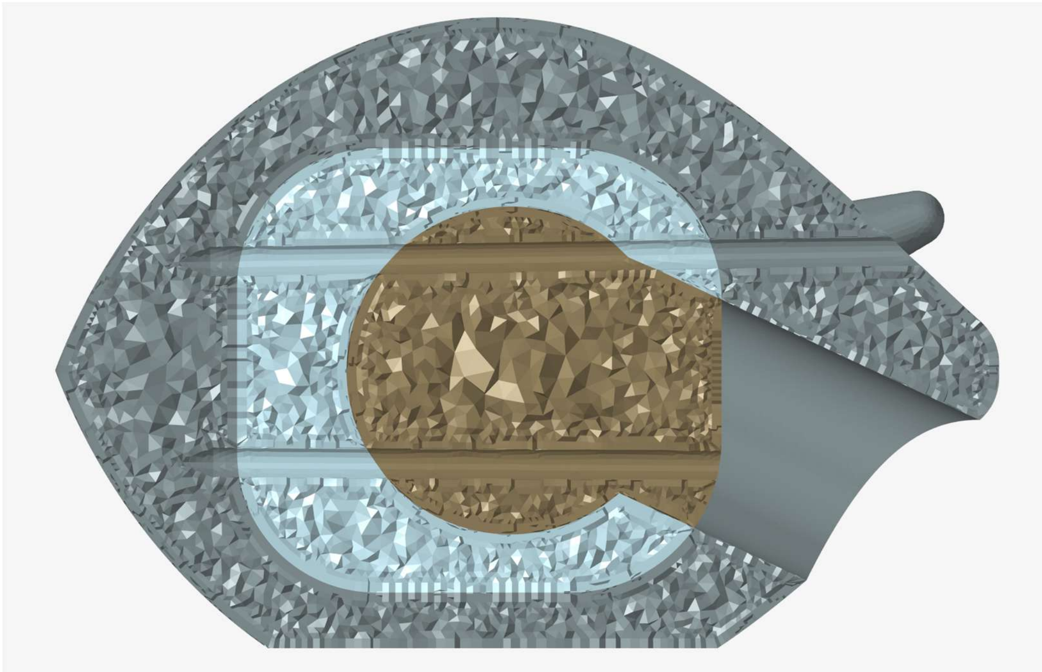


Figure 13: Section view of mesh

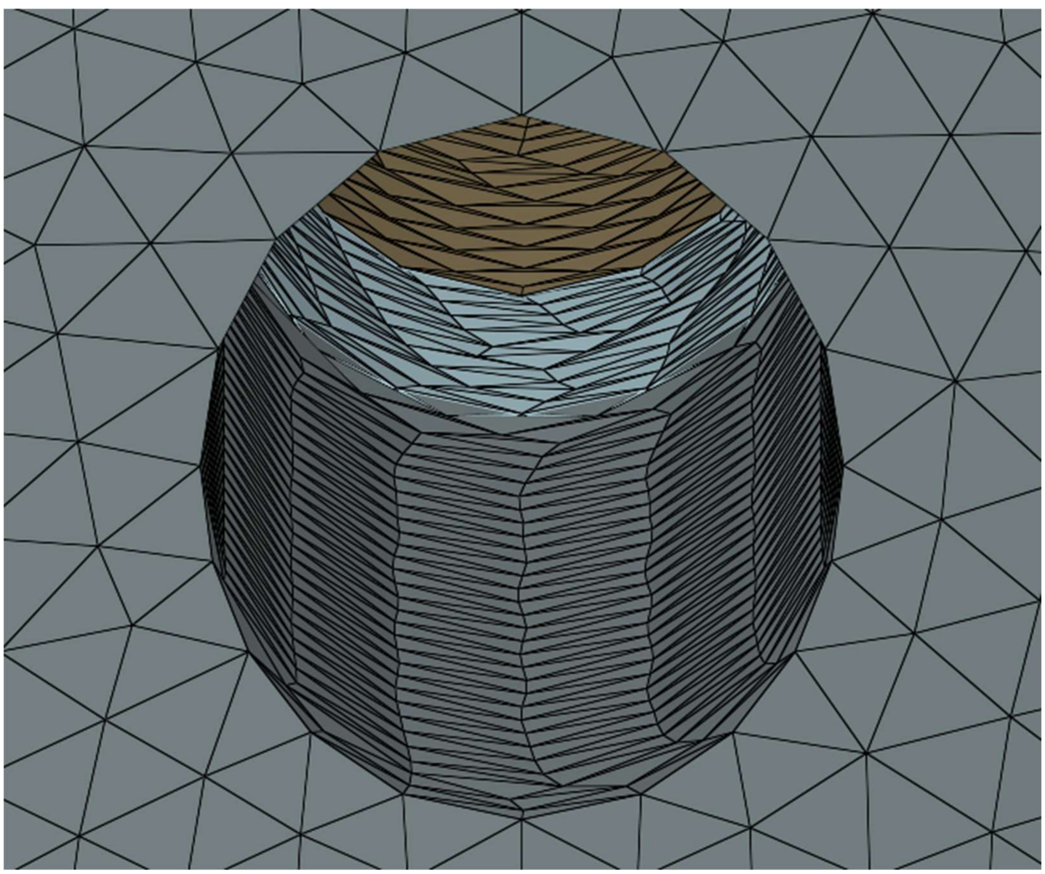


Figure 14: Close-up of mesh around skewer hole

The mesh around the skewer holes have approximately 20 elements each, as seen in Figure 14. There are 5 boundary layers for each different material, meaning between materials there are 10 boundary layers. This provides sufficient contact between materials. The first boundary layer, as shown in Figure 10, has an aspect ratio of approximately 6.6, which is larger than desired, however, is the largest aspect ratio in the entire mesh.

Material Property Assignment

Material properties were assigned to the mesh as shown in Figure 15, where grey represents turkey, blue represents ducken, and brown represents stuffing.

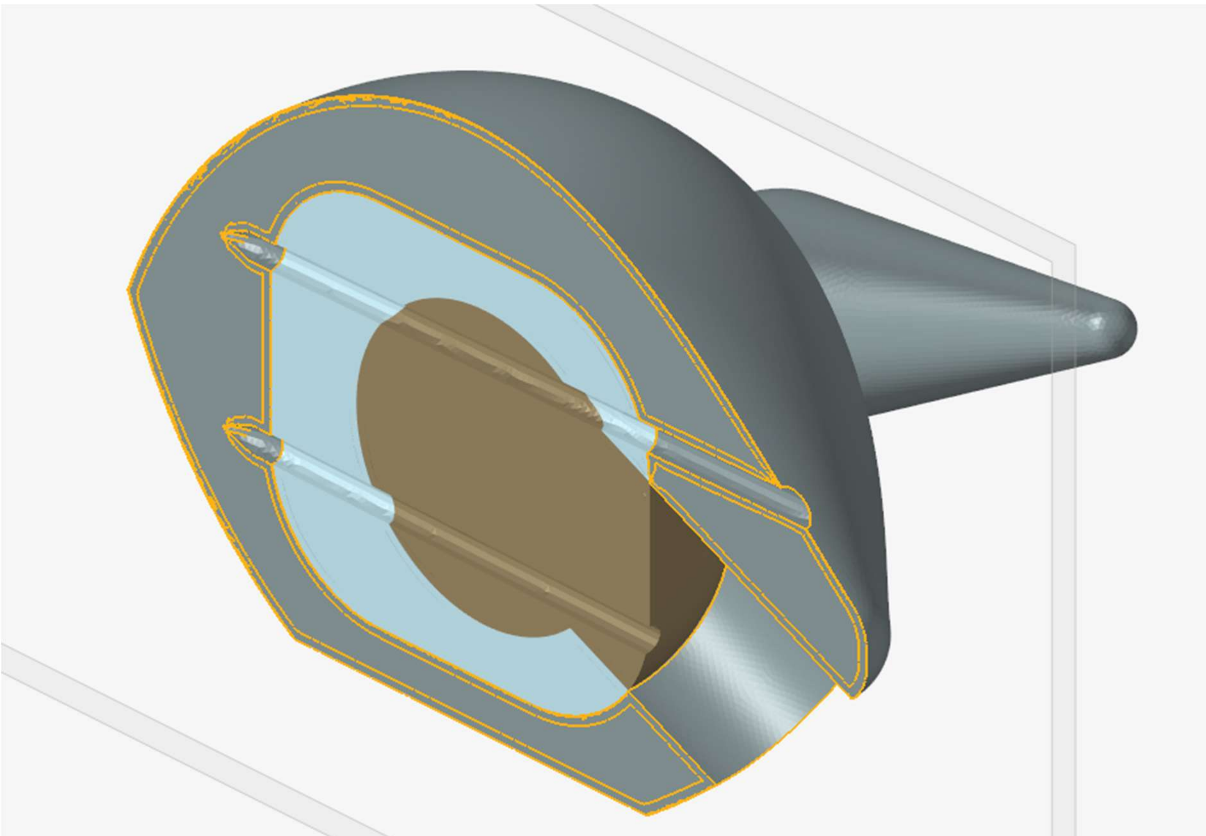


Figure 15: Material assignments, color coded

Boundary Conditions

To simulate the heat input from the skewers, a heat flux boundary condition was used. The value of this heat flux was calculated by taking the desired power consumption of the pair of skewers and dividing it by the combined surface area of two skewers as shown in Eqn. 1.7. This boundary condition was applied to every surface where the skewers touch the turkey, ducken, and stuffing as shown in Figure 16 below.

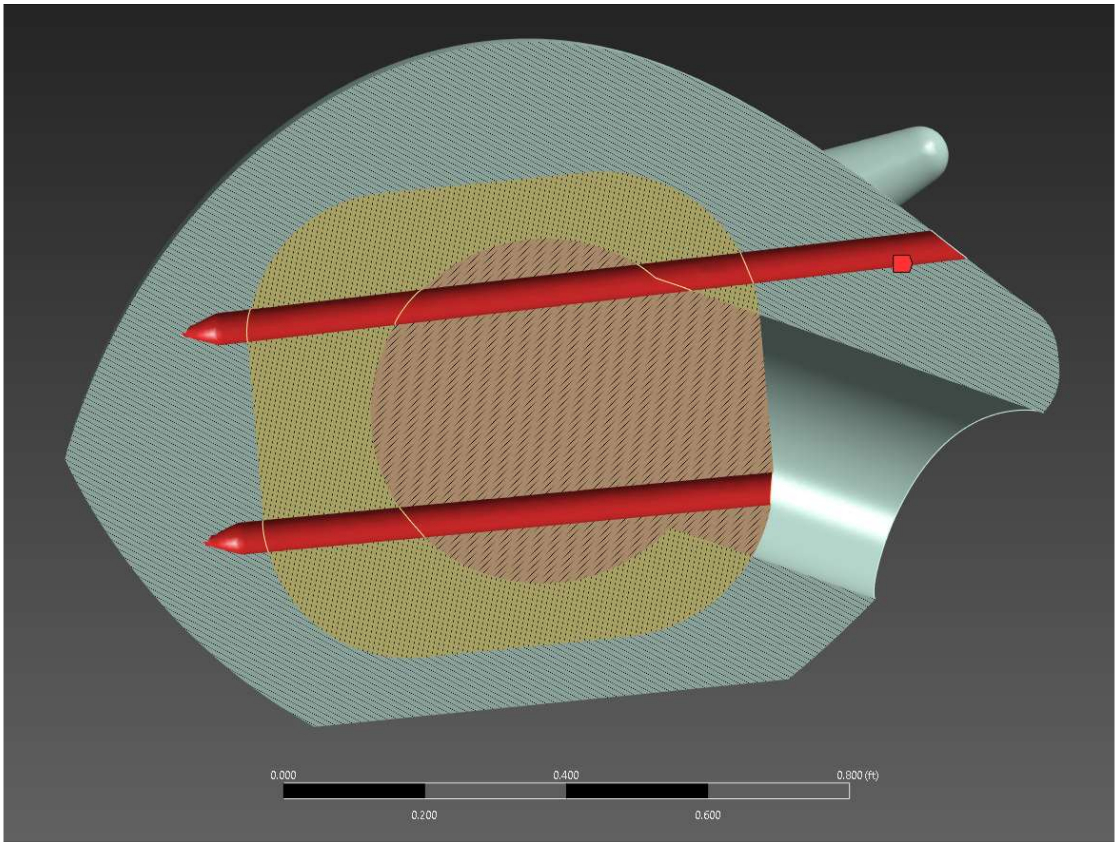


Figure 16: Skewer boundary, heat flux application location

To simulate the effects of the oven, convection, and radiation boundary conditions were applied to all surfaces exposed to the oven including the exterior turkey surface, the turkey wings, the base of the turkey, and the surfaces of the stuffing and cavity which are exposed to open air. The film coefficient used for convection was $7.19 * 10^{-4} \frac{BTU}{s*ft^2}$ based on the work of Cernela, et al, who conducted an analysis of heat transfer in commercially available ovens. For radiation, an emissivity of 0.9 was used based on the work of Ibarra, et al, who measured the emissivity of raw and cooked chicken meat. The locations where these boundary conditions are applied can be seen in Figure 17 and Figure 18.

The turducken is to be placed in the oven on a tray with fine grating to minimize the effect of conduction on the turkey's bottom. Instead, the turkey bottom will experience both convection and radiation. If needed the user should place a thin tray on the lower oven level to catch any moisture lost during cooking. This tray is considered far enough from the turducken such that the convection and radiation from the oven deems the potential radiation and convection from this tray to be negligible.

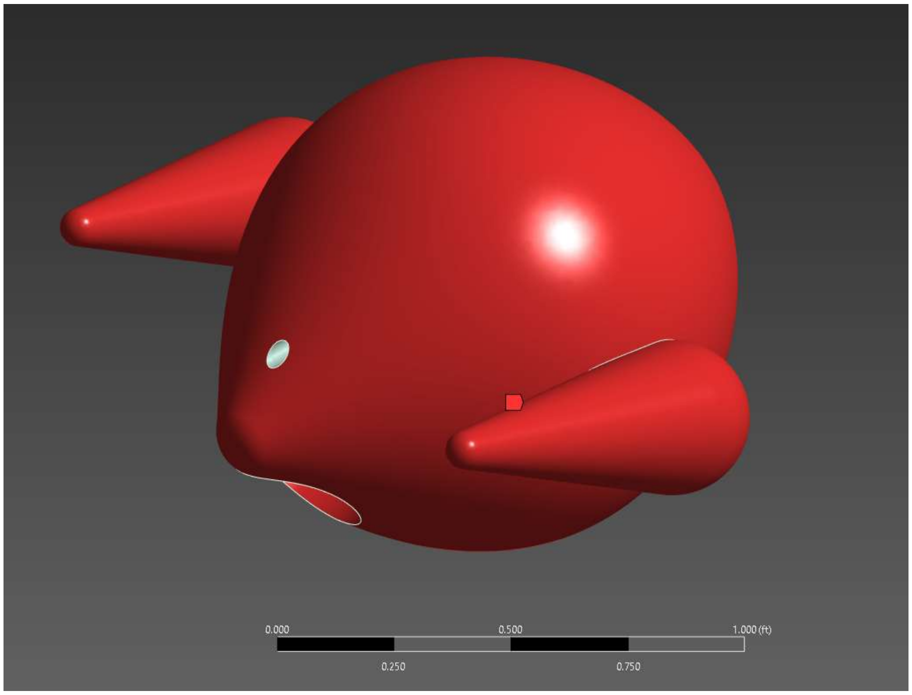


Figure 17: Turkey outer surface, back

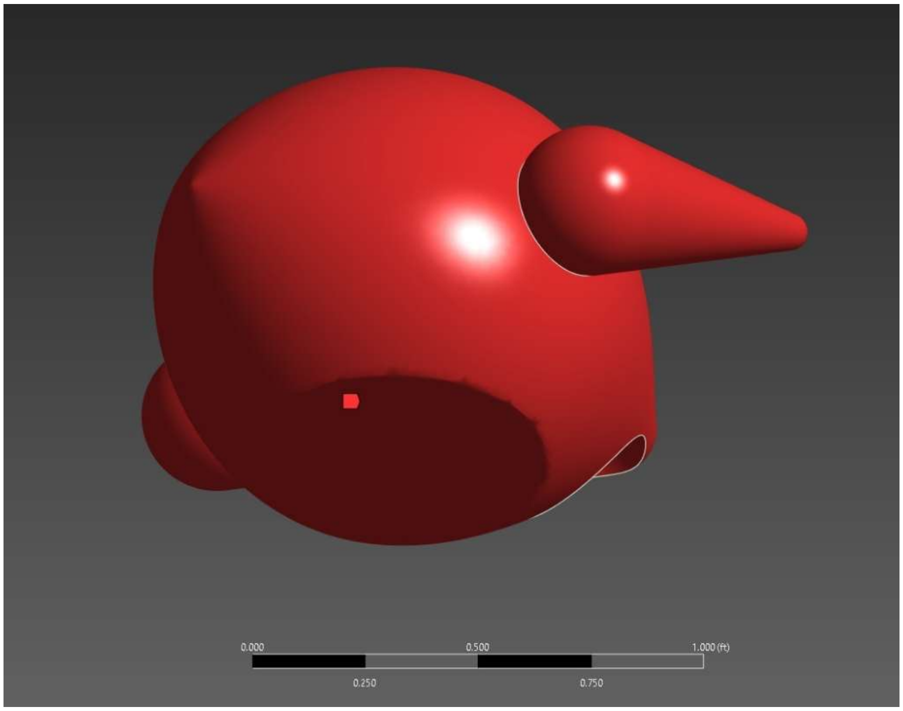


Figure 18: Turkey outer surface, front

Results

Cooking Procedure

The turducken is cooked with the total skewer power consumption of 125 W, 62.5 W each. The skewers are first inserted and turned on for 4.25 hours at room temperature, followed by 2.5 hours of oven at 350 °F with the skewers still running, and ending with 2 hours of rest at room temperature with the skewers turned off. This yields a total cooking time of 8.75 hours. These timings are shown in Figure 19 and Table 11.

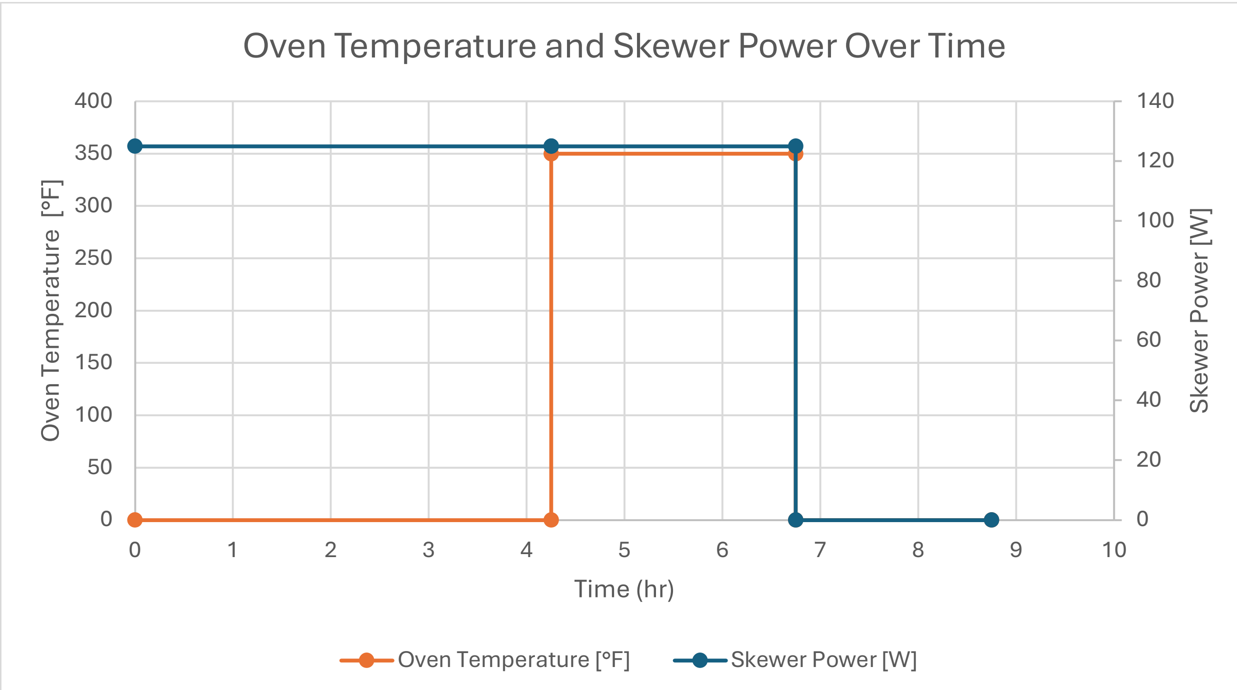


Figure 19: Graph of oven temperature and skewer power over time

Table 11: Skewer and Oven On/Off Timings Throughout Cooking Process

Time (hr)	Skewers	Oven
0	On	Off
4.25	On	On
6.75	Off	Off
8.75	End	End

Internal Temperature Over Time

Figure 20 shows the minimum, average, and maximum temperature of the turducken throughout the cooking process.

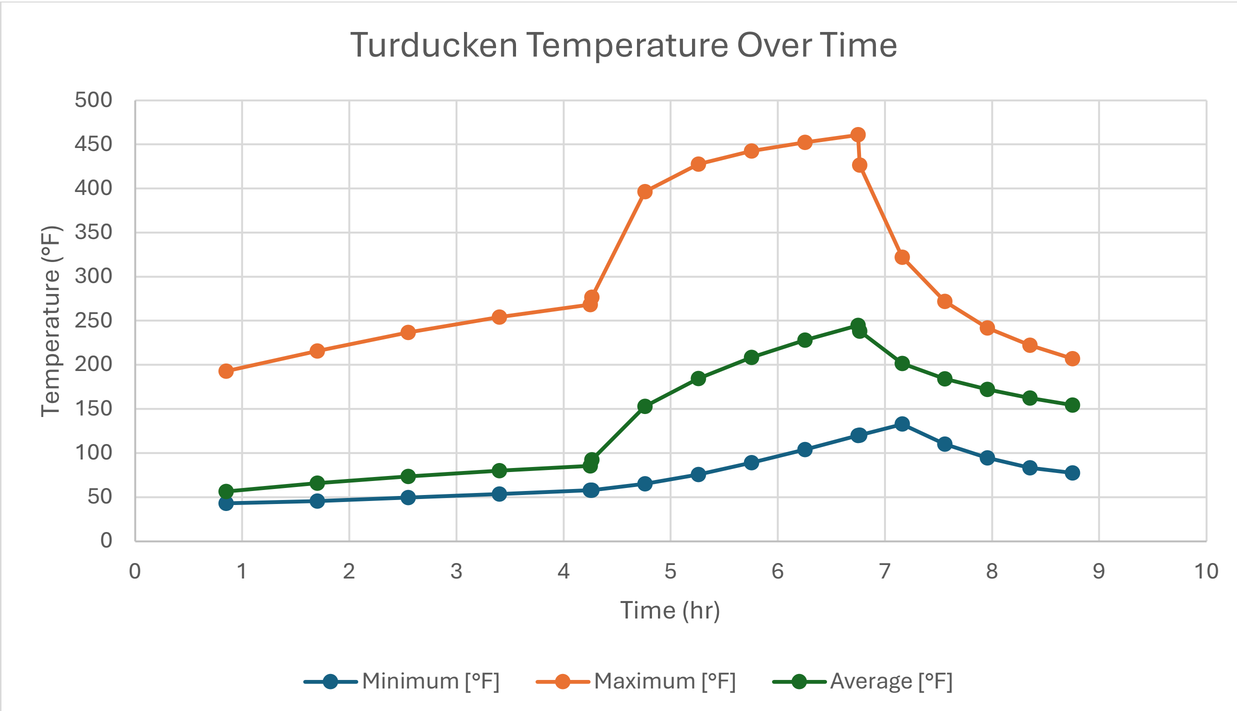


Figure 20: Turducken temperature over time

Figure 20 was constructed by extracting the maximum, minimum, and average temperatures over time to excel to create a clearer plot.

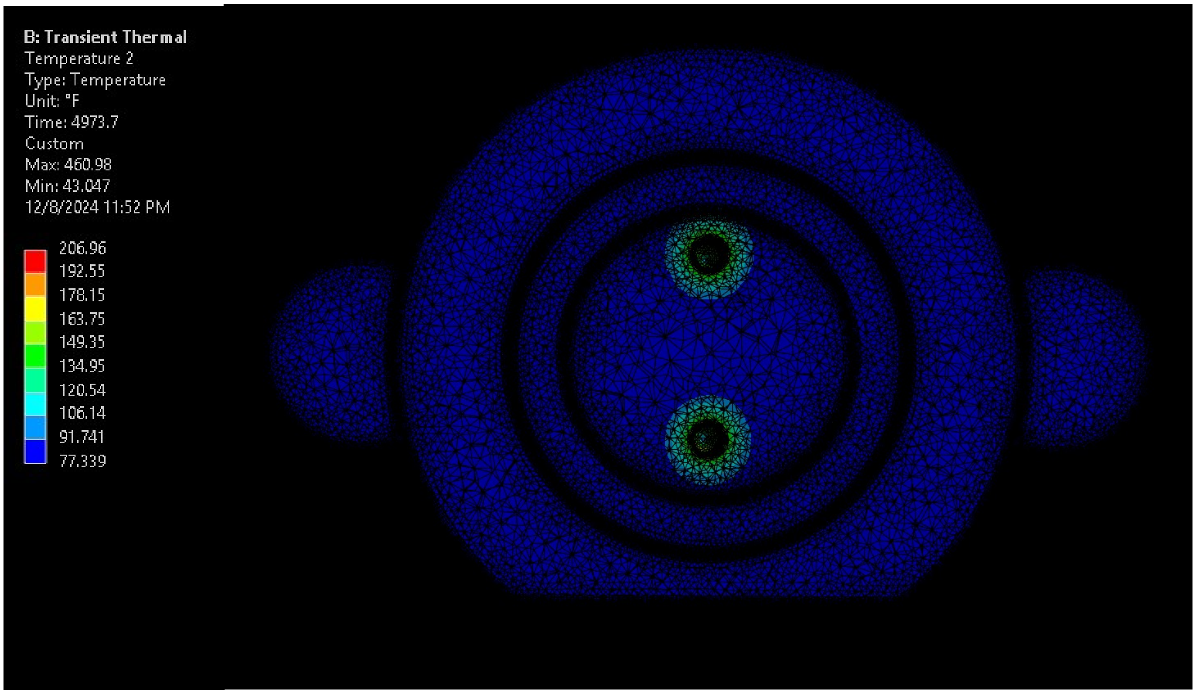


Figure 21: Turducken cross section during skewer operation

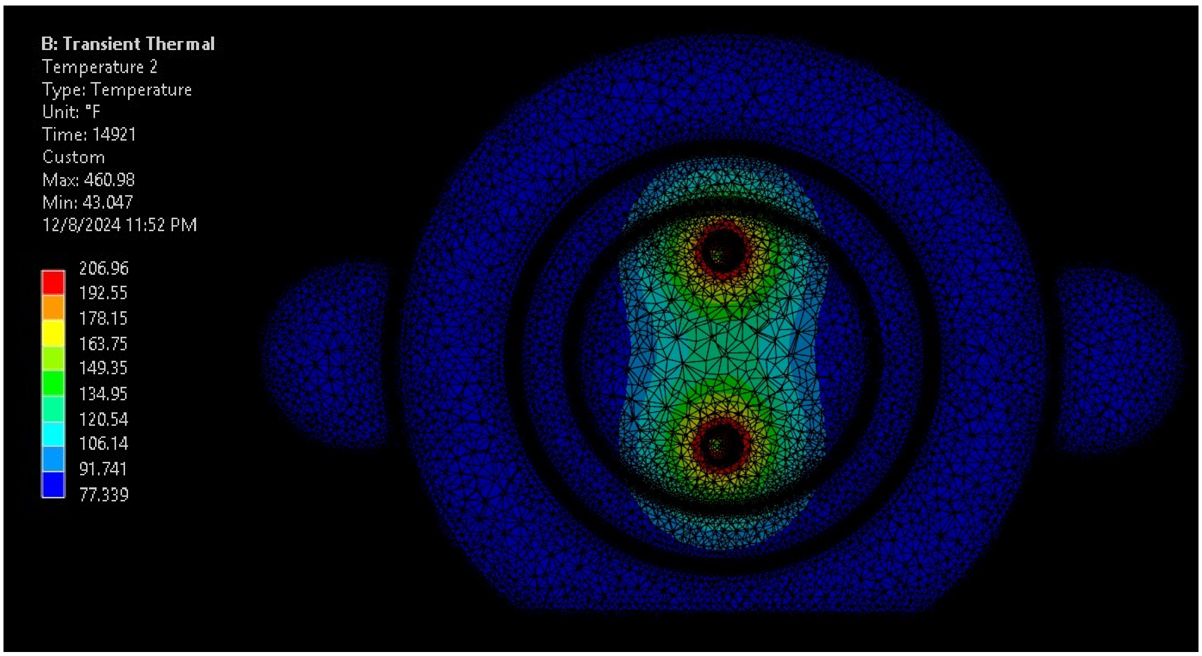


Figure 22:Turducken cross section at end of skewer operation

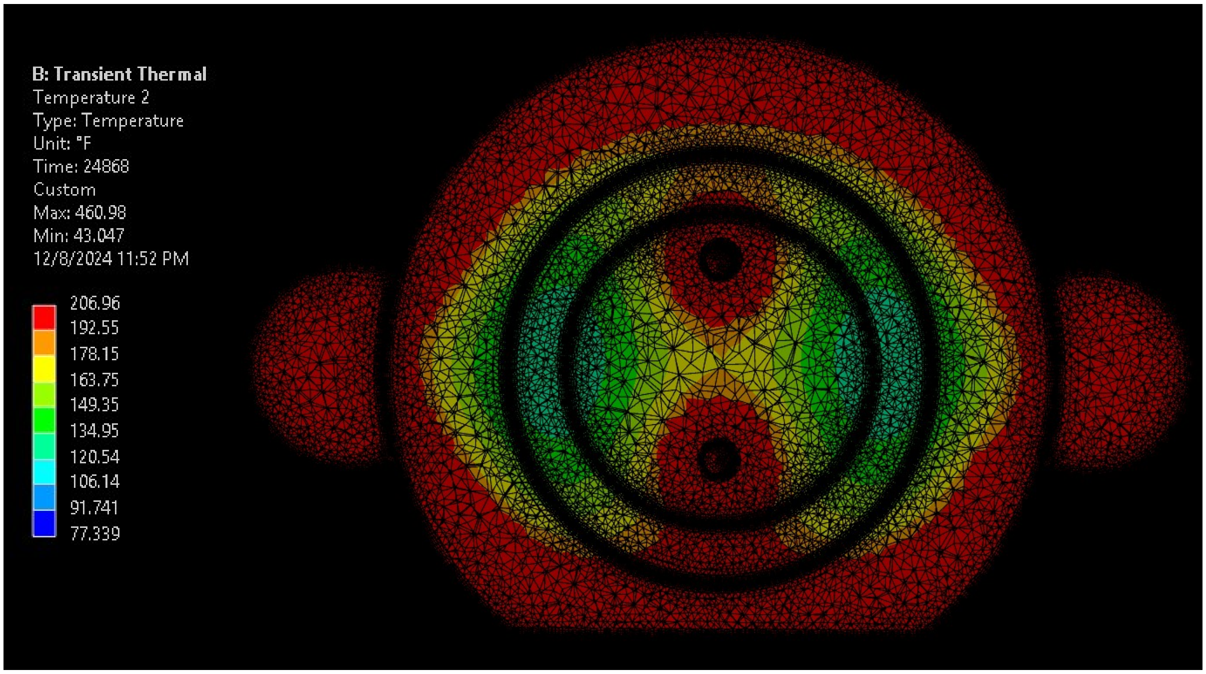


Figure 23: Turducken cross section during oven operation:

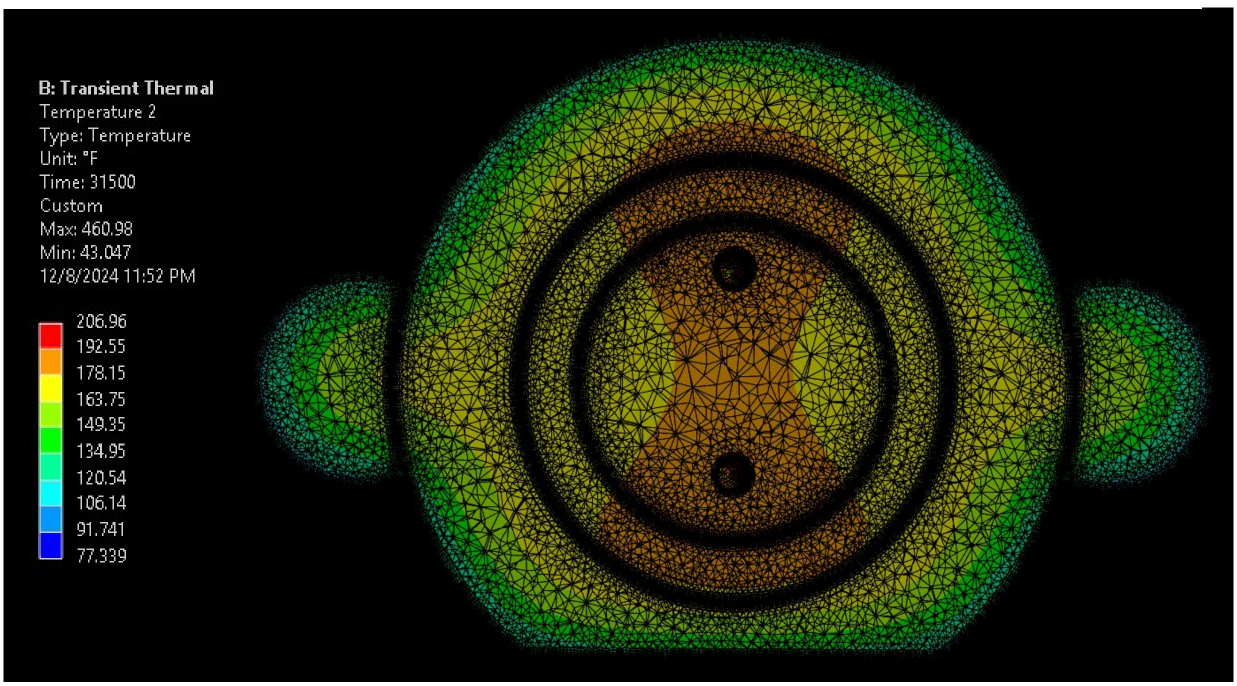


Figure 24: Turducken cross section after resting

At the end of the cooking process, all of the ducken and stuffing, as seen by the smaller concentric circles in Figure 24, has cooked to an adequate serving temperature of at least ~165°F. To achieve this the outer turkey layer has consequently cooled down, however the entire turducken has been cooked at some point as will be shown in the maximum nodal temperatures over time in the following section.

Thermal Gradient at End of Cooking Process

At the end of the cooking process, after the turducken is allowed to cool, the maximum temperature is 206 °F, which occurs at the intersection between the skewer and the ducken. The thermal gradient at the end of this process is shown in Figure 24 and Figure 23.

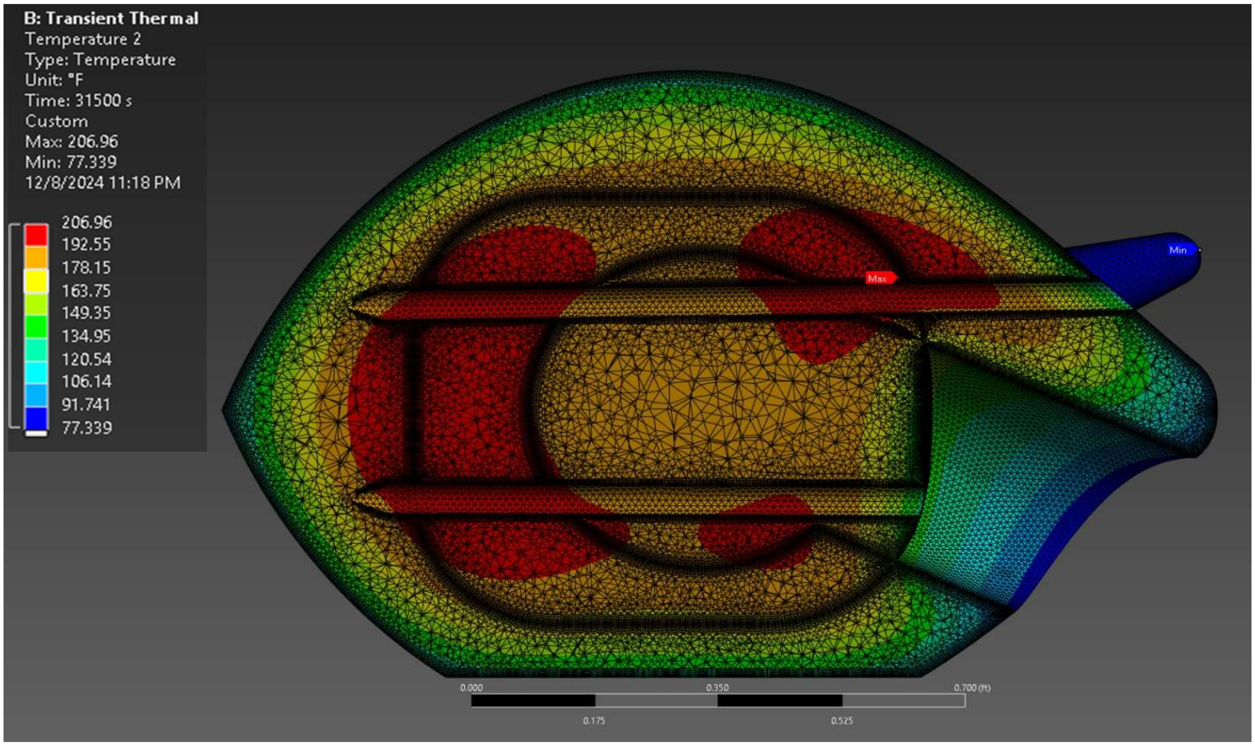


Figure 23: Turducken cross section after resting

Maximum and Minimum Temperatures over Time

The maximum and minimum temperatures of the turducken throughout the entire cooking process are 460.98°F and 165.21°F respectively as shown in their respective cross sections in Figure 24 and Figure 25 below.

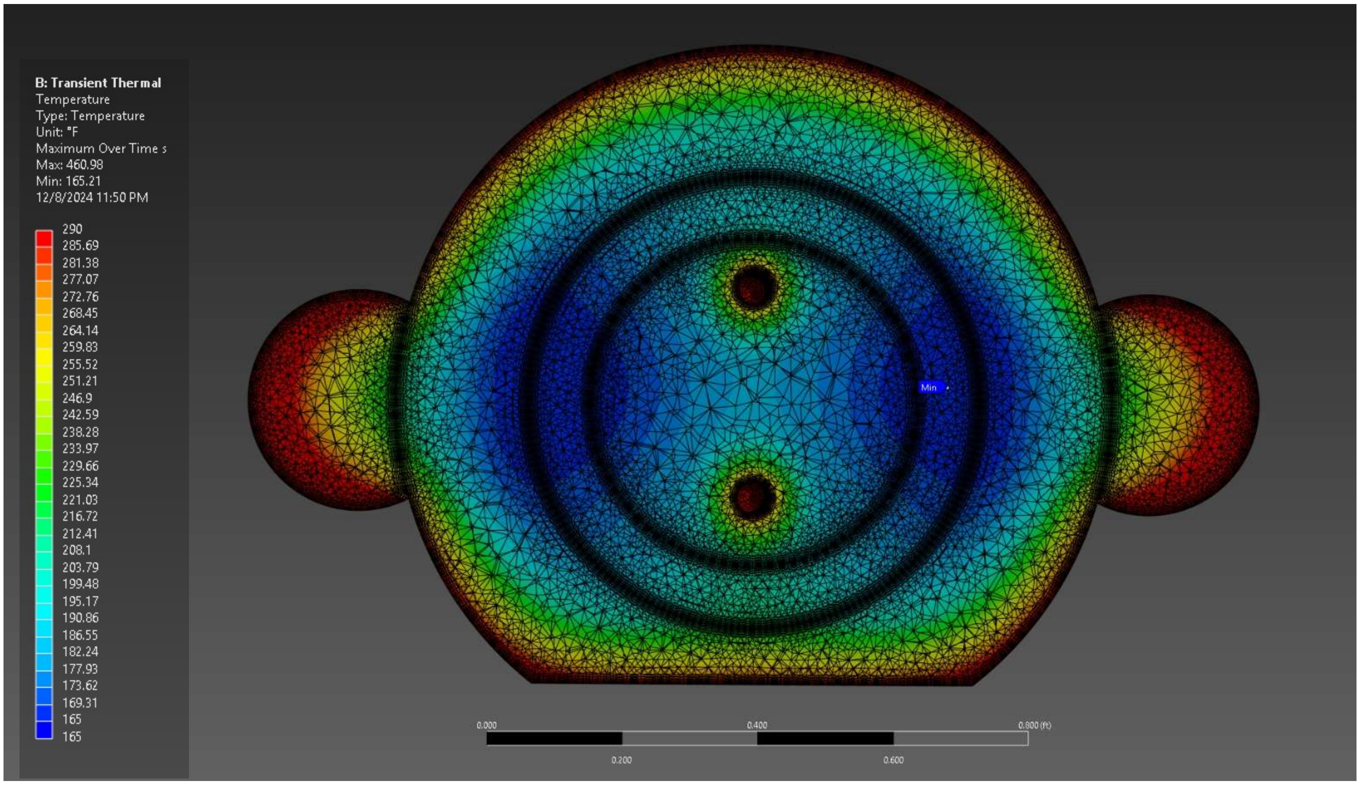


Figure 24: Maximum temperature over time, side view

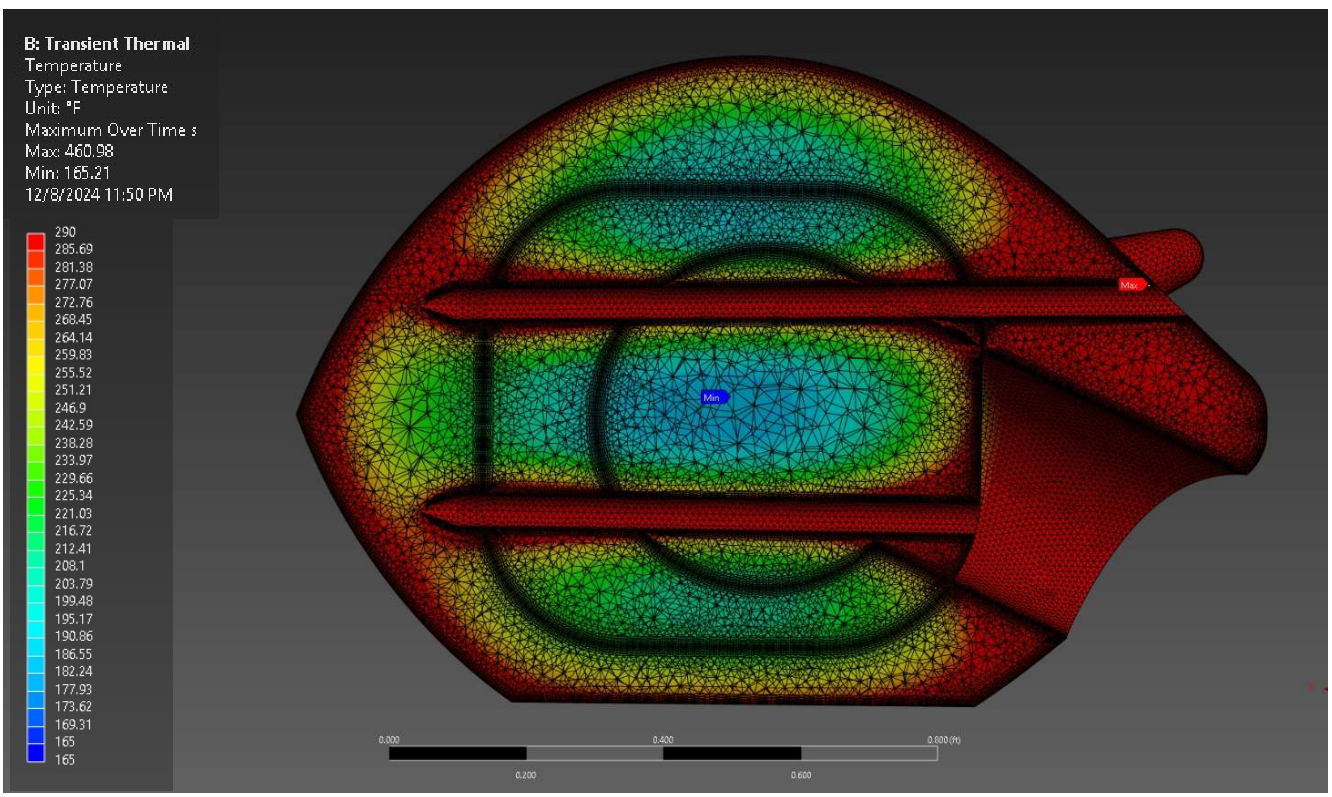


Figure 25: Maximum temperature over time, front view

Burning & Undercooked

ANSYS Mechanical Analysis

The burned volume of turducken can be found by evaluating the proportion of nodes which reach a temperature above the burn point of 290 °F. As the elements are not of relatively constant dimensions, measuring burnt percentage by nodes is not accurate but still a good estimate. As seen in Figure 24, the burnt nodes over time are concentrated in the turkey boundary layer which has a relatively high nodal concentration and covers the greatest relative circumference. This means that the calculated burnt percentage based off nodes is a very liberal estimate as it counts far more burnt nodes per volume compared to unburnt nodes towards the turducken center. By extracting the text file of the maximum nodal temperatures over time to excel, the undercooked percentage was confirmed to be 0% as expected since the minimum temperature over time was 165.21°F as shown in Figure 24. The burnt percentage based off nodes was 31.85%, meaning the burnt percentage of the turducken based off volume is expected to be far less. This is proved in the following APDL analysis.

Ansys APDL Analysis

Before exporting to APDL, a volume solution was found and exported as a .txt file, which was then ported into an excel sheet. The temperature solution at a specific time step was then exported to APDL, where an element table was created with all elements. The Element table was then exported to a .txt, which then was imported into the same excel with the volume solution. The volumes of the elements with an average temperature above 290°F were then summed and found as a fraction of the total volume. The imported Turducken in APDL can be seen in Figure 26 and Figure 27.

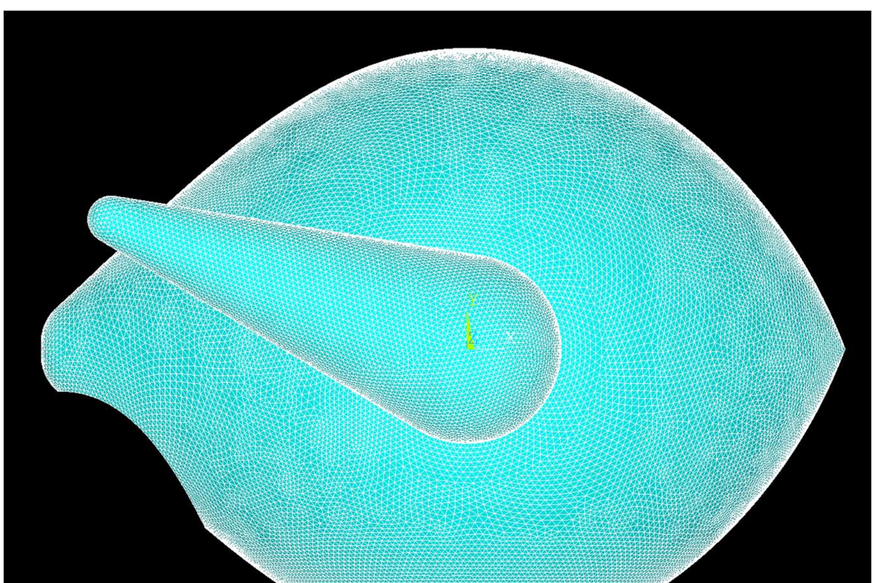


Figure 26: Imported .db Turducken File in APDL

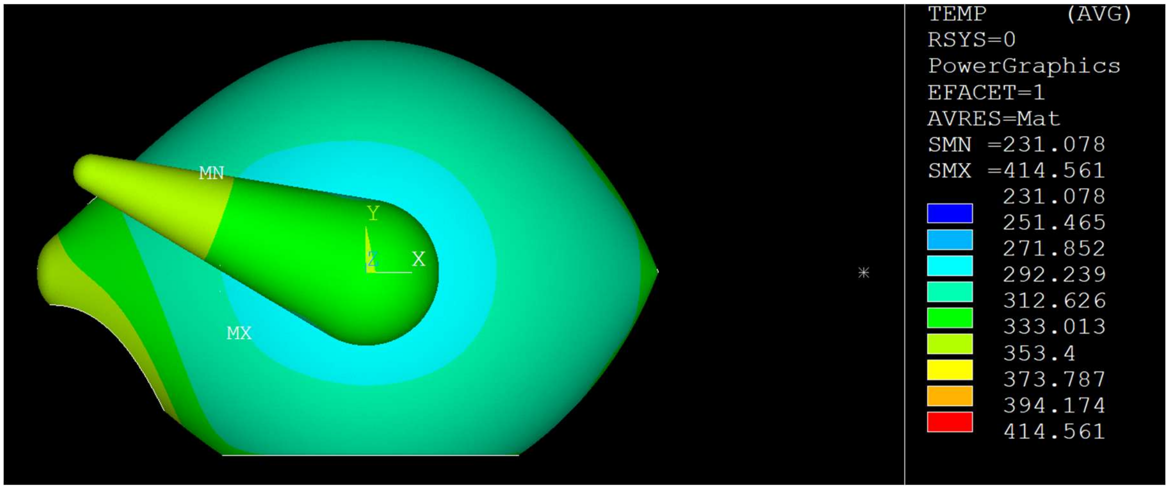


Figure 27: Gradient of the temperature throughout the turducken volume on APDL

When finding the burnt percentage based on volume in excel the result was 2.22%. Unfortunately, only around 900,000 elements were exported of the 5 million total and we could not find which elements these were. 2% is clearly unrealistic and likely means the elements chosen were farther towards the turducken center than on the boundary. This approach, if done properly, will yield a more accurate burn percentage since determining by volume is more accurate than by node as explained in the Ansys Mechanical Analysis section above.

Conclusion

How Did FEA Help?

Utilizing Finite Element Analysis to simulate a transient thermal model was helpful in determining if the deliverables of the project and subsequent product were met. This ensured that all parts of the bird met the minimum safe cooking temperature of 165°F while minimizing parts that could reach the burning threshold of 290°F. FEA allowed the consideration of complex geometries such as those of a turkey which would have been impossible with traditional hand calculations. The result provided reassurance that the skewer design was robust and met the requirements. Furthermore, the result provides guidelines for real life applications of the product, namely, the user instruction manual, which provides cooking instructions.

User Instructions

Cooking Instructions

1. Make your stuffing! Mix ¾ cups of butter, two large celery sticks, ½ cups of onion, and nine cups of potatoes into a homogenous mixture, and stuff into your turducken.
2. Place stuffed Turducken into refrigerator for 1-2 hours to ensure the entire turducken is at an initial temperature of 42°F
3. Remove from refrigerator. Note each skewer has 2 notches. One skewer must be inserted in the turkey approximately 13.5 inches above the opening for the stuffing, up to the double notch.
4. The second skewer is to be inserted into the stuffing, up to the single notch, approximately 10 inches. Ensure both skewers are inserted at an angle between the angle the wings make with the horizontal and the horizontal.
5. Turn on the skewers and wait 4 hours
6. Preheat oven to 350 °F and wait 15 minutes
7. Place Turducken in oven on grated tray for 2 hours and 30 minutes (Place tray on oven bottom if desired to catch any moisture lost)
8. Take Turducken out of oven and turn off the skewers
9. Allow the Turducken to rest for 2 hours
10. Enjoy!

Design Time

An average of ~50 hours per person for a 4-person team leads to a total of approximately 200 hours spent.

Appendix A: Hand Calculation Verifications

Ingredient Properties

The density, thermal conductivity, specific heat, and thermal diffusivity formulas from the ASHRAE 2006 handbook can be found in the following tables for each nutrient:

Table 12: Formulas for Nutrient Densities (lb/ft^3) Dependent on Temperature (^{o}F)

Nutrient	Formula	Eqn.
Water	$\rho_{H_2O}(T) = 62.174 + (0.0047425)T - (7.2397 * 10^{-8})T^2$	(A.1)
Protein	$\rho_{protein}(T) = 83.599 - (0.017979)T$	(A.2)
Fat	$\rho_{fat}(T) = 58.246 - (0.014482)T$	(A.3)
Carbs	$\rho_{carbs}(T) = 100.17 - (0.010767)T$	(A.4)
Ash	$\rho_{ash}(T) = 151.62 - (0.0097329)T$	(A.5)

Table 13: Formulas for Nutrient Thermal Conductivities (Btu/(hr·ft· °F) Dependent on Temperature (°F)

Nutrient	Formula	Eqn.
Water	$k_{H_2O}(T) = 0.31064 + (6.4226 * 10^{-4})T - (1.1955 * 10^{-6})T^2$	(A.6)
Protein	$k_{protein}(T) = 0.090535 + (4.1486 * 10^{-4})T - (4.8467 * 10^{-7})T^2$	(A.7)
Fat	$k_{fat}(T) = 0.10722 - (8.6581 * 10^{-5})T - (3.1652 * 10^{-8})T^2$	(A.8)
Carbs	$k_{carbs}(T) = 0.10133 + (4.9478 * 10^{-4})T - (7.7238 * 10^{-7})T^2$	(A.9)
Ash	$k_{ash}(T) = 0.17553 + (4.8292 * 10^{-4})T - (5.1839 * 10^{-7})T^2$	(A.10)

Table 14: Formulas for Nutrient Specific Heats (Btu/(lb·°F) Dependent on Temperature (°F)

Nutrient	Formula	Eqn.
Water	$c_{H_2O}(T) = 0.99827 - (3.7879 * 10^{-5})T + (4.0347 * 10^{-7})T^2$	(A.11)
Protein	$c_{protein}(T) = 0.47442 + (1.6661 * 10^{-4})T - (9.6784 * 10^{-8})T^2$	(A.12)
Fat	$c_{fat}(T) = 0.4673 + (2.1815 * 10^{-4})T - (3.5391 * 10^{-7})T^2$	(A.13)
Carbs	$c_{carbs}(T) = 0.36114 + (2.8843 * 10^{-4})T - (4.3788 * 10^{-7})T^2$	(A.14)
Ash	$c_{ash}(T) = 0.25266 + (2.681 * 10^{-4})T - (2.7141 * 10^{-7})T^2$	(A.15)

Table 15: Formulas for Nutrient Thermal Diffusivities (ft^2/hr) Dependent on Temperature (°F)

Nutrient	Formula	Eqn.
Water	$\alpha_{H_2O}(T) = 0.0046428 + (1.5289 * 10^{-5})T - (2.873 * 10^{-8})T^2$	(A.16)
Protein	$\alpha_{protein}(T) = 0.002317 + (1.1364 * 10^{-5})T - (1.7516 * 10^{-8})T^2$	(A.17)
Fat	$\alpha_{fat}(T) = 0.0038358 - (2.4128 * 10^{-7})T - (4.579 * 10^{-10})T^2$	(A.18)
Carbs	$\alpha_{carbs}(T) = 0.0027387 + (1.3198 * 10^{-5})T - (2.7769 * 10^{-8})T^2$	(A.19)
Ash	$\alpha_{ash}(T) = 0.0045565 + (8.9716 * 10^{-6})T - (1.4644 * 10^{-8})T^2$	(A.20)

The following formulas represent the thermal conductivity and specific heat equivalents to the density example in Eqn. 1.1.

$$k_{ingredient}(T) = \sum \rho_i(T) x_i(T) \tag{A.21}$$

$$c_{ingredient}(T) = \sum \rho_i(T) x_i(T) \tag{A.22}$$

Note that Eqn. A.21 refers to thermal conductivity and Eqn. A.22 refers to specific heat. In both equations and Eqn. 1.1 "i" includes water, protein, fat, carbohydrates, and ash.

Imported Property Plots

The following figures show the density, thermal conductivity, and specific heat of the turkey, ducken, and stuffing as they vary from 42°F to 500°F.

Turkey Property Plots

Figure 28: Turkey Density (lb/ft^3) *over Temperature Range 42*°F *to 500*°F

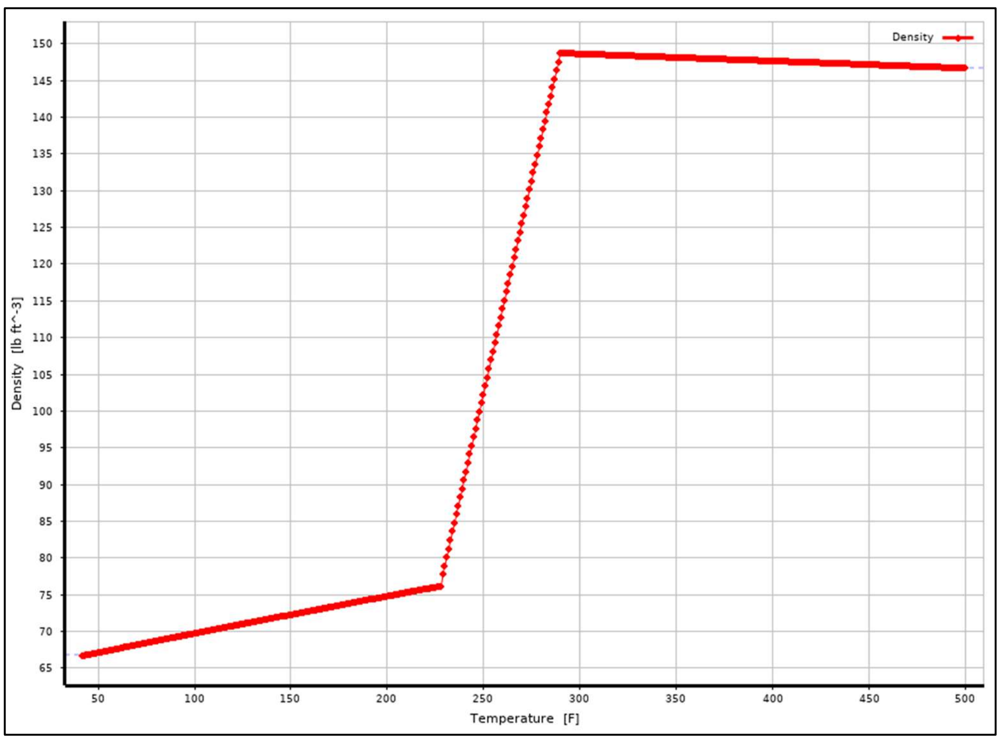


Figure 2: Turkey Thermal Conductivity (Btu/(hr·ft· °F) over Temperature Range of 42℉ to 500℉

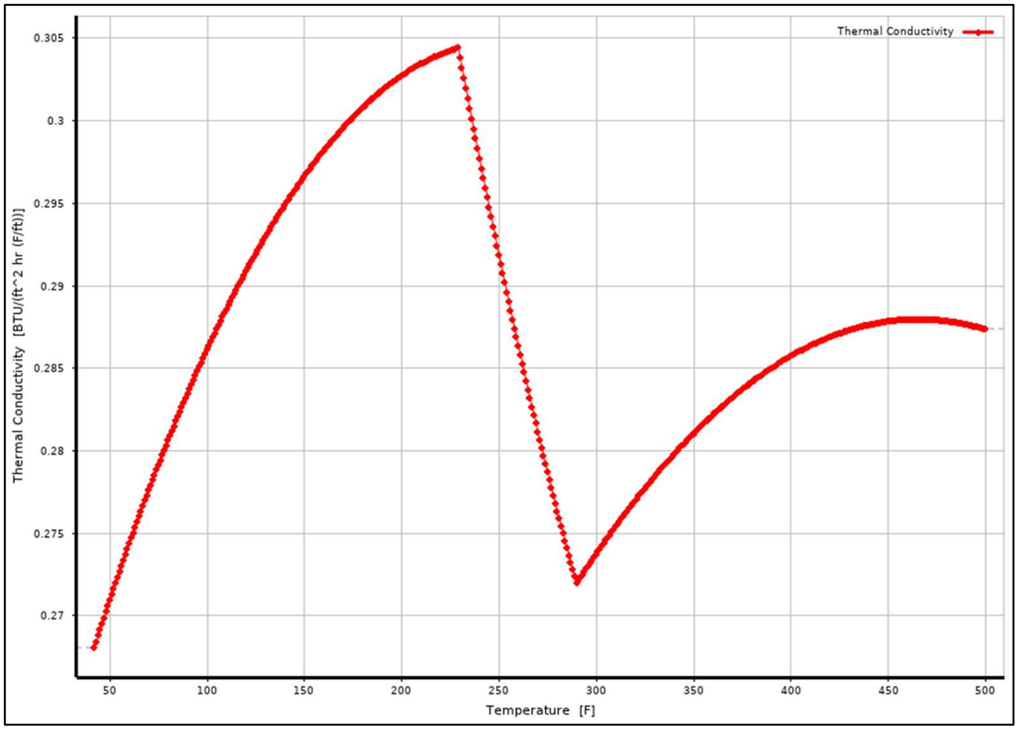
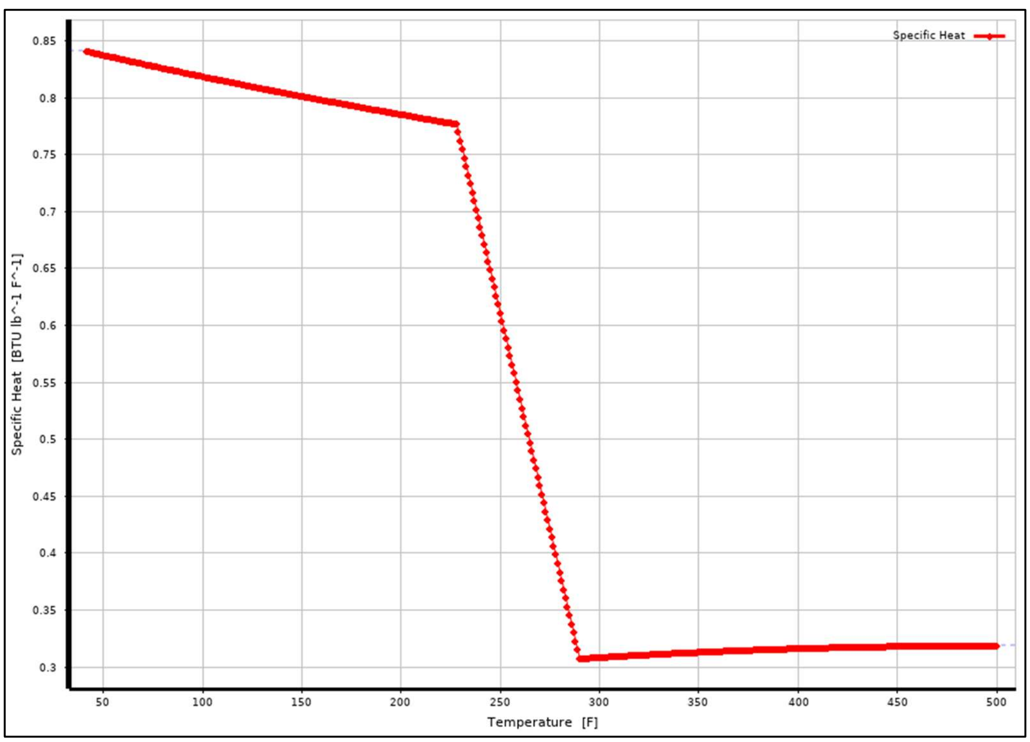


Figure 3: Turkey Specific Heat (Btu/(lb·℉) over Temperature Range of 42℉ to 500℉



Ducken Property Plots

Figure 4: Ducken Density (lb/ft^3) over Temperature Range 42℉ to 500℉

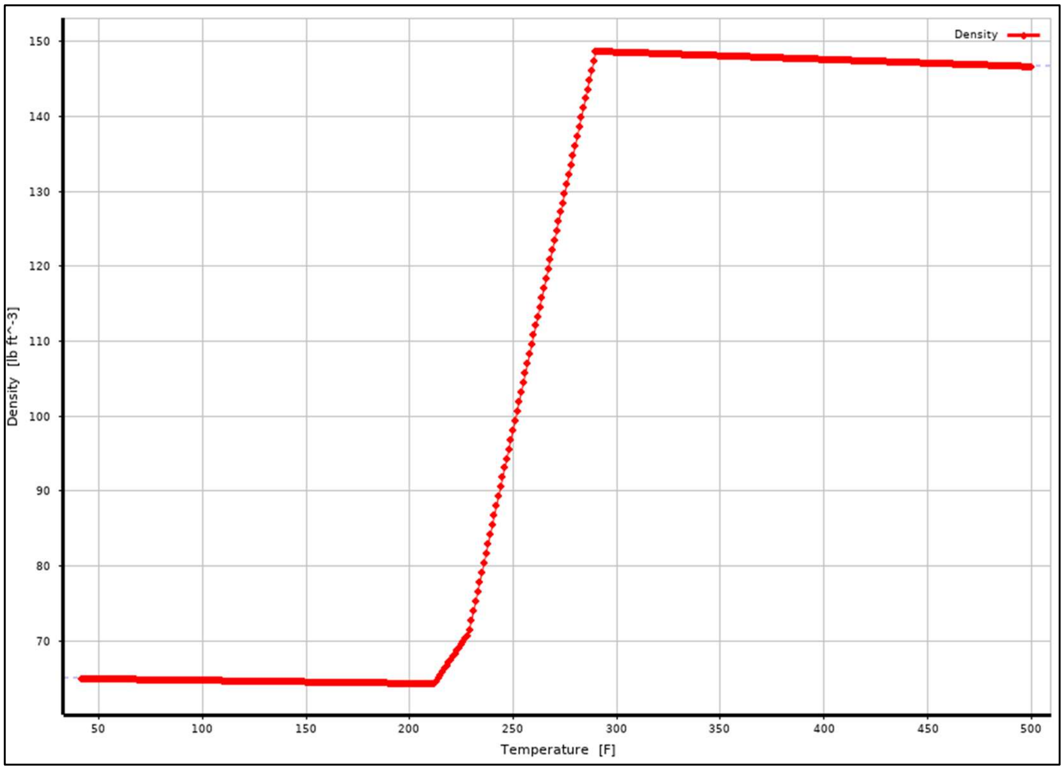


Figure 5: Ducken Thermal Conductivity (Btu/(hr·ft· °F) over Temperature Range of 42℉ to 500℉

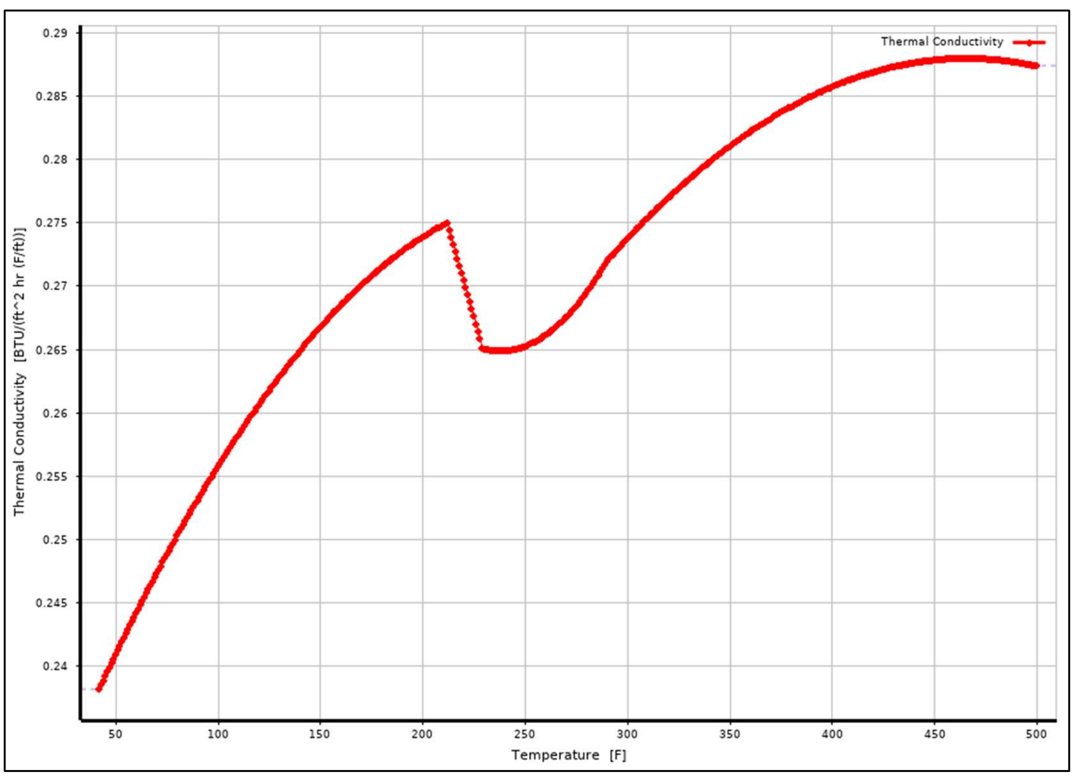
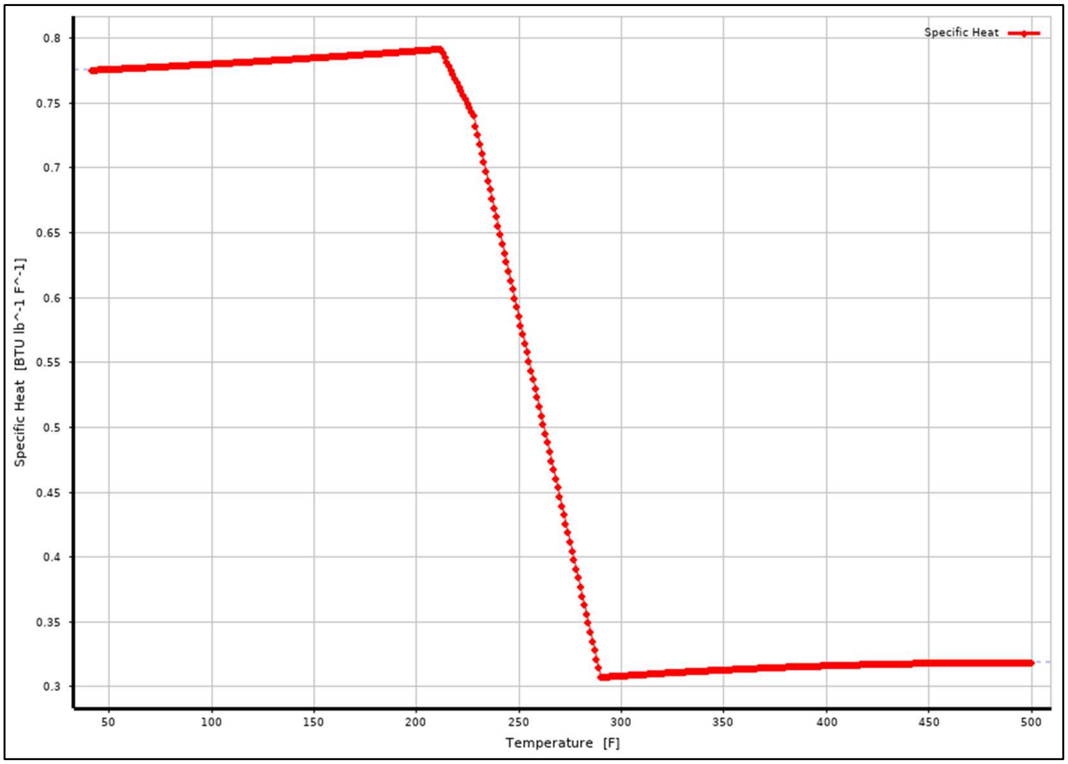


Figure 6: Ducken Specific Heat (Btu/(lb·°F) over Temperature Range of 42°F to 500°F



Stuffing Property Plots

Figure 7: Stuffing Density (lb/ft^3) *over Temperature Range 42*°F *to 500*°F

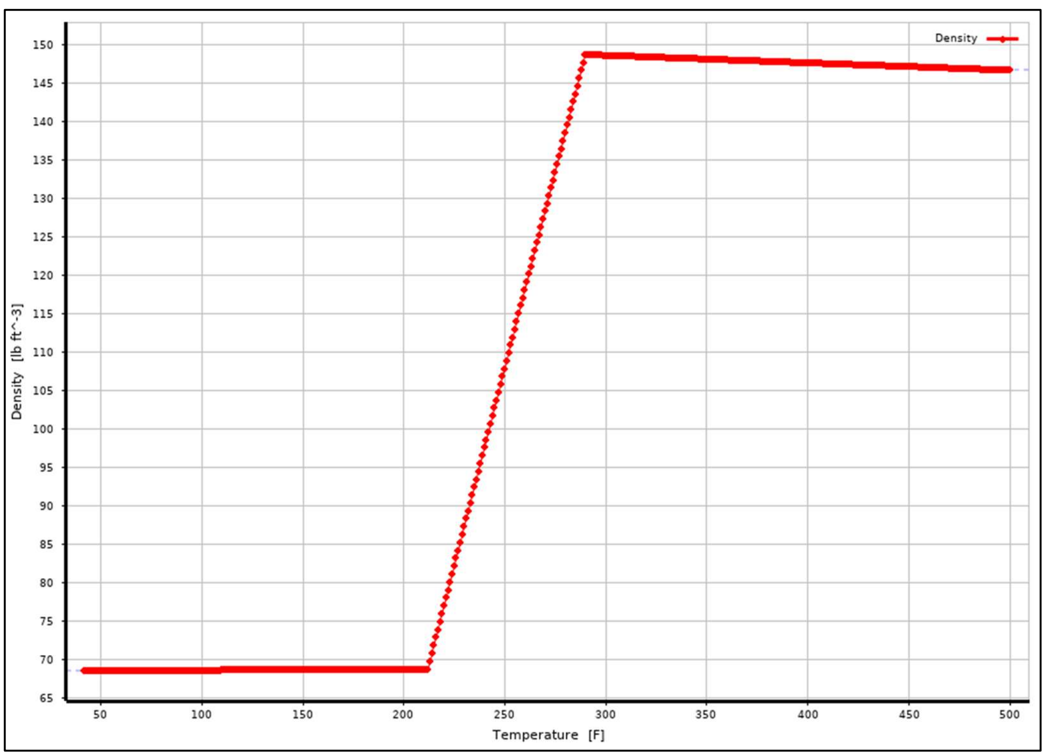


Figure 8: Stuffing Thermal Conductivity (Btu/(hr·ft· °F) over Temperature Range of 42°F to 500°F

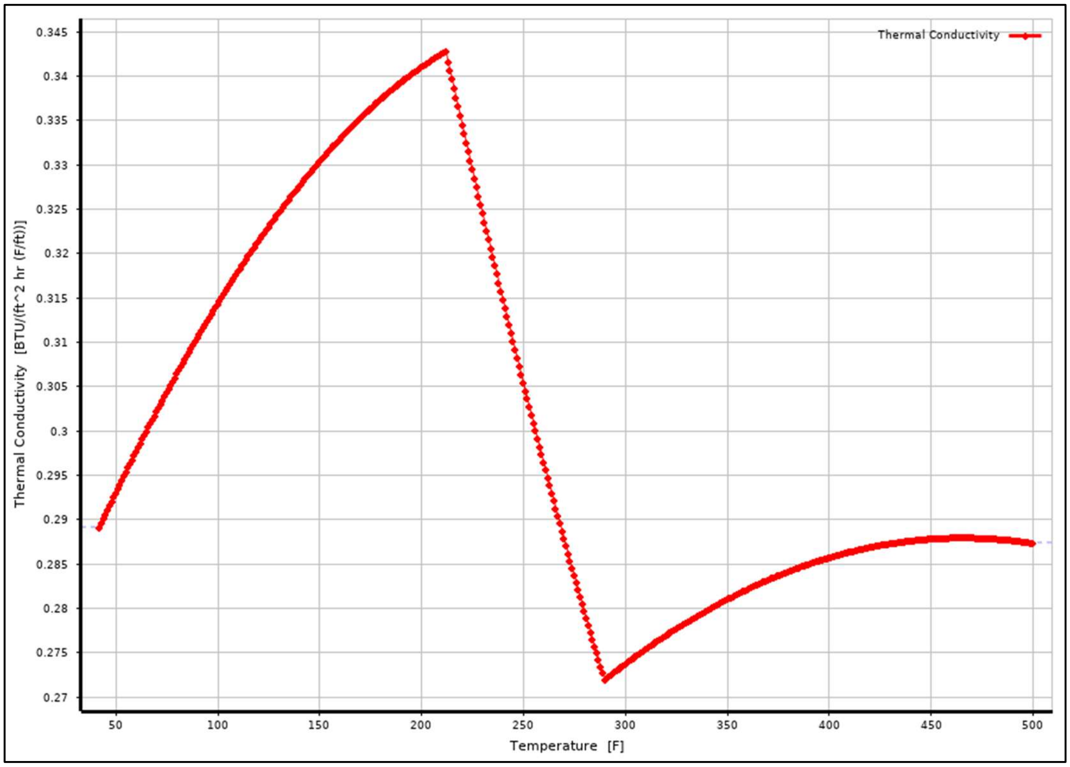
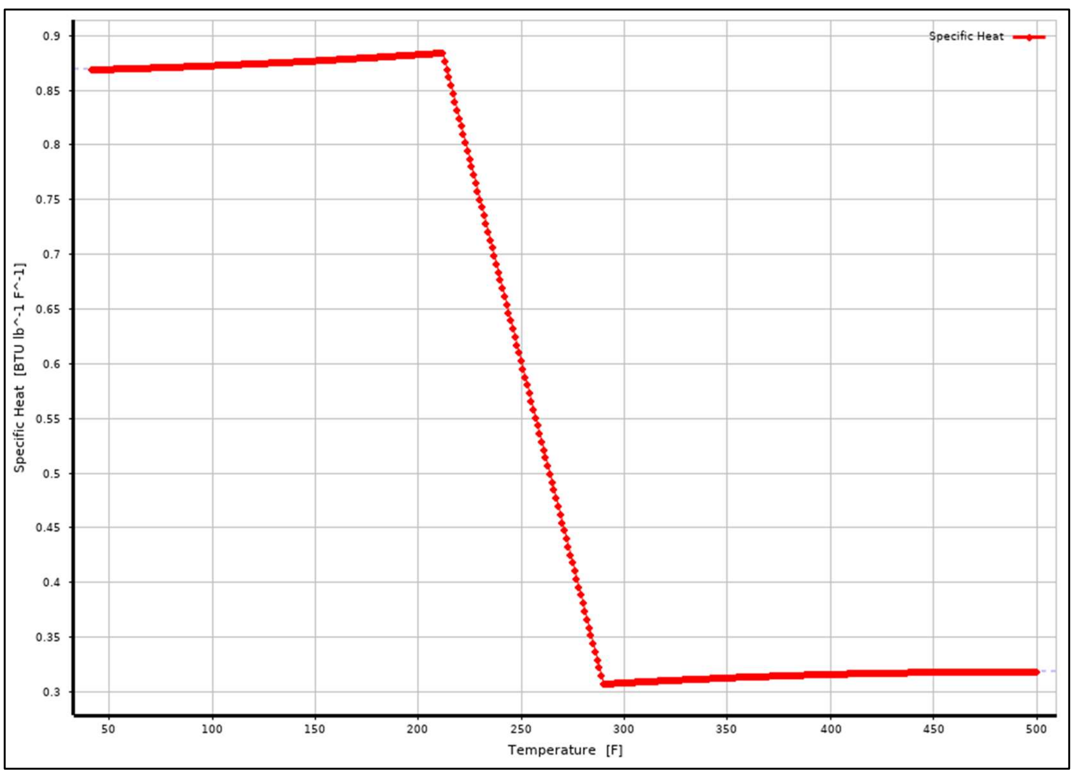


Figure 9: Stuffing Specific Heat (Btu/(lb·°F) over Temperature Range of 42°F to 500°F



Ingredient Compositions Throughout Temperature Range

Non-poultry Compositions

Table 16: Composition of Celery from 42°F Onwards

	$\mathbf{42}$°F $-$ $\mathbf{212}$°F	$\mathbf{212}$°F $-$ $\mathbf{290}$°F	$\mathbf{290}$°F $+$
$\boldsymbol{x_{H_2O,ce}}(\%)$	94.64	$94.64 - 1.213(T - 212)$	0
$\boldsymbol{x_{protein,ce}}$ (%)	0.75	$0.75 - 0.0096(T - 212)$	0
$\boldsymbol{x_{fat,ce}}$ (%)	0.14	$0.14 - 0.0018(\mathrm{T} - 212)$	0
$\boldsymbol{x_{carbs,ce}}$ (%)	3.65	$3.65 - 0.0468(T - 212)$	0
$\boldsymbol{x_{ash,ce}}$ (%)	0.82	$0.82 + 1.27(T - 212)$	100

Table 17: Composition of Onion from 42°F Onwards

	$\mathbf{42}$°F $-$ $\mathbf{212}$°F	$\mathbf{212}$°F $-$ $\mathbf{290}$°F	$\mathbf{290}$°F $+$
$\boldsymbol{x_{H_2O,o}}(\%)$	89.68	$89.68 - 1.15(T - 212)$	0
$\boldsymbol{x_{protein,o}}$ (%)	1.16	$1.16 - 0.015(T - 212)$	0
$\boldsymbol{x_{fat,o}}$ (%)	0.16	$0.16 - 0.0021(\mathrm{T} - 212)$	0
$\boldsymbol{x_{carbs,o}}$ (%)	8.63	$8.63 - 0.1106(T - 212)$	0
$\boldsymbol{x_{ash,o}}$ (%)	0.37	$0.37 + 1.277(T - 212)$	100

Table 18: Composition of Potato from 42°F Onwards

	$\mathbf{42}$°F $-$ $\mathbf{212}$°F	$\mathbf{212}$°F $-$ $\mathbf{290}$°F	$\mathbf{290}$°F $+$
$\boldsymbol{x_{H_2O,p}}(\%)$	78.96	$78.96 - 1.01(T - 212)$	0
$\boldsymbol{x_{protein,p}}$ (%)	2.07	$2.07 - 0.0265(T - 212)$	0
$\boldsymbol{x_{fat,p}}$ (%)	0.10	$0.1 - 0.0013(\mathrm{T} - 212)$	0
$\boldsymbol{x_{carbs,p}}$ (%)	17.98	$17.98 - 0.2305(T - 212)$	0
$\boldsymbol{x_{ash,p}}$ (%)	0.89	$0.89 + 1.271(T - 212)$	100

*Note that the composition of butter is included in the body of the text in Table 2: Composition of Butter from 42°F Onwards.

Poultry Compositions

Table 19: Composition of Chicken from 42℉ Onwards

	$\mathbf{42}$℉ $-$ $\mathbf{212}$℉	$\mathbf{212}$℉ $-$ $\mathbf{228}$℉	$\mathbf{228}$℉ $-$ $\mathbf{290}$℉	$\mathbf{290}$℉ $+$
$x_{H_2O,ch}(\%)$	65.99	$65.99-0.47(T-212)$	$58.5-0.94(T-228)$	0
$x_{protein,ch}$ (%)	18.60	18.60	$18.6-0.3(T-228)$	0
$x_{fat,ch}$ (%)	15.06	15.06	$15.06-0.243(T-228)$	0
$x_{carbs,ch}$ (%)	0	0	0	0
$x_{ash,ch}$ (%)	0.68	$0.68+0.47(T-212)$	$8.18+1.481(T-228)$	100

Table 20: Composition of Duck from 42℉ Onwards

	$\mathbf{42}$℉ $-$ $\mathbf{212}$℉	$\mathbf{212}$℉ $-$ $\mathbf{228}$℉	$\mathbf{228}$℉ $-$ $\mathbf{290}$℉	$\mathbf{290}$℉ $+$
$x_{H_2O,d}(\%)$	48.50	$48.5-0.47(T-212)$	$56-0.903(T-228)$	0
$x_{protein,d}$ (%)	11.49	11.49	$11.49-0.185(T-228)$	0
$x_{fat,d}$ (%)	39.34	39.34	$39.34-0.635(T-228)$	0
$x_{carbs,d}$ (%)	0	0	0	0
$x_{ash,d}$ (%)	0.88	$0.88+0.47(T-212)$	$8.38+1.478(T-228)$	100

*Note that the composition of turkey is included in the body of the text in Table 3.

Fourier Number

$$F_o = \frac{\alpha t}{L^2} \tag{A.21}$$

The formula above solves for the Fourier number F_o where α is the thermal diffusivity of the material with unit area over time, t is the time, and L is the characteristic length.

Appendix B: Python Code for Heat Transfer Calculations

```
import sympy as sp
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt

#Inputs
Time_stop=10
rad=0
T_inf=70
T_oven=350
T_0=42
Q_in=125*3.412  # converting W to btu/hr

#Material Properties at 42 Degrees

#Thermal Conductivity  BTU/(hr*ft*F)
k_s=0.29
k_d=0.24
k_t=0.27
k=[k_t, k_t, k_d, k_d, k_s, k_s]

#Specific Heat  BTU/(lb*F)
c_s=0.87
c_d=0.78
c_t=0.84

#Density  (lb/ft^3)
p_s=68.65
p_d=65.05
p_t=66.78

#Dimensions ft
r_s=0.23
#r_s=0.115
r_d=0.36
#r_d=0.295
r_t=0.66
#r_t=0.49

r=[r_s/2, r_s, (r_d+r_s)/2, r_d, ((r_t+r_d)/2), r_t]
```

```
#Film Ceofficient  BTU/(hr*ft^2*F)
h=7.1911e-4*3600

#Room Resistance from freezing (42 F)
e=0.9 #Thermal Resistivity for meat
s= 1.714*10**-9 #Stefan-Boltzmann constant
h_rad_room=e*s*(42**2+T_inf**2)*(42+T_inf)
R_conv_room=1/(h*4*np.pi*r_t**2)
R_rad_room=1/(h_rad_room*(2*np.pi*r_t**2))

#Oven Resistance (Assuming turkey skin is defrosted to 70 F)
h_rad_oven=e*s*(70**2+T_inf**2)*(70+T_inf)
R_conv_oven=1/(h*4*np.pi*r_t**2)
R_rad_oven=1/(h_rad_oven*(2*np.pi*r_t**2))

#Room Resistance at rest (Assuming turkey skin is 350 F)
h_rad_rest=e*s*(350**2+T_inf**2)*(350+T_inf)
R_conv_rest=1/(h*4*np.pi*r_t**2)
R_rad_rest=1/(h_rad_rest*(2*np.pi*r_t**2))

#Thermal Resisitivies
#Oven
R_oven=np.zeros(6)
R_oven[0]=1/(1/R_conv_oven+1/R_rad_oven)
for i in range(1, 6):
  R_oven[i]=(1/(4*np.pi*k[i]))*(1/r[i-1]-1/r[i])

V=np.zeros(6)
V[0]=(4/3)*np.pi*r[0]**3
for i in range(1, 6):
  V[i]=(4/3)*np.pi*(r[i]**3-r[i-1]**3)

#Room
R_room=np.zeros(6)
R_room[0]=1/(1/R_conv_room+1/R_rad_room)
for i in range(1, 6):
  R_room[i]=(1/(4*np.pi*k[i]))*(1/r[i-1]-1/r[i])

V=np.zeros(6)
V[0]=(4/3)*np.pi*r[0]**3
for i in range(1, 6):
  V[i]=(4/3)*np.pi*(r[i]**3-r[i-1]**3)
```

```
#Rest
R_rest=np.zeros(6)
R_rest[0]=1/(1/R_conv_rest+1/R_rad_rest)
for i in range(1, 6):
  R_rest[i]=(1/(4*np.pi*k[i]))*(1/r[i-1]-1/r[i])

V = np.zeros(6)
V[0]=(4/3)*np.pi*r[0]**3
for i in range(1, 6):
  V[i]=(4/3)*np.pi*(r[i]**3-r[i-1]**3)

#Capacitances
C1=p_t*V[4]*c_t # turkey capacitance
C2=p_d*V[2]*c_d #ducken capacitance
C3=p_s*V[0]*c_s #stuffing capacitance
print(C1,C2,C3)

#Skewers on

#ODE
def skewer_ode(t, T):
    T1, T2, T3, T4, T5, T6 = T
    # Differential equations for capacitive nodes
    dT1_dt=(T_inf-T1)/R_room[0]-(T1-T2)/R_room[1]
    dT2_dt=(1/C1)*(((T1-T2)/R_room[1])-((T2-T3)/R_room[2])+(Q_in/5))
    dT3_dt=(T2-T3)/R_room[2]-(T3-T4)/R_room[3]
    dT4_dt=(1/C2)*(((T3-T4)/R_room[3])-((T4-T5)/R_room[4])+(2*Q_in/5))
    dT5_dt=(T4-T5)/R_room[4]-(T5-T6)/R_room[5]
    dT6_dt=(1/C3)*(((T5-T6)/R_room[5])+(2*Q_in/5))
    return [dT1_dt, dT2_dt,dT3_dt, dT4_dt,dT5_dt, dT6_dt]

#Time span
t_span=(0, 5)  #time range in hours
t_eval=np.linspace(0, 5, 100)  #evaluation points

initial_conditions=[T_0]*6

#Solve ODE
solution=solve_ivp(skewer_ode, t_span, initial_conditions, t_eval=t_eval)

#Results
t = solution.t
T1, T2, T3, T4, T5, T6 = solution.y
```

```
#Plot
plt.plot(t, T1, label="Turkey Boundary (T1)", color="red")
plt.plot(t, T2, label="Turkey Inside (T2)", color="blue")
plt.plot(t, T3, label="Ducken Bounday(T3)", color="green")
plt.plot(t, T4, label="Ducken (T4)", color="orange")
plt.plot(t, T5, label="Stuffing Boundary (T5)", color="purple")
plt.plot(t, T6, label="Stuffing (T6)", color="indigo")
plt.axhline(165, color="green", linestyle="--", label="165°F (Safe Temp)")
plt.axhline(290, color="purple", linestyle="--", label="290°F (Burn Temp)")
plt.xlabel("Time (hr)")
plt.ylabel("Temperature (°F)")
plt.legend()
plt.show()

final_time = 4.25
final_index = np.argmin(np.abs(solution.t - final_time))

print(f"Final values after skewer cooking:")
print(f"T1: {solution.y[0][final_index]}")
print(f"T2: {solution.y[1][final_index]}")
print(f"T3: {solution.y[2][final_index]}")
print(f"T4: {solution.y[3][final_index]}")
print(f"T5: {solution.y[4][final_index]}")
print(f"T6: {solution.y[5][final_index]}")

#After Oven is On

#ODE After Oven Turns on
def oven_ode(t, T):
    T1,T2,T3,T4,T5,T6 = T
    #Differential equations for capacitive nodes
    dT1_dt=(T_oven-T1)/R_oven[0]-(T1-T2)/R_oven[1]
    dT2_dt=(1/C1)*(((T1-T2)/R_oven[1])-((T2-T3)/R_oven[2])+(Q_in/5))
    dT3_dt=(T2-T3)/R_oven[2]-(T3-T4)/R_oven[3]
    dT4_dt=(1/C2)*(((T3-T4)/R_oven[3])-((T4-T5)/R_oven[4])+(2*Q_in/5))
    dT5_dt=(T4-T5)/R_oven[4]-(T5-T6)/R_oven[5]
    dT6_dt=(1/C3)*(((T5-T6)/R_oven[5])+(2*Q_in/5))
    return [dT1_dt, dT2_dt,dT3_dt, dT4_dt,dT5_dt, dT6_dt]
```

```python
#Solve ODE for the second phase
solution_after_oven=solve_ivp(oven_ode, t_span, [solution.y[0][final_index], solution.y[1][final_index], solution.y[2][final_index], solution.y[3][final_index], solution.y[4][final_index], solution.y[5][final_index]], t_eval=t_eval)

#Set initial conditions for the second simulation to the final values of the first simulation
t_after_oven=solution_after_oven.t
T1_after, T2_after, T3_after, T4_after, T5_after, T6_after=solution_after_oven.y

#Extract results
t_after_oven=solution_after_oven.t
T1_after, T2_after, T3_after, T4_after, T5_after, T6_after=solution_after_oven.y

#Plot
plt.plot(t_after_oven, T1_after, label="Turkey Boundary (T1)", color="red")
plt.plot(t_after_oven, T2_after, label="Turkey Inside (T2)", color="blue")
plt.plot(t_after_oven, T3_after, label="Ducken Boundary (T3)", color="green")
plt.plot(t_after_oven, T4_after, label="Ducken (T4)", color="orange")
plt.plot(t_after_oven, T5_after, label="Stuffing Boundary (T5)", color="purple")
plt.plot(t_after_oven, T6_after, label="Stuffing (T6)", color="indigo")
plt.axhline(165, color="green", linestyle="--", label="165°F (Safe Temp)")
plt.axhline(290, color="purple", linestyle="--", label="290°F (Burn Temp)")
plt.xlabel("Time (hr)")
plt.ylabel("Temperature (°F)")
plt.legend()
plt.show()

#Print Final Values
final_time=2.5
final_index_after_oven=np.argmin(np.abs(solution_after_oven.t - final_time))

print("\nFinal values after oven is on:")
print(f"T1: {solution_after_oven.y[0][final_index_after_oven]}")
print(f"T2: {solution_after_oven.y[1][final_index_after_oven]}")
print(f"T3: {solution_after_oven.y[2][final_index_after_oven]}")
print(f"T4: {solution_after_oven.y[3][final_index_after_oven]}")
print(f"T5: {solution_after_oven.y[4][final_index_after_oven]}")
print(f"T6: {solution_after_oven.y[5][final_index_after_oven]}")
```

```python
#Resting After Cooking

#ODE During Rest
def rest_ode(t, T):
    T1,T2,T3,T4,T5,T6=T
    #Differential equations for capacitive nodes
    dT1_dt=(T_inf-T1)/R_room[0]-(T1-T2)/R_room[1]
    dT2_dt=(1/C1)*(((T1-T2)/R_room[1])-((T2-T3)/R_room[2]))
    dT3_dt=(T2-T3)/R_room[2]-(T3-T4)/R_room[3]
    dT4_dt=(1/C2)*(((T3-T4)/R_room[3])-((T4-T5)/R_room[4]))
    dT5_dt=(T4-T5)/R_room[4]-(T5-T6)/R_room[5]
    dT6_dt=(1/C3)*(((T5-T6)/R_room[5]))
    return [dT1_dt, dT2_dt,dT3_dt, dT4_dt,dT5_dt, dT6_dt]

#Solve ODE for the third phase
solution_after_rest=solve_ivp(rest_ode, t_span, [solution_after_oven.y[0][final_index_after_oven], solution_after_oven.y[1][final_index_after_oven], solution_after_oven.y[2][final_index_after_oven], solution_after_oven.y[3][final_index_after_oven], solution_after_oven.y[4][final_index_after_oven], solution_after_oven.y[5][final_index_after_oven]], t_eval=t_eval)

#Set initial conditions for the second simulation to the final values of the first simulation
t_after_rest=solution_after_rest.t
T1_after, T2_after, T3_after, T4_after, T5_after, T6_after = solution_after_rest.y

#Extract results
t_after_rest=solution_after_rest.t
T1_after, T2_after, T3_after, T4_after, T5_after, T6_after = solution_after_rest.y

#Plot
plt.plot(t_after_rest, T1_after, label="Turkey Boundary (T1)", color="red")
plt.plot(t_after_rest, T2_after, label="Turkey Inside (T2)", color="blue")
plt.plot(t_after_rest, T3_after, label="Ducken Boundary (T3)", color="green")
plt.plot(t_after_rest, T4_after, label="Ducken (T4)", color="orange")
plt.plot(t_after_rest, T5_after, label="Stuffing Boundary (T5)", color="purple")
plt.plot(t_after_rest, T6_after, label="Stuffing (T6)", color="indigo")
plt.axhline(165, color="green", linestyle="--", label="165°F (Safe Temp)")
plt.axhline(290, color="purple", linestyle="--", label="290°F (Burn Temp)")
plt.xlabel("Time (hr)")
plt.ylabel("Temperature (°F)")
plt.legend()
plt.show()

#Print Final Values
final_time=2
final_index_after_rest=np.argmin(np.abs(solution_after_rest.t-final_time))

print("\nFinal values after resting:")
print(f"T1: {solution_after_rest.y[0][final_index_after_rest]}")
print(f"T2: {solution_after_rest.y[1][final_index_after_rest]}")
print(f"T3: {solution_after_rest.y[2][final_index_after_rest]}")
print(f"T4: {solution_after_rest.y[3][final_index_after_rest]}")
print(f"T5: {solution_after_rest.y[4][final_index_after_rest]}")
print(f"T6: {solution_after_rest.y[5][final_index_after_rest]}")
```

Appendix C: References

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