EE 381 – 5 Points

Project 4

Hypothesis Testing Project   Name, I.D. #, and Date: ______________________________

Worksheet and Project Instructions.

There is an established system in use that works 50% of the time. A new system is available and it is argued that the new system is an improvement with a probability that it works exceeding 50%. Both the old and the new systems are expensive to run and are time consuming to operate. Thus, only a small number of trials can be performed. Since there will be a finite number of trails, the probability in each trial is constant, there are two outcomes, and each trial is independent we are considering a Binomial R.V.

Hypothesis

State the hypothesis: ____

Binomial Distribution

In the analysis that follows we'll assume 18 trials total. Thus, $n = 18$ in the binomial distribution

$$P(\{X = x\}) = {}_nC_x \; p^x (1-p)^{(n-x)}$$

Using Python produce a plot or bar graph or both of the binomial distribution with $n = 18$ and $p = 0.5$.

Critical Value ($C.V.$)

We want to determine a number between 0 and 18 that will be the value sufficient for us to decide to reject the null hypothesis even when it is true.

$$P(\{X \geq C.V.\}|\{H_0 \equiv T\}) = P(\{X \geq C.V.\}|\{p = \tfrac{1}{2}\})$$

We can try some choices of the critical value by writing a program in Python. If we try a critical value of 11 then we need to evaluate the statement

$$P\left(\{X \geq 11\}\middle|\left\{p = \frac{1}{2}\right\}\right) = \sum_{x=11}^{18} {}_{18}C_x \; (\tfrac{1}{2})^x (1 - \tfrac{1}{2})^{(18-x)}$$

This can be done for other values, as well.

Write Python code to determine the probability for each of the choices of X.

What choice of the critical value most closely fits $P\big(\{X \geq c.v\}\big|\{p = \tfrac{1}{2}\}\big) = 0.05$?

Having determined the $C.V.$ we have the probability of rejecting the null hypothesis when it is true. This is the probability of a type I error or α error. In general, the probability of committing a Type I error is referred to as a test's level of significance, and is denoted by α. Usually in hypothesis tests the value of α is chosen a priori. The value customarily used is 5% but this is not absolute.

The mistake of accepting the null hypothesis when the alternative hypothesis is true is referred to as type II error.

Type II or β error

To determine the β error we need to calculate its probability for a specific value of the parameter in the new system. This would be done for, potentially, every possible value of that parameter. This can be simplified since we are looking for values greater than 50%. Conveying an improvement over the old system.

Using Python construct a list of $\tilde{p}$ values ranging from 0.55 through to 1.00 in steps of 0.05. Then using the $C.V.$ you determined calculate the β probabilities with a repetition statement coded in Python. The equation of interest is:

$$\beta = P(\{X \leq C.V. -1\}|\{p = \tilde{p}\}) = \sum_{x=0}^{C.V.-1} {}_{18}C_x \ (\tilde{p})^x (1-\tilde{p})^{(18-x)}$$

The β is the probability that we accept the null hypothesis, H_0, when the alternative probability, H_1 , is true. Using Python produce several plots or bar graphs or both of the binomial distribution with $n = 18$ and $p > 0.5$.

The Power of the Test.

From above β is the probability we accept the null hypothesis when the alternative hypothesis is true. Thus, $1-\beta$ is the probability that we reject the null hypothesis when the alternative hypothesis is true. The quantity $1-\beta$ is the *power* of the test. From the work above you can see that $1-\beta$ is a function of the values of the parameter $\tilde{p}$.

Using Python create a plot of $1-\beta$ verses $\tilde{p}$. This is a power curve.

Submit the completed lab project the week of 4/15/18.

Inform the author of any errors.

You can work with other students on this project as long as you list all names and sources.