A Problem For Paul

November 2, 2012

The Problem

Given a 2-blade B and a vector v, find the vector v' which is the orthogonal reflection of v about B.

One Possible Solution

Start by drawing a picture!

Now, without loss of generality, we can write B as

$$B = a \wedge b,$$

where a and b are a pair of orthogonal vectors, where b is unit-length, and where v is orthogonal to the vector a.

It follows that $|a| = |B|$. Why?

We will now prove that

$$v' = -BvB^{-1}.$$

To begin, notice that $B = ab$. Why?

Next, notice that $B^{-1} = ba/|a|^2$. Why?

We then notice that $bvb = v'$. Why?

Now notice that $-av'a/|a|^2 = v'$. Why?

We now see that

$$-BvB^{-1} = -\frac{abvba}{|a|^2} = -\frac{av'a}{|a|^2} = v'.$$

And there we have it!

Another Possible Solution

Notice that $v_\parallel = (v \cdot B)B^{-1}$ and $v_\perp = (v \wedge B)B^{-1}$. We then have

$$\begin{aligned}
v' &= a_\parallel - a_\perp \\
&= (a \cdot B)B^{-1} - (a \wedge B)B^{-1} \\
&= -(B \cdot a)B^{-1} - (B \wedge a)B^{-1} \\
&= -(B \cdot a + B \wedge a)B^{-1} \\
&= -BaB^{-1},
\end{aligned}$$

which confirms the earlier result!