CONFORMAL GEOMETRY AND DYNAMICS
An Electronic Journal of the American Mathematical Society
Volume 00, Pages 000–000 (Xxxx XX, XXXX)
S 1088-4173(XX)0000-0

A SYSTEM FOR READING AND WRITING THE LANGUAGE OF CONFORMAL GEOMETRIC ALGEBRA

SPENCER T. PARKIN

Abstract. This paper is the first of its kind to treat, not the development or use of conformal geometric algebra (CGA), but a systematic method for reading and writing in the language of CGA. A consensus on the terminology and semantics used in such a system is needed by anyone writing or reading in this language so that analytical ideas can be properly and universally communicated in the language. Different ways of looking at the language and different interpretations have led to confusion between those who are knowledgable on the subject, the present author among them. The main result of this paper is a system for communicating in the language of CGA that brings together and disambiguates the current and common phrases and terminologies used when communicating ideas in CGA. Of particular significance is a rigorous treatment of what is meant by an imaginary geometry in the conformal model.

1. The Three Principle Geometric Representations

We begin by setting forth the three principle geometric representations used by the conformal model.

(1) A subset of n-dimensional Euclidean space $\mathbb{R}^n$.
(2) A blade taken from a geometric algebra $\mathbb{G}$ upon which the conformal model may be imposed.
(3) A set of parameters consisting of scalars and Euclidean vectors taken from $\mathbb{G}$.

To be able to communicate effectly in the language of CGA, one must master the relationships between all three of these fundamental representations of geometry. The first of these (1) is generated by the second (2). The third (3) generates representations of the form (2) through composition as well as is generated by representations of the form (2) through decomposition.

Working with geometry in the representation (1) is often impractical and does not easily lend itself to geometric analysis. This is where geometric algebra goes to work for us in the representation (2). The representation (3) is what ultimately makes CGA a useful analytical tool as it is a set of parameters that collectively and completely characterize a certain type of geometry. For example, a center, radius and normal are enough to uniquely characterize a 2-dimensional circle in 3-dimensional space.

We concern ourselves now with rigorously defining how these three geometric representations are related to one another in the conformal model.

2010 *Mathematics Subject Classification.* Primary .

2. The Direct And Dual Representations

For the remainder of this paper, we will let $\mathbb{G}$ denote our conformal geometric algebra, $\mathbb{V}$ the $(n+2)$-dimensional vector space generating this algebra, and $\mathbb{V}^n$ the n-dimensional Euclidean vector sub-space of $\mathbb{V}$ from which representations of the form (1) are produced.

E-mail address: spencer.parkin@gmail.com