Examination of Equ. 14.6

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On page 403, it is claimed that the direct form of a real round is given by

$$\Sigma = \left(o + \frac{1}{2}\rho^2\infty\right) A_k,$$

where A_k is a purely Euclidean blade. I will show that this is *also* the dual form of an imaginary round. Consider the simple case of $k = 1$, and let A_k be a unit Euclidean vector. If Σ as a dual imaginary round, then there cannot exist a Euclidean vector x such that $p(x) \cdot \Sigma = 0$. Letting $\sigma = o + \frac{1}{2}\rho^2\infty$, we see that

$$\begin{aligned} p(x) \cdot \Sigma &= p(x) \cdot (\sigma \wedge A_k) \\ &= (p(x) \cdot \sigma)A_k - (p(x) \cdot A_k)\sigma. \end{aligned}$$

Now, since $\Sigma \neq 0$, it is clear that A_k and σ are linearly independent, so that $p(x) \cdot \Sigma = 0$ if and only if $p(x) \cdot \sigma = 0$ and $p(x) \cdot A_k = 0$. Then since A_k is a real dual plane, clearly there exists x such that $p(x) \cdot A_k = 0$. However, notice that σ is an imaginary dual round! It follows that there does not exist x such that $p(x) \cdot \sigma = 0$ and therefore there is no x such that $p(x) \cdot \Sigma = 0$, which is what we wanted to show.

A similar argument can be given for the cases when $k > 1$.