CONFORMAL GEOMETRY AND DYNAMICS
An Electronic Journal of the American Mathematical Society
Volume 00, Pages 000–000 (Xxxx XX, XXXX)
S 1088-4173(XX)0000-0

# THE QUADRIC MODEL OF GEOMETRIC ALGEBRA

SPENCER T. PARKIN

Abstract. A great achievement of the conformal model of geometric algebra is that the elements of computations are representative of geometry, and may therefore be thought of as geometry. A limitation of the model, however, is its inability to represent all quadric surfaces. Set forth in this paper, the quadric model of geometric algebra, while maintaining the idea of geometries as elements of computation, overcomes this limitation at the expense of added complexity and dimension.

## 1. Representing Quadric Surfaces

We begin with a description of all $D$-dimensional quadric surfaces as given in [1]. Letting $\mathbb{V}^{D+1}$ be a $(D+1)$-dimensional vector space, we may describe a $D$-dimensional quadric surface as the set of all projective points $p \in \mathbb{V}^{D+1}$ such that

$$0 = \sum_{i=0}^{D}\sum_{j=0}^{D} \alpha_{ij}(p \cdot e_i)(p \cdot e_j), \tag{1.1}$$

where $\{e_i\}_{k=0}^{D}$ is an orthonormal basis generating $\mathbb{V}^{D+1}$, and the set of scalars $\{\alpha_{ij}\}$ characterize the surface. We will let $p/(p\cdot e_0)$ be the homogenization of $p$ as a point in $D$-dimensional Euclidean space. In this way, all such Euclidean points satisfying equation (1.1) are those on the surface of the $D$-dimensional quadric it represents.

The equation (1.1) has a matrix form where, in matrix algebra, we might consider the matrix as a representative of the quadric geometry and therefore may also consider matrix operations as geometric operations. Seeking a more basis independent formulation, however, we will appeal to geometric algebra.

Let $\{e_i\}_{k=0}^{2D+1}$ be an orthonormal basis generating the $2(D+1)$-dimensional Euclidean vector space $\mathbb{V}^{2(D+1)}$ with $\mathbb{V}^{D+1}$ as a $(D+1)$-dimensional vector sub-space, and let $\mathbb{G}\left(\mathbb{V}^{2(D+1)}\right)$ denote the geometric algebra generated by this vector space. Then, barrowing from the ideas in [2], if we define the versor $R$ as a rotor given by

$$R = 2^{-D/2}\prod_{i=0}^{D}(1 - e_i e_{i+D+1}), \tag{1.2}$$

then the equation (1.1) may be rewritten as

$$0 = pRp\tilde{R} \cdot G, \tag{1.3}$$

2010 *Mathematics Subject Classification.* Primary .

where

$$G = \sum_{i=0}^{D} \sum_{j=0}^{D} \alpha_{ij} e_i e_{j+D+1}. \tag{1.4}$$

In the form (1.3) we may consider the bivector $G$ as a representative of the quadric surface. Immediately we see here that, unlike the conformal model, elements of the form $pRp\tilde{R}$ do not represent points in the manner set forth by equation (1.3). To see this, simply realize that the inner product square of any non-zero 2-blade is non-zero. There are no null blades in our geometric algebra. Nevertheless, we have succeeded here in finding a form of element in a geometric algebra that, under a given definition, represents the variety of algebraic varieties known as quadric surfaces.

## 2. Constructing Quadric Surfaces

## 3. Operations On Quadric Surfaces

In the conformal model of geometric algebra, all geometries are represented by blades, and therefore, the inner and outer products have closure in the set of all geometric representatives. This, however, is not true of the quadric model, and so we cannot take advantage of the same results in geometric algebra that allowed for intersecting and combining geometries. In this model, only addition and subtraction have closure in the set of geometric representatives, which is the set of all bivectors.

## References

1. *Quadric*, `http://en.wikipedia.org/wiki/Quadric`.
2. C. Doran and D. Hestenes, *Lie groups as spin groups*, J. Math. Phys. **34** (1993), 8.

*E-mail address*: spencer.parkin@disney.com