A Problem For Paul

October 31, 2012

The Problem

Given a 2-blade B and a vector v, find the vector v' which is the orthogonal reflection of v about B.

One Possible Solution

Start by drawing a picture!

Now, without loss of generality, we can write B as

$$B = a \wedge b$$
,

where a and b are a pair of orthogonal vectors, where b is unit-length, and where v is orthogonal to the vector a.

It follows that |a| = |B|. Why?

We will now prove that

$$v' = -BvB^{-1}.$$

To begin, notice that B = ab. Why?

Next, notice that $B^{-1} = ba/|a|^2$. Why?

We then notice that bvb = v'. Why?

Now notice that $-av'a/|a|^2 = v'$. Why?

We now see that

$$-BvB^{-1} = -\frac{abvba}{|a|^2} = -\frac{av'a}{|a|^2} = v'.$$

And there we have it!