Abstract Algebra Exercises

September 8, 2012

Chapter 1

Exercise 5

For $n \geq 3$, describe the elements of D_n. How many elements does D_n have?

    The group D_n, when $n \geq 3$, will have n rotation operations and n reflections operations. So the group will have order $2n$. The group D_2 has a 2 rotation and 2 reflection operations that are the same, so it must have order 2. The group D_1 has order 1.

Exercise 6

In D_n, explain geometrically why a reflection followed by a reflection must be a rotation.

    Rotations preserve the winding order of the n-gon, but reflections do not. An even number of reflection will leave the winding order of the n-gon invariant. Then since the rotations are the set of all winding preserving operations, two successive reflections must be a rotation.

Exercise 7

In D_n, explain geometrically why a rotation followed by a rotation must be a rotation.

    Because the set of all rotations in D_n forms its own sub-group.

Exercise 8

In D_n, explain geometrically why a rotation and a reflection taken together in either order must be a reflection.

An odd number of reflections combined with any number of rotations does not preserve winding order. The only non-winding-order-preserving operations are the reflections. So any rotation and reflection combination must be a reflection.