

©Copyright 2023

Spencer Wallace

Planetesimal Accretion Beyond the Solar System: An Application to Systems of Tightly-Packed Inner Planets

Spencer Wallace

A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

2023

Reading Committee:

Thomas Quinn, Chair

Rory Barnes

Eric Agol

Program Authorized to Offer Degree:

Astronomy

University of Washington

Abstract

Planetesimal Accretion Beyond the Solar System: An Application to Systems of
Tightly-Packed Inner Planets

Spencer Wallace

Chair of the Supervisory Committee:
Chair Thomas Quinn
Department of Chair

This sample dissertation is an aid to students who are attempting to format their theses with L^AT_EX, a sophisticated text formatter widely used by mathematicians and scientists everywhere.

- It describes the use of a specialized macro package developed specifically for thesis production at the University. The macros customize L^AT_EX for the correct thesis style, allowing the student to concentrate on the substance of his or her text.¹
- It demonstrates the solutions to a variety of formatting challenges found in thesis production.
- It serves as a template for a real dissertation.

¹See Appendix A to obtain the source to this thesis and the class file.

TABLE OF CONTENTS

	Page
List of Figures	ii
Chapter 1: Introduction	1
Chapter 2: Formation of Planetary Embryos in the Inner Solar System	2
2.1 Introduction	2
2.2 Simulations	6
2.3 Results	12
Chapter 3: Detecting Cold Jupiters via Collisional Grinding of Planetesimals	13
Chapter 4: Formation of Planetary Embryos at Short Orbital Periods	14
Chapter 5: In-Situ Formation of STIPs	15
Bibliography	16
Appendix A: Where to find the files	24

LIST OF FIGURES

Figure Number		Page
2.1	Evolution of the maximum (solid curve) and mean (dashed curve) planetesimal mass in the $N=4000$ and $N=10^6$ particle simulations. At early times, the maximum mass grows more quickly than the mean mass, which is indicative of runaway growth. After a few thousand years, the separation between the curves becomes a constant factor, signalling the start of oligarchic growth. .	9
2.2	Snapshots from the low and high resolution models in the $a - e$ plane. On the left, the light dots represent individual planetesimals, while the contours on the right hand plots represent curves of constant number density. The contour levels are the same between all panels and correspond to 7.8×10^6 , 1.6×10^7 , 2.3×10^7 , and 3.1×10^7 planetesimals per AU per unit eccentricity. The black circles denote the configuration of the 6 largest bodies in the simulation, with the area of the circles scaled to the mass of the body. The horizontal error bars are scaled to 5 times the Hill radius of the bodies.	10
2.3	Cumulative number of bodies in each mass bin for the low and high resolution runs, shown at the end of runaway growth (top row) and the end of the simulation (bottom row). The dashed line indicates a slope of -1.5, which is characteristic of runaway growth.	11

ACKNOWLEDGMENTS

The author wishes to express sincere appreciation to University of Washington, where he has had the opportunity to work with the T_EX formatting system, and to the author of T_EX, Donald Knuth, *il miglior fabbro*.

DEDICATION

to my dear wife, Joanna

Chapter 1

INTRODUCTION

Chapter 2

FORMATION OF PLANETARY EMBRYOS IN THE INNER SOLAR SYSTEM

2.1 *Introduction*

The standard scenario of terrestrial planet formation involves the pairwise accretion of small rocky bodies, called planetesimals, that condense from dust out of the protostellar disc [76]. This accretion process can be broken into a series of distinct stages. First, dust particles settle toward the midplane of the disc and clump together via gravitational instability [23, 89], streaming instabilities [38, 37], turbulent concentration [10, 16, 15, 27] or direct sticking [67, 87, 20, 39]. These formation models predict a wide variation in the initial size of planetesimals, ranging from hundreds of meters up to a few hundred kilometers in diameter.

After this condensation process ends, growth continues via pairwise collisions between planetesimals. Above about 1 km in size, gravitational focusing becomes effective and the collision rate strongly increases. This marks the beginning of the intermediate stage of accretion. During this phase, runaway growth [19, 43, 5] quickly increases the size of the largest bodies. Eventually, the largest bodies grow massive enough to heat the small planetesimals, which then inhibits the runaway growth effect. This marks the transition into the phase of oligarchic growth in which the handful of large bodies all tend to grow at the same rate. This produces a bimodal size distribution of planetesimals, with many small bodies and a handful of very large bodies. During this phase, a combination of scattering events between the oligarchs and dynamical friction from the small planetesimals places the oligarchs on nearly circular orbits that are spaced apart by approximately 5-10 Hill radii [44]. Eventually, the oligarchs accrete most of the available material in the vicinity of their orbit, which eventually throttles their growth rate. On much longer time scales, the large bodies perturb

each other onto crossing orbits, forming Mars to Earth sized bodies via occasional collisions. This phase of late stage accretion is highly chaotic and takes much longer to play out than the previous stages [11, 72].

Generally, planetesimal accretion models fail to produce a configuration of planets that resembles that of the Solar System. Planets produced in the vicinity of Mars are systematically too massive [84, 71, 61, 34] and the terrestrial planets that form are too eccentric compared to their Solar System counterparts [11, 2, 9]. In addition, the present day water content of Earth, along with the large D/H ratio of Venus [18] does not match with models of solids that condensed out of the Solar nebula at 1 AU.

These issues all have proposed solutions, which generally involve altering the initial conditions for late-stage accretion simulations. The condition of the Solar System at the beginning of the late stage accretion phase is poorly constrained. This is mostly due to the fact that planetesimal formation, and therefore the intermediate accretion phase which produces the planetary embryos, is not well understood. Although the present day population of small Solar System bodies has continued to evolve since the end of the intermediate accretion phase, the current size frequency distribution (SFD) of asteroid belt and Kuiper belt objects contains some clues about the accretion history. [57] argued that the SFD of asteroid belt objects larger than 100 km in diameter has been largely unchanged, aside from a size independent depletion factor. [19] did a long term stability analysis of small Solar System objects and found that small bodies left over from accretion should still be largely unperturbed. For these reasons, it should be possible to connect observables in the Solar System to planetesimal formation theories by modeling only the intermediate stages of accretion.

There are two common ways to model planetesimal growth. A powerful approach is to use statistical methods to track the evolution of large groups of planetesimals. This is known as the particle-in-a-box method [24, 85]. The evolution of growth is followed by tracking planetesimals in discrete bins of mass and semi-major axis. This removes the need to calculate the motion of every individual body and allows very large collections of planetesimals to be followed. Unfortunately, the dynamics that governs the evolution of

these bodies does not always naturally emerge with this approach. As an example, [80] found that a careful treatment of three body encounters led to a different prediction for the initial size distribution of planetesimals near the asteroid belt. Additionally, the particle-in-a-box method is not well suited for studying oligarchic growth because the largest mass bins, which dominate the evolution in this stage, contain small numbers of bodies. Non-gravitational effects such as gas drag and fragmentation require extra care to implement self-consistently [49], although it has been successfully done [86, 9]. To alleviate some of these issues while still being able to model large populations, a newer hybrid approach, in which large bodies are treated as single entities and planetesimals are treated as statistical ensembles has been developed [82, 40, 51, 59].

The most reliable and straightforward approach is to use N-body methods to follow the evolution and growth of the planetesimals [48]. By tracking the individual motions of bodies, the dynamics governing their evolution naturally emerges, no matter what the distribution of bodies looks like. However, N-body simulations involving collision detection are extremely computationally expensive, which severely limits both the resolution and number of timesteps that can be achieved. This is why there are very few studies of runaway and oligarchic with direct N-body simulations in the literature [43, 44, 45, 5]. Instead, N-body methods are most commonly used to study late stage accretion, where the self-gravity and collisional evolution of the residual planetesimals is largely unimportant [11, 2, 9, 65, 57].

To date, we are not aware of any N-body simulations that resolve both the runaway and oligarchic growth phase with more than about 10^4 particles. At this resolution, stochasticity likely has a significant influence on dynamical friction and resonances may not be sufficiently resolved. [72] found that insufficient planetesimal resolution during the oligarchic growth phase limits the effectiveness of dynamical friction felt by the oligarchs, producing a population of embryos with unrealistically high eccentricities. This idea has also been applied to planet migration through a disc of planetesimals. [14] showed that resonances make a significant contribution to the dynamical friction torque exerted by the disc on the planet. This phenomenon requires the planetesimals to be finely resolved [8]. Dynamical friction

is also facilitated via resonances in galactic dynamics [54]. [83, 83] examined the effect of N-body particle counts on galaxy dynamics and showed that resonant interactions between the bar and halo were only effective with sufficient particle phase space coverage.

For these reasons, we motivate the need for a high resolution N-body simulation of planetesimal growth. We do this to better understand the effect that dynamical friction has on the intermediate stages of terrestrial planet growth and to examine what predictions a high resolution model makes about the residual population of planetesimals in the Solar System. In particular, resonances which are only effective with fine enough resolution, may have an important influence on planetesimal growth. In this paper, we investigate planetesimal evolution during the runaway and oligarchic growth phases with a direct N-body model. We begin by simulating an annulus of planetesimals with similar resolution to [44] to validate our model. We then run the same configuration with 100x more particles to better understand the effects of resolution on the intermediate stages of terrestrial planet growth.

In section 2.2, we describe the simulation code that we use and provide a detailed description of the collision model. We also summarize the initial conditions used. In section 2.3, we present the results of the low and high resolution simulation of terrestrial planet growth and highlight differences between the two. We find that a bump develops in the mass distribution of the high resolution simulation, which does not appear in the low resolution run. This feature manifests itself shortly after oligarchic growth commences and we infer that it is produced by extra heating via mean motion resonances between the oligarchs and small planetesimals. To further demonstrate this effect, we re-run the high resolution simulation with a narrower annulus (depopulating many of the resonances) and show that this reduces the prominence of the bump in the mass distribution. In section ??, we present a set of collisionless simulations of a planetary embryo embedded in an annulus of planetesimals. This more clearly demonstrates the differences in dynamical behavior between the low and high resolution models. In section ??, we present two more simulations of planetesimal growth at intermediate resolutions and demonstrate that the location of the bump is sensitive to the initial planetesimal mass. Finally, section ?? connects these new results to our present

understanding of terrestrial planet growth and we discuss the additional steps necessary to use our results to constrain the initial size of planetesimals in the Solar System.

2.2 Simulations

2.2.1 Numerical Methods

All of the simulations described in this paper were performed with the highly parallel N-body code CHANGA . CHANGA is written in the CHARM++ programming language and has been shown to perform well when using up to half a million processors [55] simultaneously. Gravitational forces are calculated using a modified Barnes-Hut tree algorithm with hexadecapole order expansions of the moments. For all of the simulations described in this paper, a node opening criterion of $\Theta_{BH} = 0.7$ was used. [75] was able to resolve mean motion resonances in a terrestrial disc using a similar tree code with the same node opening criterion, so we expect that our model should properly handle resonance effects as well. The equations of motion are integrated using a kick-drift-kick leapfrog scheme. For more information about the implementation of CHANGA see [36].

2.2.2 Collision Model

CHANGA is a smoothed particle hydrodynamics code originally designed for cosmology simulations. In order to use CHANGA to study planetesimal coagulation, we implemented a hard-body collision model that treats particles as solid objects with a fixed radius, rather than as smooth tracers of a fluid with a characteristic softening length. This work was largely based off of the hard body collision implementation in PKDGRAV, which is described in [74] and [75].

Collisions are predicted at the beginning of each drift step by extrapolating the positions of the particles forward using the velocities calculated during the first kick. For each particle, the closest 64 neighbors are considered in the collision search. Because CHANGA is a tree code, the nearest neighbors of a particle can be retrieved in $\mathcal{O}(N \log N)$ time. After

extrapolating the positions forward and checking for overlap with any neighbors, the earliest collision time t_{coll} is stored for each particle.

After the prediction phase, particles with t_{coll} less than the time step size ΔT must have their collisions resolved. For simplicity, all collisions in our simulations result in perfect accretion. [28] showed that excluding the effects of fragmentation and collisional rebound does not qualitatively change the growth modes of the planetesimals. A merger between two particles of mass m_1 and m_2 results in a single particle of mass $M = m_1 + m_2$, with the radius set to conserve density. The position and velocity of the resulting particle is set to the centre of mass position and velocity of the colliders at the moment of contact. The resulting merged particle is then drifted to the end of the step. If multiple collisions are predicted during a time step, the earliest collision is resolved first. Because resolving a collision can result in another imminent collision, collisions must be resolved one by one, with a new prediction check being run each time.

As in [43, 44] we accelerate the accretion process by artificially inflating the physical radius of the bodies by a factor of f . This technique reduces the accretion time-scale by approximately a factor of f^2 , significantly reducing the number of timesteps that must be integrated. Additionally, inflating the particle radii allows us to use a smaller annulus with less planetesimals. The reason we cannot use an arbitrarily skinny annulus is because the edges tend to expand outward due to the unrealistic boundary conditions, decreasing the surface density. The time-scale for this expansion is set by the two body relaxation time-scale, which scales with N . Reducing the accretion time-scale by increasing f allows us to study planetesimal growth with a smaller, less computationally expensive annulus.

We must be careful, however, to choose a value of f that is not too large. The rms eccentricity and inclination of the planetesimal disc grows as gravitational encounters transform energy due to Keplerian shear into random motions. By accelerating the growth rate, we cause the transitions between growth modes to happen early, when the disc is less dynamically excited. This discrepancy is partly compensated for by the fact that our model ignores the effects of gas drag, which would damp the eccentricities and inclinations of the plan-

etesimals. We adopt $f = 6$ for our calculations, which reduces the amount of gravitational scattering by less than 10% of its true value and does not qualitatively change the modes of planetesimal growth [44].

As previously mentioned, simulation (ii) is meant to be compared with the results of [44]. This allows us to both validate our collision model and also have a baseline to compare our simulations to. In both cases, the planetesimals are distributed randomly in an annulus centred at 1 AU with a width $\Delta a = 0.085$ AU around a $1 M_{\odot}$ star. This annulus width was chosen so that the required particle count is minimized without boundary effects influencing planetesimal growth. The surface mass density of the ring is set to 10 g cm^{-2} , which approximately corresponds to the minimum-mass solar nebula model [26] at 1 AU. In case (i), the eccentricities and inclinations are taken from a Rayleigh distribution [32] with $\langle e^2 \rangle^{1/2} = 2\langle i^2 \rangle^{1/2} = 4h/a$, where h is the mutual Hill radius. To match the same eccentricity and inclination dispersion with larger planetesimals in case (ii), we use $\langle e^2 \rangle^{1/2} = 2\langle i^2 \rangle^{1/2} = 0.635h/a$. In both simulations, the planetesimals are given an internal density of 2 g cm^{-3} . We use fixed timesteps with $\Delta T = 0.0025$ years and evolve the simulations for 20,000 years. Large timesteps diminish the effectiveness of gravitational focusing, which inhibits the collision rate. We find that the collision rate is converged below $\Delta T = 0.0025$ years. The integration time was chosen to be comparable to the orbital repulsion and Hill radius growth time-scale [42] so that the effects of oligarchic growth are fully realized by the end of the simulation.

It is worth noting that simulation (i) was a very computationally expensive undertaking. In total, the full 20,000 years of integration required approximately 130,000 CPU hours and over 50 days of wall clock time.

2.2.3 Initial Conditions

We begin by using our collision model to perform two sets of calculations:

- (i) 10^6 equal mass bodies with $m = 1.2 \times 10^{21} \text{ g}$
- (ii) 4000 equal mass bodies with $m = 3 \times 10^{23} \text{ g}$

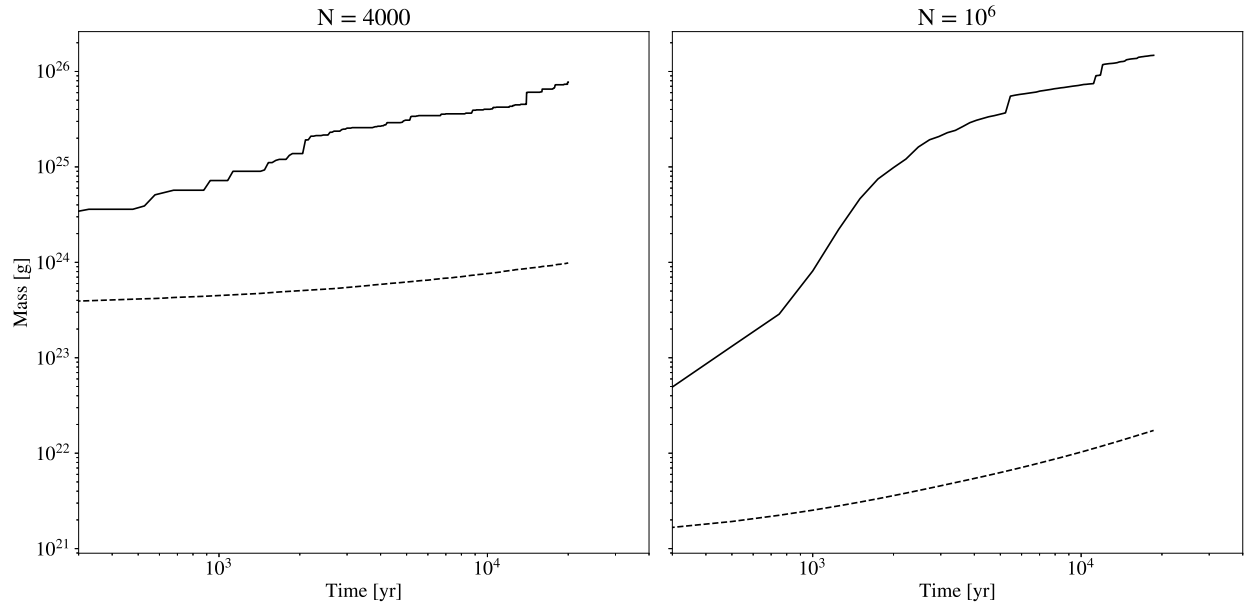


Figure 2.1: Evolution of the maximum (solid curve) and mean (dashed curve) planetesimal mass in the $N=4000$ and $N=10^6$ particle simulations. At early times, the maximum mass grows more quickly than the mean mass, which is indicative of runaway growth. After a few thousand years, the separation between the curves becomes a constant factor, signalling the start of oligarchic growth.

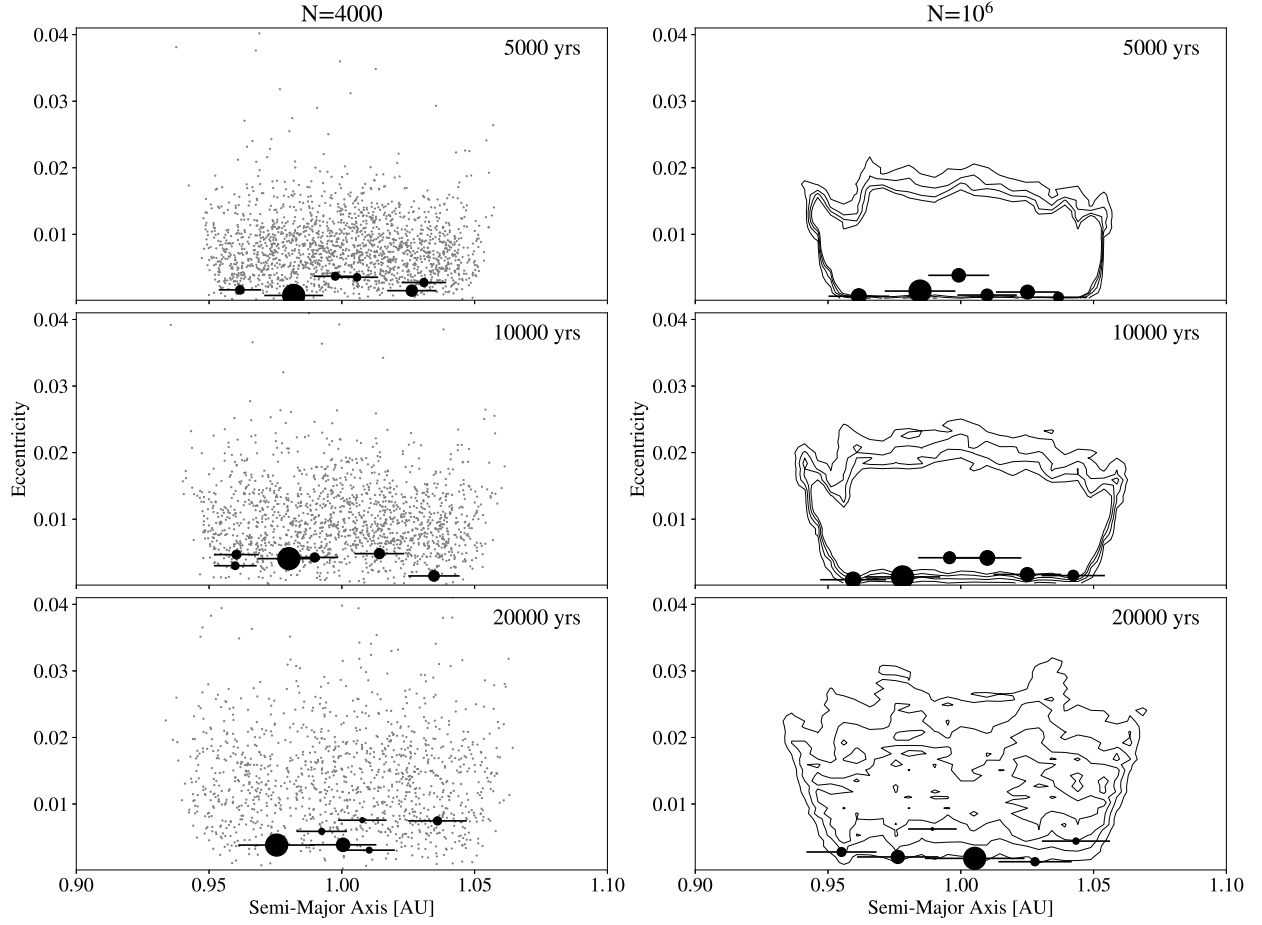


Figure 2.2: Snapshots from the low and high resolution models in the $a - e$ plane. On the left, the light dots represent individual planetesimals, while the contours on the right hand plots represent curves of constant number density. The contour levels are the same between all panels and correspond to 7.8×10^6 , 1.6×10^7 , 2.3×10^7 , and 3.1×10^7 planetesimals per AU per unit eccentricity. The black circles denote the configuration of the 6 largest bodies in the simulation, with the area of the circles scaled to the mass of the body. The horizontal error bars are scaled to 5 times the Hill radius of the bodies.

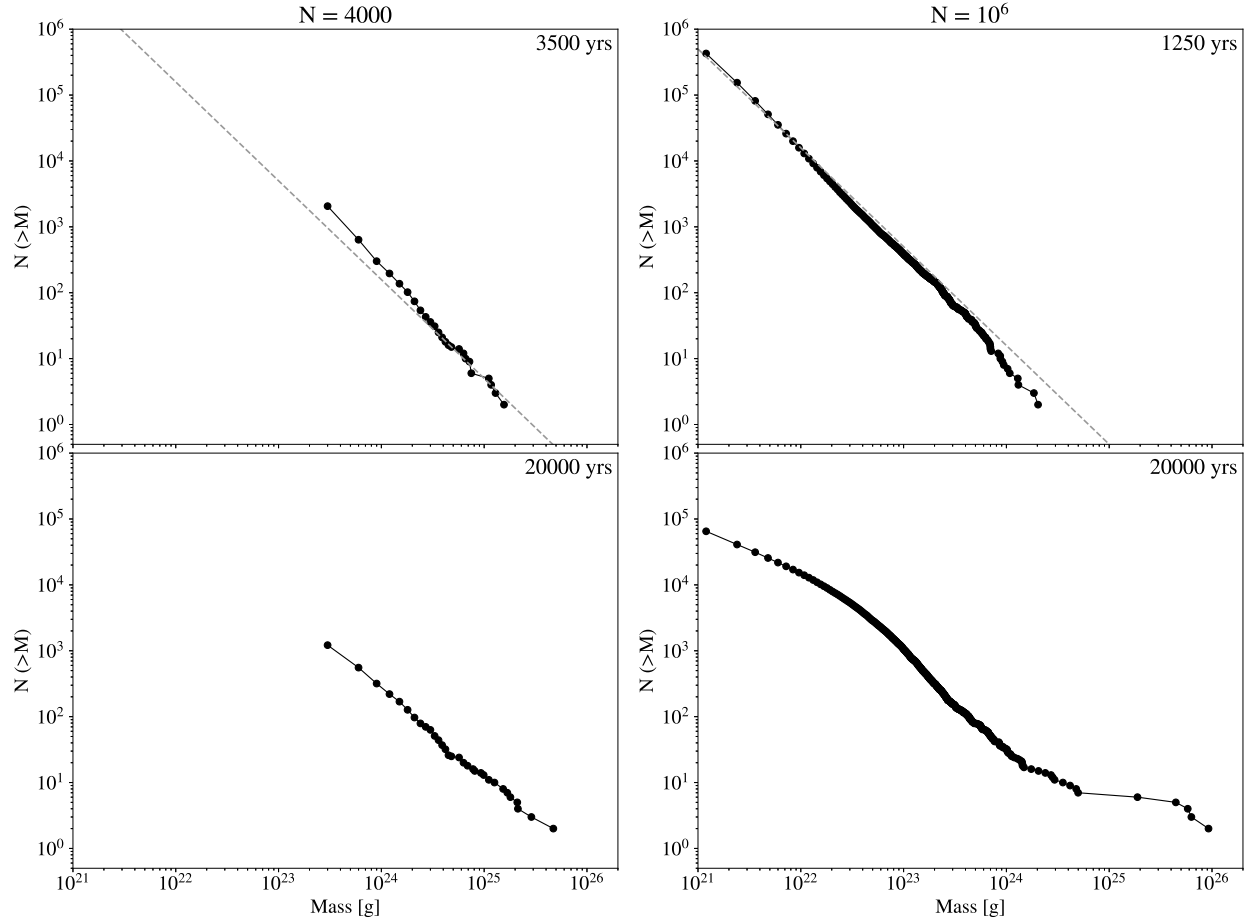


Figure 2.3: Cumulative number of bodies in each mass bin for the low and high resolution runs, shown at the end of runaway growth (top row) and the end of the simulation (bottom row). The dashed line indicates a slope of -1.5, which is characteristic of runaway growth.

2.3 Results

2.3.1 Low vs High Resolution

We begin by comparing the evolution of growth between the low resolution ($N=4000$) and high resolution ($N=10^6$) models. As planetesimals collide and grow, gravitational focusing becomes increasingly effective and the relative growth rate increases with mass [24]. Figure 2.1 shows the evolution of the average and maximum planetesimal mass in both simulations. Runaway growth at early times is evident from the fact that the maximum mass grows more quickly than the mean mass. Eventually, the largest bodies begin to dynamically heat the neighboring planetesimals, which slows the growth rate of the largest bodies.

Chapter 3

DETECTING COLD JUPITERS VIA COLLISIONAL GRINDING OF PLANETESIMALS

Chapter 4

FORMATION OF PLANETARY EMBRYOS AT SHORT ORBITAL PERIODS

Chapter 5

IN-SITU FORMATION OF STIPS

BIBLIOGRAPHY

- [1] I. Adachi, C. Hayashi, and K. Nakazawa. The gas drag effect on the elliptical motion of a solid body in the primordial solar nebula. *Progress of Theoretical Physics*, 56:1756–1771, December 1976.
- [2] C. B. Agnor, R. M. Canup, and H. F. Levison. On the Character and Consequences of Large Impacts in the Late Stage of Terrestrial Planet Formation. *Icarus*, 142:219–237, November 1999.
- [3] Y. Alibert, C. Mordasini, W. Benz, and C. Winisdoerffer. Models of giant planet formation with migration and disc evolution. *A&A*, 434(1):343–353, Apr 2005.
- [4] P. J. Armitage. *Astrophysics of Planet Formation*. October 2013.
- [5] R. Barnes, T. R. Quinn, J. J. Lissauer, and D. C. Richardson. N-Body simulations of growth from 1 km planetesimals at 0.4 AU. *Icarus*, 203:626–643, October 2009.
- [6] C. Baruteau, A. Crida, S. J. Paardekooper, F. Masset, J. Guilet, B. Bitsch, R. Nelson, W. Kley, and J. Papaloizou. Planet-Disk Interactions and Early Evolution of Planetary Systems. In Henrik Beuther, Ralf S. Klessen, Cornelis P. Dullemond, and Thomas Henning, editors, *Protostars and Planets VI*, page 667, Jan 2014.
- [7] W. F. Bottke, D. D. Durda, D. Nesvorný, R. Jedicke, A. Morbidelli, D. Vokrouhlický, and H. Levison. The fossilized size distribution of the main asteroid belt. *Icarus*, 175:111–140, May 2005.
- [8] Adrián Brunini, Pablo J. Santamaría, Héctor R. Viturro, and Rodolfo G. Cionco. On the number of particles in N-body simulations of planet disk interaction. *Planetary and Space Science*, 55:2121–2127, November 2007.
- [9] J. E. Chambers. Making More Terrestrial Planets. *Icarus*, 152:205–224, August 2001.
- [10] J. E. Chambers. Planetesimal formation by turbulent concentration. *Icarus*, 208:505–517, August 2010.
- [11] J. E. Chambers and G. W. Wetherill. Making the Terrestrial Planets: N-Body Integrations of Planetary Embryos in Three Dimensions. *Icarus*, 136:304–327, December 1998.

- [12] S. Chandrasekhar. Dynamical Friction. I. General Considerations: the Coefficient of Dynamical Friction. *ApJ*, 97:255, March 1943.
- [13] Subrahmanyan Chandrasekhar. *Principles of stellar dynamics*. 1960.
- [14] R. G. Cionco and A. Brunini. Orbital migrations in planetesimal discs: N-body simulations and the resonant dynamical friction. *MNRAS*, 334:77–86, July 2002.
- [15] J. N. Cuzzi, R. C. Hogan, and W. F. Bottke. Towards initial mass functions for asteroids and Kuiper Belt Objects. *Icarus*, 208:518–538, August 2010.
- [16] J. N. Cuzzi, R. C. Hogan, and K. Shariff. Toward Planetesimals: Dense Chondrule Clumps in the Protoplanetary Nebula. *ApJ*, 687:1432–1447, November 2008.
- [17] Stanley F. Dermott, Renu Malhotra, and Carl D. Murray. Dynamics of the Uranian and Saturnian satellite systems: A chaotic route to melting Miranda? *Icarus*, 76(2):295–334, Nov 1988.
- [18] T. M. Donahue, J. H. Hoffman, R. R. Hodges, and A. J. Watson. Venus was wet - A measurement of the ratio of deuterium to hydrogen. *Science*, 216:630–633, May 1982.
- [19] M. Duncan, T. Quinn, and S. Tremaine. The long-term evolution of orbits in the solar system - A mapping approach. *Icarus*, 82:402–418, December 1989.
- [20] P. Garaud, F. Meru, M. Galvagni, and C. Olczak. From Dust to Planetesimals: An Improved Model for Collisional Growth in Protoplanetary Disks. *ApJ*, 764:146, February 2013.
- [21] P. Goldreich and S. Tremaine. The excitation of density waves at the Lindblad and corotation resonances by an external potential. *ApJ*, 233:857–871, Nov 1979.
- [22] P. Goldreich and S. Tremaine. Disk-satellite interactions. *ApJ*, 241:425–441, Oct 1980.
- [23] P. Goldreich and W. R. Ward. The Formation of Planetesimals. *ApJ*, 183:1051–1062, August 1973.
- [24] R. Greenberg, W. K. Hartmann, C. R. Chapman, and J. F. Wacker. Planetesimals to planets - Numerical simulation of collisional evolution. *Icarus*, 35:1–26, July 1978.
- [25] Evgeni Grishin and Hagai B. Perets. Application of Gas Dynamical Friction for Planetesimals. I. Evolution of Single Planetesimals. *ApJ*, 811:54, Sep 2015.

- [26] C. Hayashi. Structure of the Solar Nebula, Growth and Decay of Magnetic Fields and Effects of Magnetic and Turbulent Viscosities on the Nebula. *Progress of Theoretical Physics Supplement*, 70:35–53, 1981.
- [27] P. F. Hopkins. Jumping the gap: the formation conditions and mass function of ‘pebble-pile’ planetesimals. *MNRAS*, 456:2383–2405, March 2016.
- [28] S. Ida, E. Kokubo, and J. Makino. The Origin of Anisotropic Velocity Dispersion of Particles in a Disc Potential. *MNRAS*, 263:875, Aug 1993.
- [29] S. Ida and D. N. C. Lin. Toward a Deterministic Model of Planetary Formation. I. A Desert in the Mass and Semimajor Axis Distributions of Extrasolar Planets. *ApJ*, 604(1):388–413, Mar 2004.
- [30] S. Ida and J. Makino. Scattering of planetesimals by a protoplanet - Slowing down of runaway growth. *Icarus*, 106:210, November 1993.
- [31] Shigeru Ida, Geoffrey Bryden, D. N. C. Lin, and Hidekazu Tanaka. Orbital Migration of Neptune and Orbital Distribution of Trans-Neptunian Objects. *ApJ*, 534(1):428–445, May 2000.
- [32] Shigeru Ida and Junichiro Makino. N-Body simulation of gravitational interaction between planetesimals and a protoplanet . I. velocity distribution of planetesimals. *Icarus*, 96:107–120, March 1992.
- [33] Satoshi Inaba, Hidekazu Tanaka, Kiyoshi Nakazawa, George W. Wetherill, and Eiichiro Kokubo. High-Accuracy Statistical Simulation of Planetary Accretion: II. Comparison with N-Body Simulation. *Icarus*, 149(1):235–250, Jan 2001.
- [34] A. Izidoro, S. N. Raymond, A. Morbidelli, and O. C. Winter. Terrestrial planet formation constrained by Mars and the structure of the asteroid belt. *MNRAS*, 453:3619–3634, November 2015.
- [35] R. Jedicke, J. Larsen, and T. Spahr. *Observational Selection Effects in Asteroid Surveys*, pages 71–87. 2002.
- [36] P. Jetley, F. Gioachin, L. Mendes, C. Kale, and T. Quinn. Massively parallel cosmological simulations with ChaNGa. *Proceedings of IEEE International Parallel and Distributed Processing Symposium*, 2008.
- [37] A. Johansen, M.-M. Mac Low, P. Lacerda, and M. Bizzarro. Growth of asteroids, planetary embryos, and Kuiper belt objects by chondrule accretion. *Science Advances*, 1:1500109, April 2015.

- [38] A. Johansen, J. S. Oishi, M.-M. Mac Low, H. Klahr, T. Henning, and A. Youdin. Rapid planetesimal formation in turbulent circumstellar disks. *Nature*, 448:1022–1025, August 2007.
- [39] A. Kataoka, H. Tanaka, S. Okuzumi, and K. Wada. Fluffy dust forms icy planetesimals by static compression. *A&A*, 557:L4, September 2013.
- [40] Scott J. Kenyon and Benjamin C. Bromley. Terrestrial Planet Formation. I. The Transition from Oligarchic Growth to Chaotic Growth. *AJ*, 131(3):1837–1850, Mar 2006.
- [41] David R. Kirsh, Martin Duncan, Ramon Brasser, and Harold F. Levison. Simulations of planet migration driven by planetesimal scattering. *Icarus*, 199(1):197–209, Jan 2009.
- [42] E. Kokubo and S. Ida. Orbital evolution of protoplanets embedded in a swarm of planetesimals. *Icarus*, 114:247–257, April 1995.
- [43] E. Kokubo and S. Ida. On Runaway Growth of Planetesimals. *Icarus*, 123:180–191, September 1996.
- [44] E. Kokubo and S. Ida. Oligarchic Growth of Protoplanets. *Icarus*, 131:171–178, January 1998.
- [45] E. Kokubo and S. Ida. Formation of Protoplanet Systems and Diversity of Planetary Systems. *ApJ*, 581:666–680, December 2002.
- [46] E. Kokubo and S. Ida. Dynamics and accretion of planetesimals. *Progress of Theoretical and Experimental Physics*, 2012(1):01A308, October 2012.
- [47] Eiichiro Kokubo and Shigeru Ida. Formation of Protoplanets from Planetesimals in the Solar Nebula. *Icarus*, 143:15–27, January 2000.
- [48] M. Lecar and S. J. Aarseth. A numerical simulation of the formation of the terrestrial planets. *ApJ*, 305:564–579, June 1986.
- [49] Z. M. Leinhardt. Terrestrial Planet Formation: A Review and Current Directions. In D. Fischer, F. A. Rasio, S. E. Thorsett, and A. Wolszczan, editors, *Extreme Solar Systems*, volume 398 of *Astronomical Society of the Pacific Conference Series*, page 225, 2008.
- [50] Z. M. Leinhardt and D. C. Richardson. Planetesimals to Protoplanets. I. Effect of Fragmentation on Terrestrial Planet Formation. *ApJ*, 625:427–440, May 2005.

- [51] H. F. Levison, M. J. Duncan, and E. Thommes. A Lagrangian Integrator for Planetary Accretion and Dynamics (LIPAD). *AJ*, 144:119, October 2012.
- [52] Jack J. Lissauer and Glen R. Stewart. Growth of Planets from Planetesimals. In Eugene H. Levy and Jonathan I. Lunine, editors, *Protostars and Planets III*, page 1061, Jan 1993.
- [53] Y. Lithwick. After Runaway: The Trans-Hill Stage of Planetesimal Growth. *ApJ*, 780:22, January 2014.
- [54] D. Lynden-Bell and A. J. Kalnajs. On the generating mechanism of spiral structure. *MNRAS*, 157:1, January 1972.
- [55] H. Menon, L. Wesolowski, and G. et al. Zheng. Adaptive Techniques for Clustered N-Body Cosmological Simulations. *Computational Astrophysics and Cosmology*, 2:1, 2015.
- [56] Y. Miguel, O. M. Guilera, and A. Brunini. The diversity of planetary system architectures: contrasting theory with observations. *MNRAS*, 417(1):314–332, Oct 2011.
- [57] A. Morbidelli, W. F. Bottke, D. Nesvorný, and H. F. Levison. Asteroids were born big. *Icarus*, 204:558–573, December 2009.
- [58] A. Morbidelli, J. I. Lunine, D. P. O’Brien, S. N. Raymond, and K. J. Walsh. Building Terrestrial Planets. *Annual Review of Earth and Planetary Sciences*, 40:251–275, May 2012.
- [59] R. Morishima. A particle-based hybrid code for planet formation. *Icarus*, 260:368–395, November 2015.
- [60] R. Morishima. Onset of oligarchic growth and implication for accretion histories of dwarf planets. *Icarus*, 281:459–475, January 2017.
- [61] R. Morishima, J. Stadel, and B. Moore. From planetesimals to terrestrial planets: N-body simulations including the effects of nebular gas and giant planets. *Icarus*, 207:517–535, June 2010.
- [62] C. D. Murray and S. F. Dermott. *Solar System Dynamics*. February 2000.
- [63] R. A. Murray-Clay and E. I. Chiang. Brownian Motion in Planetary Migration. *ApJ*, 651:1194–1208, November 2006.

- [64] Y. Nakagawa, M. Sekiya, and C. Hayashi. Settling and growth of dust particles in a laminar phase of a low-mass solar nebula. *Icarus*, 67:375–390, September 1986.
- [65] D. P. O’Brien, A. Morbidelli, and H. F. Levison. Terrestrial planet formation with strong dynamical friction. *Icarus*, 184:39–58, September 2006.
- [66] K. Ohtsuki, G. R. Stewart, and S. Ida. Evolution of Planetesimal Velocities Based on Three-Body Orbital Integrations and Growth of Protoplanets. *Icarus*, 155:436–453, February 2002.
- [67] S. Okuzumi, H. Tanaka, H. Kobayashi, and K. Wada. Rapid Coagulation of Porous Dust Aggregates outside the Snow Line: A Pathway to Successful Icy Planetesimal Formation. *ApJ*, 752:106, June 2012.
- [68] C. W. Ormel, C. P. Dullemond, and M. Spaans. Accretion among preplanetary bodies: The many faces of runaway growth. *Icarus*, 210:507–538, November 2010.
- [69] Andrew Pontzen, Rok Roškar, Greg Stinson, and Rory Woods. pynbody: N-Body/SPH analysis for python. Astrophysics Source Code Library, May 2013.
- [70] R. R. Rafikov. Dynamical Evolution of Planetesimals in Protoplanetary Disks. *AJ*, 126:2529–2548, November 2003.
- [71] S. N. Raymond, D. P. O’Brien, A. Morbidelli, and N. A. Kaib. Building the terrestrial planets: Constrained accretion in the inner Solar System. *Icarus*, 203:644–662, October 2009.
- [72] S. N. Raymond, T. Quinn, and J. I. Lunine. High-resolution simulations of the final assembly of Earth-like planets I. Terrestrial accretion and dynamics. *Icarus*, 183:265–282, August 2006.
- [73] Sean N. Raymond, Thomas Quinn, and Jonathan I. Lunine. Making other earths: dynamical simulations of terrestrial planet formation and water delivery. *Icarus*, 168:1–17, March 2004.
- [74] D. C. Richardson. Tree Code Simulations of Planetary Rings. *MNRAS*, 269:493, July 1994.
- [75] D. C. Richardson, T. Quinn, J. Stadel, and G. Lake. Direct Large-Scale N-Body Simulations of Planetesimal Dynamics. *Icarus*, 143:45–59, January 2000.

- [76] V. S. Safronov and E. V. Zvjagina. Relative Sizes of the Largest Bodies during the Accumulation of Planets. *Icarus*, 10:109–115, January 1969.
- [77] A. Shannon, Y. Wu, and Y. Lithwick. Conglomeration of Kilometer-sized Planetesimals. *ApJ*, 801:15, March 2015.
- [78] Hidekazu Tanaka, Taku Takeuchi, and William R. Ward. Three-Dimensional Interaction between a Planet and an Isothermal Gaseous Disk. I. Corotation and Lindblad Torques and Planet Migration. *ApJ*, 565(2):1257–1274, Feb 2002.
- [79] S. J. Weidenschilling. Aerodynamics of solid bodies in the solar nebula. *MNRAS*, 180:57–70, July 1977.
- [80] S. J. Weidenschilling. Initial sizes of planetesimals and accretion of the asteroids. *Icarus*, 214:671–684, August 2011.
- [81] S. J. Weidenschilling and D. R. Davis. Orbital resonances in the solar nebula: Implications for planetary accretion. *Icarus*, 62(1):16–29, Apr 1985.
- [82] S. J. Weidenschilling, D. Spaute, D. R. Davis, F. Marzari, and K. Ohtsuki. Accretional Evolution of a Planetesimal Swarm. *Icarus*, 128(2):429–455, Aug 1997.
- [83] M. D. Weinberg and N. Katz. The bar-halo interaction - II. Secular evolution and the religion of N-body simulations. *MNRAS*, 375:460–476, February 2007.
- [84] G. W. Wetherill. An alternative model for the formation of the asteroids. *Icarus*, 100:307–325, December 1992.
- [85] G. W. Wetherill and G. R. Stewart. Accumulation of a swarm of small planetesimals. *Icarus*, 77:330–357, February 1989.
- [86] G. W. Wetherill and G. R. Stewart. Formation of planetary embryos - Effects of fragmentation, low relative velocity, and independent variation of eccentricity and inclination. *Icarus*, 106:190, November 1993.
- [87] F. Windmark, T. Birnstiel, C. Güttler, J. Blum, C. P. Dullemond, and T. Henning. Planetesimal formation by sweep-up: how the bouncing barrier can be beneficial to growth. *A&A*, 540:A73, April 2012.
- [88] O. C. Winter and C. D. Murray. Resonance and chaos. I. First-order interior resonances. *A&A*, 319:290–304, March 1997.

- [89] A. N. Youdin and F. H. Shu. Planetesimal Formation by Gravitational Instability. *ApJ*, 580:494–505, November 2002.

Appendix A

WHERE TO FIND THE FILES

The `uwthesis` class file, `uwthesis.cls`, contains the parameter settings, macro definitions, and other \TeX commands which allow \LaTeX to format a thesis. The source to the document you are reading, `uwthesis.tex`, contains many formatting examples which you may find useful. The bibliography database, `uwthesis.bib`, contains instructions to Bib \TeX to create and format the bibliography. You can find the latest of these files on:

- My page.

`https://staff.washington.edu/fox/tex/thesis.shtml`

- CTAN

`http://tug.ctan.org/tex-archive/macros/latex/contrib/uwthesis/`

(not always as up-to-date as my site)

VITA

Jim Fox is a Software Engineer with IT Infrastructure Division at the University of Washington. His duties do not include maintaining this package. That is rather an avocation which he enjoys as time and circumstance allow.

He welcomes your comments to `fox@uw.edu`.