Literature Review for Causal Inference with Continuous and Multi-valued Treatments

Spencer Woody

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1 Introduction

1.1 Hirano and Imbens (2005)

Posit existence of potential outcomes $Y_i(t)$, the unit-level dose-response function for $t \in \mathcal{T} = [t_0, t_1]$.

Goal: average dose-response function $\mu(t) = \mathrm{E}[Y_i(t)]$

Assumption 1 (weak unconfoundedness): $Y(t) \perp T \mid X$ for all $t \in \mathcal{T}$.

Generalized propensity score: Let $r(t, x)$ be the conditional density of the treatment given the covariates:

$$r(t, x) = f_{T|X}(t \mid x)$$

The generalized propensity score is $R = r(T, X)$

1.2 Imbens (2000)

1.3 Imai and van Dyk (2004)

Propose a generalized propensity score function (conditional density), and then subclassify units based on $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_{\hat{\psi}}(\mathbf{X})$ which uniquely characterizes the propensity score function.

Results:

Propensity function is a balancing score (see also Hirano & Rubin):

$$p(\mathbf{T}^A \mid X) = p\{\mathbf{T}^A \mid X, e(\cdot \mid X)\} = p\{\mathbf{T}^A \mid e(\cdot \mid X)\}$$

Strong ignorability of Treatment Assignment Given the Propensity Function

$$p\{Y(\mathbf{t}^P)\} = \int p(Y(\mathbf{t}) \mid \mathbf{T}^A = \mathbf{t}^P, \boldsymbol{\theta})p(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}$$

Subclassify into J groups based on $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_{\hat{\psi}}(\mathbf{X})$

Use parametric model $p_\phi\{Y(\mathbf{t}^P) \mid \mathbf{T}^A = \mathbf{t}^P\}$. Approximate the dose response using

$$\begin{aligned} p\{Y(\mathbf{t}^P)\} &= \int p(Y(\mathbf{t}) \mid \mathbf{T}^A = \mathbf{t}^P, \boldsymbol{\theta})p(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \\ &\approx \sum_{j=1}^{J} p_{\hat{\phi}_j}\{Y(\mathbf{t}^P) \mid \mathbf{T}^A = \mathbf{t}^P\}W_j \end{aligned}$$

with weights W_j

1.4 Moodie and Stephens (2012)

1.5 Papadogeorgou and Dominici (2018)

References

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