STA 360: Homework 3

January 27, 2014

Professor Dunson

Spencer Woody

Problem 1

Suppose we are only given random drawings $\theta^{(0)}, \ldots, \theta^{(n)}$ from the $\text{Ga}(\hat{a}, \hat{b})$ distribution. Then we can look at the 2.5% quantile and 97.5% quantile of these random drawings (in R we use the `quantile` command) in order to estimate θ_L and θ_U.

Problem 2

Now we suppose that $a = b = 1$, and $\sum y_i = 325, n = 250$, so our posterior distribution is Ga(326, 251).

(a) If we use the inverse CDF function (`qgamma` in R), then we can calculate the 95% credible interval. In this case, our credible interval is (1.161627, 1.443528).

(b) If we are only allowed to use random samples from the gamma distribution (`rgamma` in R), then we simply find the quantiles of these samples. If we draw 250 random samples, we have a 95% credible interval of (1.168850, 1.451595).

(c) Our MLE estimate is $\sum y_i / n = 1.3$. This is contained within both credible intervals we created. Also, the intervals from parts (a) and (b) are approximately equal.

R code is shown on the following page.

```
############################################################
######### Created by Spencer Woody on 20 Aug 2016 #########
############################################################

a <- 1
b <- 1

sumofy <- 325
n <- 250

ahat <- 1 + 325
bhat <- 1 + n

# Part a
lower1 = qgamma(0.025, ahat, bhat)
upper1 = qgamma(0.975, ahat, bhat)
sprintf('Our 95%% credible interval is (%f, %f)',lower1, upper1)

# Part b
samples = rgamma(250, ahat, bhat)
lower2 = quantile(samples, 0.025)
upper2 = quantile(samples, 0.975)
sprintf('Our 95%% credible interval is (%f, %f)', lower2, upper2)

# Part c
MLE = sumofy / n
print(MLE)

# END
```