Chapter 4 - Karnaugh Maps

Three Variable K-Maps

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1. Use a k-map to find a minimum SOP expression for : $F(A,B,C) = \Sigma m(0,1,3,4,6)$

Truth Table

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0

KMAP

		0	1
		C'	C
00	A'B'	1	1
01	A'B	0	1
11	AB	1	0
10	AB'	1	0

KMAP Legend

		0	1
		C'	C
00	A'B'	0	1
01	A'B	2	3
11	AB	6	7
10	AB'	4	5

Minimum SOP form

To find the minimum SOP expression, first group all adjacent 1s:

GROUPS: $[0,1],[1,3],[6,4] = A'B' + A'C + AC'$

$A'B'+A'C+AC'$
= $A'(B'+C)$ + AC' (Distributive law)

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3. Identify all of the implicants contained in $F(A,B,C) = A \oplus B + B'C'$.

Truth Table

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

KMAP

		0	1
		C'	C
00	A'B'	1	1
01	A'B	1	1
11	AB	1	1
10	AB'	1	1

KMAP Legend

		0	1
		C'	C
00	A'B'	0	1
01	A'B	2	3
11	AB	6	7
10	AB'	4	5

IMPLICANTS = $\text{groups\_of\_one}([0],[2],[3],[4],[5]), \text{groups\_of\_two}([0,2],[2,3],[0,4],[4,5])$

PRIME IMPLICANTS = $[0,2],[2,3],[0,4],[4,5] = A'C' + A'B + B'C + AB'$

ESSENTIAL PRIME IMPLICANTS= $[0,2],[2,3],[4,5] = A'C' + A'B + AB'$

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5. $F(A,B,C) = A'C + AB'$ and $G(A,B,C) = (A' + C')(B' + C)(A + C)$ are equivalent expressions. Use a k-map to determine which term(s) must be don't cares.

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F(A, B, C) = A´C + AB
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		0	1
		C'	C
00	A'B'	0	1
01	A'B	0	1
11	AB	0	0
10	AB'	1	1

		0	1
		C'	C
00	A'B'	0	1
01	A'B	2	3
11	AB	6	7
10	AB'	4	5

$(A'+C')(B'+C)(A+C)$

		0	1
		C'	C
00	A'B'	0	1
01	A'B	0	1
11	AB	0	0
10	AB'	1	0

		0	1
		C'	C
00	A'B'	0	1
01	A'B	2	3
11	AB	6	7
10	AB'	4	5

Therefore, since minterm 5 is the only one that is not shared, it must be a don't care.

Four Variable K-Maps

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1. Use a k-map to find a minimum SOP expression for $F(A,B,C,D) = \Pi M(0,3,4,8,9,10,14)$

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	1	0	1	0
01	A'B	1	0	0	0
11	AB	0	0	0	0
10	AB'	1	0	1	0

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	1	3	2
01	A'B	4	5	7	6
11	AB	12	13	15	14
10	AB'	8	9	11	10

group ones to find F

Prime Implicants $= [0,4],[8,0],[11,3]$

Minimum SOP:

$F = A'C'D' + B'C'D' + B'CD$

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3. Use a k-map to find a minimum POS expression for $F(A,B,C,D) = \Sigma m(1,3,4,5,6,12,14,15)$

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	1	1	0
01	A'B	1	1	0	1
11	AB	1	0	1	1
10	AB'	0	0	0	0

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	1	3	2
01	A'B	4	5	7	6
11	AB	12	13	15	14
10	AB'	8	9	11	10

group zeros to find F'

Prime Implicants $= [8,9,11,10],[9,13],[7],[0,2,8,10]$

$F' = AB' + AC'D + A'BCD + B'D'$

Solve for F to find POS using demorgans law

$F'' = (AB' + AC'D + A'BCD + B'D')$

$F'' = (A' + B)(A + C' + D')(A + B' + C' + D')(B + D)$

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4. A sensor is capable of determining whether or not a car is speeding (driving faster than the speed limit) or driving dangerously (driving 10 m.p.h. or more above the speed limit).

The sensor receives the codes given in table 4.6, where AB corresponds to the speed limit and CD corresponds to the speed of the vehicle. Use a k-map to solve for F (as either minimum SOP or minimum POS), which indicates if the car is speeding.

AB	Speed Limit	CD	Car's Speed
00	45 m.p.h.	00	< 45 m.p.h.
01	55 m.p.h.	01	46–55 m.p.h.
10	65 m.p.h.	10	56–65 m.p.h.
11	unused	11	66–75 m.p.h.

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	1	1	1
01	A'B	0	0	1	1
11	AB	x	x	x	x
10	AB'	0	0	1	0

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	1	3	2
01	A'B	4	5	7	6
11	AB	12	13	15	14
10	AB'	8	9	11	10

From KMAP:

$F = [3,2,7,6],[3,7,11,15],[1,3,2] = A'C + CD + A'B'$

Prime Implicant Tables

1.

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	0	0	1
01	A'B	0	1	1	1
11	AB	0	0	0	1
10	AB'	0	0	0	1

		00	01	11	10
		C'D'	C'D	CD	CD'
00	A'B'	0	1	3	2
01	A'B	4	5	7	6
11	AB	12	13	15	14
10	AB'	8	9	11	10

		2	5	6	7	10	14
[5,7]	A'BD	0	1	0	1	0	0
[10,14,6,2]	CD'	1	0	1	0	1	1

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PI Table
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Therefore the minimum SOP expression is $A'BD + CD'$

		00	01	11	10
		D'E'	D'E	DE	DE'
000	A'B'C'	0	0	0	0
001	A'B'C	0	1	x	0
011	A'BC	0	x	1	0
010	A'BC'	0	0	1	1
110	ABC'	0	0	0	1
111	ABC	0	0	x	1
101	AB'C	0	0	0	0
100	AB'C'	0	0	0	0

		00	01	11	10
		D'E'	D'E	DE	DE'
000	A'B'C'	0	1	3	2
001	A'B'C	4	5	7	6
011	A'BC	12	13	15	14
010	A'BC'	8	9	11	10
110	ABC'	24	25	27	26
111	ABC	28	29	31	30
101	AB'C	20	21	23	22
100	AB'C'	16	17	19	18

		5	10	11	15	26	30
[5,7,13,15]	A'CE	1	0	0	1	0	0
[11,10]	A'BC'D	0	1	1	0	0	0
[15,11]	A'BDE	0	0	1	1	0	0
[26,30]	ABDE'	0	0	0	0	1	1
[31,30]	ABCD	0	0	0	0	0	1
[5]	A'B'CD'E	1	0	0	0	0	0

Therefore, the minimum SOP expression for this would be $A'CE + A'BC'D + ABDE'$

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5. Identify the essential PIs by using a PI table for ΠM (1, 2, 3, 4, 5, 12, 13, 15, 16, 17, 20, 28, 29)ΠD(6, 14, 18, 19, 22, 30).

		00	01	11	10
		D'E'	D'E	DE	DE'
000	A'B'C'	0	1	1	1
001	A'B'C	1	1	0	x
011	A'BC	1	1	1	x
010	A'BC'	0	0	0	0
110	ABC'	0	0	0	0
111	ABC	1	1	0	x
101	AB'C	1	0	0	x
100	AB'C'	1	1	x	x

		00	01	11	10
		D'E'	D'E	DE	DE'
000	A'B'C'	0	1	3	2
001	A'B'C	4	5	7	6
011	A'BC	12	13	15	14
010	A'BC'	8	9	11	10
110	ABC'	24	25	27	26
111	ABC	28	29	31	30
101	AB'C	20	21	23	22
100	AB'C'	16	17	19	18

PT with only EPIs (with only one 1 in at leas one column)

		1	2	3	4	5	12	13	15	16	17	20	28	29	num_terms_looped
[4,5,12,13]	A'CD'	0	0	0	1	1	1	1	0	0	0	0	0	0	4
[12,13,28,29]	BCD'	0	0	0	0	0	1	1	0	0	0	0	1	1	4
[12,13,15,14]	A'BC	0	0	0	0	0	1	1	1	0	0	0	0	0	3
[2,3,18,19]	B'C'D	0	1	1	0	0	0	0	0	0	0	0	0	0	2
[1,3,17,19]	B'C'E	1	0	1	0	0	0	0	0	0	1	0	0	0	3
[16,18,20,22]	AB'E'	0	0	0	0	0	0	0	0	1	0	1	0	0	2

Therefore the EPIs are $A'CD', BCD', A'BC, B'C'D, B'C'E, AB'E'$