Instructor's Supplement Problems

Chapter 03

1. Represent the electrical network shown in Figure I-3.1 in state space, where $i_R(t)$ is the output. [Section: 3.4]

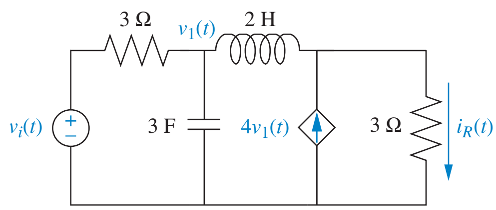

FIGURE I-3.1

2. Consider the mechanical system of Figure I-3.2. If the spring is nonlinear, and the force, F_s, required to stretch the spring is $F_s = 2x_1^2$, represent the system in state space linearized about $x_1 = 1$ if the output is x_2. [Section: 13.]

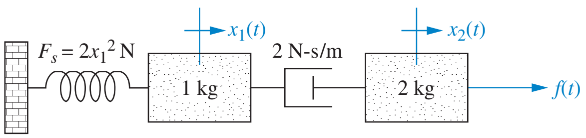

FIGURE I-3.2

3. In the past, Type-1 diabetes patients had to inject themselves with insulin three to four times a day. New delayed-action insulin analogues, such as insulin Glargine, require a single daily dose. A similar procedure to the one described in the Pharmaceutical Drug Absorption case study of this chapter is used to find a model for the concentration-time evolution of plasma for insulin Glargine. For a specific patient, state-space model matrices are given by (*Tarín, 2007*)

$$\mathbf{A} = \begin{bmatrix} -0.435 & 0.209 & 0.02 \\ 0.268 & -0.394 & 0 \\ 0.227 & 0 & -0.02 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{C} = [0.0003 \quad 0 \quad 0]; \quad \mathbf{D} = 0$$

where the state vector is given by

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The state variables are

x_1 = insulin amount in plasma compartment
x_2 = insulin amount in liver compartment
x_3 = insulin amount in interstitial (in body tissue) compartment

The system's input is u = external insulin flow. The system's output is y = plasma insulin concentration.

a. Find the system's transfer function.

b. `Verify your result using MATLAB.`

4. A linear, time-invariant model of the hypothalamic-pituitary-adrenal axis of the endocrine system with five state variables has been proposed as follows (*Kyrylov, 2005*):

$$\frac{dx_0}{dt} = a_{00}x_0 + a_{02}x_2 + d_0$$

$$\frac{dx_1}{dt} = a_{10}x_0 + a_{11}x_1 + a_{12}x_2$$

$$\frac{dx_2}{dt} = a_{20}x_0 + a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4$$

$$\frac{dx_3}{dt} = a_{32}x_2 + a_{33}x_3$$

$$\frac{dx_4}{dt} = a_{42}x_2 + a_{44}x_4$$

where each of the state variables represents circulatory concentrations as follows:

x_0 = corticotropin-releasing hormone
x_1 = corticotropin
x_2 = free cortisol
x_3 = albumin-bound cortisol
x_4 = corticosteroid-binding globulin
d_0 = an external generating factor

Express the system in the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$.

5. Experiments to identify precision grip dynamics between the index finger and thumb have been performed using a ball-drop experiment. A subject holds a device with a small receptacle into which an object is dropped, and the response is measured (*Fagergren*, *2000*). Assuming a step input, it has been found that the response of the motor subsystem together with the sensory system is of the form

$$G(s) = \frac{Y(s)}{R(s)} = \frac{s + c}{(s^2 + as + b)(s + d)}$$

Convert this transfer function to a state-space representation.

Bibliography

Fagergren, A., Ekeberg, Ö., and Forssberg, H. Precision Grip Force Dynamics: A System Identification Approach. *IEEE Transactions on Biomedical Engineering*, vol. 47, no. 10, 2000, pp. 1366–1375.

Kyrylov, V., Severyanova, L. A., and Vieira, A. Modeling Robust Oscillatory Behavior of the Hypothalamic-Pituitary-Adrenal Axis. *IEEE Transactions on Biomedical Engineering*, vol. 52, no. 12, 2005, pp. 1977–1983.

Tarin, C., Teufel, E., Picó, J., Bondia, J., and Pfleiderer, H. J. Comprehensive Pharmacokinetic Model of Insulin Glargine and Other Insulin Formulations. *IEEE Transactions on Biomedical Engineering*, vol. 52, no. 12, 2005, pp. 1994–2005.