Instructor's Supplement Problems

Chapter 06

1. `Use MATLAB to find the pole locations for the system of Problem 6 in the text problems.` MATLAB **ML**

2. `Use MATLAB and the Symbolic Math Toolbox to generate a Routh table to solve Problem 3 in the text problems.` Symbolic Math **SM**

3. Consider the unity-feedback system of Figure P6.3 in the text problems with

$$G(s) = \frac{1}{4s^2(s^2+2)}$$

a. Using the Routh–Hurwitz criterion, find the region of the s-plane where the poles of the closed-loop system are located.

b. `Use MATLAB to verify your answer and find the poles of T(s). Indicate whether this system is stable, unstable, or marginally stable. [Section: 6.3]` MATLAB **ML**

4. Given the unity-feedback system of Figure P6.3 in the text problems with

$$G(s) = \frac{84}{s(s^7+5s^6+12s^5+25s^4+45s^3+50s^2+82s+60)}$$

tell how many poles of the closed-loop transfer function lie in the right half-plane, in the left half-plane, and on the $j\omega$-axis. [Section: 6.3]

5. `Repeat Problem 11in the text problems using MATLAB.` MATLAB **ML**

6. For the system of Figure I-6.1, tell how many closed-loop poles are located in the right half-plane, in the left half-plane, and on the $j\omega$-axis. Notice that there is positive feedback. [Section: 6.3]

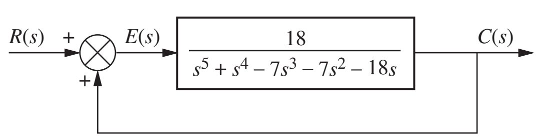

FIGURE I-6.1

7. Determine if the unity-feedback system of Figure P6.3 in the text problems with

$$G(s) = \frac{K(s^2+1)}{(s+1)(s+2)}$$

can be unstable. [Section: 6.4]

8. `Repeat Problem 16 in the text problems using MATLAB.` MATLAB **ML**

9. Repeat Problem 25 in the text problems for [Section: 6.4]

$$G(s) = \frac{K(s-1)(s-2)}{(s+2)(s^2+2s+2)}$$

10. `Repeat Problem 27 in the text problems using MATLAB.` MATLAB **ML**

11. For the unity-feedback system of Figure P6.3 in the text problems with

$$G(s) = \frac{K(s+2)}{(s^2+1)(s+4)(s-1)}$$

find the range of K for which there will be only two closed-loop, right-half-plane poles. [Section: 6.4]

12. Find the value of K in the system of Figure I-6.2 that will place the closed-loop poles as shown. [Section: 6.4]

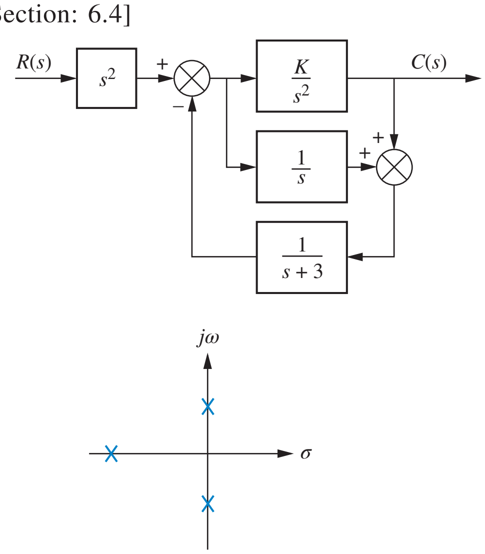

FIGURE I-6.2 Closed-loop system with pole plot

13. The closed-loop transfer function of a system is

$$T(s) = \frac{s^2 + K_1 s + K_2}{s^4 + K_1 s^3 + K_2 s^2 + 5s + 1}$$

Determine the range of K_1 in order for the system to be stable. What is the relationship between K_1 and K_2 for stability? [Section: 6.4]

14. For the transfer function below, find the constraints on K_1 and K_2 such that the function will have only two $j\omega$ poles. [Section: 6.4]

$$T(s) = \frac{K_1 s + K_2}{s^4 + K_1 s^3 + s^2 + K_2 s + 1}$$

15. An interval polynomial is of the form

$$P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \cdots$$

with its coefficients belonging to intervals $x_i \le a_i \le y_i$, where x_i, y_i are prescribed constants. Kharitonov's theorem says that an interval polynomial has all its roots in the left half-plane if each one of the following four polynomials has its roots in the left half-plane (*Minichelli, 1989*):

$$K_1(s) = x_0 + x_1 s + y_2 s^2 + y_3 s^3 + x_4 s^4 + x_5 s^5 + y_6 s^6 + \cdots$$
$$K_2(s) = x_0 + y_1 s + y_2 s^2 + x_3 s^3 + x_4 s^4 + y_5 s^5 + y_6 s^6 + \cdots$$
$$K_3(s) = y_0 + x_1 s + x_2 s^2 + y_3 s^3 + y_4 s^4 + x_5 s^5 + x_6 s^6 + \cdots$$
$$K_4(s) = y_0 + y_1 s + x_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + x_6 s^6 + \cdots$$

Use Kharitonov's theorem and the Routh–Hurwitz criterion to find if the following polynomial has any zeros in the right half-plane.

$$P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3$$
$$2 \le a_0 \le 4; \quad 1 \le a_1 \le 2; \quad 4 \le a_2 \le 6; \quad a_3 = 1$$

16. The read/write head assembly arm of a computer hard disk drive (HDD) can be modeled as a rigid rotating body with inertia I_b. Its dynamics can be described with the transfer function

$$P(s) = \frac{X(s)}{F(s)} = \frac{1}{I_b s^2}$$

where $X(s)$ is the displacement of the read/write head and $F(s)$ is the applied force (*Yan, 2003*). Show that if the HDD is controlled in the configuration shown in Figure P6.9 in the text problems, the arm will oscillate and cannot be positioned with any precision over a HDD track. Find the oscillation frequency.

17. A system is represented in state space as

State Space
SS

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 2 & -4 \\ 1 & -4 & 3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \mathbf{x}$$

Determine how many eigenvalues are in the right half-plane, in the left half-plane, and on the $j\omega$-axis. [Section: 6.5]

18. A Butterworth polynomial is of the form

$$B_n(s) = 1 + (-1)^n \left(\frac{s}{\omega_c}\right)^{2n}, \; n > 0$$

Use the Routh–Hurwitz criteria to find the zeros of a Butterworth polynomial for:

a. $n = 1$;

b. $n = 2$

Bibliography

Minnichelli, R. J., Anagnost, J. J., and Desoer, C. A. An Elementary Proof of Kharitonov's Stability Theorem with Extensions. *IEEE Transactions on Automatic Control*, vol. 34, 1989, pp. 995–998.

Yan, T., and Lin, R. Experimental Modeling and Compensation of Pivot Nonlinearly in Hard Disk Drives. *IEEE Transactions on Magnetics*, vol. 39, 2003, pp. 1064–1069.