Instructor's Supplement Problems

Chapter 07

1. What is the steady-state error for a step input of 15 units applied to the unity-feedback system of Figure P7.1 in the text problems, where [Section: 7.3]

$$G(s) = \frac{1020(s+13)(s+26)(s+33)}{(s+65)(s+75)(s+91)}$$

2. A Type 3 unity-feedback system has $r(t) = 10t^3$ applied to its input. Find the steady-state position error for this input if the forward transfer function is [Section: 7.3]

$$G(s) = \frac{1030(s^2+8s+23)(s^2+21s+18)}{s^3(s+6)(s+13)}$$

3. For the system shown in Figure I-7.1, [Section: 7.4]

a. What value of K will yield a steady-state error in position of 0.01 for an input of $(1/10)t$?

b. What is the K_v for the value of K found in Part **a**?

c. What is the minimum possible steady-state position error for the input given in Part **a**?

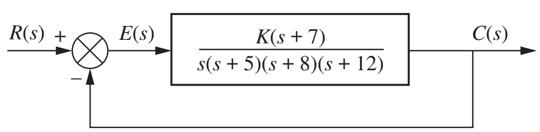

FIGURE I-7.1

4. Find the value of K for the unity-feedback system shown in Figure P7.1 in the text problems, where

$$G(s) = \frac{K(s+3)}{s^2(s+7)}$$

if the input is $10t^2u(t)$, and the desired steady-state error is 0.061 for this input. [Section: 7.4]

5. For the unity-feedback system of Figure P7.1 in the text problems, where

$$G(s) = \frac{K(s+13)(s+19)}{s(s+6)(s+9)(s+22)}$$

find the value of K to yield a steady-state error of 0.4 for a ramp input of $27tu(t)$. [Section: 7.4]

6. For the unity-feedback system of Figure P7.1 in the text problems, where

$$G(s) = \frac{K}{s(s+4)(s+8)(s+10)}$$

find the minimum possible steady-state position error if a unit ramp is applied. What places the constraint upon the error?

7. Given the unity-feedback control system of Figure P7.1 in the text problems, where

$$G(s) = \frac{K}{s^n(s+a)}$$

find the values of n, K, and a in order to meet specifications of 12% overshoot and $Kv = 110$. [Section: 7.4]

8. Repeat Problem 26 in the text problems for the system shown in Figure I-7.2. [Section: 7.3]

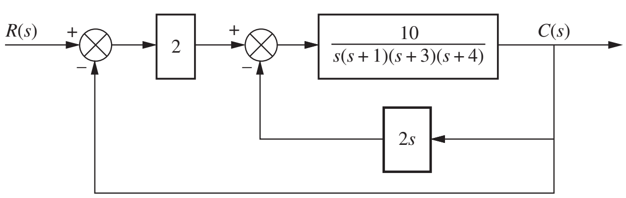

FIGURE I-7.2

MATLAB
ML

9. For the system shown in Figure I-7.3, use MATLAB to find the following: [Section: 7.3]

a. The system type

b. K_p, K_v, and K_a

c. The steady-state error for inputs of $100u(t)$, $100tu(t)$, and $100t^2u(t)$

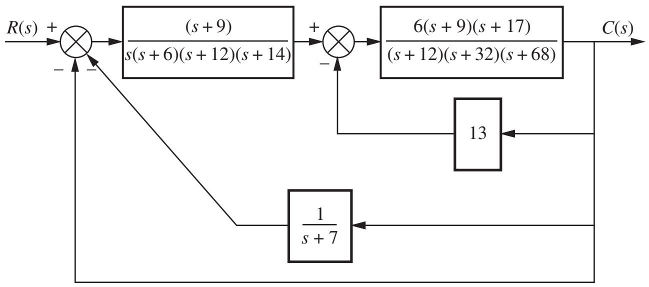

FIGURE I-7.3

10. The transfer function from elevator deflection to altitude change in a Tower Trainer 60 Unmanned Aerial Vehicle is

$$P(s) = \frac{h(s)}{\delta(s)_e} = \frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s}$$

An autopilot is built around the aircraft as shown in Figure I-7.4, with $F(s) = H(s) = 1$ and

$$G(s) = \frac{0.00842(s + 7.895)(s^2 + 0.108s + 0.3393)}{(s + 0.07895)(s^2 + 4s + 8)}$$

(*Barkana, 2005*). The steady-state error for a ramp input in this system is $e_{ss} = 25$. Find the slope of the ramp input.

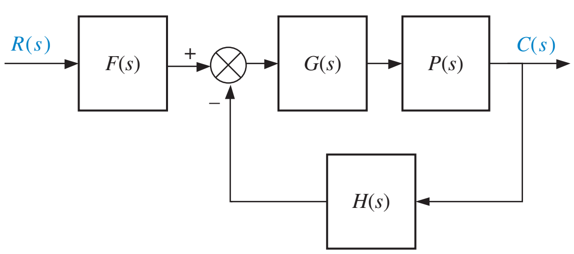

FIGURE I-7.4

11. Find the total steady-state error due to a unit step input and a unit step disturbance in the system of Figure I-7.5. [Section: 7.5]

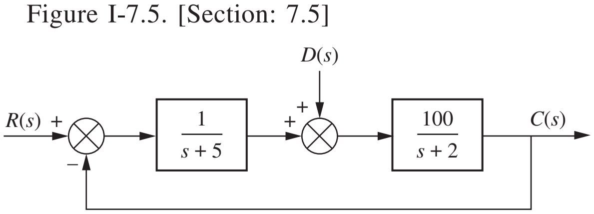

FIGURE I-7.5

12. For the system shown in Figure I-7.6, [Section: 7.6]

a. What is the system type?

b. What is the appropriate static error constant?

c. What is the value of the appropriate static error constant?

d. What is the steady-state error for a unit step input?

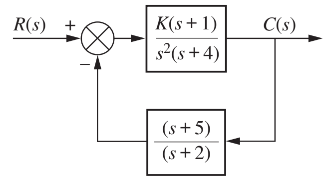

FIGURE I-7.6

13. a. Show that the sensitivity to plant changes in the system of Figure I-7.4 is

$$S_{T:P} = \frac{P}{T}\frac{\delta T}{\delta P} = \frac{1}{1 + L(s)}$$

where $L(s) = G(s)P(s)H(s)$ and

$$T(s) = \frac{C(s)}{R(s)} = \frac{F(s)}{H(s)} \cdot \frac{L(s)}{1 + L(s)}.$$

b. Show that $S_{T:P}(s) + \dfrac{T(s)H(s)}{F(s)} = 1$ for all values of s.

14. In Figure I-7.4, $P(s) = \dfrac{5}{s}$, $H(s) = 1$,

$$T(s) = \frac{C(s)}{R(s)} = \frac{200K}{(s + 1)(s + 3)(s^2 + 2s + 20)}$$

and

$$S_{T:P} = \frac{P}{T}\frac{\delta T}{\delta P} = \frac{s^2 + 2s}{s^2 + 2s + 20}$$

a. Find $F(s)$ and $G(s)$.

b. Find the value of K that will result in zero steady-state error for a unit step input.

15. For each of the following closed-loop systems, find the steady-state error for unit step and unit ramp inputs. Use both the final value theorem and input substitution methods. [Section: 7.8]

a. $\dot{\mathbf{x}} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} r;\ y = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}\mathbf{x}$

b. $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r;\ y = [1 \quad 0 \quad 0]\mathbf{x}$

c. $\dot{\mathbf{x}} = \begin{bmatrix} -9 & -5 & -1 \\ 1 & 0 & -2 \\ -3 & -2 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} r;\ y = [1 \quad -2 \quad 4]\mathbf{x}$

16. Glycolysis is a feedback process through which living cells use glucose to generate adenosine triphosphate (ATP), necessary for cell operations. A linearized glycolysis model (*Chandra, 2011*) is given by

$$\begin{bmatrix} \dot{\Delta x} \\ \dot{\Delta y} \end{bmatrix} = \begin{bmatrix} -k & a+g+h \\ (q+1)k & -qa-g(q+1)+qh \end{bmatrix}\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\delta$$

where δ is the perturbation (disturbance input) on ATP production, Δy is the change in ATP level (output). $\alpha > 0$ is the cooperativity of ATP binding to PFK, $g > 0$ is the feedback strength of ATP on PK (PFK and PK are two different types of glycolytic enzymes), $k > 0$ is the intermediate reaction rate, $q > 0$ is the autocatalytic stoichiometry, and $h > 0$ is the feedback strength of ATP on the PFK enzyme.

a. Since in this system δ is a disturbance input, zero steady-state error is achieved when $\frac{\Delta y}{\delta} = 0$. Show that in steady state ($\dot{\Delta x} = 0$, $\dot{\Delta y} = 0$), $\frac{\Delta y}{\delta} = \frac{1}{a-h}$.

b. Use the Routh–Hurwitz stability criterion to show that the system will be closed-loop stable as long as

$$0 < h - a < \frac{k + g(q+1)}{q}$$

c. Assuming that h is the only parameter of choice for steady-state error adjustments, show that zero steady-state error is not achievable.

17. Packet information flow in a router working under TCP/IP can be modeled using the linearized transfer function

$$P(s) = \frac{Q(s)}{f(s)} = \frac{\frac{C^2}{2N}e^{-sR}}{\left(s + \frac{2N}{R^2C}\right)\left(s + \frac{1}{R}\right)}$$

where

C = link capacity (packets/second)

N = load factor (number of TCP sessions)

Q = expected queue length

R = round trip time (second)

p = probability of a packet drop

The objective of an active queue management (AQM) algorithm is to automatically choose a packet-drop probability, p, so that the queue length is maintained at a desired level. This system can be represented by the block diagram of Figure I-7.4 with the plant model in the $P(s)$ block, the AQM algorithm in the $G(s)$ block, and $F(s) = H(s) = 1$. Several AQM algorithms are available, but one that has received special attention in the literature is the random early detection (RED) algorithm. This algorithm can be approximated with $G(s) = \frac{LK}{s+K}$, where L and K are constants (*Hollot, 2001*). Find the value of L required to obtain a 10% steady-state error for a unit step input when $C = 3750$ packets/s, $N = 50$ TCP sessions, $R = 0.1$ s, and $K = 0.005$.

Bibliography

Barkana, I. Classical and Simple Adaptive Control of Nonminimum Phase Autopilot Design. *Journal of Guidance, Control, and Dynamics*, vol. 28, 2005, pp. 631–638.

Chandra, F. A., Buzi, G., and Doyle, J. C. Glycolytic Oscillations and Limits on Robust Efficiency. *Science,* vol. 333, American Association for the Advancement of Science, July 8, 2011, pp. 187–192.

Hollot, C. V., Misra, V., Towsley, D., and Gong, W. A Control Theoretic Analysis of RED. *Proceedings of IEEE INFOCOM*, 2001, pp. 1510–1519.