# Instructor's Supplement Problems

## Chapter 09

**1.** Repeat Problem 3 in the text problems for $G(s)=\dfrac{K}{s(s+3)(s+7)}$. [Section: 9.2]

**2.** Redo Problem 5 in the text problems using MATLAB in the following way:

MATLAB ML

**a.** MATLAB will generate the root locus for the uncompensated system along with the 0.707 damping ratio line. You will interactively select the operating point. MATLAB will then inform you of the coordinates of the operating point, the gain at the operating point, as well as the estimated $\%OS$, $T_s$, $T_p$, $\zeta$, $\omega_n$, and $K_p$ represented by a second-order approximation at the operating point.

**b.** MATLAB will display the step response of the uncompensated system.

**c.** Without further input, MATLAB will calculate the compensated design point and will then ask you to input a value for the PD compensator zero from the keyboard. MATLAB will respond with a plot of the root locus showing the compensated design point. MATLAB will then allow you to keep changing the PD compensator value from the keyboard until a root locus is plotted that goes through the design point.

**d.** For the compensated system, MATLAB will inform you of the coordinates of the operating point, the gain at the operating point, as well as the estimated $\%OS$, $T_s$, $T_p$, $\zeta$, $\omega_n$, and $K_p$ represented by a second-order approximation at the operating point.

**e.** MATLAB will then display the step response of the compensated system.

**3.** Design a PD controller for the system shown in Figure I-9.1 to reduce the settling time by a factor of 4 while continuing to operate the system with 20.5% overshoot. Compare the performance of the compensated system to that of the uncompensated system. Summarize the results in a table similar to that in Example 9.7.

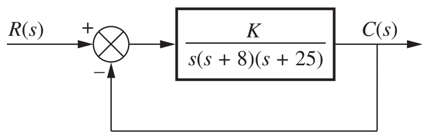


**FIGURE I-9.1**

**4.** For the unity-feedback system of Figure P9.1 in the text problems with

$$G(s)=\frac{K}{s(s+1)(s^2+10s+26)}$$

do the following: [Section: 9.3]

**a.** Find the settling time for the system if it is operating with 15% overshoot.

**b.** Find the zero of a compensator and the gain, $K$, so that the settling time is 7 seconds. Assume that the pole of the compensator is located at $-15$.

**c.** Use MATLAB or any other computer program to simulate the system's step response to test the compensator.

MATLAB ML

**5.** Redo Problem 15 in the text problems using a lag-lead compensator and MATLAB in the following way:

MATLAB ML

**a.** MATLAB will generate the root locus for the uncompensated system along with the 0.5 damping-ratio line. You will interactively select the operating point. MATLAB will then proceed to inform you of the coordinates of the operating point, the gain at the operating point, as well as the estimated $\%OS$, $T_s$, $T_p$, $\zeta$, $\omega_n$, and $K_p$ represented by a second-order approximation at the operating point.

*(problem continues)*

(*continued*)

**b.** MATLAB will display the step response of the uncompensated system.

**c.** Without further input, MATLAB will calculate the compensated design point and will then ask you to input a value for the lead compensator pole from the keyboard. MATLAB will respond with a plot of the root locus showing the compensated design point. MATLAB will then allow you to keep changing the lead compensator pole value from the keyboard until a root locus is plotted that goes through the design point.

**d.** For the compensated system, MATLAB will inform you of the coordinates of the operating point, the gain at the operating point, as well as the estimated %OS, $T_s$, $T_p$, $\zeta$, $\omega_n$, and $K_p$ represented by a second-order approximation at the operating point.

**e.** MATLAB will then display the step response of the compensated system.

**f.** Change the compensator's zero location a few times and collect data on the compensated system to see if any other choices of the compensator zero yield advantages over the original design.

**g.** Using the steady-state error of the uncompensated system, add a lag compensator to yield an improvement of 30 times over the uncompensated system's steady-state error, with minimal effect on the designed transient response. Have MATLAB plot the step response. Try several values for the lag compensator's pole and see the effect on the step response.

**6.** The unity-feedback system shown in Figure P9.1, with

$$G(s) = \frac{K}{(s^2 + 4s + 8)(s + 10)}$$

is to be designed to meet the following specifications:

Overshoot: Less than 22%

Settling time: Less than 1.6 seconds

$K_p = 15$

Do the following: [Section: 9.4]

**a.** Evaluate the performance of the uncompensated system operating at approximately 10% overshoot.

**b.** Design a passive compensator to meet the desired specifications.

**c.** Use MATLAB to simulate the compensated system. Compare the response with the desired specifications.

MATLAB
ML

**7.** Repeat Problem 19 in the text problems for a peak time of 1.047 seconds, a damping ratio of 0.5, zero steady-state error for a step input, and with

$$G(s) = \frac{K}{(s+1)^2}$$

**8.** The room temperature of an 11 $m^2$ room is to be controlled by varying the power of an indoor radiator. For this specific room, the open-loop transfer function from radiator power, $\dot{Q}(s)$, to temperature, $T(s)$, is (*Thomas, 2005*)

$$G(s) = \frac{T(s)}{\dot{Q}(s)} = \frac{(1 \times 10^{-6})s^2 + (1.314 \times 10^{-9})s + (2.66 \times 10^{-13})}{s^3 + 0.00163s^2 + (5.272 \times 10^{-7})s + (3.538 \times 10^{-11})}$$

The system is assumed to be in the closed-loop configuration shown in Figure P9.1.

**a.** For a unit-step input, calculate the steady-state error of the system.

**b.** Try using the procedure of Section 9.2 to design a PI controller to obtain zero steady-state error for step inputs without appreciably changing the transient response. Then explain why it is not possible to do so.

**c.** Design a PI controller of the form $G_c(s) = \dfrac{K(s+z)}{s}$ that will reduce the step-response error to zero while not changing significantly the transient response. (Hint: Place the zero of the compensator in a position where the closed-loop poles of the uncompensated root locus will not be affected significantly.)

**d.** Use Simulink to simulate the systems of Parts **b** and **c** and to verify the correctness of your design in Part **c**.

Simulink
SL

**9.** Figure I-9.2 shows a two-tank system. The liquid inflow to the upper tank can be controlled using a valve and is represented by $F_0$. The upper tank's outflow equals the lower tank's inflow and is represented by $F_1$. The outflow of the lower tank is $F_2$. The objective of the design is to control the liquid level, $y(t)$, in the lower tank. The open-loop transmission for this system is $\dfrac{Y(s)}{F_o(s)} = \dfrac{a_2 a_3}{s^2 + (a_1 + a_4)s + a_1 a_4}$ (*Romagnoli, 2006*). The system will be controlled in a loop analogous to that of Figure P9.1, where the lower liquid level will be measured and compared to a set point. The resulting error will be fed to a controller, which in turn will open or close the valve feeding the upper tank.

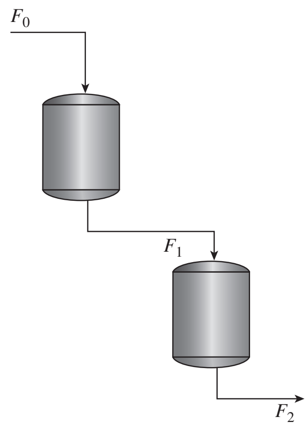


**FIGURE I-9.2**[1]

**a.** Assuming $a_1 = 0.04$, $a_2 = 0.0187$, $a_3 = 1$, and $a_4 = 0.227$, design a lag compensator to obtain a step-response steady-state error of 10% without affecting the system's transient response appreciably.

**b.** `Verify your design through MATLAB simulations.` MATLAB **ML**

**10.** You are given the motor whose transfer function is shown in Figure I-9.3(*a*).

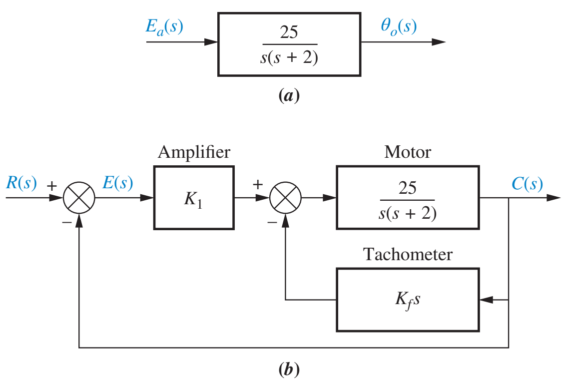


**FIGURE I-9.3**

**a.** If this motor were the forward transfer function of a unity-feedback system, calculate the percent overshoot and settling time that could be expected.

**b.** You want to improve the closed-loop response. Since the motor constants cannot be changed and you cannot use a different motor, an amplifier and tachometer are inserted into the loop as shown in Figure I-9.3(*b*). Find the values of $K_1$ and $K_f$ to yield a percent overshoot of 15% and a settling time of 0.5 second.

**c.** Evaluate the steady-state error specifications for both the uncompensated and the compensated systems.

**11.** Given the system in Figure I-9.4, find the values of $K$ and $K_f$ so that the closed-loop system will have a 4.32% overshoot and the minor loop will have a damping ratio of 0.8. Compare the expected performance of the system without tachometer compensation to the expected performance with tachometer compensation.

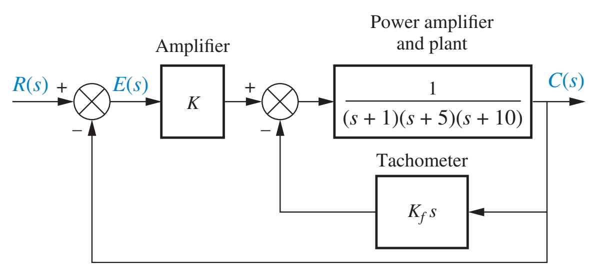


**FIGURE I-9.4**

**12.** A coordinate measuring machine (CMM) measures coordinates on three-dimensional objects. The accuracy of CMMs is affected by temperature changes as well as by mechanical resonances due to joint elasticity. These resonances are more pronounced when the machine has to go over abrupt changes of dimension, such as sharp corners at high speed. Each of the machine links

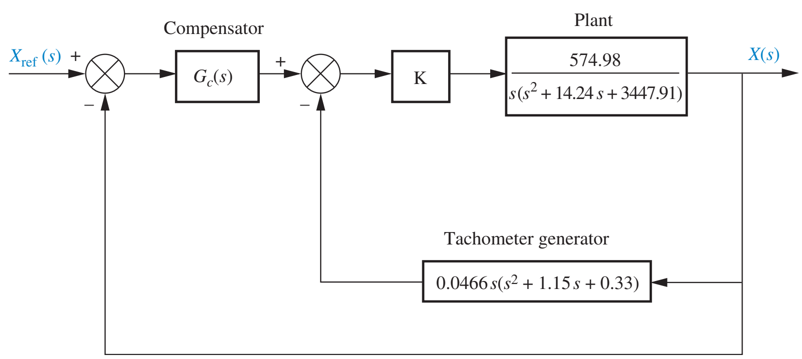


**FIGURE I-9.5**

[1] Romagnoli, J.A., and Palazoglu, A. *Introduction to Process Control*, CRC Press, Boca Raton, 2006. P. 44, Figure 3.4.

can be controlled in a closed-loop configuration, such as the one shown in Figure I-9.5, designed for a specific machine with prismatic (sliding) links. In the figure, $X_{ref}(s)$ is the commanded position and $X(s)$ is the actual position. The minor loop uses a tachometer generator to obtain the joint speed, while the main loop controls the joint's position (*Özel, 2003*).

**a.** Find the value of $K$ that will result in a minor loop with $\zeta = 0.5$.

**b.** Use a notch filter compensator, $G_c(s)$, for the external loop so that it results in a closed-loop damping factor of $\zeta = 0.7$ with $T_s \approx 4$ seconds.

**c.** Use MATLAB to simulate the compensated system's closed-loop step response. MATLAB **ML**

**13.** An X-4 quadrotor flyer is designed as a small-sized unmanned autonomous vehicle (UAV) that flies mainly indoors and can help in search and recognizance missions. To minimize mechanical problems and for simplicity, this aircraft uses fixed pitch rotors with specially designed blades. Therefore, for thrust it is necessary to add a fifth propeller. A simplified design of the thrust control design can be modeled as in Figure P9.1 with $G(s) = G_c(s)P(s)$ where

$$P(s) = \frac{1.90978\left(\frac{s}{0.43}+1\right)}{\left(\frac{s}{9.6}+1\right)\left(\frac{s}{0.54}+1\right)}$$

represents the dynamics of the thruster rotor gain, the motor, and the battery dynamics. Initially, the system is designed using a proportional compensator given by $G_c(s) = 3$ (*Pounds, 2009*).

**a.** Calculate the resulting steady-state error for a unit-step input.

**b.** Design a lag compensator to yield half the steady-state error of the proportional compensator, without appreciably affecting the system's transient response.

**c.** Use MATLAB to simulate the original design and the lag compensated design. Verify your results. MATLAB **ML**

**14.** A *metering pump* is a pump capable of delivering a precise flow rate of fluid. Most metering pumps consist of an electric motor that varies the strike length of a shaft, allowing more or less fluid to pass through its body. The control of such a valve has been considered and the open-loop transfer function of the pump was found to be (*Yu, 2011*)

$$\frac{U(s)}{Y(s)} = \frac{1.869}{s^2 + 12.32s + 0.4582}$$

where the output of the system, $Y(s)$, represents the liquid flow, and the input, $U(s)$, is the command signal to the motor that varies the pump's plunger strike length.

**a.** Design a PID controller to reduce the steady-state error to zero with a maximum 5% overshoot and a 20% reduction in uncompensated settling time.

**b.** Find the characteristics of the uncompensated system.

**c.** Design a PID controller to reduce the steady-state error to zero; achieve a maximum 5% overshoot and a 20% reduction in uncompensated settling time.

**d.** Simulate the step response for the compensated system to verify the validity of your design.

# Bibliography


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