Chapter 1 Problems

SS Student solution available in interactive e-text.

1. A rotating *potentiometer* is a simple sensor that can be used to measure angular displacements; see Figure P1.1. The resistance between A and C is fixed, but the resistance from A to B and from B to C changes with the angular position of the wiper arm. If it takes 15 turns to move the wiper arm from A to C, find an equivalent block diagram for the system showing the input $\theta(t)$, the voltage output $v_o(t)$, and inside the block, the constant gain by which the input is multiplied to obtain the output. A PowerPoint animation is available for instructors at www.wiley.com/go/Nise/ControlSystemsEngineering8e. See *Potentiometer* [Section 1.4: Introduction to a Case Study].

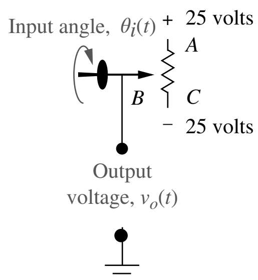

FIGURE P1.1 Potentiometer

2. A temperature control system operates by sensing the difference between the thermostat setting and the actual temperature and then opening a fuel valve an amount proportional to this difference. Draw a functional closed-loop block diagram similar to Figure 1.8(*d*) identifying the input and output transducers, the controller, and the plant. Further, identify the input and output signals of all subsystems previously described [Section 1.4: Introduction to a Case Study].

3. We can build a control system that will automatically adjust a motorcycle's radio volume as the noise generated by the motorcycle changes. The noise generated by the motorcycle increases with speed. As the noise increases, the system increases the volume of the radio. Assume that the amount of noise can be represented by a voltage generated by the speedometer cable, and the volume of the radio is controlled by a dc voltage (*Hogan, 1988*). If the dc voltage represents the desired volume disturbed by the motorcycle noise, draw the functional block diagram of the automatic volume control system, showing the input transducer, the volume control circuit, and the speed transducer as blocks. Also, show the following signals: the desired volume as an input, the actual volume as an output, and voltages representing speed, desired volume, and actual volume. An animation PowerPoint presentation (PPT) demonstrating this system is available for instructors at www.wiley.com/go/Nise/ControlSystemsEngineering8e. See *Motorcycle* [Section 1.4: Introduction to a Case Study].

4. A dynamometer is a device used to measure torque and speed and to vary the load on rotating devices. The dynamometer operates as follows to control the amount of torque: A hydraulic actuator attached to the axle presses a tire against a rotating flywheel. The greater the displacement of the actuator, the greater the force that is applied to the rotating flywheel. A strain gage load cell senses the force. The displacement of the actuator is controlled by an electrically operated valve whose displacement regulates fluid flowing into the actuator (*D'Souza, 1988*). Draw a functional block diagram of a closed-loop system that uses the described dynamometer to regulate the force against the tire during testing. Show all signals and systems. Include amplifiers that power the valve, the valve, the actuator and load, and the tire [Section 1.4: Introduction to a Case Study]. **SS**

5. The vertical position, $x(t)$, of a grinding wheel is controlled by a closed-loop system. The input to the system is the desired depth of grind, and the output is the actual depth of grind. The difference between the desired depth and the actual depth drives the motor, resulting in a force applied to the work. This force results in a feed velocity for the grinding wheel (*Jenkins, 1997*). Draw a closed-loop functional block diagram for the grinding process, showing the input, output, force, and grinder feed rate [Section 1.4: Introduction to a Case Study]. **SS**

6. The human eye has a biological control system that varies the pupil diameter to maintain constant light intensity to the retina. As the light intensity increases, the optical nerve sends a signal to the brain, which commands internal eye muscles to decrease the pupil's eye diameter. When the light intensity decreases, the pupil diameter increases.
   a. Draw a functional block diagram of the light-pupil system indicating the input, output, and intermediate signals; the sensor; the controller; and the actuator [Section 1.4: Introduction to a Case Study].
   b. Under normal conditions the incident light will be larger than the pupil. If the incident light is smaller than the diameter of the pupil, the feedback path is broken (*Bechhoefer, 2005*). Modify your block diagram from Part **a.** to show where the loop is broken. What will happen if the narrow beam of light varies in intensity, such as in a sinusoidal fashion?
   c. It has been found (*Bechhoefer, 2005*) that it takes the pupil about 300 milliseconds to react to a change in the incident light. If light shines off center to the retina,

describe the response of the pupil with delay present and then without delay present.

7. A Segway®[1] Personal Transporter (PT) (Figure P1.2) is a two-wheeled vehicle in which the human operator stands vertically on a platform. As the driver leans left, right, forward, or backward, a set of sensitive gyroscopic sensors sense the desired input. These signals are fed to a computer that amplifies them and commands motors to propel the vehicle in the desired direction. One very important feature of the PT is its safety: The system will maintain its vertical position within a specified angle despite road disturbances, such as uphills and downhills or even if the operator over-leans in any direction. Draw a functional block diagram of the PT system that keeps the system in a vertical position. Indicate the input and output signals, intermediate signals, and main subsystems (http://segway.com).

FIGURE P1.2 The Segway Personal Transporter (PT)

8. In humans, hormone levels, alertness, and core body temperature are synchronized through a 24-hour circadian cycle. Daytime alertness is at its best when sleep/wake cycles are in sync with the circadian cycle. Thus alertness can be easily affected with a distributed work schedule, such as the one to which astronauts are subjected. It has been shown that the human circadian cycle can be delayed or advanced through light stimulus. To ensure optimal alertness, a system is designed to track astronauts' circadian cycles and increase the quality of sleep during missions. Core body temperature can be used as an indicator of the circadian cycle. A computer model with optimum circadian body temperature variations can be compared to an astronaut's body temperatures. Whenever a difference is detected, the astronaut is subjected to a light stimulus to advance or delay the astronaut's circadian cycle (*Mott*, *2003*). Draw a functional block diagram of the system. Indicate the input and output signals, intermediate signals, and main subsystems.

9. Tactile feedback is an important component in the learning of motor skills such as dancing, sports, and physical rehabilitation. A suit with white dots recognized by a vision system to determine arm joint positions with millimetric precision was developed. This suit is worn by both teacher and student to provide position information. (*Lieberman*, *2007*). If there is a difference between the teacher's positions and that of the student, vibrational feedback is provided to the student through eight strategically placed vibrotactile actuators in the wrist and arm. This placement takes advantage of a sensory effect known as *cutaneous rabbit* that tricks the subject to feel uniformly spaced stimuli in places where the actuators are not present. These stimuli help the student adjust to correct the motion. In summary, the system consists of an instructor and a student having their movements followed by the vision system. Their movements are fed into a computer that finds the differences between their joint positions and provides proportional vibrational strength feedback to the student. Draw a block diagram describing the system design.

10. Moored floating platforms are subject to external disturbances such as waves, wind, and currents that cause them to drift. There are certain applications, such as diving support, drilling pipe-laying, and tanking between ships in which precise positioning of moored platforms is very important (*Muñoz-Mansilla*, *2011*). Figure P1.3 illustrates a tethered platform in which side thrusters are used for positioning. A control system is to be designed in which the objective is to

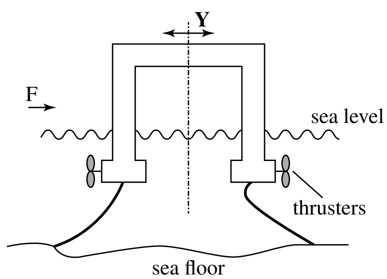

FIGURE P1.3 Tethered platform using side thrusters for positioning[2]

[1] Segway is a registered trademark of Segway, Inc. in the United States and/or other countries.

[2] Muñoz-Mansilla, R., Aranda, J., Diaz, J. M., Chaos, D., and Reinoso, A. J., Applications of QFT Robust Control Techniques to Marine Systems. *9th IEEE International Conference on Control and Automation*. December 19–21, 2011, pp. 378–385. (Figure 3, p. 382).

minimize the drift, Y, and an angular deviation from the vertical axes, ϕ (not shown). The disturbances acting on the system's outputs are the force, F, and the torque, M, caused by the external environment. In this problem, the plant will have one input, the force delivered by the thrusters (F_u) and two outputs, Y and ϕ. Note also that this is a disturbance attenuation problem, so there is no command input. Draw a block diagram of the system indicating the disturbances F and M, the control signal F_u, and the outputs Y and ϕ. Your diagram should also have blocks for a controller, the one-input two-output plant, and a block indicating how the disturbances affect each of the outputs.

11. Figure P1.4 shows the topology of a photo-voltaic (PV) system that uses solar cells to supply electrical power to a residence with hybrid electric vehicle loads (*Gurkavnak, 2009*). The system consists of a PV array to collect the sun's rays, a battery pack to store energy during the day, a dc/ac inverter to supply ac power to the load, and a bidirectional dc/dc converter to control the terminal voltage of the solar array according to a maximum power point tracking (MPPT) algorithm. In case of sufficient solar power (solar insolation), the dc/dc converter charges the battery and the solar array supplies power to the load through the dc/ac inverter. With less or no solar energy (solar non-insolation), power is supplied from the battery to the load through the dc/dc converter and the dc/ac inverter. Thus, the dc/dc converter must be bidirectional to be able to charge and discharge the battery. With the MPPT controller providing the reference voltage, the converter operates as a step-up converter (boost) to discharge the battery if the battery is full or a step-down (buck) converter, which charges the battery if it is not full.[3]

In Figure P1.4, the Inverter is controlled by the Power Manager and Controller through the Current Controller. The Power Manager and Controller directs the Inverter to take power either from the battery, via the Bidirectional Converter, or the solar array, depending upon the time of day and the battery state of charge (SOC). Draw the following two functional block diagrams for this system:

a. A diagram that illustrates the conversion of solar irradiation into energy stored in the battery. In that diagram, the input is the solar irradiance, $r(t)$, and the output is the battery voltage, $v_b(t)$.

b. The main diagram, in which the input is the desired output voltage, $v_r(t)$, and the output is the actual inverter output voltage, $v_o(t)$.

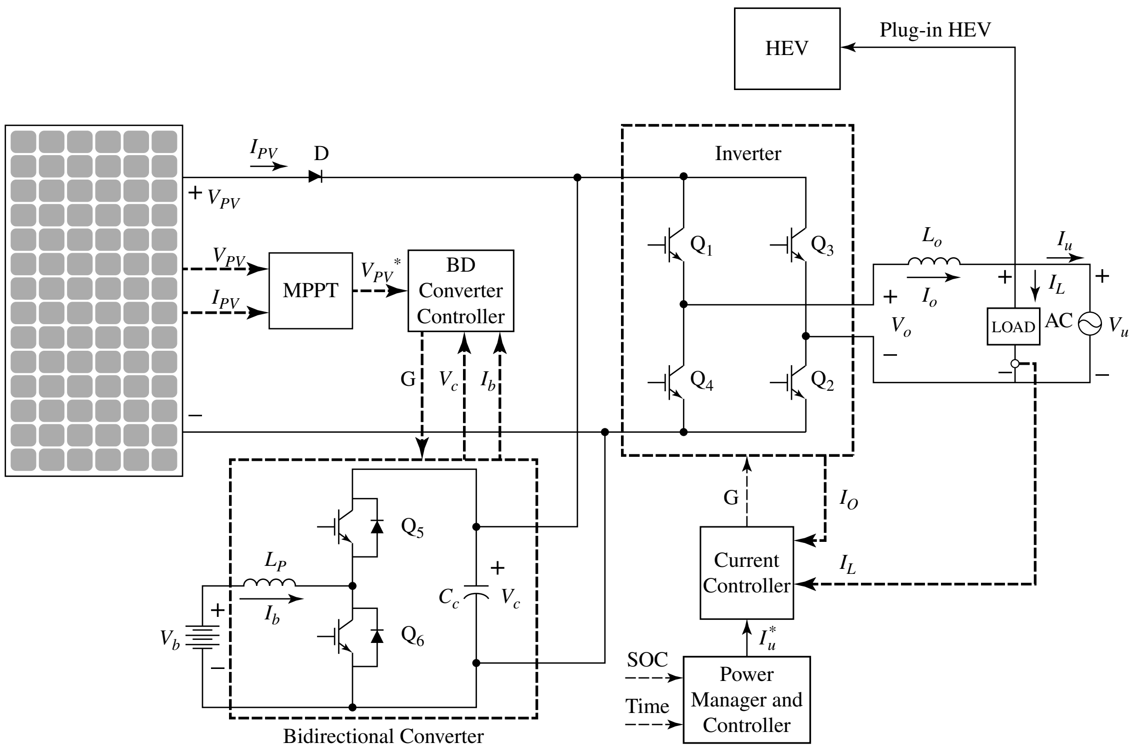

FIGURE P1.4 Proposed solar powered residential home with plug-in hybrid electric vehicle (PHEV) loads[4]

[3] For a description of all other operational scenarios, refer to the above-listed reference.

[4] Gurkaynak, Y., Li, Z., and Khaligh, A. A Novel Grid-tied, Solar Powered Residential Home with Plug-in Hybrid Electric Vehicle (PHEV) Loads. *IEEE Vehicle Power and Propulsion Conference 2009*, pp. 813–816. (Figure 1, p. 814).

Both of these functional block diagrams should show their major components, including the PV array, MPPT controller, dc/dc converter, battery pack, dc/ac inverter, current controller, and the Power Manager and Controller. Show all signals, including intermediate voltages and currents, time of day, and the SOC of the battery.

12. Oil drilling rigs are used for drilling holes for identification of oil or natural gas sources and for extraction. An oil drilling system can be thought of as a drill inside a straw, which is placed inside a glass. The straw assembly represents the drill string, the drill surrounded by fluid, and the glass represents the volume, the annulus, around the drill string through which slurry and eventually oil will flow as the drilling progresses.

Assume that we want to control the drill pressure output, $P_d(t)$, with a reference voltage input, $V_d(t)$. A control loop model (*Zhao*, *2007*) consists of a drill-pressure controller, drill motor subsystem, pulley subsystem, and drill stick subsystem. The output signal of the latter, the drill pressure, $P_d(t)$, is measured using a transducer, which transmits a negative feedback voltage signal, $V_b(t)$, to the drill pressure controller. That signal is compared at the input of the controller to the reference voltage, $V_r(t)$, Based on the error, $e(t) = V_r(t) - V_b(t)$, the drill pressure controller sets the desired drill speed, ω_d, which is the input to the drill motor subsystem whose output is the actual drill speed, ω_a, which is the input to the pulley subsystem. The output of the pulley system drives the drill stick subsystem. The drill stick subsystem may be severely affected by environmental conditions, which may be represented as disturbances acting between the pulley and stick subsystems.

Draw a functional block diagram for the above system, showing its major components as well as all signals.

SS 13. Given the electric network shown in Figure P1.5. [Review]

a. Write the differential equation for the network if $v(t) = u(t)$, a unit step.

b. Solve the differential equation for the current, $i(t)$, if there is no initial energy in the network.

c. Make a plot of your solution if $R/L = 1$.

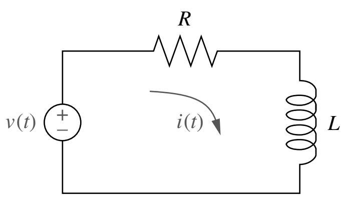

FIGURE P1.5 *RL* network

14. Repeat Problem 13 using the network shown in Figure P1.6. Assume $R = 1\,\Omega, L = 0.5\,\text{H}$, and $1/LC = 16$. [Review]

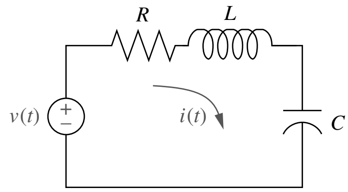

FIGURE P1.6 *RLC* network

15. Assuming zero initial conditions, use classical methods to find solutions for the following differential equations: [Review]

a. $\dfrac{dx}{dt} + 5x = 2\cos 3t$

b. $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 2x = 2\sin t$

c. $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 20x = 5u(t)$

16. Solve the following differential equations using classical methods and the given initial conditions: [Review]

a. $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 2x = \sin 2t$ SS

$x(0) = 2;\ \dfrac{dx}{dt}(0) = -3$

b. $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + x = 5e^{-2t} + t$

$x(0) = 2;\ \dfrac{dx}{dt}(0) = 1$

c. $\dfrac{d^2x}{dt^2} + 4x = t^2$

$x(0) = 1;\ \dfrac{dx}{dt}(0) = 2$

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

17. **Control of HIV/AIDS.** As of 2012, the number of people living worldwide with Human Immunodeficiency Virus/Acquired Immune Deficiency Syndrome (HIV/AIDS) was estimated at 35 million, with 2.3 million new infections per year and 1.6 million

deaths due to the disease (*UNAIDS*, *2013*). Currently there is no known cure for the disease, and HIV cannot be completely eliminated in an infected individual. Drug combinations can be used to maintain the virus numbers at low levels, which helps prevent AIDS from developing. A common treatment for HIV is the administration of two types of drugs: reverse transcriptase inhibitors (RTIs) and protease inhibitors (PIs). The amount in which each of these drugs is administered is varied according to the amount of HIV viruses in the body (*Craig*, *2004*). Draw a block diagram of a feedback system designed to control the amount of HIV viruses in an infected person. The plant input variables are the amount of RTIs and PIs dispensed. Show blocks representing the controller, the system under control, and the transducers. Label the corresponding variables at the input and output of every block.

18. **Hybrid vehicle.** The use of hybrid cars is becoming increasingly popular. A hybrid electric vehicle (HEV) combines electric machine(s) with an internal combustion engine (ICE), making it possible (along with other fuel consumption–reducing measures, such as stopping the ICE at traffic lights) to use smaller and more efficient gasoline engines. Thus, the efficiency advantages of the electric drivetrain are obtained, while the energy needed to power the electric motor is stored in the onboard fuel tank and not in a large and heavy battery pack.

There are various ways to arrange the flow of power in a hybrid car. In a serial HEV (Figure P1.7), the ICE is not connected to the drive shaft. It drives only the generator, which charges the batteries and/or supplies power to the electric motor(s) through an inverter or a converter.

The HEVs sold today are primarily of the parallel or split-power variety. If the combustion engine can turn the drive wheels as well as the generator, then the vehicle is referred to as a *parallel* hybrid, because both an electric motor and the ICE can drive the vehicle. A parallel hybrid car (Figure P1.8) includes a relatively small battery pack (electrical storage) to put out extra power to the electric motor when fast acceleration is needed. See (*Bosch*, *5th ed., 2000*), (*Bosch*, *7th ed., 2007*), (*Edelson*, *2008*), and (*Anderson*, *2009*) for more detailed information about HEV.

As shown in Figure P1.9, split-power hybrid cars utilize a combination of series and parallel drives (*Bosch*, *5th ed., 2007*). These cars use a planetary gear (3) as a split-power transmission to allow some of the ICE power to be applied mechanically to the drive. The other part is converted into electrical energy through the alternator (7) and the inverter (5) to feed the electric motor (downstream of the transmission) and/or to charge the high-voltage battery (6). Depending upon driving conditions, the ICE, the electric motor, or both propel the vehicle.

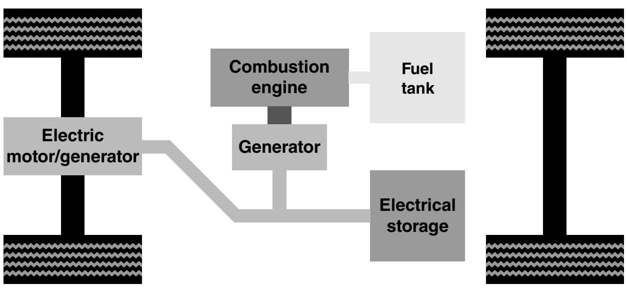

FIGURE P1.7 Serial hybrid-electric vehicle[5]

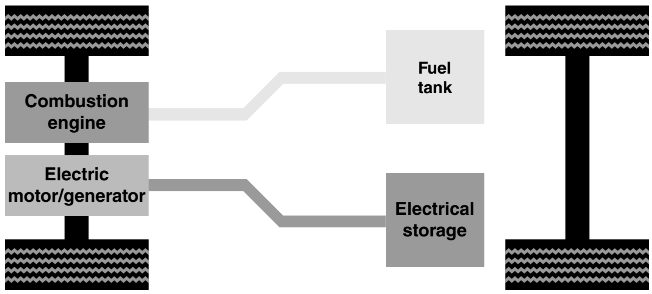

FIGURE P1.8 Parallel hybrid drive[6]

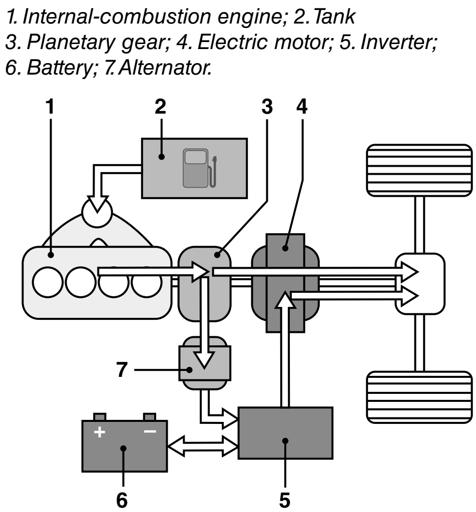

FIGURE P1.9 Split-power hybrid electric vehicle[7]

[5] Mark Looper, www.Altfuels.org. Alternative Drivetrains, July 2005, http://www.altfuels.org/backgrnd/altdrive.html. Last accessed 10/13/2009.

[6] Mark Looper, www.Altfuels.org. Alternative Drivetrains, July 2005, http://www.altfuels.org/backgrnd/altdrive.html. Last accessed 10/13/2009.

[7] Robert Bosch GmbH, *Bosch Automotive Handbook,* 7th ed. John Wiley & Sons, Ltd., UK, 2007.

Draw a functional block diagram for the cruise (speed) control system of:

a. A serial hybrid vehicle, showing its major components, including the speed sensor, electronic control unit (ECU), inverter, electric motor, and vehicle dynamics; as well as all signals, including the desired vehicle speed, actual speed, control command (ECU output), controlled voltage (inverter output), the motive force (provided by the electric motor), and running resistive force;[8]

b. A parallel hybrid vehicle, showing its major components, which should include also a block that represents the accelerator, engine, and motor, as well as the signals (including accelerator displacement and combined engine/motor motive force);

c. A split-power HEV, showing its major components and signals, including, in addition to those listed in Parts **a** and **b**, a block representing the planetary gear and its control, which, depending upon driving conditions, would allow the ICE, the electric motor, or both to propel the vehicle, that is, to provide the necessary total motive force.

19. Parabolic trough collector. A set of parabolic mirrors can be used to concentrate the sun's rays to heat a fluid flowing in a pipe positioned at the mirrors' focal points (*Camacho*, *2012*). The heated fluid, such as oil, for example, is transported to a pressurized tank to be used to create steam to generate electricity or power an industrial process. Since the solar energy varies with time of day, time of year, cloudiness, humidity, etc., a control system has to be developed in order to maintain the fluid temperature constant. The temperature is mainly controlled by varying the amount of fluid flow through the pipes, but possibly also with a solar tracking mechanism that tilts the mirrors at appropriate angles.

Assuming fixed mirror angles, draw the functional block diagram of a system to maintain the fluid temperature a constant. The desired and actual fluid temperature difference is fed to a controller followed by an amplifier and signal conditioning circuit that varies the speed of a fluid circulating pump. Label the blocks and links of your diagram, indicating all the inputs to the system, including external disturbances such as solar variations, cloudiness, humidity, etc.

[8] These include the aerodynamic drag, rolling resistance, and climbing resistance. The aerodynamic drag is a function of car speed, whereas the other two are proportional to car weight.

Chapter 2 Problems

1. Derive the Laplace transform for the following time functions: [Section: 2.2]
   a. $u(t)$
   b. $tu(t)$
   c. $\sin \omega t\, u(t)$
   d. $\cos \omega t\, u(t)$
2. Using the Laplace transform pairs of Table 2.1 and the Laplace transform theorems of Table 2.2, derive the Laplace transforms for the following time functions: [Section: 2.2]
   a. $e^{-at}\sin \omega t\, u(t)$
   b. $e^{-at}\cos \omega t\, u(t)$
   c. $t^3 u(t)$
3. Repeat Problem 14 in Chapter 1, using Laplace transforms. Assume zero initial conditions. [Sections: 2.2; 2.3]

SS
4. Repeat Problem 15 in Chapter 1, using Laplace transforms. Assume that the forcing functions are zero prior to $t = 0-$. [Section: 2.2]
5. Using Laplace transforms, solve the differential equations in Chapter 1, Problem 16 for the following initial conditions: (a) $x(0)=2, x'(0)=-2$; (b) $x(0)=1; x'(0)=1$; (c) $x(0)=0, x'(0)=1$. Assume that all input functions are zero for $t<0$ and note that $x'(0)=\frac{dx}{dt}(0)$. [Section 2.2]
6. Use MATLAB and the Symbolic Math Toolbox to find the inverse Laplace transform of the following frequency functions: [Section: 2.2]

Symbolic Math
SM

   a. $G(s) = \dfrac{(s^2+3s+10)(s+5)}{(s+3)(s+4)(s^2+2s+100)}$
   b. $G(s) = \dfrac{s^3+4s^2+2s+6}{(s+8)(s^2+8s+3)(s^2+5s+7)}$

SS
7. A system is described by the following differential equation:

$$\frac{d^3y}{dt^3}+3\frac{d^2y}{dt^2}+5\frac{dy}{dt}+y=\frac{d^3x}{dt^3}+4\frac{d^2x}{dt^2}+6\frac{dx}{dt}+8x$$

   Find the expression for the transfer function of the system, $Y(s)/X(s)$. [Section: 2.3]
8. Write the differential equation that corresponds to each of the following transfer functions. [Section: 2.3]
   a. $\dfrac{X(s)}{F(s)} = \dfrac{10}{s^2+7s+80}$
   b. $\dfrac{X(s)}{F(s)} = \dfrac{100}{(s+3)(s+17)}$
   c. $\dfrac{X(s)}{F(s)} = \dfrac{s-8}{s^3+10s^2-7s+30}$
9. Write the differential equation for the system shown in Figure P2.1. [Section: 2.3] SS

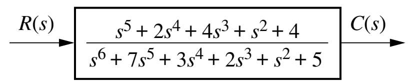

FIGURE P2.1

10. Assume the input to the block in Figure P2.2 is $r(t) = 10t^4$. Write the equivalent differential equation for the system depicted in the block diagram. [Section: 2.3]

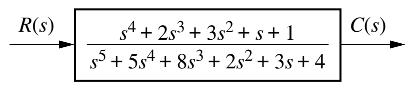

FIGURE P2.2

11. A system is described by the following differential equation: [Section 2.3]

$$\frac{d^2x}{dt^2}+4\frac{dx}{dt}+5x=1$$

   with the initial conditions $x(0)=1,\ \dot{x}(0)=-1$. Show a block diagram of the system, giving its transfer function and all pertinent inputs and outputs. (Hint: the initial conditions will show up as added inputs to an effective system with zero initial conditions.)
12. Use MATLAB to generate the transfer function: [Section: 2.3] SS

MATLAB
ML

$$G(s)=\frac{5(s+15)(s+26)(s+72)}{s(s+55)(s^2+5s+30)(s+56)(s^2+27s+52)}$$

   in the following ways:
   a. the ratio of factors;
   b. the ratio of polynomials.
13. Use MATLAB to generate the partial-fraction expansion of the following function: [Section: 2.3] SS

$$F(s)=\frac{10^4(s+5)(s+70)}{s(s+45)(s+55)(s^2+7s+110)(s^2+6s+95)}$$

14. For each of the circuits in Figure P2.3, find the transfer function, $G(s) = V_o(s)/V_i(s)$. [Section: 2.4]

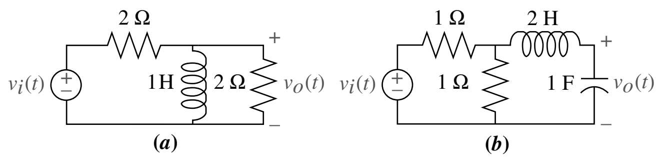

FIGURE P2.3

SS 15. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each network shown in Figure P2.4. Solve the problem using mesh analysis. [Section: 2.4]

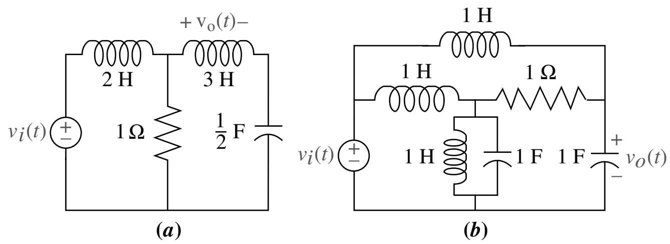

FIGURE P2.4

16. Repeat Problem 15 using nodal equations. [Section: 2.4]

17. For each of the circuits shown in Figure P2.5, find the corresponding transfer function $G(s) = V_o(s)/V_i(s)$. [Section: 2.4]

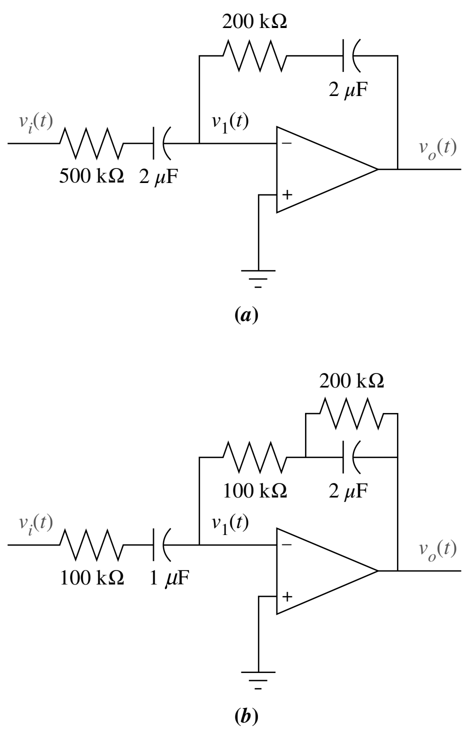

FIGURE P2.5

18. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each operational amplifier circuit shown in Figure P2.6. [Section: 2.4] SS

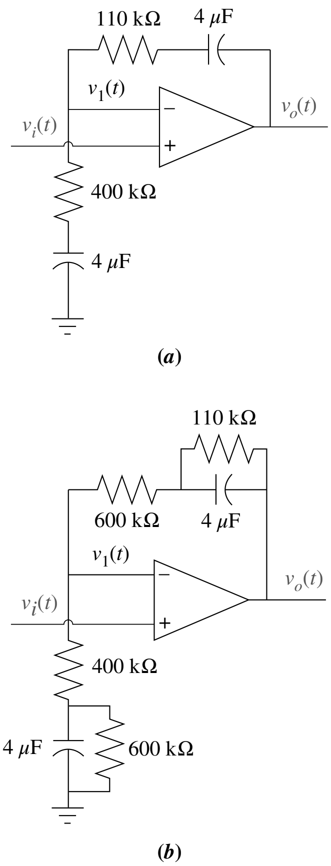

FIGURE P2.6

19. For the translational mechanical system of Figure P2.7, find the transfer function, $X_1(s)/F(s)$. [Section: 2.5]

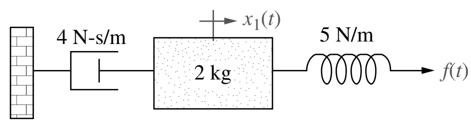

FIGURE P2.7

20. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical network shown in Figure P2.8. [Section: 2.5] SS

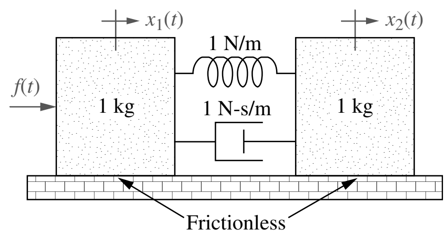

FIGURE P2.8

21. Find the tranfer function, $G(s) = X_2(s)/F(s)$, for the system shown in Figure P2.9 [Section: 2.5]

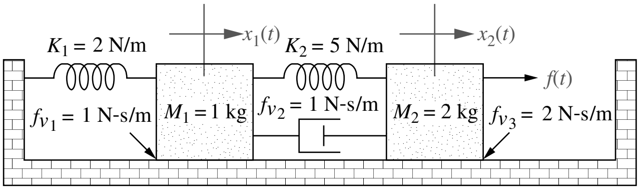

FIGURE P2.9

22. Find the transfer function, $X_3(s)/F(s)$, for each system shown in Figure P2.10. [Section: 2.5]

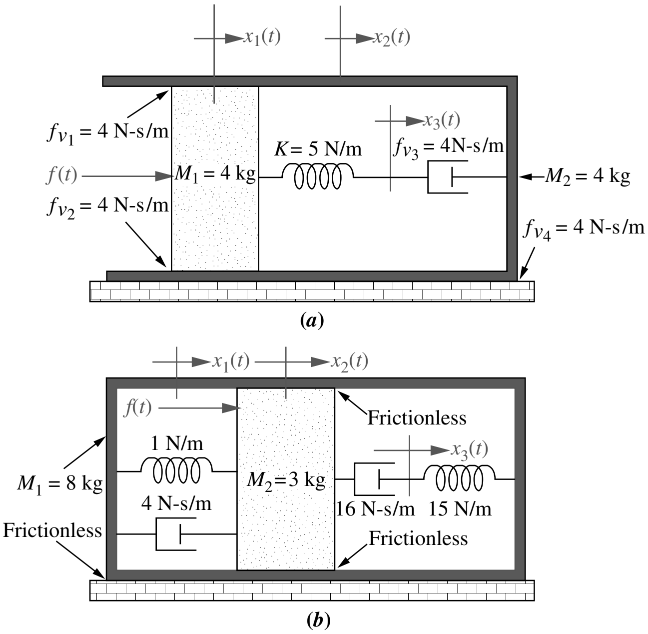

FIGURE P2.10

23. Write, but do not solve, the equations of motion for the translational mechanical system shown in Figure P2.11. [Section: 2.5]

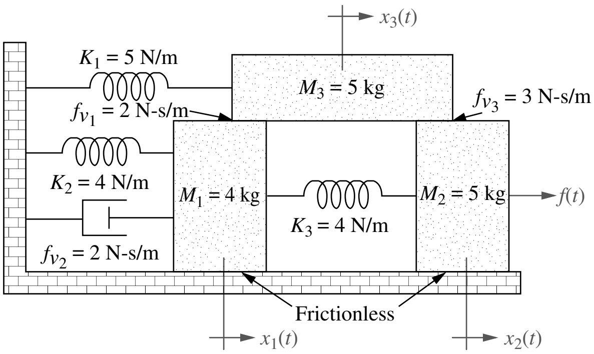

FIGURE P2.11

24. For the unexcited (no external force applied) system of Figure P2.12, do the following:
    a. Write the differential equation that describes the system.
    b. Assuming initial conditions $x(0) = x_0$ and $\dot{x}(0) = x_1$, write a Laplace transform expression for $X(s)$.
    c. Find $x(t)$ by obtaining the inverse Laplace transform from the result in Part **c**.
    d. What will be the oscillation frequency in Hz for this system?

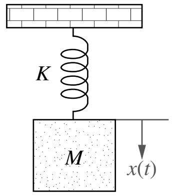

FIGURE P2.12

25. For each of the rotational mechanical systems shown in Figure P2.13, write, but do not solve, the equations of motion. [Section: 2.6]

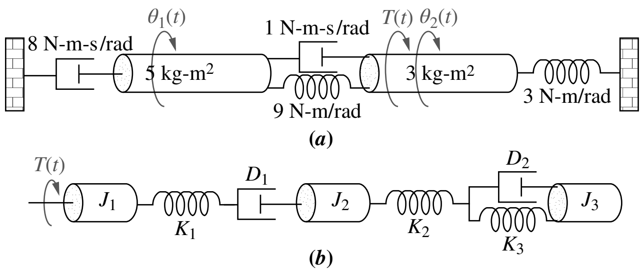

FIGURE P2.13

26. Calculate the transfer function $G(s) = \theta_2(s)/T(s)$ for the stystem of Figure P2.14. [Section: 2.6]

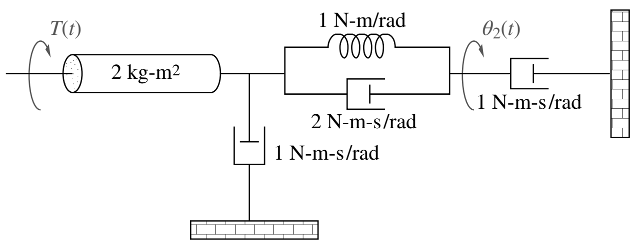

FIGURE P2.14

27. For the rotational mechanical system with gears shown in Figure P2.15, find the transfer function,

$G(s) = \theta_3(s)/T(s)$. The gears have inertia and bearing friction as shown. [Section: 2.7]

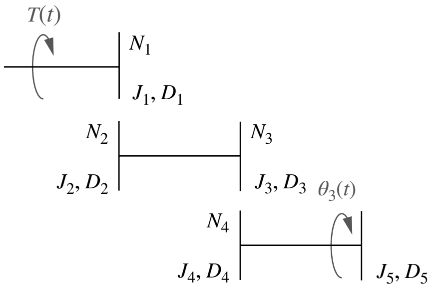

FIGURE P2.15

SS **28.** For the rotational system shown in Figure P2.16, find the transfer function, $G(s) = \theta_2(s)/T(s)$. [Section: 2.7]

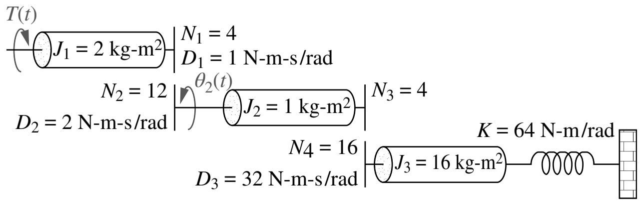

FIGURE P2.16

29. Obtain the transfer function, $G(s) = \theta_2(s)/T(s)$, for the system of Figure P2.17. [Section: 2.7]

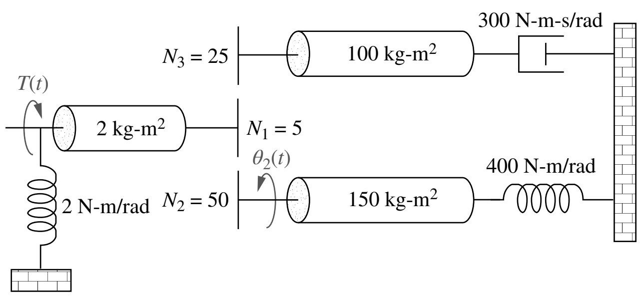

FIGURE P2.17

30. For the rotational system of Figure P2.18, find the transfer function, $G(s) = \theta_2(s)/T(s)$. [Section: 2.7]

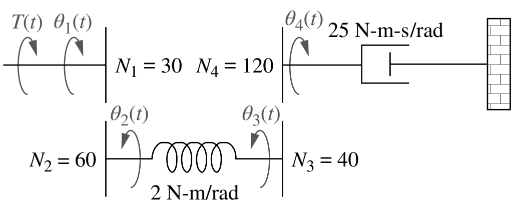

FIGURE P2.18

31. For the rotational system shown in Figure P2.19, find the transfer function, $G(s) = \theta_L(s)/T(s)$. [Section: 2.7]

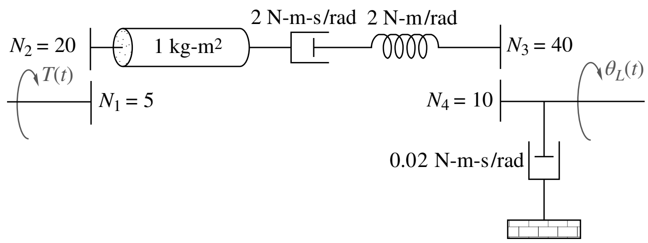

FIGURE P2.19

32. Given the rotational system shown in Figure P2.20, find the transfer function, $G(s) = \theta_6(s)/\theta_1(s)$. [Section: 2.7]

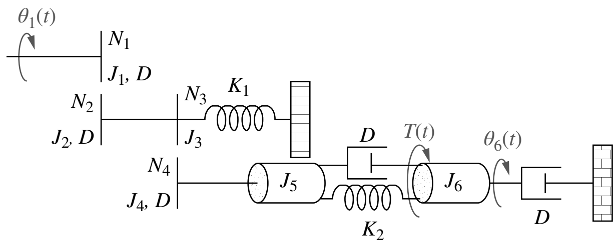

FIGURE P2.20

33. For the combined translational and rotational system shown in Figure P2.21, find the transfer function, $G(s) = X(s)/T(s)$. [Sections: 2.5; 2.6; 2.7]

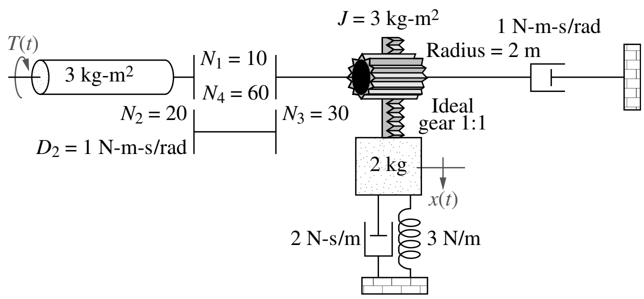

FIGURE P2.21

34. In Figure P2.22, a load is driven with a motor whose torque–speed characteristic is shown in the Figure. Determine the transfer fuction, $G(s) = \theta_L(s)/ E_a(s)$

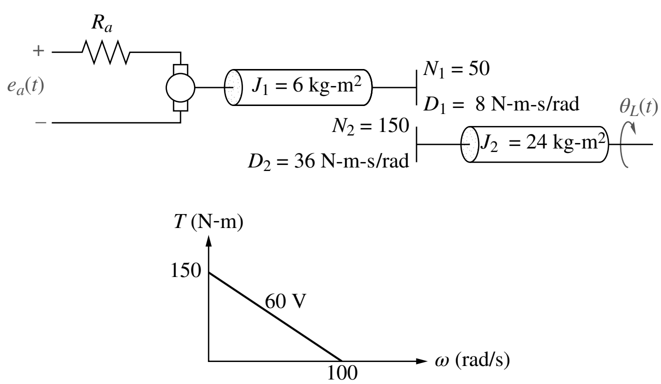

FIGURE P2.22

SS **35.** The motor whose torque–speed characteristics are shown in Figure P2.23 drives the load shown in the diagram. Some of the gears have inertia. Find the transfer function, $G(s) = \theta_2(s)/E_a(s)$. [Section: 2.8]

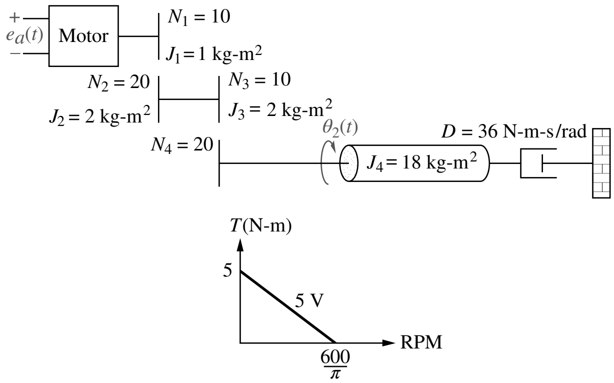

FIGURE P2.23

36. In this chapter, we derived the transfer function of a dc motor relating the angular displacement output to the armature voltage input. Often we want to control the output torque rather than the displacement. Derive the transfer function of the motor that relates output torque to input armature voltage. [Section: 2.8]

37. Find the transfer function, $G(s) = X(s)/E_a(s)$, for the system shown in Figure P2.24. [Sections: 2.5–2.8]

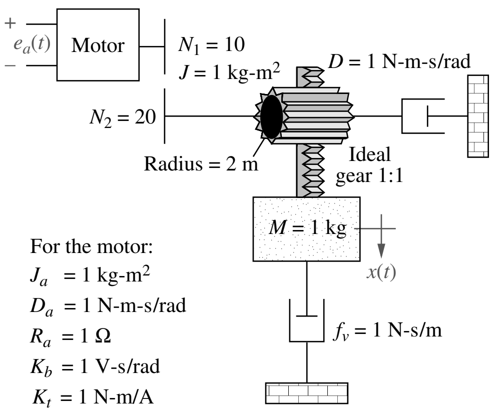

FIGURE P2.24

SS **38.** Find the series and parallel analogs for the translational mechanical system shown in Figure 2.20 in the text. [Section: 2.9]

39. Find the series and parallel analogs for the rotational mechanical systems shown in Figure P2.13(b) in the problems. [Section: 2.9]

40. A system's output, c, is related to the system's input, r, by the straight-line relationship, $c = 5r + 7$. Is the system linear? [Section: 2.10]

41. Consider the differential equation SS

$$\frac{d^3x}{dt^3} + 10\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 15x = f(x)$$

where $f(x)$ is the input and is a function of the output, x. If $f(x) = 3e^{-5x}$, linearize the differential equation for x near 0. [Section: 2.10]

42. For the translational mechanical system with a nonlinear spring shown in Figure P2.25, find the transfer function, $G(s) = X(s)/F(s)$, for small excursions around $f(t) = 1$. The spring is defined by $x_s(t) = 1 - e^{-f_s(t)}$, where $x_s(t)$ is the spring displacement and $f_s(t)$ is the spring force. [Section: 2.10]

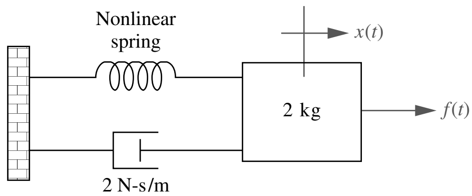

FIGURE P2.25

43. Enzymes are large proteins that biological systems use to increase the rate at which reactions occur. For example, food is usually composed of large molecules that are hard to digest; enzymes break down the large molecules into small nutrients as part of the digestive process. One such enzyme is amylase, contained in human saliva. It is commonly known that if you place a piece of uncooked pasta in your mouth its taste will change from paper-like to sweet as amylase breaks down the carbohydrates into sugars. Enzyme breakdown is often expressed by the following relation:

$$S + E \underset{k_1}{\overset{k_s}{\rightleftarrows}} C \xrightarrow{k_1} P$$

In this expression a substrate (S) interacts with an enzyme (E) to form a combined product (C) at a rate k_1. The intermediate compound is reversible and gets disassociated at a rate k_{-1}. Simultaneously some of the compound is transformed into the final product (P) at a rate k_2. The kinetics describing this reaction are known as the Michaelis–Menten equations and consist of four nonlinear differential equations. However, under some conditions these equations can be simplified. Let E_0 and S_0 be the initial concentrations of enzyme and substrate, respectively. It is generally accepted that under some energetic conditions or when the enzyme concentration is very big ($E_0 \gg S_0$), the kinetics for this reaction are given by

$$\frac{dS}{dt} = k_\psi(\tilde{K}_s C - S)$$

$$\frac{dC}{dt} = k_\psi(S - \tilde{K}_M C)$$

$$\frac{dP}{dt} = k_2 C$$

where the following constant terms are used (*Schnell*, *2004*):

$$k_\psi = k_1 E_0$$
$$\tilde{K}_s = \frac{k-1}{k_\psi}$$

and

$$\tilde{K}_M = \tilde{K}_s + \frac{k_2}{k_\psi}$$

a. Assuming the initial conditions for the reaction are $S(0) = S_0$, $E(0) = E_0$, $C(0) = P(0) = 0$, find the Laplace transform expressions for S, C, and P: $\mathscr{L}\{S\}$, $\mathscr{L}\{C\}$, and $\mathscr{L}\{P\}$, respectively.

b. Use the final theorem to find $S(\infty)$, $C(\infty)$, and $P(\infty)$.

44. Humans are able to stand on two legs through a complex feedback system that includes several sensory inputs—equilibrium and visual along with muscle actuation. In order to gain a better understanding of the workings of the postural feedback mechanism, an individual is asked to stand on a platform to which sensors are attached at the base. Vibration actuators are attached with straps to the individual's calves. As the vibration actuators are stimulated, the individual sways and movements are recorded. It was hypothesized that the human postural dynamics are analogous to those of a cart with a balancing standing pole attached (inverted pendulum). In that case, the dynamics can be described by the following two equations:

$$\begin{aligned} J\frac{d^2\theta}{dt^2} &= mgl\sin\theta(t) + T_{\text{bal}} + T_d(t) \\ T_{\text{bal}}(t) &= -mgl\sin\theta(t) + kJ\theta(t) - \eta J\dot{\theta}(t) \\ &\quad -\rho J\int_0^t \theta(t)dt \end{aligned}$$

where m is the individual's mass; l is the height of the individual's center of gravity; g is the gravitational constant; J is the individual's equivalent moment of inertia; η, ρ, and k are constants given by the body's postural control system; $\theta(t)$ is the individual's angle with respect to a vertical line; $T_{\text{bal}}(t)$ is the torque generated by the body muscles to maintain balance; and $T_d(t)$ is the external torque input disturbance. Find the transfer function $\frac{\Theta(s)}{T_d(s)}$ (*Johansson*, *1988*).

45. In order to design an underwater vehicle that has the characteristics of both a long-range transit vehicle (torpedo-like) and a highly maneuverable low-speed vehicle (boxlike), researchers have developed a thruster that mimics that of squid jet locomotion (*Krieg*, *2008*). It has been demonstrated there that the average normalized thrust due to a command step input, $U(s) = \frac{T_{ref}}{s}$ is given by:

$$T(t) = T_{ref}(1 - e^{-\lambda t}) + a\sin(2\pi f t)$$

where T_{ref} is the reference or desired thrust, λ is the system's damping constant, a is the amplitude of the oscillation caused by the pumping action of the actuator, f is the actuator frequency, and $T(t)$ is the average resulting normalized thrust. Find the thruster's transfer function $\frac{T(s)}{U(s)}$. Show all steps.

46. The Gompertz growth model is commonly used to model tumor cell growth. Let $v(t)$ be the tumor's volume, then

$$\frac{dv(t)}{dt} = \lambda e^{-\alpha t} v(t)$$

where λ and α are two appropriate constants (*Edelstein-Keshet*, *2005*).

a. Verify that the solution to this equation is given by $v(t) = v_0 e^{\lambda/\alpha(1-e^{-\alpha t})}$, where v_0 is the initial tumor volume.

b. This model takes into account the fact that when nutrients and oxygen are scarce at the tumor's core, its growth is impaired. Find the final predicted tumor volume (let $t \to \infty$).

c. For a specific mouse tumor, it was experimentally found that $\lambda = 2.5$ days, $\alpha = 0.1$ days with $v_0 = 50 \times 10^{-3}$ mm^3 (*Chignola*, *2005*). Use any method available to make a plot of $v(t)$ vs. t.

d. Check the result obtained in Part **b** with the results from the graph found in Part **c.**

47. A muscle hanging from a beam is shown in Figure P2.26(a) (*Lessard*, *2009*). The α-motor neuron can be used to electrically stimulate the muscle to contract and pull the mass, m, which under static conditions causes the muscle to stretch. An equivalent mechanical system to this setup is shown in Figure P2.36(b). The force F_{iso} will be exerted when the muscle contracts. Find an expression for the displacement $X_1(s)$ in terms of $F_1(s)$ and $F_{iso}(s)$.

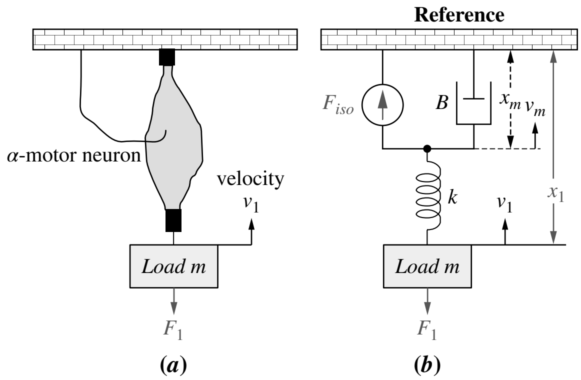

FIGURE P2.26 **a.** Motor neuron stimulating a muscle;[1] **b.** equivalent circuit[2]

48. A three-phase ac/dc converter supplies dc to a battery charging system or dc motor (*Graovac*, *2001*). Each phase has an ac filter represented by the equivalent circuit in Figure P2.27.

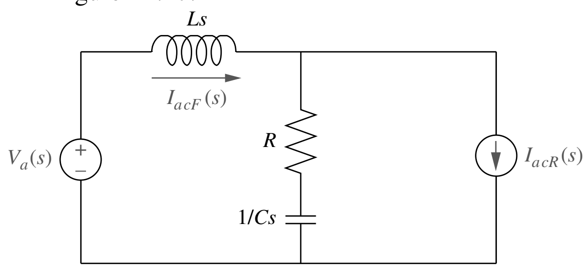

FIGURE P2.27 AC filter equivalent circuit for a three-phase ac/dc converter

Derive that the inductor current in terms of the two active sources is

$$I_{acF}(s) = \frac{1 + RCs}{LCs^2 + RCs + 1} I_{acR}(s) + \frac{Cs}{LCs^2 + RCs + 1} V_a(s)$$

49. A photovoltaic system is used to capture solar energy to be converted to electrical energy. A control system is used to pivot the solar platform to track the sun's movements in order to maximize the captured energy. The system consists of a motor and load similar to that discussed in Section 2.8. A model has been proposed (*Agee*, *2012*) that is different from the model developed in the chapter in the following ways: (1) the motor inductance was not neglected and (2) the load, in addition to having inertia and damping, has a spring. Find the transfer function, $\theta_m(s)/E_a(s)$, for this augmented system assuming all load impedances have already been reflected to the motor shaft.

[1] Lessard, C. D. Basic Feedback Controls in Biomedicine, Morgan & Claypool, San Rafael, CA, 2009. Figure 2.8, p. 12.
[2] Lessard, C. D. Basic Feedback Controls in Biomedicine, Morgan & Claypool, San Rafael, CA, 2009 Figure 2.9, p. 13.

50. In a paint mixing plant, two tanks supply fluids to a mixing cistern. The height, h, of the fluid in the cistern is dependent upon the difference between the input mass flow rate, q, and the output flow rate, q_e. A nonlinear differential equation describing this dependency is given by (*Schiop*, *2010*)

$$\frac{dh}{dt} + \frac{A_e}{A}\sqrt{2gh} = \frac{q}{\rho A}$$

where A = cross-sectional area of the cistern, A_e = cross-sectional area of the exit pipe, g = acceleration due to gravity, and ρ = liquid density.

a. Linearize the nonlinear equation about the equilibrium point (h_0, q_0) and find the transfer function relating the output cistern fluid level, $H(s)$, to the input mass flow rate, $Q(s)$.

b. The color of the liquid in the cistern can be kept constant by adjusting the input flow rate, q, assuming the input flow's color is specifically controlled. Assuming an average height, h_{av}, of the liquid in the cistern, the following equation relates the net flow of color to the cistern to the color in the cistern.

$$e_1 q - e q_e = \frac{d}{dt}(\rho A e h_{av})$$

where e_1 = fractional part of flow representing color into the cistern, and e = fractional part of the cistern representing color in the cistern. Assume that the flow out of the cistern is constant and use the relationship, $q_e = \rho A_e\sqrt{2gh_{av}}$, along with the given equation above to find the transfer function, $E(s)/Q(s)$, that relates the color in the cistern to the input flow rate.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

51. **Control of HIV/AIDS.** HIV inflicts its damage by infecting healthy CD4 + T cells (a type of white blood cell) that are necessary to fight infection. As the virus embeds in a T cell and the immune system produces more of these cells to fight the infection, the virus propagates in an opportunistic fashion. As we now develop a simple HIV model, refer to Figure P2.28. Normally T cells are produced at a rate s and die at a rate d. The HIV virus is present in the bloodstream in the infected individual. These viruses in the bloodstream, called *free viruses*, infect healthy T cells at a rate β. Also, the viruses reproduce through the T cell multiplication process or otherwise at a rate k. Free viruses die at a rate c. Infected T cells die at a rate μ.

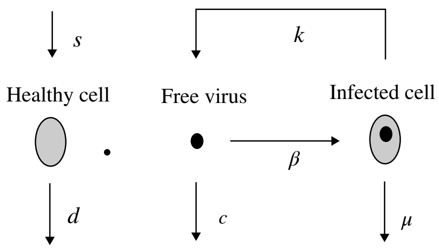

FIGURE P2.28[3]

A simple mathematical model that illustrates these interactions is given by the following equations (*Craig*, *2004*):

$$\frac{dT}{dt} = s - dT - \beta T v$$
$$\frac{dT^*}{dt} = \beta T v - \mu T^*$$
$$\frac{dv}{dt} = kT^* - cv$$

where

T = number of healthy T cells
T^* = number of infected T cells
v = number of free viruses

a. The system is nonlinear; thus linearization is necessary to find transfer functions as you will do in subsequent chapters. The nonlinear nature of this model can be seen from the above equations. Determine which of these equations are linear, which are nonlinear, and explain why.

b. The system has two equilibrium points. Show that these are given by

$$(T_0,\ T_0^*,\ v_0) = \left(\frac{s}{d},\ 0,\ 0\right)$$

and

$$(T_0,\ T_0^*,\ v_0) = \left(\frac{c\mu}{\beta k},\ \frac{s}{\mu} - \frac{cd}{\beta k},\ \frac{sk}{c\mu} - \frac{d}{\beta}\right)$$

52. Hybrid vehicle. Problem 18 in Chapter 1 discusses the cruise control of serial, parallel, and split-power hybrid electric vehicles (HEVs). The functional block diagrams developed for these HEVs indicated that the speed of a vehicle depends upon the balance between the motive forces (developed by the gasoline engine and/or the electric motor) and running resistive forces. The resistive forces include the aerodynamic drag, rolling resistance, and climbing resistance. Figure P2.29 illustrates the running resistances for a car moving uphill (*Bosch*, *2007*).

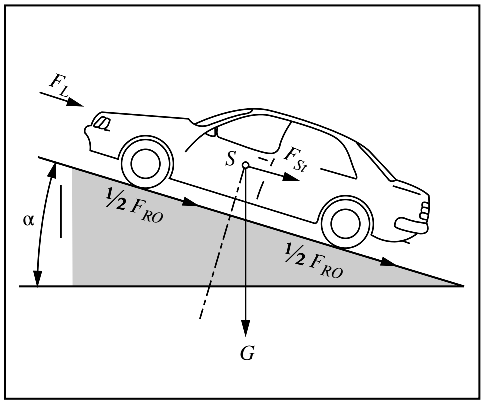

FIGURE P2.29 Running resistances[4]

The total running resistance, F_w, is calculated as $F_w = F_{Ro} + F_L + F_{St}$, where F_{Ro} is the rolling resistance, F_L is the aerodynamic drag, and F_{St} is the climbing resistance. The aerodynamic drag is proportional to the square of the sum of car velocity, v, and the head-wind velocity, v_{hw}, or $v + v_{hw}$. The other two resistances are functions of car weight, G, and the gradient of the road (given by the gradient angle, α), as seen from the following equations:

$$F_{Ro} = fG\cos\alpha = fmg\cos\alpha$$

where

f = coefficient of rolling resistance
m = car mass, in kg
g = gravitational acceleration, in m/s^2
$F_L = 0.5\rho C_w A(v + v_{\mathrm{hw}})^2$.

and

ρ = air density, in kg/m^3
C_w = coefficient of aerodynamic drag
A = largest cross-section of the car, in kg/m^2
$F_{St} = G\sin\alpha = mg\sin\alpha$.

[3] Craig, I. K., Xia, X., and Venter, J. W. *Introducing HIV/AIDS Education Into the Electrical Engineering Curriculum at the University of Pretoria.* IEEE Transactions on Education, vol. 47, no. 1, February 2004, pp. 65–73. Fig. 1, p. 66. IEEE transactions on education by INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS; IEEE EDUCATION GROUP; IEEE EDUCATION SOCIETY Reproduced with permission of INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, in the format Republish in a book via Copyright Clearance Center.

[4] Robert Bosch GmbH, *Bosch Automotive Handbook*, 7th ed. John Wiley & Sons Ltd. UK, 2007. P. 430. Figure at bottom left.

The motive force, F, available at the drive wheels is:

$$F = \frac{Ti_{tot}}{r}\eta_{tot} = \frac{P\eta_{tot}}{v}$$

where

$$\begin{aligned} T &= \text{motive torque} \\ P &= \text{motive power} \\ i_{tot} &= \text{total transmission ratio} \\ r &= \text{tire radius} \\ \eta_{tot} &= \text{total drive-train efficiency.} \end{aligned}$$

The surplus force, $F - F_w$, accelerates the vehicle (or retards it when $F_w > F$). Letting $a = \dfrac{F - F_w}{k_m \cdot m}$, where a is the acceleration and k_m is a coefficient that compensates for the apparent increase in vehicle mass due to rotating masses (wheels, flywheel, crankshaft, etc.):

a. Show that car acceleration,[5] a, may be determined from the equation:

$$F = fmg\cos\alpha + mg\sin\alpha + 0.5\rho C_w A(v + v_{hw})^2 + k_m ma$$

b. Assuming constant acceleration and using the average value for speed, find the average motive force, F_{av} (in N), and power, P_{av} (in kW) the car needs to accelerate from 40 to 60 km/h in 4 seconds on a level road, $(\alpha = 0°)$, under windless conditions, where $v_{hw} = 0$. You are given the following parameters: $\mathrm{m} = 1590\,\mathrm{kg}$, $\mathrm{A} = 2\,\mathrm{m}^2$, $f = 0.011$, $\rho = 1.2\,\mathrm{kg/m}^3$, $C_w = 0.3$, $\eta_{tot} = 0.9$, $k_m = 1.2$. Furthermore, calculate the additional power, P_{add}, the car needs after reaching 60 km/h to maintain its speed while climbing a hill with a gradient $\alpha = 5°$.

c. The equation derived in Part **a** describes the nonlinear car motion dynamics where $F(t)$ is the input to the system, and $v(t)$ the resulting output. Given that the aerodynamic drag is proportional to v^2 under windless conditions, linearize the resulting equation of motion around an average speed, $v_o = 50$ km/h, when the car travels on a level road,[6] where $\alpha = 0°$. (Hint: Expand $v^2 - v_0^2$ in a truncated Taylor series). Write that equation of motion and represent it with a block diagram in which the block G_v represents the vehicle dynamics. The output of that block is the car speed, $v(t)$, and the input is the excess motive force, $F_e(t)$, defined as: $F_e = F - F_{St} - F_{Ro} + F_o$, where F_o is the constant component of the linearized aerodynamic drag.

d. Use the equation in Part **c** to find the vehicle transfer function: $G_v(s) = V(s)/F_e(s)$.

53. Parabolic trough collector. In a significant number of cases, the open-loop transfer function from fluid flow to fluid temperature in a parabolic trough collector can be approximated (*Camacho, 2012*) by:

$$P(s) = \frac{K}{1 + \tau s}e^{-sT}$$

a. Write an analytic expression for the unit step response of the open-loop system assuming that $h(t)$ represents the output temperature and $q(t)$ the input fluid flow.

b. Make a sketch of the unit step response of the open-loop system. Indicate on your figure the time delay, the settling time, the initial and final values of the response, and the value of the response when $t = \tau + T$.

c. Call the output temperature $h(t)$ and the input fluid flow $q(t)$. Find the differential equation that represents the open-loop system.

[5] Other quantities, such as top speed, climbing ability, etc., may also be calculated by manipulation from that equation.

[6] Note that on a level road the climbing resistance, $F_{st} = 0$, since $\sin\alpha = \sin 0° = 0$.

Chapter 3 Problems

1. Write a state-space representation for the system in Figure P3.1. Assume that the system's output is $v_o(t)$. [Section: 3.4]

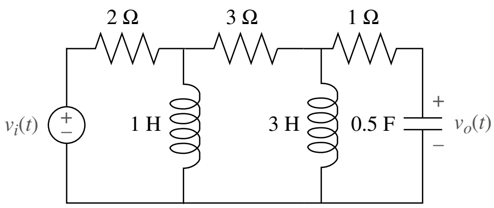

FIGURE P3.1

2. For the circuit of Figure P3.2, the output is across the 2Ω resistor. Find a state-space representation. [Section: 3.4]

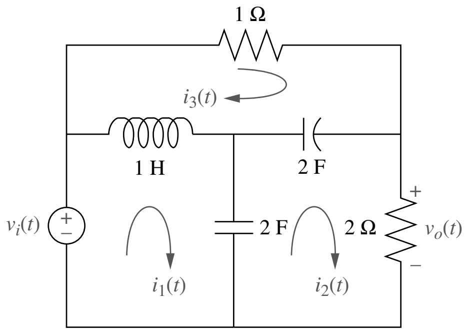

FIGURE P3.2

3. Find a state-space representation for the system in Figure P3.3. Assume the output is $x_1(t)$. [Section: 3.4]

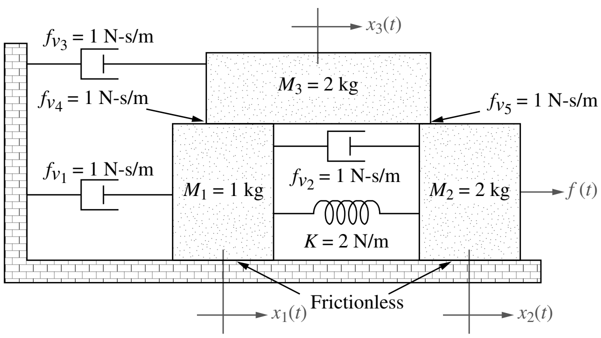

FIGURE P3.3

4. Find a state-space representation for the system in Figure P3.4. Assume the output is $x_2(t)$. [Section: 3.4]

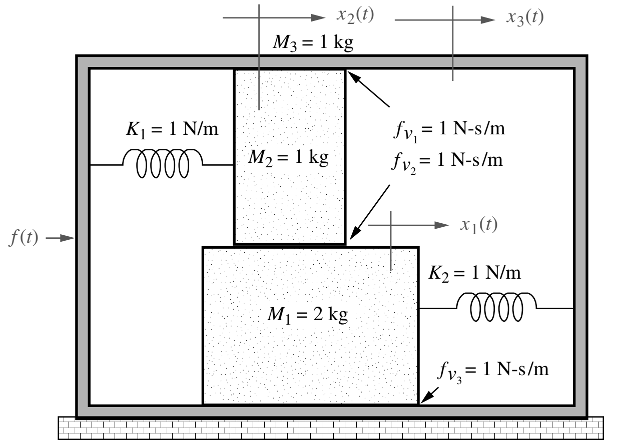

FIGURE P3.4

5. Assuming $\theta_1(t)$ is the output of the rotational system of Figure P3.5, find a state-space representation. [Section: 3.4]

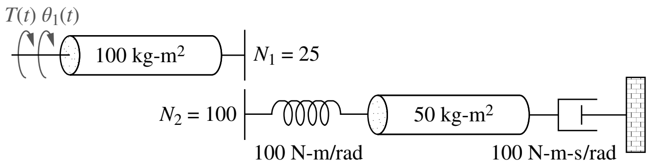

FIGURE P3.5

6. Represent the system shown in Figure P3.6 in state space where the output is $\theta_L(t)$. [Section: 3.4] SS

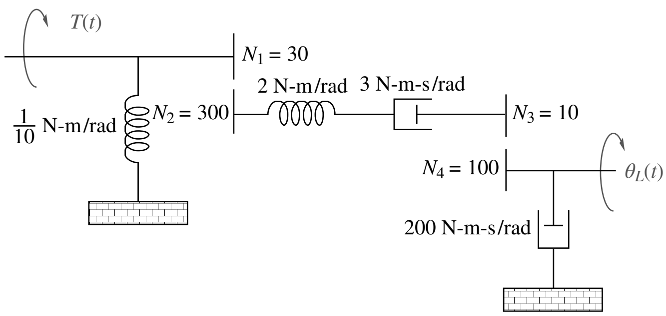

FIGURE P3.6

7. Show that the system of Figure 3.7 in the text yields a fourth-order transfer function if we relate the displacement of either mass to the applied force, and a third-order one if we relate the velocity of either mass to the applied force. [Section: 3.4] SS

8. For each of the system of Figure P3.7 find a state-space representation in phase-variable form. [Section: 3.5]

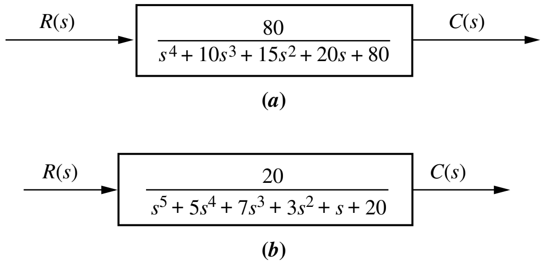

FIGURE P3.7

9. Repeat Problem 8 using MATLAB. [Section: 3.5] MATLAB **ML**

10. Express each one of the systems in Figure P3.8 in state space phase-variable form. [Section: 3.5]

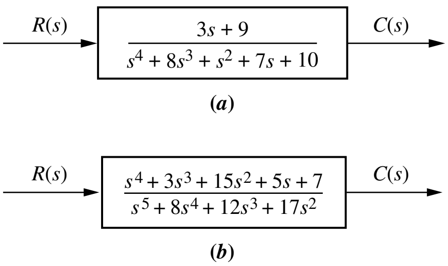

FIGURE P3.8

SS 11. Repeat Problem 10 using MATLAB. [Section: 3.5] MATLAB **ML**

12. Find a vector-matrix state-space representation for the transfer function. [Section: 3.5]

$$T(s) = \frac{s(s-3)}{(s+1)(s^2+3s+10)}$$

13. For each one of the following systems in state space, find the corresponding transfer function $G(s) = Y(s)/R(s)$. [Section: 3.6]

a. $$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 23 \end{bmatrix} r$$

$$y = [1 \quad 0 \quad 0]\mathbf{x}$$

b. $$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 2 & -6 \\ -4 & -5 & 0 \\ 3 & -3 & 7 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} r$$

$$y = [2 \quad 0 \quad 2]\mathbf{x}$$

c. $$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 8 & 7 \\ 5 & -4 & 2 \\ -9 & -3 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix} r$$

$$y = [7 \quad 2 \quad 1]\mathbf{x}$$

14. Use MATLAB to find the transfer function, $G(s)=Y(s)/R(s)$, for each of the following systems represented in state space: [Section: 3.6] MATLAB **ML**

a. $$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -7 & -9 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 5 \\ 8 \\ 2 \end{bmatrix} r$$

$$y = [1 \quad 3 \quad 6 \quad 6]\mathbf{x}$$

b. $$\dot{\mathbf{x}} = \begin{bmatrix} 3 & 1 & 0 & 4 & -2 \\ -3 & 5 & -5 & 2 & -1 \\ 0 & 1 & -1 & 2 & 8 \\ -7 & 6 & -3 & -4 & 0 \\ -6 & 0 & 4 & -3 & 1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2 \\ 7 \\ 8 \\ 5 \\ 4 \end{bmatrix} r$$

$$y = [1 \quad -2 \quad -9 \quad 7 \quad 6]\mathbf{x}$$

15. Repeat Problem 14 using MATLAB, the Symbolic Math Toolbox, and Eq. (3.73). [Section: 3.6] Symbolic Math **SM**

16. A missile in flight, as shown in Figure P3.9, is subject to four forces: thrust, lift, drag, and gravity. The missile flies at an angle of attack, α, from its longitudinal axis, creating lift. For steering, the body angle from vertical, ϕ, is controlled by rotating the engine at the tail. The transfer function relating the body angle, ϕ, to the angular displacement, δ, of the engine is of the form **SS**

$$\frac{\Phi(s)}{\delta(s)} = \frac{K_a s + K_b}{K_3 s^3 + K_2 s^2 + K_1 s + K_0}$$

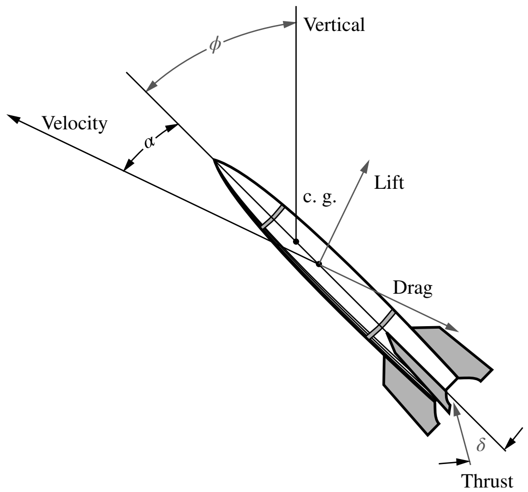

FIGURE P3.9 Missile

Represent the missile steering control in state space. [Section: 3.5]

SS **17.** Given the dc servomotor and load shown in Figure P3.10, represent the system in state space, where the state variables are the armature current, i_a, load displacement, θ_L, and load angular velocity, ω_L. Assume that the output is the angular displacement of the armature. Do not neglect armature inductance. [Section: 3.4]

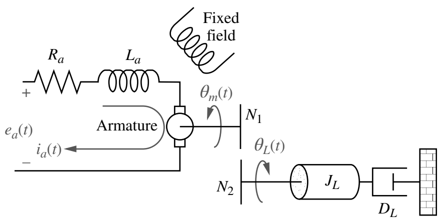

FIGURE P3.10 Motor and load

18. Image-based homing for robots can be implemented by generating heading command inputs to a steering system based on the following guidance algorithm. Suppose the robot shown in Figure P3.11(a) is to go from point R to a target, point T, as shown in Figure P3.11(b). If $\mathbf{R}_x$, $\mathbf{R}_y$, and $\mathbf{R}_z$ are vectors from the robot to each landmark, X, Y, Z, respectively, and $\mathbf{T}_x$, $\mathbf{T}_y$, and $\mathbf{T}_z$ are vectors from the target to each landmark, respectively, then heading commands would drive the robot to minimize $\mathbf{R}_x - \mathbf{T}_x$, $\mathbf{R}_y - \mathbf{T}_y$, and $\mathbf{R}_z - \mathbf{T}_z$ simultaneously, since the differences will be zero when the robot arrives at the target (*Hong*, *1992*). If Figure P3.11(c) represents the control system that steers the robot, represent each block—the controller, wheels, and vehicle—in state space. An animation PowerPoint presentation (PPT) demonstrating this system is available for instructors at www.wiley.com/go/Nise/ControlSystemsEngineering8e. See *Robot*. [Section: 3.5]

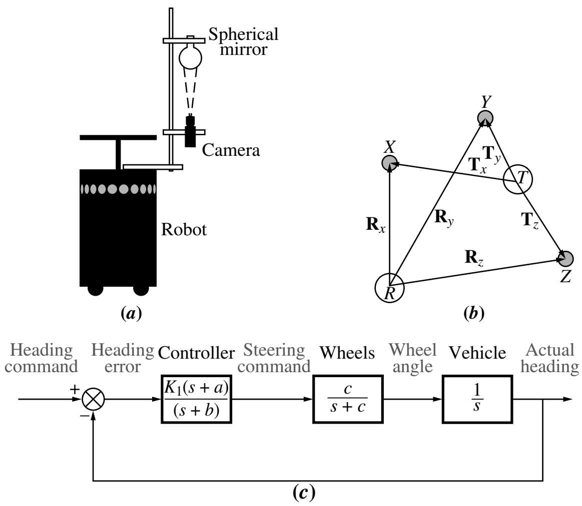

FIGURE P3.11 **a.** Robot with television imaging system;[1] **b.** vector diagram showing concept behind image-based homing;[1] **c.** heading control system

[1] Hong, J.; Tan, X.; Pinette, B.; Weiss, R.; and Riseman, E. M. Image-Based Homing, *IEEE Control Systems*, Feb. 1992, pp. 38–45. © 1992 IEEE.

19. Modern robotic manipulators that act directly upon their target environments must be controlled so that impact forces as well as steady-state forces do not damage the targets. At the same time, the manipulator must provide sufficient force to perform the task. In order to develop a control system to regulate these forces, the robotic manipulator and target environment must be modeled. Assuming the model shown in Figure P3.12, represent in state space the manipulator and its environment under the following conditions (*Chiu*, *1997*). [Section: 3.5]

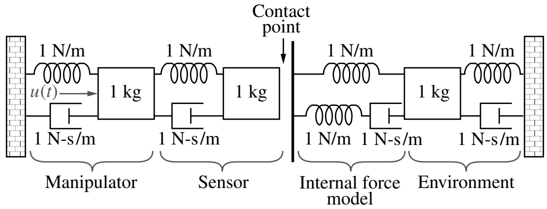

FIGURE P3.12 Robotic manipulator and target environment[2]

a. The manipulator is not in contact with its target environment.

b. The manipulator is in constant contact with its target environment.

20. In this chapter, we described the state-space representation of single-input, single-output systems. In general, systems can have multiple inputs and multiple outputs. An autopilot is to be designed for a submarine as shown in Figure P3.13 to maintain a constant depth under severe wave disturbances. We will see that this system has two inputs and two outputs and thus the scaler u becomes a vector, $\mathbf{u}$, and the scaler y becomes a vector, $\mathbf{y}$, in the state equations.

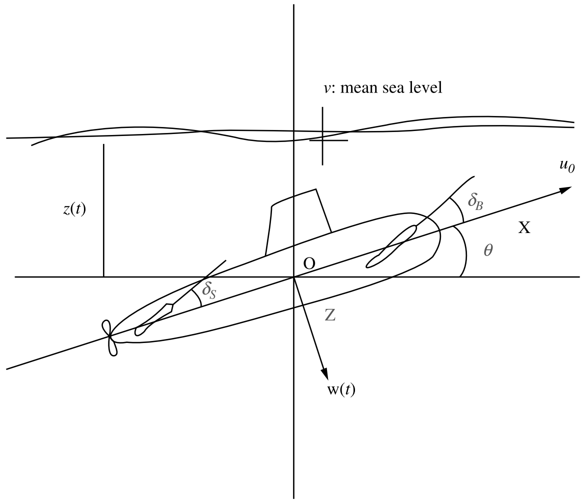

FIGURE P3.13[3]

[2] Based on Chiu, D. K., and Lee, S. Design and Experimentation of a Jump Impact Controller. *IEEE Control Systems*, June 1997, Figure 1, p. 99. 1997 IEEE.

[3] Liceaga-Castro E., van der Molen G.M. Submarine H$^\infty$ Depth Control Under Wave Disturbances. *IEEE Trans. on Control Systems Technology*, Vol. 3 No. 3, 1995. Figure 1, p. 339.

It has been shown that the system's linearized dynamics under neutral buoyancy and at a given constant speed are given by (*Liceaga-Castro*, *2009*):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$
$$\mathbf{y} = \mathbf{Cx}$$

where

$$\mathbf{x} = \begin{bmatrix} w \\ q \\ z \\ \theta \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} z \\ \theta \end{bmatrix}; \quad \mathbf{u} = \begin{bmatrix} \delta_B \\ \delta_S \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -0.038 & 0.896 & 0 & 0.0015 \\ 0.0017 & -0.092 & 0 & -0.0056 \\ 1 & 0 & 0 & -3.086 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} -0.0075 & -0.023 \\ 0.0017 & -0.0022 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and where

w = the heave velocity
q = the pitch rate
z = the submarine depth
θ = the pitch angle
δ_B = the bow hydroplane angle
δ_S = the stern hydroplane angle

Since this system has two inputs and two outputs, four transfer functions are possible.

a. Use MATLAB to calculate the system's matrix transfer function. MATLAB **ML**

b. Using the results from Part **a**, write the transfer function $\frac{z(s)}{\delta_B(s)}$, $\frac{z(s)}{\delta_S(s)}$, $\frac{\theta(s)}{\delta_B(s)}$, and $\frac{\theta(s)}{\delta_S(s)}$.

21. Show that the following three state-space representations will result in the same transfer function. Thus, in general, the state-space representation of a system is not unique as will be discussed in Chapter 5.

a. $\dot{x} = -10x + 4u$
$y = 9x$

b. $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix}u$$
$$y = \begin{bmatrix} 9 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c. $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}u$$
$$y = \begin{bmatrix} 9 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

22. Figure P3.14 shows a schematic description of the global carbon cycle (*Li*,). In the figure, $m_A(t)$ represents the amount of carbon in gigatons (GtC) present in the atmosphere of earth; $m_V(t)$ the amount in vegetation; $m_s(t)$ the amount in soil; $m_{SO}(t)$ the amount in surface ocean; and $m_{IDO}(t)$ the amount in intermediate and deep-ocean reservoirs. Let $u_E(t)$ stand for the human generated CO_2 emissions (GtC/yr). From the figure, the atmospheric mass balance in the atmosphere can be expressed as:

$$\frac{dm_A}{dt}(t) = u_E(t) - (k_{O1} + k_{L1})m_A(t) + k_{L2}m_V(t)$$
$$+k_{O2}m_{SO}(t) + k_{L4}m_S(t)$$

where the ks are exchange coefficients (yr^{-1}).

a. Write the remaining reservoir mass balances. Namely, write equations for $\frac{dm_{SO}(t)}{dt}$, $\frac{dm_{IDO}(t)}{dt}$, $\frac{dm_V(t)}{dt}$, and $\frac{dm_S(t)}{dt}$

b. Express the system in state-space form.

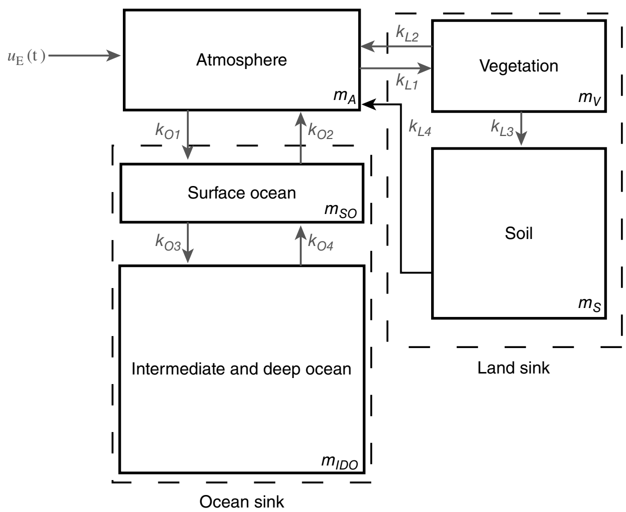

FIGURE P3.14 Global carbon cycle[4]

[4] Li, S., Jarvis, A.J., and Leedal, D.T. Are response function representations of the global carbon cycle ever interpretable? *Tellus*, vol. 61B, 2009, pp. 361–371. Fig. 1 p. 363).

23. Given the photovoltaic system described in Problem 49 in Chapter 2 (*Agee, 2012*) and defining the following state variables, system input and output as $y = x_1 = \theta_m$, $x_2 = \dot{\theta}_m$, $x_3 = i_a$, and $u = e_a$, write a state-space representation of the system in the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$, $\mathbf{y} = \mathbf{C}\mathbf{x}$.

SS **24.** A single-pole oil cylinder valve contains a spool that regulates hydraulic pressure, which is then applied to a piston that drives a load. The transfer function relating piston displacement, $X_p(s)$ to spool displacement from equilibrium, $X_v(s)$, is given by (*Qu, 2010*):

$$G(s) = \frac{X_p(s)}{X_v(s)} = \frac{K_q\omega_h^2/A_1}{s(s^2 + 2\varsigma\omega_h s + \omega_h^2)}$$

where $A_1 =$ effective area of a the valve's chamber, $K_q =$ rate of change of the load flow rate with a change in displacement, and $\omega_h =$ the natural frequency of the hydraulic system. Find the state-space representation of the system, where the state variables are the phase variables associated with the piston.

25. Figure P3.15 shows a free-body diagram of an inverted pendulum, mounted on a cart with a mass, M. The pendulum has a point mass, m, concentrated at the upper end of a rod with zero mass, a length, l, and a frictionless hinge. A motor drives the cart, applying a horizontal force, $u(t)$. A gravity force, mg, acts on m at all times. The pendulum angle relative to the y-axis, θ, its angular speed, $\dot{\theta}'$, the horizontal position of the cart, x, and its speed, x', were selected to be the state variables. The state-space equations derived were heavily nonlinear.[5] They were then linearized around the stationary point, $\mathbf{x_0} = \mathbf{0}$ and $u_0 = 0$, and manipulated to yield the following open-loop model written in perturbation form:

$$\frac{d}{dt}\delta\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u$$

However, since $\mathbf{x_0} = \mathbf{0}$ and $u_0 = 0$, then let: $\mathbf{x} = \mathbf{x}_0 + \delta\mathbf{x} = \delta\mathbf{x}$ and $u = u_0 + \delta u = \delta u$. Thus the state equation may be rewritten as (*Prasad*, *2012*):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{mg}{M} & 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{-1}{Ml} \\ 0 \\ \dfrac{1}{M} \end{bmatrix}$$

Assuming the output to be the horizontal position of $m = x_m = x + l\sin\theta = x + l\theta$ for a small angle, θ, the output equation becomes:

$$y = l\theta + x = \mathbf{C}\mathbf{x} = [\,l \quad 0 \quad 1 \quad 0\,]\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

`Given that:` M `= 2.4 kg,` m `= 0.23 kg,` l `= 0.36 m,` g `= 9.81 m/s`^{2}`, use MATLAB to find the transfer function,` $G(s) = Y(s)/U(s) = X_m(s)/U(s)$.

MATLAB ML

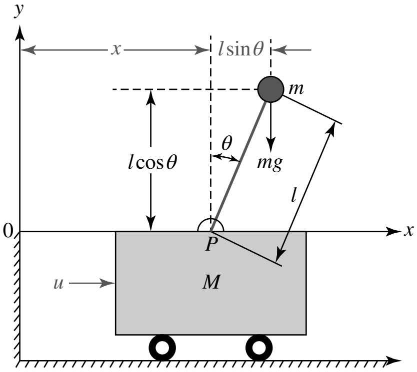

FIGURE P3.15 Motor-driven inverted pendulum cart system[6]

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

26. **Control of HIV/AIDS.** Problem 51 in Chapter 2 introduced a model for HIV infection. If retroviral drugs, RTIs and PIs as discussed in Problem 17 in Chapter 1, are used, the model is modified as follows (*Craig*, *2004*):

$$\frac{dT}{dt} = s - dT - (1 - u_1)\,\beta T v$$

$$\frac{dT^*}{dt} = (1 - u_1)\,\beta T v - \mu T^*$$

$$\frac{dv}{dt} = (1 - u_2)\,kT^* - cv$$

where $0 \le u_1 \le 1, 0 \le u_2 \le 1$ represent the effectiveness of the RTI and PI medication, respectively.

a. Obtain a state-space representation of the HIV/AIDS model by linearizing the equations about the

$$(T_0,\ T_0^*,\ v_0) = \left(\frac{c\mu}{\beta k}, \frac{s}{\mu} - \frac{cd}{\beta k}, \frac{sk}{c\mu} - \frac{d}{\beta}\right)$$

[5] As noted in the introduction to Section 3.7, the techniques for solving such nonlinear state equations are beyond the scope of this course.

[6] Prasad, L., Tyagi, B., and Gupta, H. Modeling & Simulation for Optimal Control of Nonlinear Inverted Pendulum Dynamical System using PID Controller & LQR. *IEEE Computer Society Sixth Asia Modeling Symposium*, 2012, pp. 138–143. Figure 1 p. 139. Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

equilibrium with $u_{10} = u_{20} = 0$. This equilibrium represents the asymptomatic HIV-infected patient. Note that each one of the above equations is of the form $\dot{x}_i = f_i(x_i, u_1, u_2)$, $i = 1, 2, 3$.

b. If Matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}_{T_0, T_0^*, v_0}; \qquad \mathbf{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \end{bmatrix}_{T_0, T_0^*, v_0}$$

and we are interested in the number of free HIV viruses as the system's output,

$$\mathbf{C} = [0 \quad 0 \quad 1]$$

show that

$$\mathbf{A} = \begin{bmatrix} -(d + \beta v_0) & 0 & -\beta T_0 \\ \beta v_0 & -\mu & \beta T_0 \\ 0 & k & -c \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \beta T_0 v_0 & 0 \\ -\beta T_0 v_0 & 0 \\ 0 & -kT_0^* \end{bmatrix}$$

c. Typical parameter values and descriptions for the HIV/AIDS model are shown in the following table. Substitute the values from the table into your model and write as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

Table of HIV/AIDS Model Parameters[7]

t	Time	days
d	Death of uninfected T cells	0.02/day
k	Rate of free viruses produced per infected T cell	100 counts/cell
s	Source term for uninfected T cells	$10/\text{mm}^3$/day
β	Infectivity rate of free virus particles	$2.4 \times 10^{-5}/\text{mm}^3$/day
c	Death rate of viruses	2.4/day
μ	Death rate of infected T cells	0.24/day

27. Hybrid vehicle. For Problem 18 in Chapter 1 we developed the functional block diagrams for the cruise control of serial, parallel, and split-power hybrid electric vehicles (HEV). Those diagrams showed that the engine or electric motor or both may propel the vehicle. When electric motors are the sole providers of the motive force, the forward paths of all HEV topologies are similar. In general, such a forward path can be represented (*Preitl, 2007*) by a block diagram similar to the one of Figure P3.16.

Assume the motor to be an armature-controlled dc motor. In this diagram, K_A is the power amplifier gain; $G_e(s)$ is the transfer function of the motor electric circuit and consists of a series inductor and resistor, L_a

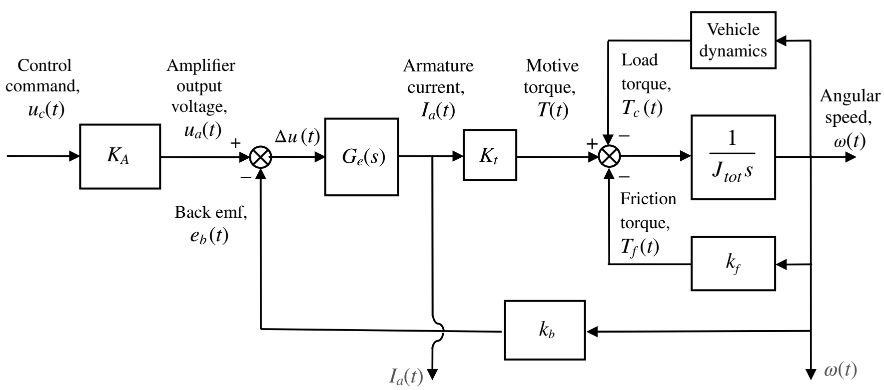

FIGURE P3.16 Block diagram representation of an HEV forward path[8]

[7] Craig, I. K., Xia, X., and Venter, J. W. *AIDS Education Into the Electrical Engineering Curriculum at the University/AIDS Education Into the Electrical Engineering Curriculum at the University of Pretoria*. IEEE Transactions on Education, vol. 47, no. 1, February 2004, pp. 65–73. Table II, p. 67. Modelling Symposium (AMS), 2012 Sixth Asia by IEEE. Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

[8] Preitl, Z., Bauer,P., and J. Bokor, J. A Simple Control Solution for Traction Motor Used in Hybrid Vehicles. *4th International Symposium on Applied Computational Intelligence and Informatics. IEEE*, 2007. Adapted from Figure 2, p. 2

and R_a, respectively; K_t is the motor torque constant; J_{tot}, is the sum of the motor inertia, J_m, the inertias of the vehicle, J_{veh}, and the two driven wheels, J_w, both of which are reflected to the motor shaft; k_f is the coefficient of viscous friction; and k_b is the back emf constant.

The input variables are $u_c(t)$, the command voltage from the electronic control unit and $T_c(t)$, the load torque. The output variables in this block diagram are the motor angular speed, $\omega(t)$, and its armature current, $I_a(t)$.

a. Write the basic time-domain equations that characterize the relationships between the state, input, and output variables for the block diagram of Figure P3.16, given that the state variables are the motor armature current, $I_a(t)$, and angular speed, $\omega(t)$.

b. Write the resulting state-space equations and then represent them in matrix form. Regard the load torque $T_c(t)$ as an extra input to the system. Thus, in your resulting state-space representation, the system will have two inputs and two outputs.

28. **Parabolic trough collector.** A transfer function model from fluid flow to fluid temperature for a parabolic trough collector was introduced in Problem 53, Chapter 2. A more detailed model for the response of this system is given under specific operation conditions (*Camacho, 2012*) by:

$$\frac{H}{Q}(s) = \frac{137.2 \times 10^{-6}}{s^2 + 0.0224s + 196 \times 10^{-6}} e^{-39s}$$

Find an appropriate state-space representation for the system.

Chapter 4 Problems

SS 1. Derive the output responses for all parts of Figure 4.7. [Section: 4.4]

2. For each one of the systems in Figure P4.1, find an analytic expression for the output. Also indicate the time constant, rise time, and settling time. [Sections: 4.2, 4.3]

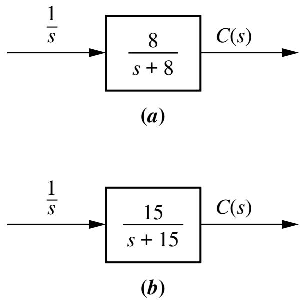

FIGURE P4.1

3. Assuming an uncharged capacitor in Figure P4.2, use Laplace tranform to find an expression for the voltage across the capacitor after the switch closes at $t = 0$. Find the time constant, rise time, and settling time for the calculated voltage. [Sections: 4.2, 4.3]

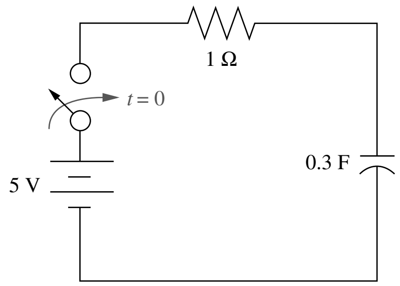

FIGURE P4.2

SS 4. `Plot the step response for Problem 3 using MATLAB. From your plots, find the time constant, rise time, and settling time.` MATLAB ML

5. For the system shown in Figure P4.3, (a) find an equation that relates settling time of the velocity of the mass to M; (b) find an equation that relates rise time of the velocity of the mass to M. [Sections: 4.2, 4.3]

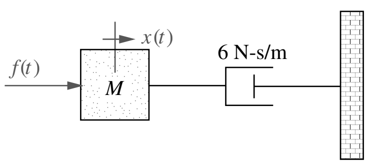

FIGURE P4.3

6. For each of the transfer functions shown below, find the locations of the poles and zeros, plot them on the s-plane, and then write an expression for the general form of the step response without solving for the inverse Laplace transform. State the nature of each response (overdamped, underdamped, and so on). [Sections: 4.3, 4.4]

a. $T(s) = \dfrac{2}{s+2}$

b. $T(s) = \dfrac{5}{(s+3)(s+6)}$

c. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

d. $T(s) = \dfrac{20}{s^2+6s+144}$

e. $T(s) = \dfrac{s+2}{s^2+9}$

f. $T(s) = \dfrac{(s+5)}{(s+10)^2}$

7. `Find the poles of T(s) using MATLAB [Section: 4.2]` MATLAB ML

$$T(s) = \frac{s^2+5s+10}{s^4+7s^3+3s^2-6s+2}$$

8. Find the transfer function and the corresponding poles for the following state-space system: [Section: 4.10]

$$\dot{\mathbf{x}} = \begin{bmatrix} 8 & -2 & 3 \\ 0 & 1 & 4 \\ 7 & 9 & 10 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \mathbf{x}; \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

9. `Repeat Problem 8 using MATLAB. [Section: 4.10]` MATLAB ML

10. Assume in Figure P4.4 that $f(t) = u(t)$. Find $x(t)$. [Section: 4.4]

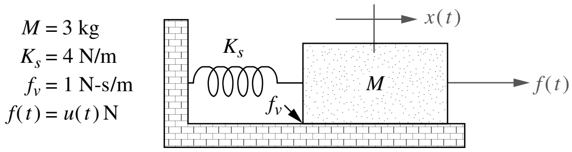

FIGURE P4.4

SS **11.** Find the damping ratio and natural frequency for each second-order system of Problem 6 and show that the value of the damping ratio conforms to the type of response (underdamped, overdamped, and so on) predicted in that problem. [Section: 4.5]

12. Using Laplace transforms, find the analytic expression for the output of a system that has a dc gain of 1, a damping ratio of 0.25, and a natural frequency of 30 rad/sec. The system is excited with a unit-step input. [Section: 4.6]

13. For each of the second-order systems that follow, find ζ, ω_n, T_s, T_p, T_r, and $\%OS$. [Section: 4.6]

SS **a.** $T(s) = \dfrac{16}{s^2 + 3s + 16}$

b. $T(s) = \dfrac{0.04}{s^2 + 0.02s + 0.04}$

c. $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$

14. Repeat Problem 13 using MATLAB. Have the computer program estimate the given specifications and plot the step responses. Estimate the rise time from the plots. [Section: 4.6]

MATLAB **ML**

15. Use MATLAB's Linear System Analyzer and obtain settling time, peak time, rise time, and percent overshoot for each of the systems in Problem 13. [Section: 4.6]

GUI Tool **GUIT**

16. Find the location of the poles of second-order systems with the following specifications: [Section: 4.6]

a. $\%OS = 15\%$; $T_s = 0.5$ second

b. $\%OS = 8\%$; $T_p = 10$ seconds

c. $T_s = 1$ seconds; $T_p = 1.1$ seconds

17. Consider the translational mechanical system of Figure P4.5. [Section: 4.6]

a. Find the transfer function $G(s) = X(s)/F(s)$.

b. Assuming a unit step as the input, calculate $\zeta, \omega_n, \%OS, T_s, T_p, T_r$, and C_{final}.

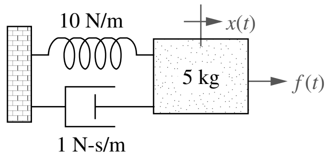

FIGURE P4.5

18. For the system shown in Figure P4.6, a step torque is applied at $\theta_1(t)$. Find: SS

a. The transfer function, $G(s) = \theta_2(s)/T(s)$

b. The percent overshoot, settling time, and peak time for $\theta_2(t)$. [Section: 4.6]

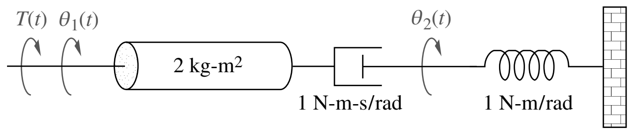

FIGURE P4.6

19. The derivation of Eq. (4.42) to calculate the settling time for a second-order system assumed an underdamped system ($\zeta < 1$). In this problem, you will calculate a similar result for a critically damped system ($\zeta = 1$).

a. Show that the unit-step response for a system with transfer function $\dfrac{C(s)}{R(s)} = \dfrac{a^2}{(s+a)^2}$ is $c(t) = 1 - e^{-at}(1 + at)$. (Note: $\mathscr{L}\left\{\dfrac{1}{(s+a)^2}\right\} = te^{-at}$. Optional: You can derive this result similarly to Example 2.2.)

b. Show that the settling time can be found by solving for T_s in $e^{-aT_s}(1 + aT_s) = 0.02$.

c. Use MATLAB to plot $e^{-x}(1 + x) = 0.02$ vs. x. Use the plot to show that $T_s = \dfrac{5.834}{a}$.

MATLAB **ML**

20. For the following transfer function with a unit-step input, find the percent overshoot, settling time, rise time, peak time, and c_{final}. $T(s) = \dfrac{300}{(s^2 + 2.4s + 9)(s + 25)}$ [Section: 4.7]

SS **21.** For each of the three unit step responses shown in Figure P4.7, find the transfer function of the system. [Sections: 4.3, 4.6]

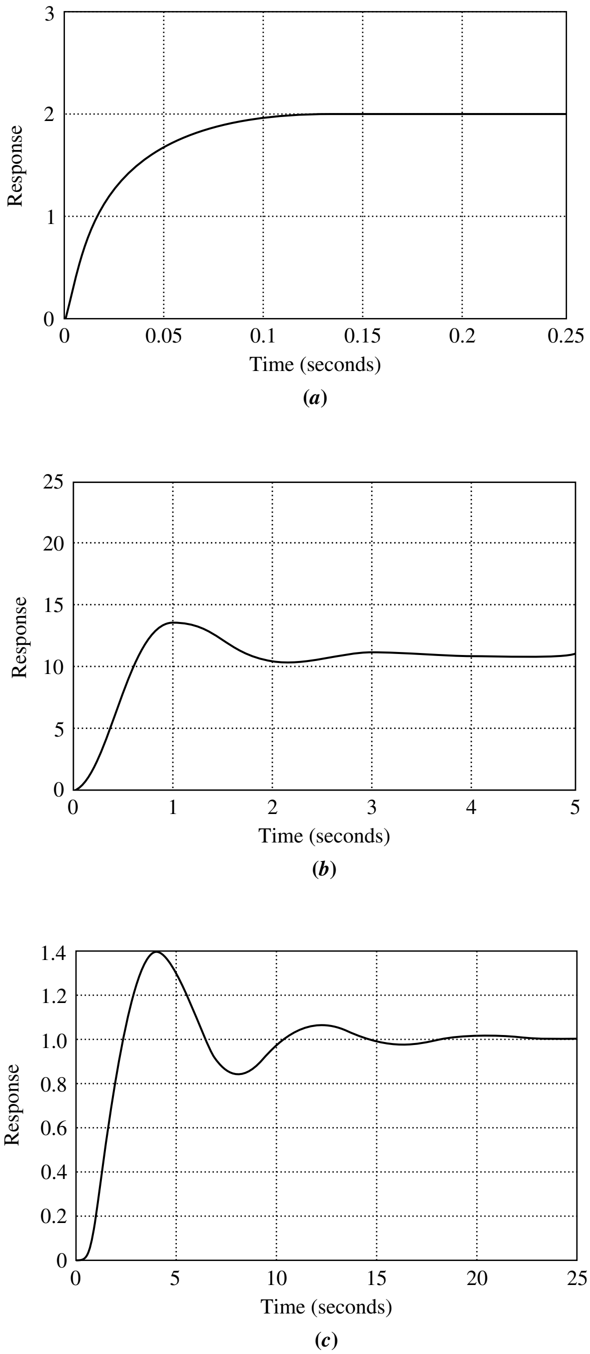

FIGURE P4.7

22. Examine each one of the following response functions to see if it is possible to cancel the zero with a pole. If it is, determine the approximate response, percent overshoot, settling time, rise time, and peak time. [Section: 4.8].

a. $C(s) = \dfrac{(s+5)}{s(s+1)(s^2+3s+10)}$

b. $C(s) = \dfrac{(s+5)}{s(s+2)(s^2+4s+15)}$

c. $C(s) = \dfrac{(s+5)}{s(s+4.5)(s^2+2s+20)}$

d. $C(s) = \dfrac{(s+5)}{s(s+4.9)(s^2+5s+20)}$

23. `Using MATLAB, plot the time response of Problem 22`**a**` and from the plot determine percent overshoot, settling time, rise time, and peak time. [Section: 4.8]` MATLAB ML

24. Find peak time, settling time, and percent overshoot for only those responses below that can be approximated as second-order responses. [Section: 4.8] SS

a. $c(t) = 0.003500 - 0.001524e^{-4t}$
$-0.001976e^{-3t}\cos(22.16t)$
$-0.0005427e^{-3t}\sin(22.16t)$

b. $c(t) = 0.05100 - 0.007353e^{-8t}$
$-0.007647e^{-6t}\cos(8t)$
$-0.01309e^{-6t}\sin(8t)$

c. $c(t) = 0.009804 - 0.0001857e^{-5.1t}$
$-0.009990e^{-2t}\cos(9.796t)$
$-0.001942e^{-2t}\sin(9.796t)$

d. $c(t) = 0.007000 - 0.001667e^{-10t}$
$-0.008667e^{-2t}\cos(9.951t)$
$-0.0008040e^{-2t}\sin(9.951t)$

25. A system is represented by the state and output equations that follow. Without solving the state equation, find the poles of the system. [Section: 4.10] State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 3 \\ -4 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}u(t)$$
$$y = [5 \quad 1]\mathbf{x}$$

26. Without solving the state equation, find [Section: 4.10] State Space SS
    a. the characteristic equation and
    b. the poles of the system for

$$\dot{\mathbf{x}} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 5 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}u(t)$$
$$y = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix}\mathbf{x}$$

SS 27. Given the following state-space representation of a system, find $Y(s)$: [Section: 4.10] State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\sin 3t$$
$$y = \begin{bmatrix} 1 & 2 \end{bmatrix}\mathbf{x};\ \mathbf{x}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

28. Given the following system represented in state space, solve for $Y(s)$ using the Laplace transform method for solution of the state equation: [Section: 4.10] State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -4 & 1 \\ 0 & 0 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}e^{-t}$$
$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\mathbf{x};\ \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

29. Use Laplace transforms to solve the following stage-space equation for $y(t)$ when the input $u(t)$ is a unit step. [Section: 4.10] State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 0 \\ -1 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}u(t)$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x};\ \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

SS 30. Solve for $y(t)$ for the following system represented in state space, where $u(t)$ is the unit step. Use the Laplace transform approach to solve the state equation. [Section: 4.10]

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -6 & 1 \\ 0 & 0 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}u(t)$$
$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\mathbf{x};\ \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

31. `Use MATLAB to plot the step response of Problem 30. [Section: 4.10]` MATLAB ML

32. `Repeat Problem 30 using MATLAB's Symbolic Math Toolbox and Eq. (4.96). In addition, run your program with an initial condition,` Symbolic Math SM

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.\ \texttt{[Section : 4.10]}$$

33. Using classical (not Laplace) methods only, solve for the state-transition matrix, the state vector, and the output of the system represented here. [Section: 4.11] State Space SS SS

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -5 \end{bmatrix}\mathbf{x};\quad y = \begin{bmatrix} 1 & 2 \end{bmatrix}\mathrm{x};$$
$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

34. Solve for $y(t)$ for the following system represented in state space, where $u(t)$ is the unit step. Use the classical approach to solve the state equation. [Section: 4.11] State Space SS SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -6 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}u(t)$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x};\ \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

35. `Repeat Problem 34 using MATLAB's Symbolic Math Toolbox and Eq. (4.109). In addition, run your program with an initial condition,`

Symbolic Math **SM**

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}. \text{[Section : 4.11]}$$

36. Using methods described in Appendix H.1, simulate the following system and plot the step response. Verify the expected values of percent overshoot, peak time, and settling time.

State Space **SS**

$$T(s) = \frac{1}{s^2 + 0.8s + 1}$$

SS **37.** A human responds to a visual cue with a physical response, as shown in Figure P4.8. The transfer function that relates the output physical response, $P(s)$, to the input visual command, $V(s)$, is (*Stefani*, *1973*).

$$G(s) = \frac{P(s)}{V(s)} = \frac{(s + 0.5)}{(s + 2)(s + 5)}$$

Do the following:

a. Evaluate the output response for a unit step input using the Laplace transform.

b. Represent the transfer function in state space.

State Space **SS**

c. `Use MATLAB to simulate the system and obtain a plot of the step response.`

MATLAB **ML**

38. Upper motor neuron disorder patients can benefit and regain useful function through the use of functional neuroprostheses. The design requires a good understanding of muscle dynamics. In an experiment to determine muscle responses, the identified transfer function was (*Zhou*, *1995*)

$$M(s) = \frac{2.5e^{-0.008s}(1 + 0.172s)(1 + 0.008s)}{(1 + 0.07s)^2(1 + 0.05s)^2}$$

Find the unit step response of this transfer function.

39. When electrodes are attached to the mastoid bones (right behind the ears) and current pulses are applied, a person will sway forward and backward. It has been found that the transfer function from the current to the subject's angle (in degrees) with respect to the vertical is given by (*Nashner*, *1974*)

$$\frac{\theta(s)}{I(s)} = \frac{5.8(0.3s + 1)e^{-0.1s}}{(s + 1)(s^2/1.2^2 + 0.6s/1.2 + 1)}$$

a. Determine whether a dominant pole approximation can be applied to this transfer function.

b. Find the body sway caused by a 250 μA pulse of 150 msec duration.

40. The response of the deflection of a fluid-filled catheter to changes in pressure can be modeled using a second-order model. Knowledge of the parameters of the model is important because in cardiovascular applications the undamped natural frequency should be close to five times the heart rate. However, due to sterility and

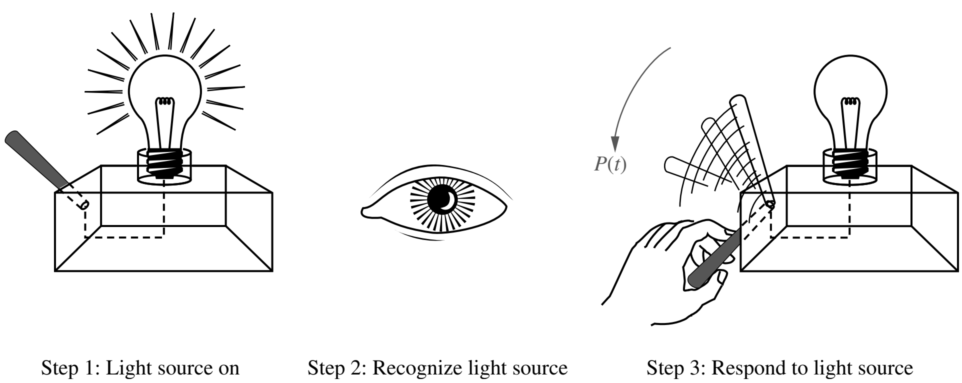

FIGURE P4.8 Steps in determining the transfer function relating output physical response to the input visual command

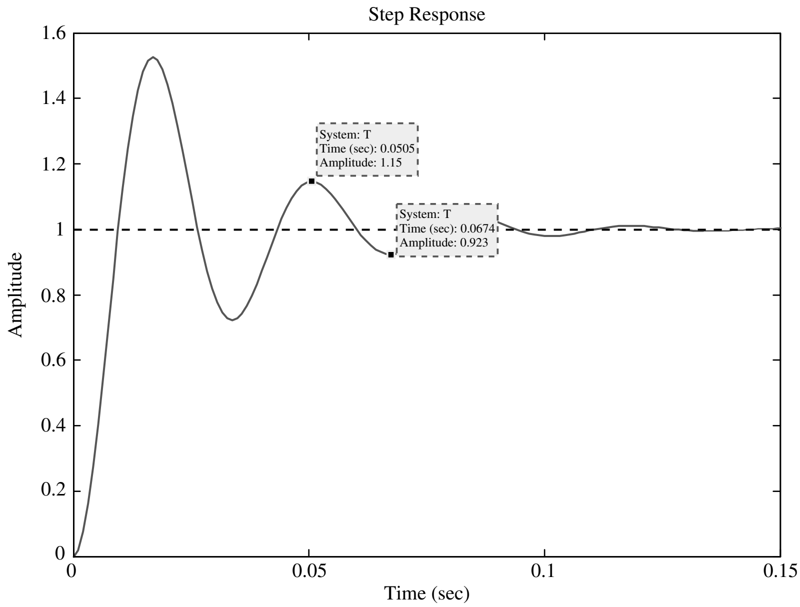

FIGURE P4.9

other considerations, measurement of the parameters is difficult. A method to obtain transfer functions using measurements of the amplitudes of two consecutive peaks of the response and their timing has been developed (*Glantz*, *1979*). Assume that Figure P4.9 is obtained from catheter measurements. Using the information shown and assuming a second-order model excited by a unit step input, find the corresponding transfer function.

41. Several factors affect the workings of the kidneys. For example, Figure P4.10 shows how a step change in arterial flow pressure affects renal blood flow in rats. In the "hot tail" part of the experiment, peripheral thermal receptor stimulation is achieved by inserting the rat's tail in heated water. Variations between different test subjects are indicated by the vertical lines. It has been argued that the "control" and "hot tail" responses are identical except for their steady-state values (*DiBona*, *2005*).

a. Using Figure P4.10, obtain the normalized ($C_{\text{final}} = 1$) transfer functions for both responses.

b. Use MATLAB to prove or disprove the assertion about the "control" and "hot tail" responses. MATLAB ML

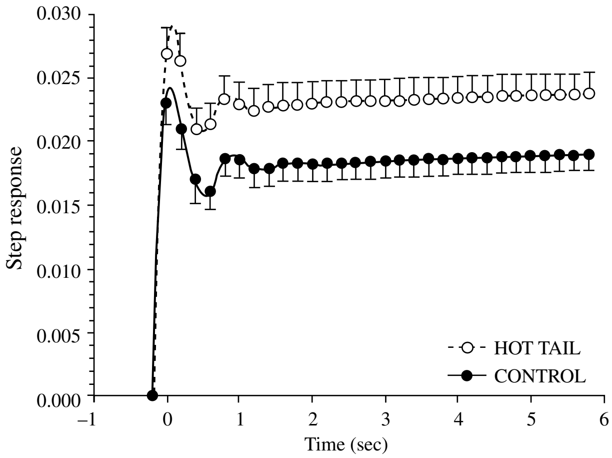

FIGURE P4.10[1]

42. The transfer function of a nanopositioning device capable of translating biological samples within a few μm uses a piezoelectric actuator and a linear variable differential transformer (LDVT) as a displacement sensor. The transfer function from input to displacement has been found to be (*Salapaka*, *2002*)

[1] DiBona G.F. Physiology in perspective: The Wisdom of the Body. Neural control of the kidney Am. J. Physiol. Regul. Integr. Comp. Physiol Vol. 289, 2005. Fig. 6 p. R639. Used with permission.

$$G(s) = \frac{9.7 \times 10^4 (s^2 - 14400s + 106.6 \times 10^6)}{(s^2 + 3800s + 23.86 \times 10^6)(s^2 + 240s + 2324.8 \times 10^3)}$$

Use a dominant-pole argument to find an equivalent transfer function with the same numerator but only three poles.

MATLAB ML

Use MATLAB to find the actual size and approximate system unit step responses, plotting them on the same graph.

Explain the differences between both responses given that both pairs of poles are so far apart.

43. At some point in their lives, most people will suffer from at least one onset of low back pain. This disorder can trigger excruciating pain and temporary disability, but its causes are hard to diagnose. It is well known that low back pain alters motor trunk patterns; thus it is of interest to study the causes for these alterations and their extent. Due to the different possible causes of this type of pain, a "control" group of people is hard to obtain for laboratory studies. However, pain can be stimulated in healthy people and muscle movement ranges can be compared. Controlled back pain can be induced by injecting saline solution directly into related muscles or ligaments. The transfer function from infusion rate to pain response was obtained experimentally by injecting a 5% saline solution at six different infusion rates over a period of 12 minutes. Subjects verbally rated their pain every 15 seconds on a scale from 0 to 10, with 0 indicating no pain and 10 unbearable pain. Several trials were averaged and the data was fitted to the following transfer function:

$$G(s) = \frac{9.72 \times 10^{-8}(s + 0.0001)}{(s + 0.009)^2(s^2 + 0.018s + 0.0001)}$$

For experimentation, it is desired to build an automatic dispensing system to make the pain level constant as shown in Figure P4.11. It follows that ideally the injection system transfer function has to be

$$M(s) = \frac{1}{G(s)}$$

to obtain an overall transfer function $M(s)G(s) \approx 1$. However, for implementation purposes, $M(s)$ must have at least one more pole than zeros (*Zedka*, *1999*). Find a suitable transfer function, $M(s)$ by inverting $G(s)$ and adding poles that are far from the imaginary axis.

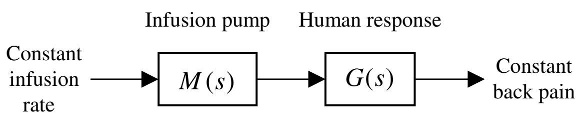

FIGURE P4.11

44. An artificial heart works in closed loop by varying its pumping rate according to changes in signals from the recipient's nervous system. For feedback compensation design, it is important to know the heart's open-loop transfer function. To identify this transfer function, an artificial heart is implanted in a calf while the main parts of the original heart are left in place. Then the atrial pumping rate in the original heart is measured while step input changes are effected on the artificial heart. It has been found that, in general, the obtained response closely resembles that of a second-order system. In one such experiment, it was found that the step response has a %OS = 30% and a time of first peak $T_p = 127$ sec (*Nakamura*, *2002*). Find the corresponding transfer function.

45. An observed transfer function from voltage potential to force in skeletal muscles is given by (*Ionescu*, *2005*)

$$T(s) = \frac{450}{(s+5)(s+20)}$$

a. Obtain the system's impulse response.

b. Integrate the impulse response to find the step response.

c. Verify the result in Part **b** by obtaining the step response using Laplace transform techniques.

46. In typical conventional aircraft, longitudinal flight model linearization results in transfer functions with two pairs of complex conjugate poles. Consequently, the natural response for these airplanes has two modes in their natural response. The "short period" mode is relatively well-damped and has a high-frequency oscillation. The "plugoid mode" is lightly damped and its oscillation frequency is relatively low. For example, in a specific aircraft the transfer function from wing elevator deflection to nose angle (angle of attack) is (*McRuer*, *1973*)

$$\frac{\theta(s)}{\delta_e(s)} = -\frac{26.12(s + 0.0098)(s + 1.371)}{(s^2 + 8.99 \times 10^{-3}s + 3.97 \times 10^{-3})(s^2 + 4.21s + 18.23)}$$

a. Find which of the poles correspond to the short period mode and which to the phugoid mode.

b. Peform a "phugoid approximation" (dominant-pole approximation), retaining the two poles and the zero closest to the; ω-axis.

c. Use MATLAB to compare the step responses of the original transfer function and the approximation. MATLAB ML

47. A crosslapper is a machine that takes as an input a light fiber fabric and produces a heavier fabric by laying the original fabric in layers MATLAB ML rotated by 90 degrees. A feedback system is required in order to maintain consistent product width and thickness by controlling its carriage velocity. The transfer function from servomotor torque, $T_m(s)$, to carriage velocity, $Y(s)$, was developed for such a machine (*Kuo, 2008*). Assume that the transfer function is:

$$G(s) = \frac{Y(s)}{T_m(s)} = \frac{33s^4 + 202s^3 + 10061s^2 + 24332s + 170704}{s^7 + 8s^6 + 464s^5 + 2411s^4 + 52899s^3 + 167829s^2 + 913599s + 1076555}$$

a. Use MATLAB to find the partial fraction residues and poles of $G(s)$.

b. Find an approximation to $G(s)$ by neglecting the second-order terms found in **a.**

c. Use MATLAB to plot on one graph the step response of the transfer function given above and the approximation found in **b.** Explain the differences between the two plots.

48. Although the use of fractional calculus in control systems is not new, in the last decade, there is increased interest in its use for several reasons. The most relevant are that fractional calculus differential equations may model certain systems with higher accuracy than integer differential equations, and that fractional calculus compensators might exhibit advantageous properties for control system design. An example of a transfer function obtained through fractional calculus is:

MATLAB
ML

$$G(s) = \frac{1}{s^{2.5} + 4s^{1.7} + 3s^{0.5} + 5}$$

This function can be approximated with an integer rational transfer function (integer powers of s) using Oustaloup's method (*Xue, 2005*). We ask you now to do a little research and consult the aforementioned reference to find and run an M-file that will calculate the integer rational transfer function approximation to $G(s)$ and plot its step response.

49. Mathematical modeling and control of pH processes are quite challenging since the processes are highly nonlinear, due to the logarithmic relationship between the concentration of hydrogen ions [H+] and pH level. The transfer function from input pH to output pH is $G_a(s) = \frac{Y_a(s)}{Y_a(s)} = \frac{14.49e^{-3.3s}}{1478.26s + 1}$, where we assume a delay of 3.3 seconds. $G_a(s)$ is a model for the anaerobic process in a wastewater treatment system in which methane bacteria need the pH to be maintained in its optimal range from 6.8 to 7.2 (*Jiayu, 2009*). Similarly, *Elarafi* (2008) used empirical techniques to model a pH neutralization plant as a second-order system with a pure delay, yielding the following transfer function relating output pH to input pH:

$$G_p(s) = \frac{Y_p(s)}{X_p(s)} = \frac{1.716 \times 10^{-5} e^{-25s}}{s^2 + 6.989 \times 10^{-3}s + 1.185 \times 10^{-6}}$$

where we assume a delay of 25 seconds.

a. Find analytical expressions for the unit-step responses $y_a(t)$ and $y_p(t)$ for the two processes, $G_a(s)$ and $G_p(s)$. (Hint: Use the time shift theorem in Table 2.2).

b. Use Simulink to plot $y_a(t)$ and $y_p(t)$ on a single graph.

Simulink
SL

50. An IPMC (ionic polymer-metal composite) is a Nafion sheet plated with gold on both sides. An IPMC bends when an electric field is applied across its thickness. IPMCs have been used as robotic actuators in several applications and as active catheters in biomedical applications. With the aim of improving actuator settling times, a state-space model has been developed for a 20 mm × 10 mm × 0.2 mm polymer sample (*Mallavarapu, 2001*):

State Space
SS

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -8.34 & -2.26 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 12.54 & 2.26 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where u is the applied input voltage and y is the deflection at one of the material's tips when the sample is tested in a cantilever arrangement.

a. Find the state-transition matrix for the system.

b. From Eq. (4.109) in the text, it follows that if a system has zero initial conditions, the system output for any input can be directly calculated from the state-space representation and the state-transition matrix using

$$y(t) = \mathbf{C}\mathbf{x}(t) = \int \mathbf{C}\Phi(t-\tau)\,\mathbf{B}u(\tau)d\tau$$

Use this equation to find the zero initial condition unit step response of the IPMC material sample.

c. Use MATLAB to verify that your step response calculation in Part b is correct. MATLAB **ML**

51. Figure P4.12 shows the free-body diagrams for planetary gear components used in the variable valve timing (VVT) system of an internal combustion engine (*Ren*, *2010*). Here an electric motor is used to drive the carrier. Analysis showed that the electric motor with planetary gear load (Figure P4.12) may be represented by the following equation:

$$\Omega_c(s) = G_e(s)E_a(s) + G_m(s)T_{cam}(s)$$

where $\Omega_c(s)$ is the output carrier angular speed in rad/s, $E_a(s)$ is the input voltage applied to the armature, and $T_{cam}(s)$ is the input load torque. The voltage input transfer function, $G_e(s)$, is

$$G_e(s) \cong \frac{K_\tau}{R_m(Js+D)+K_\tau K_m} = \frac{45}{0.2s+1}$$

and the load torque input transfer function, $G_m(s)$, is

$$G_m(s) \cong \frac{-R_m k}{R_m(Js+D)+K_\tau K_m} = \frac{-5}{0.2s+1}$$

Find an analytical expression for the output carrier angular speed, $\omega_c(t)$, if a step voltage of 100 volts is applied at $t = 0$ followed by an equivalent load torque of 10 N-m, applied at $t = 0.4$ sec.

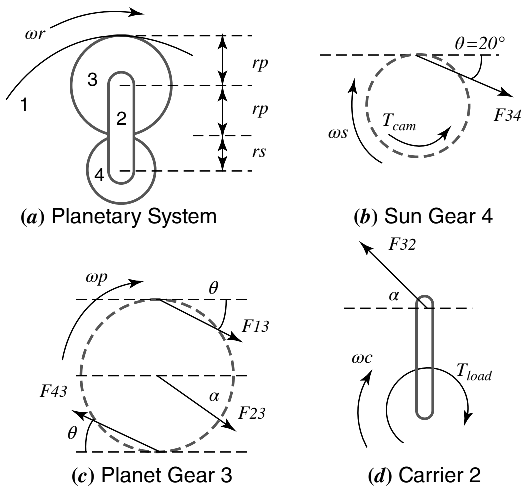

FIGURE P4.12 Free-body diagrams of planetary gear system components[2]

52. A drive system with elastically coupled load (Figure P4.13) has a motor that is connected to the load via a gearbox and a long shaft.

The system parameters are: J_M = drive-side inertia = 0.0338 kg-m^2, J_L = load-side inertia = 0.1287 kg-m^2, $K = C_T$ = torsional spring constant = 1700 N-m/rad, and D = damping coefficient = 0.15 N-m-s/rad.

This system can be reduced to a simple two-inertia model that may be represented by the following transfer function, relating motor shaft speed output, $\Omega(s)$, to electromagnetic torque input (*Thomsen, 2011*):

$$G(s) = \frac{\Omega(s)}{T_{em}(s)} = \frac{1}{s(J_M+J_L)} \cdot \frac{\dfrac{J_L}{C_T}s^2 + \dfrac{D}{C_T}s + 1}{\dfrac{J_M J_L}{C_T(J_M+J_L)}s^2 + \dfrac{D}{C_T}s + 1}$$

Assume the system is at standstill at $t = 0$, when the electromagnetic torque, T_{em}, developed by the motor changes from zero to 50 N-m. Find and plot on one graph, using MATLAB or any other program, the motor shaft speed, $\omega(t)$, rad/sec, for the following two cases: MATLAB **ML**

a. No load torque is applied and, thus, $\omega = \omega_{nl}$.

b. A load torque, $T_L = 0.2\,\omega(t)$ N-m is applied and $\omega = \omega_L$.

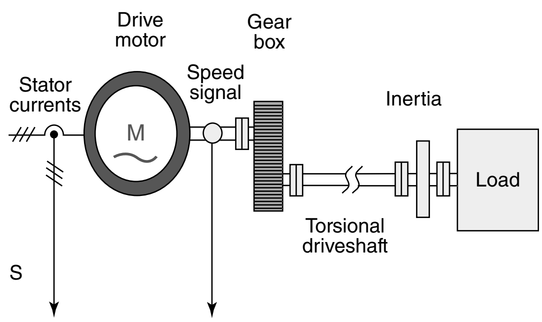

FIGURE P4.13 Partial topology of a typical motor drive system[3]

53. An inverted pendulum mounted on a motor-driven cart was presented in Problem 25 of Chapter 3. The nonlinear state-space equations representing that system were

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[2] Ren Z., and Zhu G. G. Modeling and Control of an Electric Variable Valve Timing System for SI and HCCI Combustion Mode Transition. American Control Conference, San Francisco, CA, 2011, pp. 979–984. Figure 2, p. 980.) Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

[3] Thomsen, S., Hoffmann, N., and Fuchs, F. W. "PI Control, PI-Based State Space Control, and Model-Based Predictive Control for Drive Systems With Elastically Coupled Loads—A Comparative Study." IEEE Transactions On Industrial Electronics, Vol. 58, No. 8, August 2011, pp. 3647–3657. Portion of Figure 1, p. 3648. American Control Conference (ACC), 2011 by IEEE. Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

linearized (*Prasad, 2012*) around a stationary point corresponding to the pendulum point-mass, m, being in the upright position ($x_0 = 0$ at $t = 0$), when the force applied to the cart was zero ($u_0 = 0$). The state-space model developed in that problem is

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}u$$

The state variables are the pendulum angle relative to the y-axis, θ, its angular speed, θ', the horizontal position of the cart, x, and its speed, x'. The horizontal position of m (for a small angle, θ), $x_m = x + l\sin\theta = x + l\theta$, was selected to be the output variable.

MATLAB **ML**

`Given the state-space model developed in that problem along with the output equation you developed in that problem, use MATLAB (or any other computer program) to find and plot the output,` $x_m(t)$`, in meters, for an input force,` $u(t)$`, equal to a unit impulse,` $\delta(t)$`, in Newtons.`[4]

DESIGN PROBLEMS

54. Consider the translational mechanical system shown in Figure P4.14. A 1-pound force, $f(t)$, is applied at $t = 0$. If $f_v = 1$, find K and M such that the response is characterized by a 4-second settling time and a 1-second peak time. Also, what is the resulting percent overshoot? [Section: 4.6]

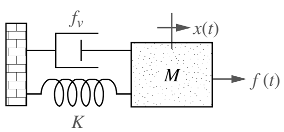


FIGURE P4.14

55. Given the translational mechanical system of Figure P4.14, where $K = 1$ and $f(t)$ is a unit step, find the values of M and f_v to yield a response with 17% overshoot and a settling time of 10 seconds. [Section: 4.6]

56. Given the system shown in Figure P4.15, find the damping, D, to yield a 30% overshoot in output angular displacement for a step input in torque. [Section: 4.6]

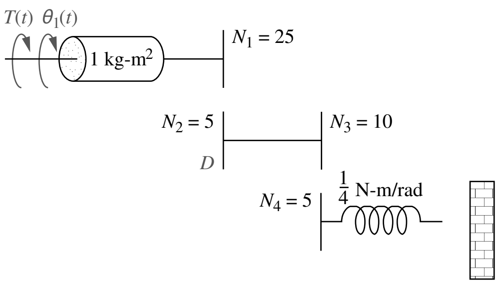


FIGURE P4.15

57. Find M and K, shown in the system of Figure P4.16, to yield $x(t)$ with 16% overshoot and 20 seconds settling time for a step input in motor torque, $T_m(t)$. [Section: 4.6]

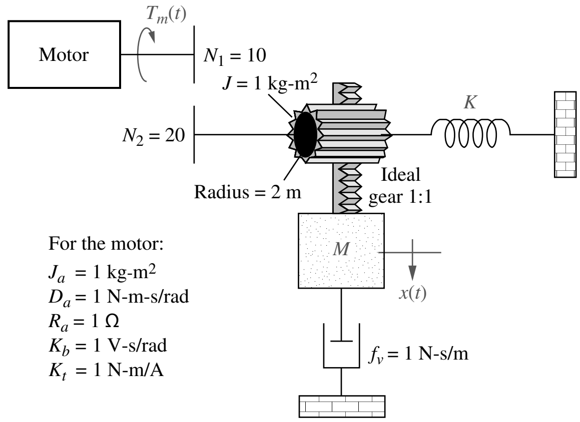


FIGURE P4.16

58. If $v_i(t)$ is a step voltage in the network shown in Figure P4.17, find the value of the resistor such that a 20% overshoot in voltage will be seen across the capacitor if $C = 10^{-6}$ F and $L = 1$ H. [Section: 4.6] **SS**

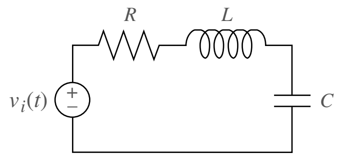


FIGURE P4.17

59. If $v_i(t)$ is a step voltage in the network shown in Figure P4.17, find the values of R and C to yield a 20% overshoot and a 1 ms settling time for $v_c(t)$ if $L = 1$ H. [Section: 4.6]

60. Given the circuit of Figure P 4.17, where $C = 10\,\mu$F, find R and L to yield 15% overshoot with a settling time of 7 ms for the capacitor voltage. The input, $v(t)$, is a unit step. [Section: 4.6]

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

61. **Control of HIV/AIDS**. In Chapter 3, Problem 26, we developed a linearized state-space model of HIV infection. The model assumed that two different drugs were used to combat the spread of the HIV virus. Since this book focuses on single-input, single-output systems, only one of the two drugs will be considered. We will assume that only RTIs are used as an input. Thus, in the equations of Chapter 3, Problem 26, $u_2 = 0$ (*Craig, 2004*). State Space **SS**

a. Show that when using only RTIs in the linearized system of Problem 26, Chapter 3, and substituting the typical parameter values given in the table of

[4] Hint: Use the command "impulseplot" over a time period from 0 to 11.0 seconds with a step of 0.1 seconds.

Problem 26**c**, Chapter 3, the resulting state-space representation for the system is given by

$$\begin{bmatrix} \dot{T} \\ \dot{T}^* \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -0.04167 & 0 & -0.0058 \\ 0.0217 & -0.24 & 0.0058 \\ 0 & 100 & -2.4 \end{bmatrix} \times \begin{bmatrix} T \\ T^* \\ v \end{bmatrix} + \begin{bmatrix} 5.2 \\ -5.2 \\ 0 \end{bmatrix} u_1$$

$$y = [0 \quad 0 \quad 1] \begin{bmatrix} T \\ T^* \\ v \end{bmatrix}$$

b. Obtain the transfer function from RTI efficiency to virus count; namely, find $\frac{Y(s)}{U_1(s)}$.

c. Assuming RTIs are 100% effective, what will be the steady-state change of virus count in a given infected patient? Express your answer in virus copies per ml of plasma. Approximately how much time will the medicine take to reach its maximum possible effectiveness?

62. Hybrid vehicle. Assume that the car motive dynamics for a hybrid electric vehicle (HEV) can be described by the transfer function

$$\frac{\Delta V(s)}{\Delta F_e(s)} = \frac{1}{1908s + 10}$$

where AV is the change of velocity in m/sec and ΔF_e is the change in excess motive force in N necessary to propel the vehicle.

a. Find an analytical expression for $\Delta v_{(t)}$ for a step change in excess motive force $\Delta F_e = 2650$ N.

b. `Simulate the system using MATLAB. Plot the expression found in Part` **a** `together with your simulated plot.` MATLAB **ML**

63. Parabolic trough collector. Figure P4.18 illustrates the results of an open-loop step-response experiment performed on a parabolic trough collector setup (*Camacho*, *2012*). In this experiment, the fluid flow on the system is suddenly decreased 1 liter/sec at $t = 0$ hours, resulting in a temperature increase as shown in Figure P4.18. Use the figure to find an *approximate* transfer function for the system. (Note: Since no further information is given on the system dynamics and due to visual approximations, several solutions are possible.)

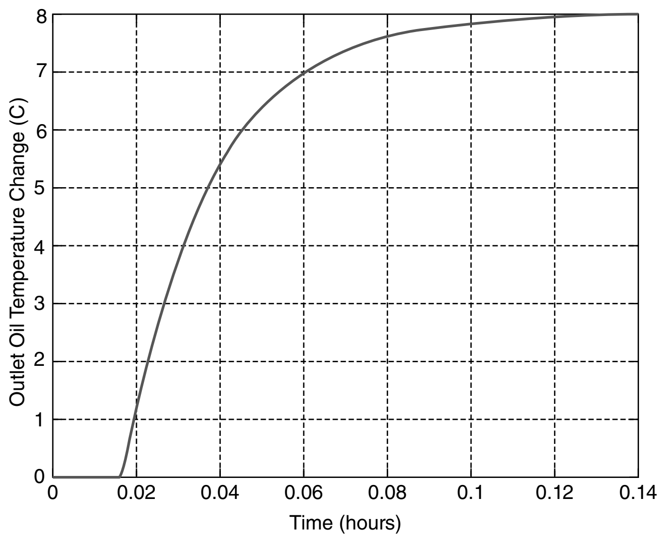


FIGURE P4.18

Chapter 5 Problems

1. Reduce the block diagram shown in Figure P5.1 to a single transfer function, $T(s) = C(s)/R(s)$. Use the following methods:
   a. Block diagram reduction [Section: 5.2]
   b. MATLAB

MATLAB ML

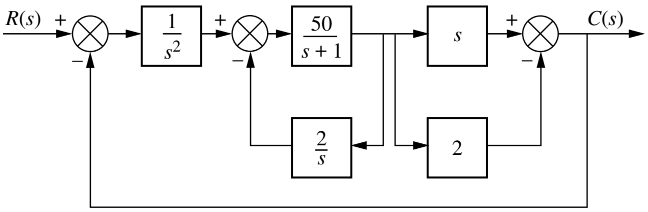


FIGURE P5.1

SS 2. Reduce the system shown in Figure P5.2 to a single transfer function, $T(s) = C(s)/R(s)$. [Section: 5.2]

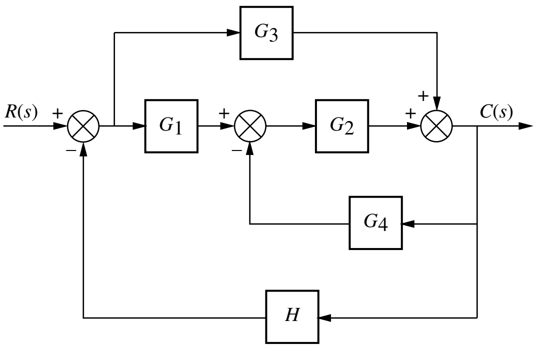


FIGURE P5.2

3. Reduce the block diagram shown in Figure P5.3 to a single block, $T(s) = C(s)/R(s)$. [Section: 5.2]

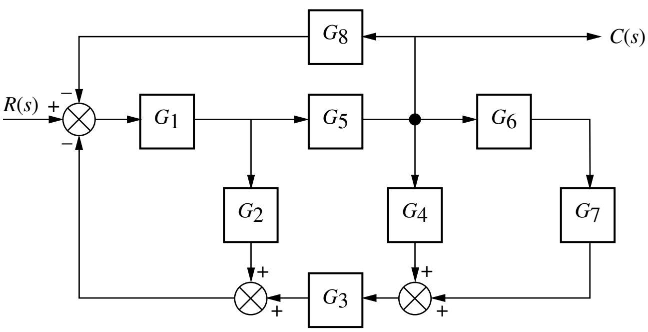


FIGURE P5.3

4. Reduce the system of Figure P5.4 to an equivalent unity-feedback system.

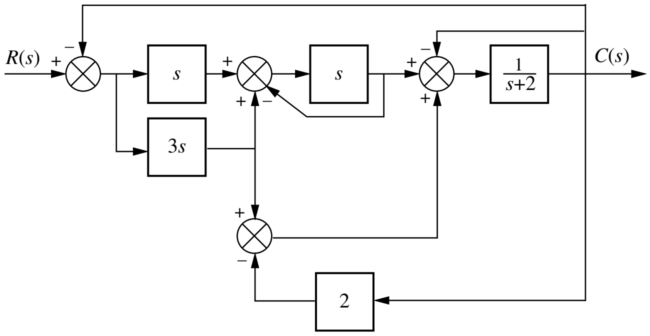


FIGURE P5.4

5. Reduce the block diagram shown in Figure P5.5 to a single transfer function, $T(s) = C(s)/R(s)$. [Section: 5.2]

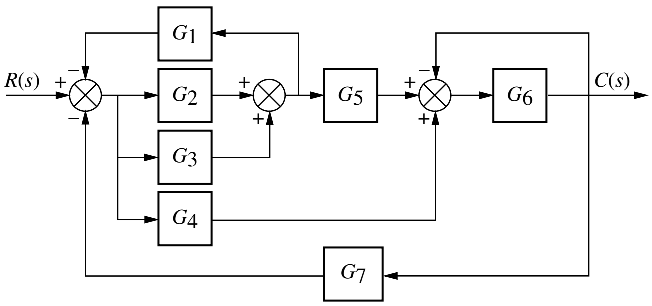


FIGURE P5.5

6. Is the system in Figure P5.6 underdamped? If so, find the percent overshoot, the settling time, and the peak time for a step input.

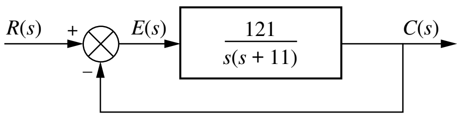


FIGURE P5.6

7. Assuming the input r(*t*) to the system in Figure P5.7 is a unit step, find the output c(*t*).

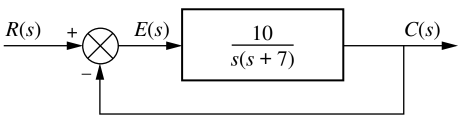


FIGURE P5.7

8. Find the closed-loop of $T(s) = C(s)/R(s)$ for the system of Figure P5.8.

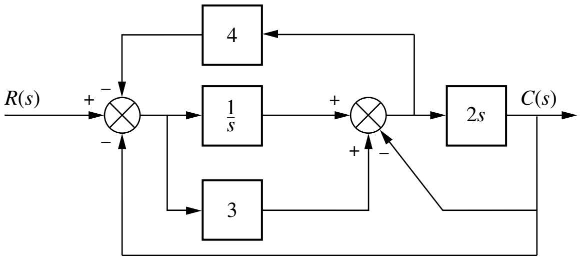


FIGURE P5.8

9. In Figure P5.9, find the value of K that will result in 15% overshoot for step inputs.

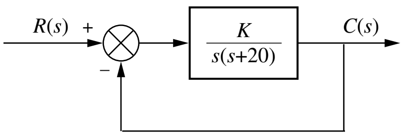


FIGURE P5.9

10. In Figure P5.10, find the values of K and α that will result in a percent overshoot of 10 and a settling time of 0.17 sec.

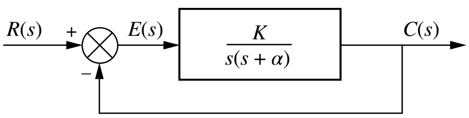


FIGURE P5.10

11. Refer to Figure P5.11. Find the value of K_1 and K_2 that will result in a step response with a peak value of 1.5 sec. and a settling time of 3 sec.

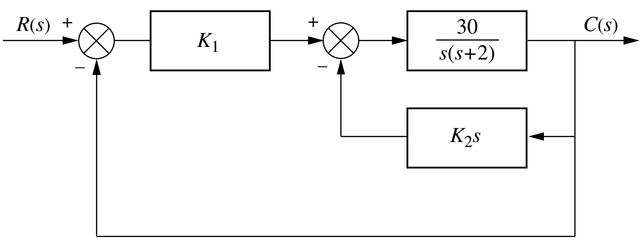


FIGURE P5.11

SS **12.** Find the following for the system shown in Figure P5.12: [Section: 5.3]

a. The equivalent single block that represents the transfer function, $T(s) = C(s)/R(s)$.

b. The damping ratio, natural frequency, percent overshoot, settling time, peak time, rise time, and damped frequency of oscillation.

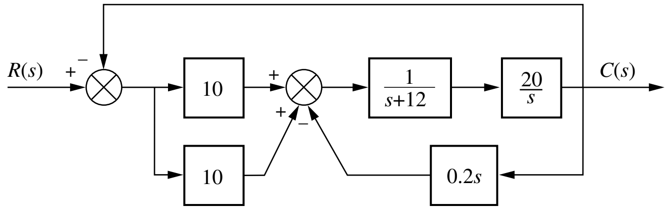


FIGURE P5.12

13. Find ζ, ω_n, percent overshoot, peak time, rise time, and settling time for the system of Figure P5.13. [Section: 5.3]

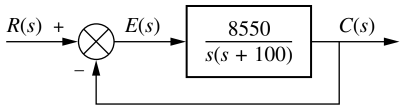


FIGURE P5.13

14. A motor and generator are set up to drive a load as shown in Figure P5.14. If the generator output voltage is $e_g(t) = K_f i_f(t)$, where $i_f(t)$ is the generator's field current, find the transfer function $G(s) = \theta_o(s)/E_i(s)$. For the generator, $K_f = 2\ \Omega$. For the motor, $K_t = 2$ N-m/A and $K_b = 2$ V-s/rad.

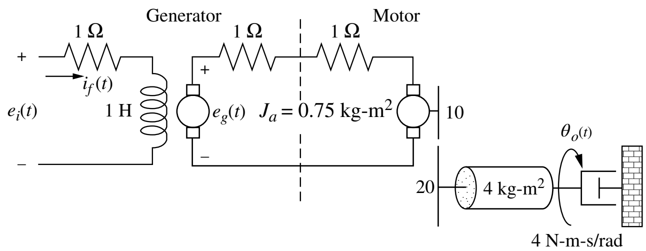


FIGURE P5.14

15. Find $G(s) = E_0(s)/T(s)$ for the system shown in Figure P5.15.

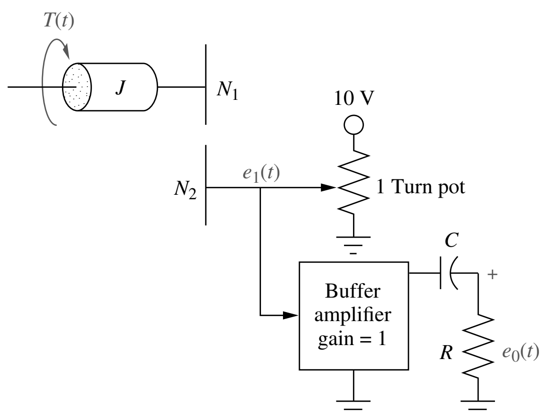


FIGURE P5.15

16. Label signals and draw a signal-flow graph for the block diagrams shown in Problem 1. [Section: 5.4]

17. Given the system below, draw a signal-flow graph and represent the system in state space in the following forms: [Section: 5.7] State Space SS

    a. Phase-variable form

    b. Cascade form

$$G(s)=\frac{200}{(s+10)(s+20)(s+30)}$$

18. Repeat Problem 17 for State Space SS

$$G(s)=\frac{20}{s(s-2)(s+5)(s+8)}$$

[Section: 5.7]

SS 19. Using Mason's rule, find the transfer function, $T(s)=C(s)/R(s)$, for the system represented in Figure P5.16. [Section: 5.5]

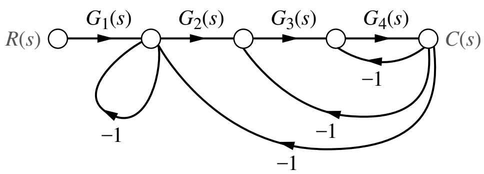


FIGURE P5.16

20. Use Mason's rule to find the transfer function of Figure 5.13 in the text. [Section: 5.5]

21. Represent the following systems in state space in Jordan canonical form. Draw the signal-flow graphs. [Section: 5.7] State Space SS

    SS a. $G(s)=\dfrac{(s+1)(s+2)}{(s+3)^2(s+4)}$

    b. $G(s)=\dfrac{(s+2)}{(s+5)^2(s+7)^2}$

    c. $G(s)=\dfrac{(s+4)}{(s+2)^2(s+5)(s+6)}$

22. Represent the systems below in state space in phase-variable form. Draw the signal-flow graphs. [Section: 5.7] State Space SS

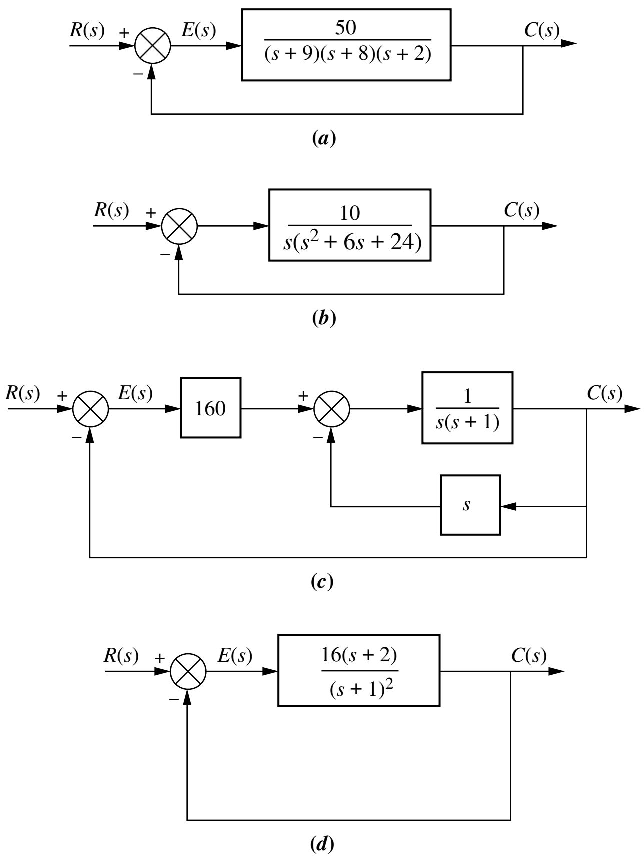
    SS a. $G(s)=\dfrac{s+3}{s^2+2s+7}$

    b. $G(s)=\dfrac{s^2+2s+6}{s^3+5s^2+2s+1}$

    c. $G(s)=\dfrac{s^3+2s^2+7s+1}{s^4+3s^3+5s^2+6s+4}$

23. Repeat Problem 22 and represent each system in controller canonical and observer canonical forms. [Section: 5.7] State Space SS

24. Represent the feedback control systems shown in Figure P5.17 in state space. When possible, represent the open-loop transfer functions separately in cascade and complete the feedback loop with the signal path from output to input. Draw your signal-flow graph to be in one-to-one correspondence to the block diagrams (as close as possible). [Section: 5.7] State Space SS



SS

FIGURE P5.17

25. Develop a state space representation for the system of Figure P5.18 State Space SS

    a. In phase-variable form

    b. In any form other than phase-variable

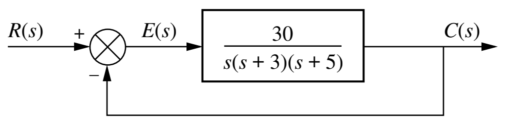


FIGURE P5.18

26. Repeat Problem 25 for the system shown in Figure P5.19. [Section: 5.7]

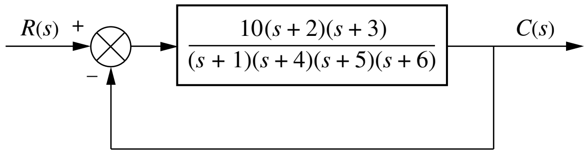


FIGURE P5.19

27. `Use MATLAB to solve Problem 26.` MATLAB **ML**

28. Find a state space-representation for the system of Figure P5.20 Use the indicated state variables $x_1(t)$, $x_3(t)$, and $x_4(t)$; the system's output is $c(t)$ and $x_2(t)$ is the state variable inside $X_1(s)/X_3(s)$. State Space **SS**

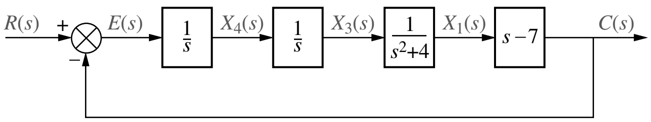


FIGURE P5.20

SS **29.** Consider the rotational mechanical system shown in Figure P5.21. State Space **SS**

a. Represent the system as a signal-flow graph.

b. Represent the system in state space if the output is $\theta_2(t)$.

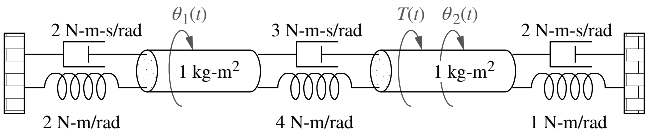


FIGURE P5.21

30. Consider the cascaded subsystems shown in Figure P5.22. If $G_1(s)$ is represented in state space as State Space **SS**

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 r$$
$$y_1 = \mathbf{C}_1\mathbf{x}_1$$

and $G_2(s)$ is represented in state space as

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 y_1$$
$$y_2 = \mathbf{C}_2\mathbf{x}_2$$

show that the entire system can be represented in state space as

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \cdots \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \left[\begin{array}{c:c} \mathbf{A}_1 & \mathbf{0} \\ \hdashline \mathbf{B}_2\mathbf{C}_1 & \mathbf{A}_2 \end{array}\right] \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \cdots \\ \mathbf{0} \end{bmatrix} r$$

$$y_2 = \left[\begin{array}{c:c} \mathbf{0} & \mathbf{C}_2 \end{array}\right] \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix}$$

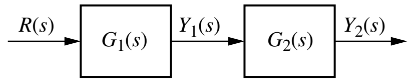


FIGURE P5.22

31. Consider the subsystems shown in Figure P5.23 and connected to form a feedback system. If $G(s)$ is represented in state space as State Space **SS**

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 e$$
$$y = \mathbf{C}_1\mathbf{x}_1$$

and $H(s)$ is represented in state space as

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 y$$
$$p = \mathbf{C}_2\mathbf{x}_2$$

show that the closed-loop system can be represented in state space as

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \cdots \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \left[\begin{array}{c:c} \mathbf{A}_1 & -\mathbf{B}_1\mathbf{C}_2 \\ \hdashline \mathbf{B}_2\mathbf{C}_1 & \mathbf{A}_2 \end{array}\right] \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \cdots \\ \mathbf{0} \end{bmatrix} r$$

$$y = \left[\begin{array}{c:c} \mathbf{C}_1 & \mathbf{0} \end{array}\right] \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix}$$

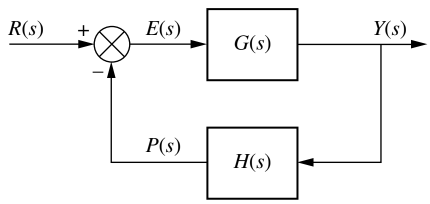


FIGURE P5.23

32. Given the system represented in state space as follows: [Section: 5.8] State Space **SS** **SS**

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & -7 & 6 \\ -8 & 4 & 8 \\ 4 & 7 & -8 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -5 \\ -7 \\ 5 \end{bmatrix} r$$

$$y = \begin{bmatrix} -9 & -9 & -8 \end{bmatrix}\mathbf{x}$$

convert the system to one where the new state vector, **z**, is

$$\mathbf{z} = \begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix}\mathbf{x}$$

33. Diagonalize following system: [Section: 5.8] State Space **SS**

$$\dot{\mathbf{x}} = \begin{bmatrix} -10 & -3 & 7 \\ 18.25 & 6.25 & -11.75 \\ -7.25 & -2.25 & 5.75 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} r$$

$$y = [1 \quad -2 \quad 4]\mathbf{x}$$

34. Diagonalize the system in Problem 33 using MATLAB. MATLAB **ML**

35. Find the closed-loop transfer function of the Unmanned Free-Swimming Submersible vehicle's pitch control system shown in Appendix A3 (*Johnson*, *1980*).

36. Use Simulink to plot the effects of nonlinearities upon the closed-loop step response of the antenna azimuth position control system shown in Appendix A2, Configuration 1. In particular, consider individually each of the following nonlinearities: saturation ($\pm$5 volts), backlash (dead-band width 0.15), deadzone (-2 to $+2$), as well as the linear response. Assume the preamplifier gain is 100 and the step input is 2 radians. Simulink **SL**

37. Figure P5.24 shows a noninverting operational amplifier.

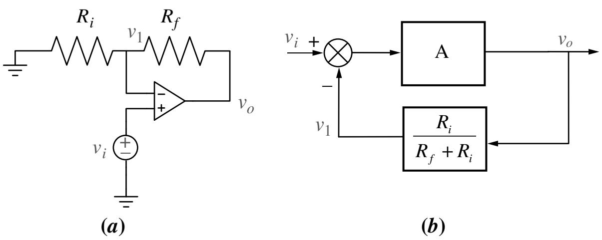


FIGURE P5.24 **a.** Noninverting amplifier; **b.** block diagram

Assuming the operational amplifier is ideal,

a. Verify that the system can be described by the following two equations:

$$v_o = A(v_i - v_o)$$

$$v_1 = \frac{R_i}{R_i + R_f} v_o$$

b. Check that these equations can be described by the block diagram of Figure P5.24(*b*).

c. Use Mason's rule to obtain the closed-loop system transfer function $\frac{V_o(s)}{V_i(s)}$.

d. Show that when $A \to \infty$, $\frac{V_o(s)}{V_i(s)} = 1 + \frac{R_f}{R_i}$.

38. Figure P5.25(*a*) shows an *n*-channel enhancement-mode MOSFET source follower circuit. Figure P5.25(*b*) shows its small-signal equivalent (where $R_i = R_1 || R_2$) (*Neamen*, *2001*). **SS**

a. Verify that the equations governing this circuit are

$$\frac{v_{in}}{v_i} = \frac{R_i}{R_i + R_s}; \quad v_{gs} = v_{in} - v_o; \quad v_o = g_m(R_s || r_o)v_{gs}$$

b. Draw a block diagram showing the relations between the equations.

c. Use the block diagram in Part **b** to find $\frac{V_o(s)}{V_i(s)}$.

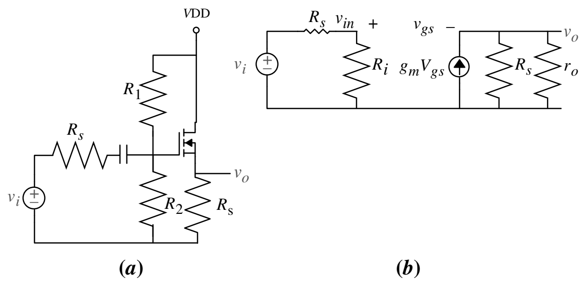


FIGURE P5.25 **a.** An *n*-channel enhancement-mode MOSFET source follower circuit; **b.** small-signal equivalent

39. A car active suspension system adds an active hydraulic actuator in parallel with the passive damper and spring to create a dynamic impedance that responds to road variations. The block diagram of Figure P5.26 depicts such an actuator with closed-loop control.

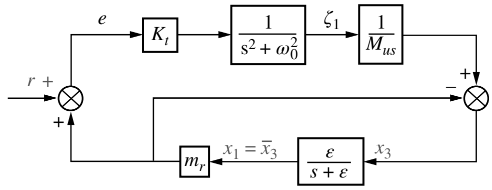


FIGURE P5.26[1]

In the figure, K_t is the spring constant of the tire, M_{US} is the wheel mass, r is the road disturbance, x_1 is the vertical car displacement, x_3 is the wheel vertical displacement, $\omega_0^2 = \frac{K_t}{M_{US}}$ is the natural frequency of the unsprung system and ε is a filtering parameter to be judiciously chosen (*Lin*, *1997*). Find the two transfer functions of interest:

[1] Lin Jung-Shan, Kanellakopoulos Ioannis, "Nonlinear Design of Active Suspensions." *IEEE Control Systems Magazine*, Vol. 17, Issue 3, June 1997 pp. 45–59. Figure 3, p. 48. IEEE control systems by IEEE CONTROL SYSTEMS SOCIETY Reproduced with permission of INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, in the format Republish in a book via Copyright Clearance Center.

a. $\dfrac{X_3(s)}{R(s)}$

b. $\dfrac{X_1(s)}{R(s)}$

40. The basic unit of skeletal and cardiac muscle cells is a *sarcomere*, which is what gives such cells a striated (parallel line) appearance. For example, one bicep cell has about 10^5 sarcomeres. In turn, sarcomeres are composed of protein complexes. Feedback mechanisms play an important role in sarcomeres and thus muscle contraction. Namely, Fenn's law says that the energy liberated during muscle contraction depends on the initial conditions and the load encountered. The following linearized model describing sarcomere contraction has been developed for cardiac muscle:

$$\begin{bmatrix} \dot{A} \\ \dot{T} \\ \dot{U} \\ \dot{SL} \end{bmatrix} = \begin{bmatrix} -100.2 & -20.7 & -30.7 & 200.3 \\ 40 & -20.22 & 49.95 & 526.1 \\ 0 & 10.22 & -59.95 & -526.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ T \\ U \\ SL \end{bmatrix} + \begin{bmatrix} 208 \\ -208 \\ -108.8 \\ -1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & 1570 & 1570 & 59400 \end{bmatrix} \begin{bmatrix} A \\ T \\ U \\ SL \end{bmatrix} - 6240u(t)$$

where

A = density of regulatory units with bound calcium and adjacent weak cross bridges (μM)

T = density of regulatory units with bound calcium and adjacent strong cross bridges (M)

U = density of regulatory units without bound calcium and adjacent strong cross bridges (M)

SL = sarcomere length (m)

The system's input is $u(t)$ = the shortening muscle velocity in meters/second and the output is $y(t)$ = muscle force output in Newtons (*Yaniv*, *2006*).

Do the following:

a. `Use MATLAB to obtain the transfer function` $\dfrac{Y(s)}{U(s)}$. MATLAB **ML**

b. `Use MATLAB to obtain a partial-fraction expansion for` $\dfrac{Y(s)}{U(s)}$. MATLAB **ML**

c. Draw a signal-flow diagram of the system in parallel form. State Space **SS**

d. Use the diagram of Part **c** to express the system in state-variable form with decoupled equations. State Space **SS**

41. An electric ventricular assist device (EVAD) has been designed to help patients with diminished but still functional heart pumping action to work in parallel with the natural heart. The device consists of a brushless dc electric motor that actuates on a pusher plate. The plate movements help the ejection of blood in systole and sac filling in diastole. System dynamics during systolic mode have been found to be:

$$\begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{P}_{ao} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -68.3 & -7.2 \\ 0 & 3.2 & -0.7 \end{bmatrix} \begin{bmatrix} x \\ v \\ P_{ao} \end{bmatrix} + \begin{bmatrix} 0 \\ 425.4 \\ 0 \end{bmatrix} e_m$$

The state variables in this model are x, the pusher plate position, v, the pusher plate velocity, and P_{ao}, the aortic blood pressure. The input to the system is e_m, the motor voltage (*Tasch*, *1990*).

a. `Use MATLAB to find a similarity transformation to diagonalize the system.` MATLAB **ML**

b. `Use MATLAB and the obtained similarity transformation of Part a to obtain a diagonalized expression for the system.` MATLAB **ML**

42. In an experiment to measure and identify postural arm reflexes, subjects hold in their hands a linear hydraulic manipulator. A load cell is attached to the actuator handle to measure resulting forces. At the application of a force, subjects try to maintain a fixed posture. Figure P5.27 shows a block diagram for the combined arm-environment system.

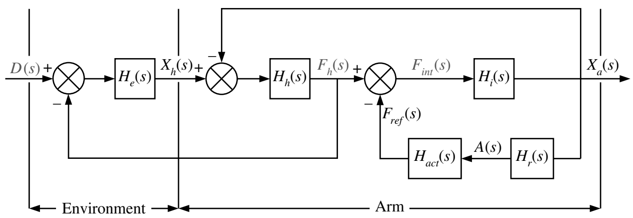


FIGURE P5.27

In the diagram, $H_r(s)$ represents the reflexive length and velocity feedback dynamics; $H_{act}(s)$ the activation dynamics; $H_i(s)$ the intrinsic act dynamics; $H_h(s)$ the hand dynamics; $H_e(s)$ the environmental dynamics; $X_a(s)$ the position of the arm; $X_h(s)$ the measured position of the hand; $F_h(s)$ the measured interaction force applied by the hand; $F_{int}(s)$ the intrinsic force; $F_{ref}(s)$ the reflexive force; $A(s)$ the reflexive activation; and $D(s)$ the external force perturbation (*de Vlugt*, *2002*).

a. Obtain a signal-flow diagram from the block diagram.

b. Find $\dfrac{F_h(s)}{D(s)}$.

43. A virtual reality simulator with haptic (sense of touch) feedback was developed to simulate the control of a submarine driven through a joystick input. Operator haptic feedback is provided through joystick position constraints and simulator movement (*Karkoub*, *2010*). Figure P5.28 shows the block diagram of the haptic feedback system in which the input u_h is the force exerted by the muscle of the human arm; and the outputs are y_s, the position of the simulator and y_j, the position of the joystick.

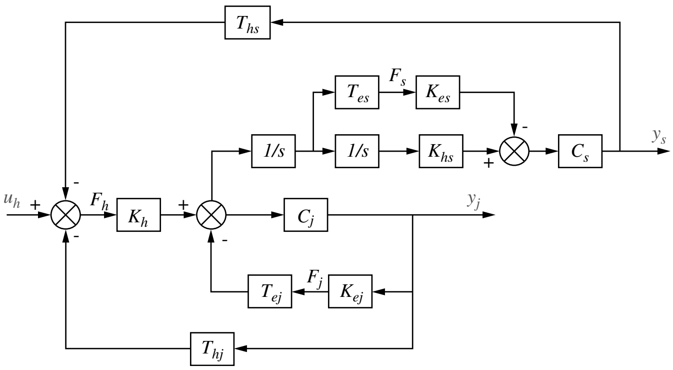


FIGURE P5.28[2]

a. Find the transfer function $\dfrac{Y_s(s)}{U_h(s)}$.

b. Find the transfer function $\dfrac{Y_j(s)}{U_h(s)}$.

44. Some medical procedures require the insertion of a needle under a patient's skin using CT scan monitoring guidance for precision. CT scans emit radiation, posing some cumulative risks for medical personnel. To avoid this problem, a remote control robot has been developed (*Piccin*, *2009*). The robot controls the needle in position and angle in the constraint space of a CT scan machine and also provides the physician with force feedback commensurate with the insertion opposition encountered by the type of tissue in which the needle is inserted. The robot has other features that give the operator the similar sensations and maneuverability as if the needle was inserted directly. Figure P5.29 shows the block diagram of the force insertion mechanism, where F_h is the input force and X_h is the output displacement. Summing junction inputs are positive unless indicated with a negative sign. By way of explanation, Z = impedance; G = transfer function; C_i = communication channel transfer functions; F = force; and X = position. Subscripts h and m refer to the master manipulator. Subscripts s and e refer to the slave manipulator.

a. Assuming $Z_h = 0, C_1 = C_s, C_2 = 1 + C_6$, and $C_4 = -C_m$, use Mason's Rule to show that the transfer function from the operators force input F_h to needle displacement X_h is given by

$$Y(s) = \frac{X_h(s)}{F_h(s)} = \frac{Z_m^{-1}C_2(1 + G_sC_s)}{1 + G_sC_s + Z_m^{-1}(c_m + C_2Z_eG_sC_s)}$$

b. Now with $Z_h \neq 0$ show that $\dfrac{X_h(s)}{F_h(s)} = \dfrac{Y(s)}{1 + Y(s)Z_h}$

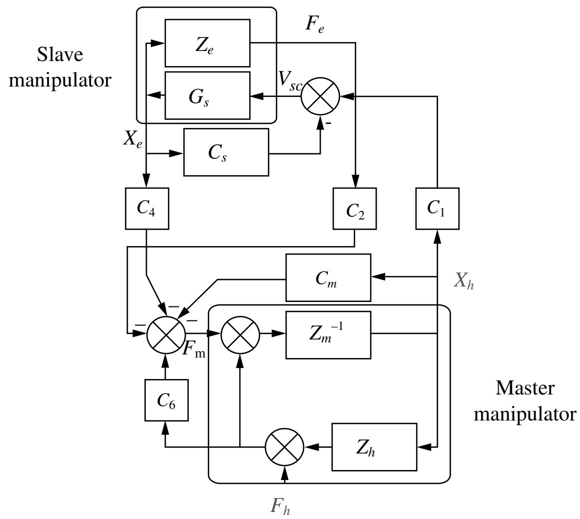


FIGURE P5.29[3]

45. Continuous casting in steel production is essentially a solidification process by which molten steel is solidified into a steel slab after passing through a mold, as shown in Figure P5.30(*a*). Final product dimensions depend mainly on the casting speed V_p (in m/min) and on the stopper position X (in %) that controls the flow of molten material into the mold (*Kong*, *1993*). A simplified model of a casting system is shown in Figure P5.30 (*b*) (*Kong*, *1993*) and (*Graebe*, *1995*). In the model, H_m = mold level (in mm); H_t = assumed constant height

[2] Karkoub M., Her, M-G., and Chen, J.M. Design and control of a haptic interactive motion simulator for virtual entertainment systems, *Robotica*, vol. 28, 2010, Figure 8, p. 53. Reproduced by permission of Cambridge University Press.

[3] Piccin, O., Barbé L., Bayle B., and Mathelin M. A Force Feedback Teleoperated Needle Insertion Device for Percutaneous Procedures. *Int. J. of Robotics Research*, vol. 28, p. 1154. Figure 14. Copyright © 2009. Reprinted by Permission of SAGE.

of molten steel in the tundish; D_z = mold thickness = depth of nozzle immerged into molten steel; and W_t = weight of molten steel in the tundish.

For a specific setting let $A_m = 0.5$ and

$$G_x(s) = \frac{0.63}{s + 0.926}$$

Also assume that the valve positioning loop may be modeled by the following second-order transfer function:

$$G_V(s) = \frac{X(s)}{Y_C(s)} = \frac{100}{s^2 + 10s + 100}$$

and the controller is modeled by the following transfer function:

$$G_C(s) = \frac{1.6(s^2 + 1.25s + 0.25)}{s}$$

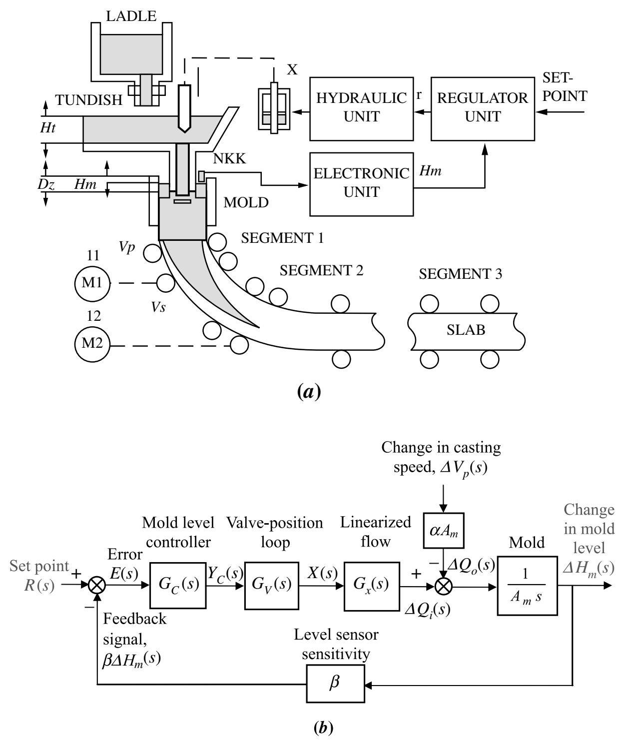


FIGURE P5.30 Steel mold process: **a.** process;[4] **b.** block diagram

The sensitivity of the mold level sensor is $\beta = 0.5$ and the initial values of the system variables at $t = 0-$ are: $R(0-) = 0; Y_C(0-) = X(0-) = 41.2; \Delta H_m(0-) = 0;$ $H_m(0-) = -75; \Delta V_p(0-) = 0;$ and $V_p(0-) = 0$. Do the following:

a. Assuming $v_p(t)$ is constant $[\Delta v_p = 0]$, find the closed-loop transfer function $T(s) = \Delta H_m(s)/R(s)$.

[4] Kong F., and de Keyser R. Identification and Control of the Mould Level in a Continuous Casting Machine. *Second IEEE Conference on Control Applications*, Vancouver, B.C., 1993. pp. 53–58. Figure 1. p. 53.

b. For $r(t) = 5\,u(t)$, $v_p(t) = 0.97\,u(t)$, and $H_m(0-) = -75$ mm, use Simulink to simulate the system. Record the time and mold level (in array format) by connecting them to **Workspace** sinks, each of which should carry the respective variable name. After the simulation ends, utilize MATLAB plot commands to obtain and edit the graph of $h_m(t)$ from $t = 0$ to 80 seconds. Simulink SL

46. It is shown in Figure 5.6(*c*) that when negative feedback is used, the overall transfer function for the system of Figure 5.6(*b*) is SS

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Develop the block diagram of an alternative feedback system that will result in the same closed-loop transfer function, $C(s)/R(s)$, with $G(s)$ unchanged and unmoved. In addition, your new block diagram must have unity gain in the feedback path. You can add input transducers and/or controllers in the main forward path as required.

47. The purpose of an Automatic Voltage Regulator is to maintain constant the voltage generated in an electrical power system, despite load and line variations, in an electrical power distribution system (*Gozde*, *2011*). Figure P5.31 shows the block

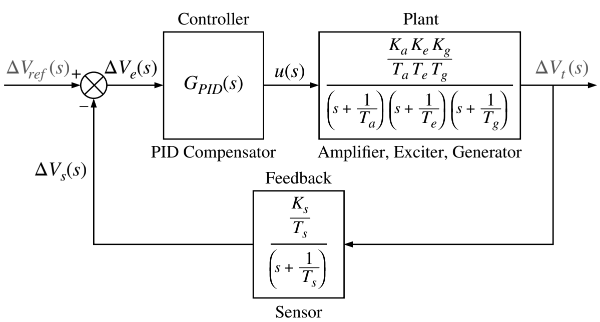


FIGURE P5.31

diagram of such a system. Assuming $K_a = 10$, $T_a = 0.1$, $K_e = 1$, $T_e = 0.4$, $K_g = 1$, $T_g = 1$, $K_s = 1$, $T_s = 0.001$, and the controller, $G_{PID}(s) = 1.6 + \frac{0.4}{s} + 0.3s$, find the closed-loop transfer function, $T(s) = \frac{\Delta V_t(s)}{\Delta V_{ref}(s)}$, of the system, expressing it as a rational function.

48. A drive system with an elastically coupled load was presented in Problem 52, Chapter 4. The mechanical part of this drive (*Thomsen*, *2011*) was reduced to a two-inertia model. Using slightly different parameters, the following transfer function results:

$$G(s) = \frac{\Omega_L(s)}{T(s)} = \frac{25(s^2 + 1.2s + 12500)}{s(s^2 + 5.6s + 62000)}$$

Here, $T(s) = T_{em}(s) - T_L(s)$, where $T_{em}(s) =$ the electromagnetic torque developed by the motor, $T_L(s)$ = the load torque, and $\Omega_L(s) =$ the load speed.

The drive is shown in Figure P5.32 as the controlled unit in a feedback control loop, where $\Omega_r(s)$ = the desired (reference) speed. The controller transfer function is $G_C(s) = K_p + \frac{K_I}{s} = 4 + \frac{0.5}{s}$ and provides an output voltage = 0 – 5.0 volts. The motor and its power amplifier have a gain, $K_M = 10$ N-m/volt.

a. Find the minor-loop transfer function, $D(s) = \frac{\Omega_L(s)}{T_{em}(s)}$ analytically or using MATLAB. MATLAB ML

b. Given that at $t = 0$, the load speed $\omega_L(t) = 0$ rad/sec and a step reference input $\omega_r(t) = 260\ u(t)$, rad/sec, is applied, use MATLAB (or any other program) to find and plot $\omega_L(t)$. Mark on the graph all of the important characteristics, such as percent overshoot, peak time, rise time, settling time, and final steady-state value.

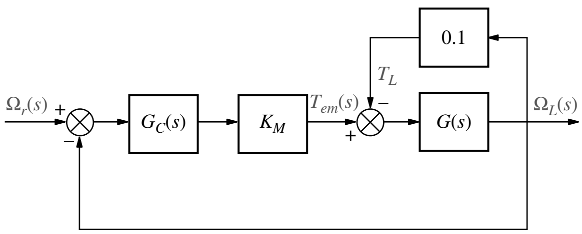


FIGURE P5.32

49. Integrated circuits are manufactured through a lithographic process on a semiconductor wafer. In lithography, similarly to chemical photography, a semiconductor wafer is covered with a photosensitive emulsion and then selectively exposed to light to form the electronic components. Due to miniaturization, this process is to be performed with nanometer accuracy and at the highest possible speed. Sophisticated apparatus and methods have been developed for this purpose. Figure P5.33 shows the block diagram of a scanner dedicated to this purpose (*Butler, 2011*). Use Mason's Rule to find:

a. The transfer function $\frac{X_{ss}(s)}{R(s)}$.

b. The transfer function $\frac{X_{ls}(s)}{R(s)}$.

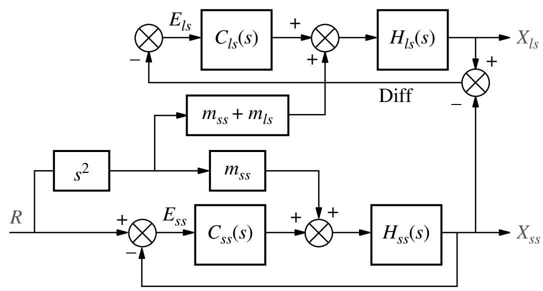


FIGURE P5.33[5]

50. In Problem 48 of Chapter 2, a three-phase ac/dc converter that supplies dc to a battery charging system (*Graovac*, *2001*) was introduced. Each phase had an ac filter represented by the equivalent circuit of Figure P2.27. You were asked to show that the following equation gives the s-domain relationship between the inductor current, $I_{acF}(s)$, and two active sources: a current source, $I_{acR}(s)$, representing a phase of the ac/dc converter, and the supply phase voltage, $V_a(s)$:

$$I_{acF}(s) = I_{acF1}(s) + I_{acF2}(s) = \frac{1 + RCs}{LCs^2 + RCs + 1} I_{acR}(s) + \frac{Cs}{LCs^2 + RCs + 1} V_a(s)$$

a. Derive an s-domain equation for $V_c(s)$.

b. Given that $R = 1\,\Omega$, $L = 1$ mH, and $C = 20\ \mu$F, $i_{acR}(t) = 10\ u(t)$ amps, $v_a(t) = 20\ t\ u(t)$ volts,[6] and assuming zero initial conditions, use Simulink to model this system and plot the inductor current, $i_{acF}(t)$, and the capacitor voltage, $v_c(t)$, over a period from 0 to 15 ms. Simulink SL

DESIGN PROBLEMS

51. The motor and load shown in Figure P5.34(*a*) are used as part of the unity-feedback system shown in Figure P5.34(*b*). Find the value of the coefficient of viscous damping, D_L, that must be used in order to yield a closed-loop transient response having a 20% overshoot.

[5] Butler, H. Position Control in Lithographic Equipment. IEEE Control Systems Magazine, October 2011, pp. 28–47. Figure 18, p. 37.

[6] Noting that a ramp is the integration of a step, we used an integrator with limits.

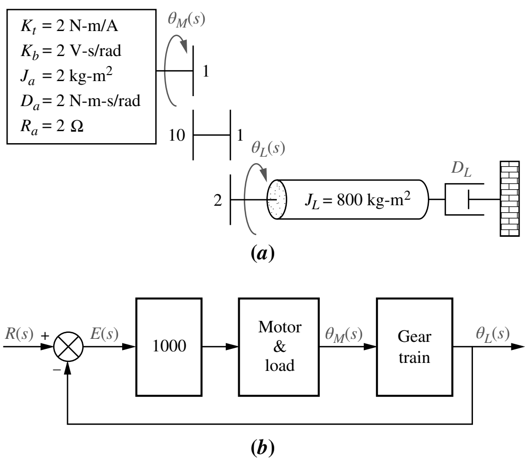


FIGURE P5.34 Position control: **a.** motor and load; **b.** block diagram

52. The system shown in Figure P5.35 will have its transient response altered by adding a tachometer. Design K and K_2 in the system to yield a damping ratio of 0.69. The natural frequency of the system before the addition of the tachometer is 10 rad/s.

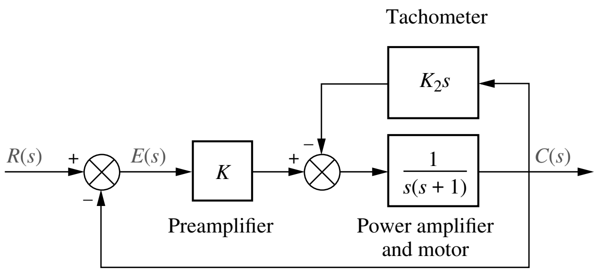


FIGURE P5.35 Position control

53. The mechanical system shown in Figure P5.36(a) is used as part of the unity feedback system shown in Figure P5.36(b). Find the values of M and D to yield 20% overshoot and 2 seconds settling time.

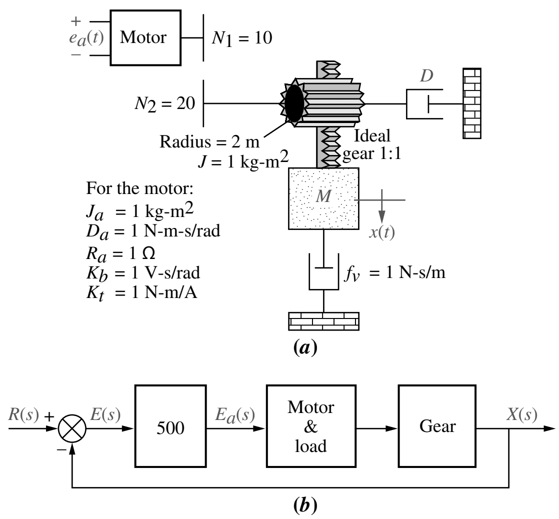


FIGURE P5.36 **a.** Motor and load; **b.** motor and load in feedback system

54. Assume ideal operational amplifiers in the circuit of Figure P5.37.
   a. Show that the leftmost operational amplifier works as a subtracting amplifier. Namely, $v_1 = v_o - v_{in}$.
   b. Draw a block diagram of the system, with the subtracting amplifier represented with a summing junction, and the circuit of the rightmost operational amplifier with a transfer function in the forward path. Keep R as a variable.
   c. Obtain the system's closed-loop transfer function.
   d. For a unit step input, obtain the value of R that will result in a settling time $T_s = 1$ msec.
   e. Using the value of R calculated in Part **d**, make a sketch of the resulting unit step response.

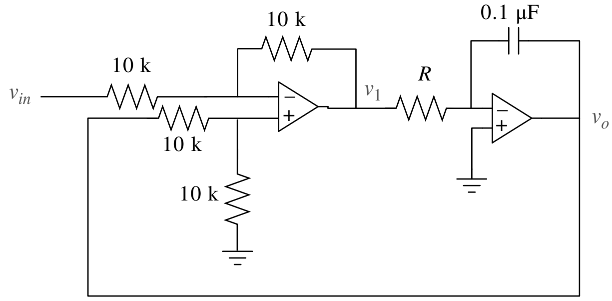


FIGURE P5.37

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

55. **Control of HIV/AIDS.** Given the HIV system of Problem 61 in Chapter 4 and repeated here for convenience (*Craig*, *2004*):

State Space **SS**

$$\begin{bmatrix} \dot{T} \\ \dot{T}^* \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -0.04167 & 0 & -0.0058 \\ 0.0217 & -0.24 & 0.0058 \\ 0 & 100 & -2.4 \end{bmatrix} \begin{bmatrix} T \\ T^* \\ v \end{bmatrix} + \begin{bmatrix} 5.2 \\ -5.2 \\ 0 \end{bmatrix} u_1$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ T^* \\ v \end{bmatrix}$$

Express the system in the following forms:

a. Phase-variable form
b. Controller canonical form
c. Observer canonical form

Finally,

d. Use MATLAB to obtain the system's diagonalized representation.

MATLAB
ML

56. Hybrid vehicle. Figure P5.38 shows the block diagram of a possible cascade control scheme for an HEV driven by a dc motor (*Preitl*, *2007*).

Let the speed controller $G_{SC}(s) = 100 + \frac{40}{s}$, the torque controller and power amp $K_A G_{TC}(s) = 10 + \frac{6}{s}$, the current sensor sensitivity $K_{CS} = 0.5$, and the speed sensor sensitivity $K_{SS} = 0.0433$. Also, following the development in previous chapters, $\frac{1}{R_a} = 1; \eta_{tot} K_t = 1.8;$ $k_b = 2;\ \ D = k_f = 0.1; \frac{1}{J_{tot}} = \frac{1}{7.226}; \frac{r}{i_{tot}} = 0.0615;$ and $\rho C_w A v_0 \frac{r}{i_{tot}} = 0.6154$.

a. Substitute these values in the block diagram, and find the transfer function, $T(s) = V(s)/R_v(s)$, using block-diagram reduction rules. [Hint: Start by moving the last $\frac{r}{i_{tot}}$ block to the right past the pickoff point.]

b. Develop a Simulink model for the original system in Figure P5.38. Set the reference signal input, $r_v(t) = 4\,u(t)$, as a step input with a zero initial value, a step time = 0 seconds, and a final value of 4 volts. Use X-Y graphs to display (over the period from 0 to 8 seconds) the response of the following variables to the step input: (1) change in car speed (m/s), (2) car acceleration (m/s^2), and (3) motor armature current (A).

Simulink
SL

MATLAB
ML

To record the time and the above three variables (in array format), connect them to four **Workspace** sinks, each of which carries the respective variable name. After the simulation ends, utilize MATLAB plot commands to obtain and edit the three graphs of interest.

57. Parabolic trough collector. Effective controller design for parabolic trough collector setups is an active area of research. One of the techniques used for controller design (*Camacho*, *2012*) is Internal Model Control (IMC). Although complete details of IMC will not be presented here, Figure P5.39(*a*) shows a block diagram for the IMC setup. Use of IMC assumes a very good knowledge of the plant dynamics. In Figure P5.39(*a*), the actual plant is $P(s)$. $\tilde{P}(s)$ is a software model that mimics the plant functions. $G(s)$ is the controller to be designed. It is also assumed that all blocks represent linear time-invariant systems and thus the superposition theorem applies to the system.

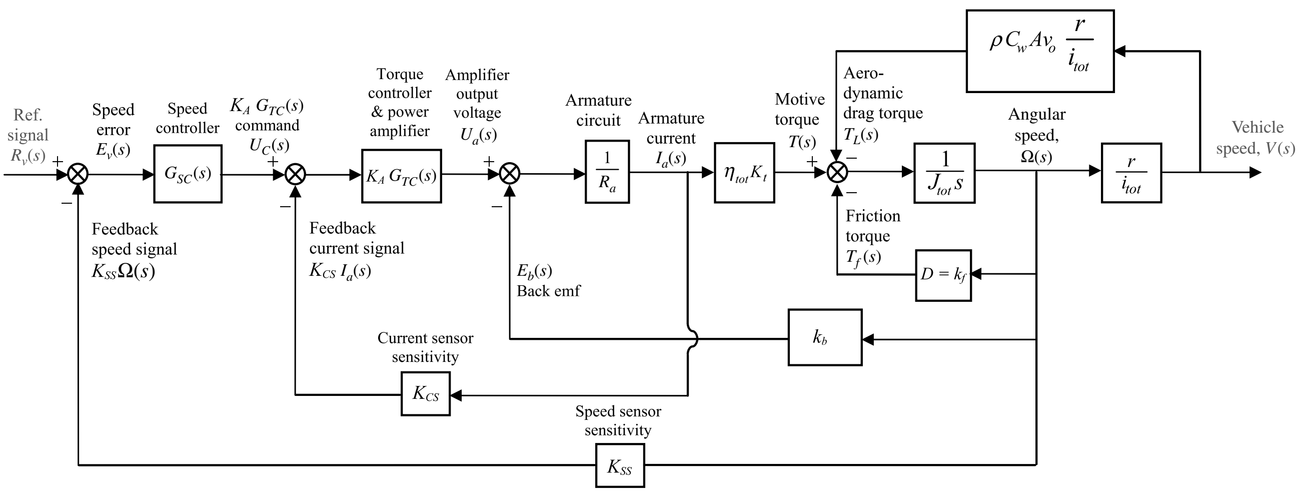


FIGURE P5.38

a. Use superposition (by assuming $D(s) = 0$) and Mason's gain formula to find the transfer function $\frac{C(s)}{R(s)}$ from command input to system output.

b. Use superposition (by assuming $R(s) = 0$) and Mason's gain formula to find the transfer function $\frac{C(s)}{D(s)}$ from disturbance input to system output.

c. Use the results of Parts **a** and **b** to find the combined output $C(s)$ due to both system inputs.

d. Show that the system of Figure P5.39(a) has the same transfer function as the system in Figure P5.39(b) when $G_C(s) = \frac{G(s)}{1 - G(s)\tilde{P}(s)}$.

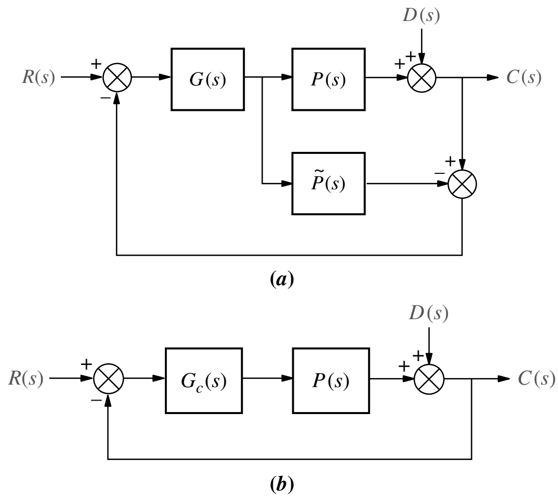


FIGURE P5.39

Chapter 6 Problems

1. Without solving for the roots, indicate the number of roots in the following polynomial that are in the left half-plane, right half-plane, and on the $j\omega$-axis. [Section: 6.2]

$$P(s) = s^5 + 4s^4 + 4s^3 + 5s^2 + 2s + 2$$

SS 2. Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis: [Section: 6.3]

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

3. Use a Routh array to find out how many poles of $T(s)$ are in the are in the left half-plane, right half-plane, and on the $j\omega$-axis. [Section: 6.3]

$$T(s) = \frac{s-2}{s^5 - 2s^4 + 4s^3 - 3s^2 + 2s - 3}$$

SS 4. The closed-loop transfer function of a system is [Section: 6.3]

$$T(s) = \frac{s^3 + 2s^2 + 7s + 21}{s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4}$$

Determine how many closed-loop poles lie in the right half-plane, in the left half-plane, and on the $j\omega$-axis.

5. In the open loop system of Figure P6.1, find out how many fo the poles are in the left half-plane, right half-plane, and on the $j\omega$-axis. [Section: 6.3]

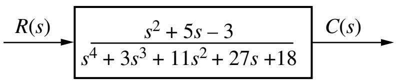


FIGURE P6.1

SS 6. How many poles are in the right half-plane, the left half-plane, and on the $j\omega$-axis for the open-loop system of Figure P6.2? [Section: 6.3]

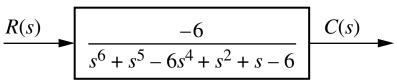


FIGURE P6.2

7. Find out if the unity-feedback system of Figure P6.3 is closed-loop stable if [Section: 6.2]

$$G(s) = \frac{584}{(s+2)(s+3)(s+4)(s+5)}$$

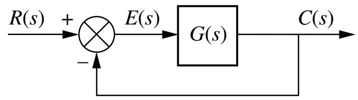


FIGURE P6.3

8. Use MATLAB to find the pole locations for the system of Problem 6. MATLAB ML SS

9. Find the range of K for closed-loop stability if in Figure P6.3. [Section: 6.4]

$$G(s) = \frac{K(s-1)}{s(s+2)(s+3)}$$

10. Using the Routh–Hurwitz criterion and the unity-feedback system of Figure P6.3 with SS

$$G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$$

tell whether or not the closed-loop system is stable. [Section: 6.2]

11. In the unity-feedback system of Figure P6.3, let

$$G(s) = \frac{10}{s(s^6 + 2s^5 - 3s^4 - 10s^3 - s^2 - 2s + 3)}$$

Find out how many poles of the closed-loop system will be in the left half-plane, right half-plane, and on the $j\omega$-axis. [Section: 6.3]

12. Consider the following Routh table. Notice that the s^5 row was originally all zeros. Tell how many roots of the original polynomial were in the right half-plane, in the left half-plane, and on the $j\omega$-axis. [Section: 6.3] SS

s^7	1	2	−1	−2
s^6	1	2	−1	−2
s^5	3	4	−1	0
s^4	1	−1	−3	0
s^3	7	8	0	0
s^2	−15	−21	0	0
s^1	−9	0	0	0
s^0	−21	0	0	0

13. Find out how many of the closed-loop poles of the system of Figure P6.4 are in the left half-plane, right half-plane, and on the $j\omega$-axis. Use the Routh–Hurwitz critera. [Section: 6.3]

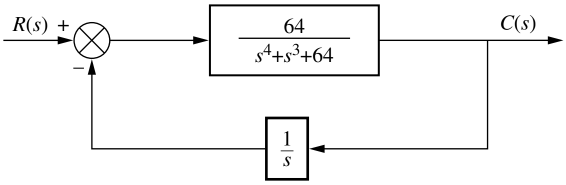


FIGURE P6.4

14. Find the rage of K for closed-loop stability in the unity-feedback system of Figure P6.3 if [Section: 6.4]

$$G(s) = \frac{K(s+10)}{s(s+2)(s+3)}$$

SS 15. In the system of Figure P6.3, let

$$G(s) = \frac{K(s-a)}{s(s-b)}$$

Find the range of K for closed-loop stability when: [Section: 6.4]

a. $a < 0, \quad b < 0$
b. $a < 0, \quad b > 0$
c. $a > 0, \quad b < 0$
d. $a > 0, \quad b > 0$

16. For the unity-feedback system of Figure P6.3 with

$$G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$$

determine the range of K for stability. [Section: 6.4]

17. `Use MATLAB and the Symbolic Math Toolbox to generate a Routh table in terms of K to solve Problem 16.` Symbolic Math **SM**

18. Find the range of K for stability for the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{K(s+4)(s-4)}{(s^2+3)}$$

19. For the unity-feedback system of Figure P6.3 with

$$G(s) = \frac{K(s+1)}{s^4(s+4)}$$

find the range of K for stability. [Section: 6.4]

20. Find the range of gain, K, to ensure stability in the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{K(s-2)(s+4)(s+5)}{(s^2+12)}$$

21. Find the range of gain, K, to ensure stability in the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{K(s+2)}{(s^2+1)(s+4)(s-1)}$$

22. Using the Routh–Hurwitz criterion, find the value of K that will yield oscillations for the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{K}{(s+77)(s+27)(s+38)}$$

23. Use the Routh–Hurwitz criterion to find the range of K for which the system of Figure P6.5 is stable. [Section: 6.4]

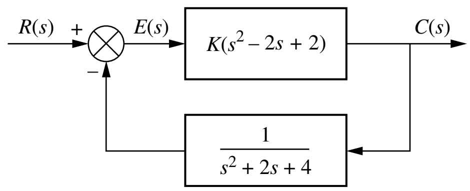


FIGURE P6.5

24. Repeat Problem 23 for the system of Figure P6.6. [Section: 6.4]

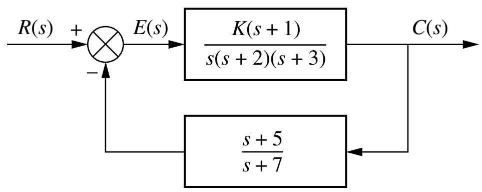


FIGURE P6.6

25. In Figure P6.3, let

$$G(s) = \frac{K(s+5)}{s(s+1)(s+3)}$$

Obtain: [Section: 6.4]

a. The range of K for closed-loop stability
b. The value of K at which the system will start oscillating
c. The frequency of oscillation in part b.

26. Consider the system of Figure P6.7. Find the range of K for closed-loop stability, the value of K that will make the system oscillate, and the oscillation frequency. [Section: 6.4]

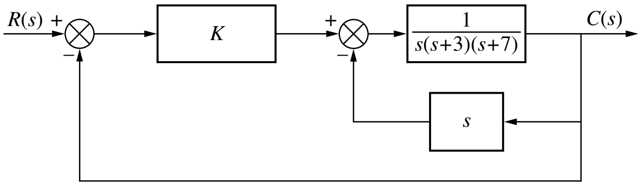


FIGURE P6.7

27. Given the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{Ks(s+2)}{(s^2 - 4s + 8)(s+3)}$$

a. Find the range of K for stability.

b. Find the frequency of oscillation when the system is marginally stable.

28. Let

$$G(s) = \frac{K}{(s+1)^3(s+5)}$$

in Figure P6.3. Then:

a. Find the range of K for closed-loop stability.

b. Find the frequency of oscillation when the system becomes marginally stable.

29. Given the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{K}{(s+49)(s^2 + 4s + 5)}$$

a. Find the range of K for stability.

b. Find the frequency of oscillation when the system becomes marginally stable.

SS **30.** Using the Routh–Hurwitz criterion and the unity-feedback system of Figure P6.3 with [Section: 6.4]

$$G(s) = \frac{K}{s(s+1)(s+2)(s+6)}$$

a. Find the range of K for stability.

b. Find the value of K for marginal stability.

c. Find the actual location of the closed-loop poles when the system is marginally stable.

31. Find the range of K to keep the system shown in Figure P6.8 stable. [Section: 6.4]

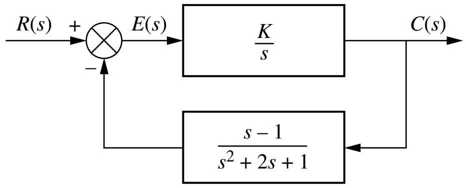


FIGURE P6.8

32. The transfer function relating the output engine fan speed (rpm) to the input main burner fuel flow rate (lb/h) in a short takeoff and landing (STOL) fighter aircraft, ignoring the coupling between engine fan speed and the pitch control command, is (*Schierman, 1992*) [Section: 6.4]

$$G(s) = \frac{1.3s^7 + 90.5s^6 + 1970s^5 + 15{,}000s^4 + 3120s^3 - 41{,}300s^2 - 5000s - 1840}{s^8 + 103s^7 + 1180s^6 + 4040s^5 + 2150s^4 - 8960s^3 - 10{,}600s^2 - 1550s - 415}$$

a. Find how many poles are in the right half-plane, in the left half-plane, and on the $j\omega$-axis.

b. Is this open-loop system stable?

33. A linearized model of a torque-controlled crane hoisting a load with a fixed rope length is SS

$$P(s) = \frac{X_T(s)}{F_T(s)} = \frac{1}{m_T}\,\frac{s^2 + \omega_0^2}{s^2(s^2 + a\omega_0^2)}$$

where $\omega_0 = \sqrt{\frac{g}{L}}$, $L =$ the rope length, $m_T =$ the mass of the car, $a =$ the combined rope and car mass, $f_T =$ the force input applied to the car, and $x_T =$ the resulting rope displacement (*Marttinen, 1990*). If the system is controlled in a feedback configuration by placing it in a loop as shown in Figure P6.9, with $K > 0$, where will the closed-loop poles be located?

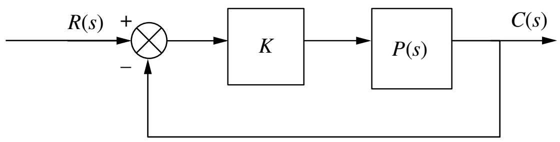


FIGURE P6.9

34. `Use MATLAB to find the eigenvalues of the following system:`

MATLAB
ML
State Space
SS

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & -4 \\ -1 & 1 & 8 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}u$$

$$y = [0 \quad 0 \quad 1]\mathbf{x}$$

State Space
SS

35. The following system in state space represents the forward path of a unity-feedback system. Use the Routh–Hurwitz criterion to determine if the closed-loop system is stable. [Section: 6.5]

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 2 \\ -5 & -4 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}u$$

$$y = [1 \quad 0 \quad 1]\mathbf{x}$$

36. `Repeat Problem 35 using MATLAB.` MATLAB ML

SS 37. An inverted pendulum, mounted on a motor-driven cart was presented in Chapter 3, Problem 25. The system's state-space model was linearized around a stationary point, $\mathbf{x_0} = \mathbf{0}$, corresponding to the pendulum point-mass, m, being in the upright position at $t = 0$, when the force applied to the cart $u_0 = 0$ (*Prasad, 2012*). We'll modify that model here to have two output variables: the pendulum angle relative to the y-axis, θ, and the horizontal position of the cart, x. The output equation becomes:

$$\mathbf{y} = \begin{bmatrix} \theta \\ x \end{bmatrix} = \mathbf{Cx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

`Using MATLAB, find out how many eigenvalues are in the right half-plane, in the left half-plane, and on the` $j\omega$`-axis. What does that tell us about the stability of that unit? [Section: 6.5]` MATLAB ML

DESIGN PROBLEMS

38. A model for an airplane's pitch loop is shown in Figure P6.10. Find the range of gain, K, that will keep the system stable. Can the system ever be unstable for positive values of K?

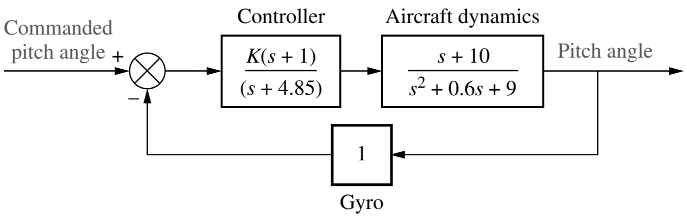


FIGURE P6.10 Aircraft pitch loop model

39. A common application of control systems is in regulating the temperature of a chemical process (Figure P6.11). The flow of a chemical reactant to a process is controlled by an actuator and valve. The reactant causes the temperature in the vat to change. This temperature is sensed and compared to a desired set-point temperature in a closed loop, where the flow of reactant is adjusted to yield the desired temperature. In Chapter 9, we will learn how a PID controller is used to improve the performance of such process control systems. Figure P6.11 shows the control system prior to the addition of the PID controller. The PID controller is replaced by the shaded box with a gain of unity. For this system, prior to the design of the PID controller, find the range of amplifier gain, K, to keep the system stable.

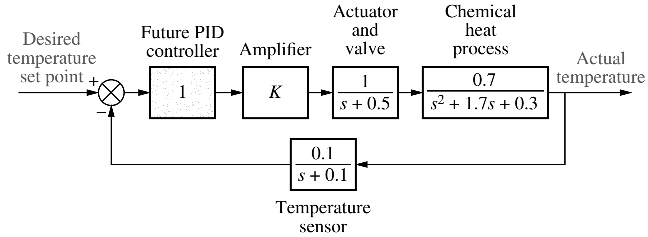


FIGURE P6.11 Block diagram of a chemical process control system

40. A transfer function from indoor radiator power, $\dot{Q}(s)$, to room temperature, $T(s)$, in an 11-m^2 room is

$$P(s) = \frac{T(s)}{\dot{Q}(s)} = \frac{1 \times 10^{-6}s^2 + 1.314 \times 10^{-9}s + 2.66 \times 10^{-13}}{s^3 + 0.00163s^2 + 5.272 \times 10^{-7}s + 3.538 \times 10^{-11}}$$

where $\dot{Q}$ is in watts and T is in °C (*Thomas, 2005*). The room's temperature will be controlled by embedding it in a closed loop, such as that of Figure P6.9. Find the range of K for closed-loop stability.

41. During vertical spindle surface grinding, adjustments are made on a multi-axis computer numerical control (CNC) machine by measuring the applied force with a dynamometer and applying appropriate corrections. This feedback force control results in higher homogeneity and better tolerances in the resulting finished product. In a specific experiment with an extremely high feed rate, the transfer function from the desired depth of cut (DOC) to applied force was

$$\frac{F(s)}{DOC(s)} = \frac{K_C}{1 + \dfrac{K_C}{ms^2 + bs + k} - \dfrac{K_C}{K_f}\left(\dfrac{1}{Ts+1}\right)}$$

where $k = 2.1 \times 10^4$ N/m, $b = 0.78$ Ns/m, $m = 1.2 \times 10^{-4}$ kg, $K_C = 1.5 \times 10^4$ N/mm, and $T = 0.004$s. The parameter K_f is varied to adjust the system. Find the range of K_f under which the system is stable (*Hekman, 1999*).

42. In order to obtain a low-cost lithium-ion battery charger, the feedback loop of Figure P6.3 is used, where $G(s) = G_c(s)P(s)$. The following transfer functions have been derived for $G(s)$ (*Tsang, 2009*):

$$P(s) = \frac{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_1 + R_2C_2)s + 1}{C_1(1 + R_2C_2)s}$$

$$G_c(s) = K_p + \frac{K_I}{s}$$

If $R_1 = 0.15\Omega$; $R_2 = 0.44\Omega$; $C_1 = 7200$F; and $C_2 = 170$F, use the Routh–Hurwitz criteria to find the range of positive K_P and K_I for which the system is closed-loop stable.

43. Figure P6.12 is a simplified and linearized block diagram of a cascade control system, which is used to control water level in a steam generator of a nuclear power plant (*Wang*, *2009*).

In this system, the level controller, $G_{LC}(s)$, is the *master* controller and the feed-water flow controller, $G_{FC}(s)$, is the *slave* controller. Using mass balance equations, the water level would ordinarily be regarded as a simple integration process of water flow. In a steam generator, however, steam flow rate and the cooling effect of feed-water change the dynamics of that process. Taking the latter into account and ignoring the much-less pronounced impact of changes in steam flow rate, a first-order lag plus time delay is introduced into the transfer function, $G_{fw}(s)$, relating the controlled level, $C(s)$, to feed-water flow rate, $Q_w(s)$ as follows:

$$G_{fw}(s) = \frac{C(s)}{Q_w(s)} = \frac{K_1 e^{-\tau_1 s}}{s(T_1 s + 1)} = \frac{2e^{-2s}}{s(25s+1)}$$
$$\approx \frac{2}{s(25s+1)(2s^2+2s+1)}$$

where $K_1 = 2$ is the process gain, $\tau_1 = 2$ is the pure time delay, and $T_1 = 25$ is the steam generator's time constant. (The expression $e^{-\tau_1 s}$ represents a time delay. This function can be represented by what is known as a *Pade approximation*. This approximation can take on many increasingly complicated forms, depending upon the degree of accuracy required. Here we use the Pade approximation, $e^{-x} \approx \dfrac{1}{1 + x + \dfrac{x^2}{2!}}$, and specific numerical values for the considered steam generator.)

The dynamic characteristics of the control valve are approximated by the transfer function

$$G_v(s) = \frac{Q_w(s)}{Y(s)} = \frac{K_v}{T_v s + 1} = \frac{1}{3s+1}$$

where K_v is the valve gain and T_v is its time constant.

Given that: $G_{FC}(s) = K_{P_{FC}} + K_{D_{FC}} s = 0.5 + 2s$ and $G_{LC}(s) = K_{P_{LC}} + K_{D_{LC}} s = 0.5 + Ks$, use the Routh–Hurwitz criterion to find the range of the level controller's derivative gain, $K_{D_{LC}} = K > 0$, that will keep the system stable.

44. Look-ahead information can be used to automatically steer a bicycle in a closed-loop configuration. A line is drawn in the middle of the lane to be followed, and an arbitrary point is chosen in the vehicle's longitudinal axis. A look-ahead offset is calculated by measuring the distance between the look-ahead point and the reference line and is used by the system to correct the vehicle's trajectory. A linearized model of a particular bicycle traveling on a straight-line path at a fixed longitudinal speed is

$$\begin{bmatrix} \dot{V} \\ \dot{r} \\ \dot{\psi} \\ \dot{Y}_g \end{bmatrix} = \begin{bmatrix} -11.7 & 6.8 & 61.6K & 7.7K \\ -3.5 & -24 & -66.9K & 8.4K \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -10 & 0 \end{bmatrix} \begin{bmatrix} V \\ r \\ \psi \\ Y_g \end{bmatrix}$$

In this model, V = bicycle's lateral velocity, r = bicycle's yaw velocity, ψ = bicycle's yaw acceleration, and Y_g = bicycle's center of gravity coordinate on the *y*-axis. K is a controller parameter to be chosen by the designer (*Özgüner*, *1995*). Use the Routh–Hurwitz citerion to find the range of K for which the system is closed-loop stable.

45. Figure P5.31 Shows the block diagram of an Automatic Voltage Regulator (*Gozde*, *2011*). Assume in this diagram the following parameter values: $K_a = 10$, $T_a = 0.1$, $K_e = 1$, $T_e = 0.4$, $K_g = 1$, $T_g = 1$, $K_s = 1$, and $T_s = 0.001$. Also assume that the PID transfer function is substituted by a simple integrator, namely $G_{PID}(s) = \dfrac{K}{s}$. Find the range of K for which the system is closed-loop stable.

46. It has been shown (*Pounds, 2011*) that an unloaded UAV helicopter is closed-loop stable and will have a characteristic equation given by

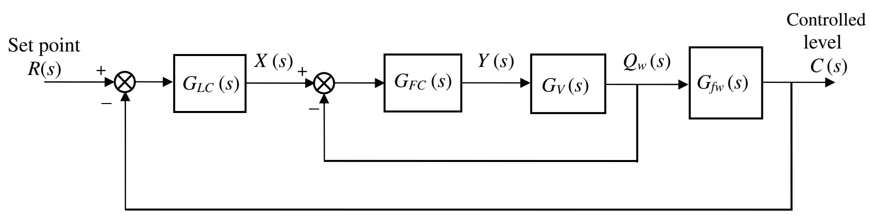


FIGURE P6.12

$$s^3 + \left(\frac{mgh}{I}(q_2 + kk_d) + q_1 g\right)s^2 + k\frac{mgh}{I}s + \frac{mgh}{I}(kk_i + q_1) = 0$$

where m is the mass of the helicopter, g is the gravitational constant, I is the rotational inertia of the helicopter, h is the height of the rotor plane above the center of gravity, q_1 and q_2 are stabilizer flapping parameters, k, k_i, and k_d are controller parameters; all constants > 0. The UAV is supposed to pick up a payload; when this occurs, the mass, height, and inertia change to m', h', and I', respectively, all still > 0. Show that the helicopter will remain stable as long as

$$\frac{m'gh'}{I'} > \frac{q_1 + kk_i - q_1 gk}{k(q_2 + kk_d)}$$

47. Figure P6.13 shows the model of the dynamics of an economic system (*Wingrove, 2012*). In this diagram x represents the rate of growth in real Gross National Product (GNP), x_0 the long-term trend (dc value) of the GNP, Δx the change over the long-term trend of the GNP, r_x the real and psychological disturbance inputs that affect the economy, r_m the random monetary inputs, and Δu fluctuations in unemployment rate. The diagram has two feedback loops: one through Friedman's model in which the economy dynamics are approximated by

$$F(s) = \frac{K_x s}{\left(\frac{s}{\omega_n}\right)^2 + 2\xi\left(\frac{s}{\omega_n}\right) + 1}$$

and a second loop through Okun's law that relates the GNP to unemployment changes. Assuming the following parameter values: $K_x = 2$ years, $\omega_n = 1.5$ rad/year, $\zeta = 0.8$, $K_u = 0.4$ and $G_x = -0.4$. Find the range of G_u for closed-loop stability.

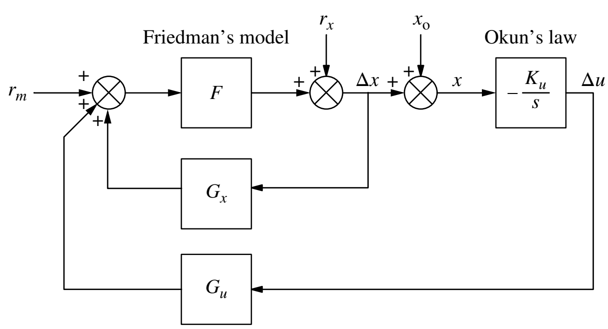


FIGURE P6.13

48. The system shown in Figure P6.14 has $G_1(s) = 1/s(s+2)\ (s+4)$. Find the following:

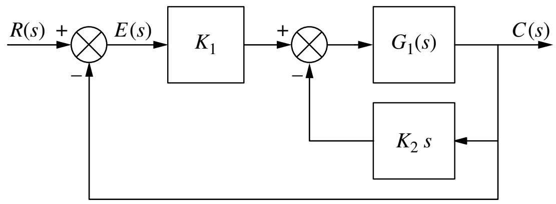


FIGURE P6.14

a. The value of K_2 for which the inner loop will have two equal negative real poles and the associated range of K_1 for system stability.

b. The value of K_1 at which the system oscillates and the associated frequency of oscillation.

c. The gain K_1 at which a real closed-loop pole is at $s = -5$. Can the step response, $c(t)$, be approximated by a second-order, underdamped response in this case? Why or why not?

d. If the response in Part **d** can be approximated as a second-order response, find the $\%OS$ and settling time, T_s, when the input is a unit step, $r(t) = u(t)$.

49. A drive system with an elastically coupled load was presented in Problems 52 and 48 in Chapters 4 and 5, respectively (*Thomsen, 2011*). That drive was shown in Figure P5.32 as the controlled unit in a feedback control system, where $\Omega_L(s)$ = the load speed and $\Omega_r(s)$ = the desired (reference) speed. If the controller transfer function is $G_C(s) = K_p + \frac{K_I}{s}$, while all of the other parameters and transfer functions are the same as in Problem 48 in Chapter 5, find the range of K_P for stability of the system if $K_I = 0.1$.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

50. Control of HIV/AIDS. The HIV infection linearized model developed in Problem 61, Chapter 4, can be shown to have the transfer function

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$

It is desired to develop a policy for drug delivery to maintain the virus count at prescribed levels. For the purpose of obtaining an appropriate $u_1(t)$, feedback will be used as shown in Figure P6.15 (*Craig, 2004*).

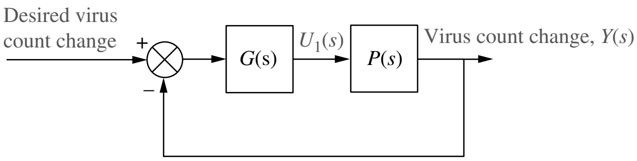


FIGURE P6.15

As a first approach, consider $G(s) = K$, a constant to be selected. Use the Routh–Hurwitz criteria to find the range of K for which the system is closed-loop stable.

51. Hybrid vehicle. Figure P6.16 shows the HEV system presented in Chapter 5, where parameter values have been substituted. It is assumed here that the speed controller has a proportional gain, K_p, to be adjusted. Use the Routh–Hurwitz stability method to find the range of positive K_p for which the system is closed-loop stable (*Graebe, 1995*).

52. Parabolic trough collector. The fluid temperature of a parabolic trough collector (*Camacho, 2012*) will be controlled by using a unity feedback structure as shown in Figure P6.9. Assume the open-loop plant transfer function is given by

$$P(s) = \frac{137.2 \times 10^{-6}}{s^2 + 0.0224s + 196 \times 10^{-6}} e^{-39s}$$

Use the Routh-Hurwitz criteria to find the range of gain K that will result in a closed-loop stable system. Note: Pure time-delay dynamics, such as the one in the transfer function of the parabolic trough collector, cannot be treated directly using the Routh–Hurwitz criterion because it is represented by a nonrational factor. However, a Padé approximation can be used for the nonrational component. The Padé approximation was introduced in Problem 6.43, but it can appear in different forms. Here, it is suggested you use a first-order approximation of the form

$$e^{-sT} \approx \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$$

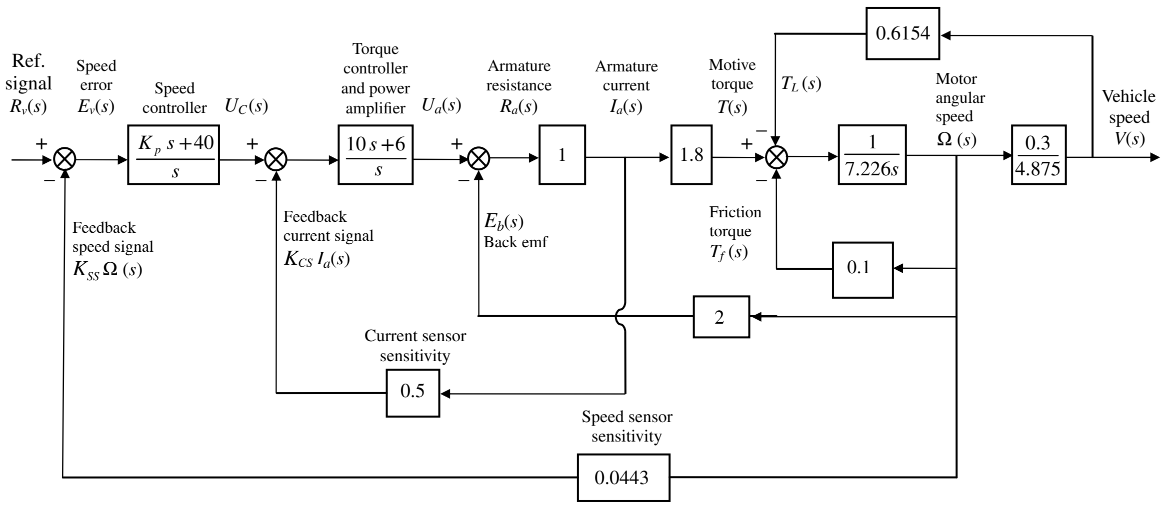


FIGURE P6.16

1. In Figure P7.1, let

$$G(s) = \frac{1350(s+2)(s+10)(s+32)}{s(s+4)(s^2+8s+32)}$$

Find the steady-state errors for the following inputs: $17u(t)$, $32tu(t)$, $48t^2u(t)$. [Section: 7.2]

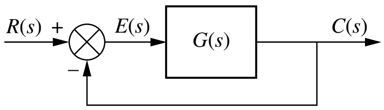


FIGURE P7.1

SS **2.** Figure P7.2 shows the ramp input $r(t)$ and the output $c(t)$ of a system. Assuming the output's steady state can be approximated by a ramp, find [Section: 7.1]

a. the steady-state error;

b. the steady-state error if the input becomes $r(t) = tu(t)$.

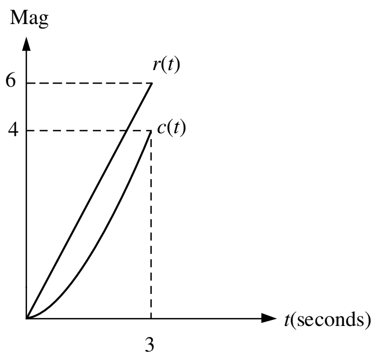


FIGURE P7.2

3. In the unity-feedback system shown of Figure P7.1,

$$G(s) = \frac{375(s+5)(s+18)(s+54)}{s^2(s+8)(s+24)}$$

Find the steady-state error when the input is $8t^2u(t)$. [Section: 7.2]

SS **4.** For the system shown in Figure P7.3, what steady-state error can be expected for the following test inputs: $10u(t)$, $10tu(t)$, $10t^2u(t)$. [Section: 7.2]

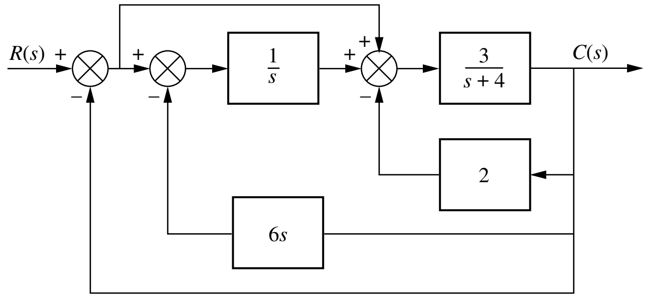


FIGURE P7.3

5. Find the steady-state error for inputs $4u(t)$, $7tu(t)$, and $5t^2\,u(t)$ for the system of Figure P7.1, when

$$G(s) = \frac{2}{(s+0.5)(s^2+1s+2)}$$

[Section: 7.3]

6. An input of $25t^3u(t)$ is applied to the input of a Type 3 unity-feedback system, as shown in Figure P7.1, where SS

$$G(s) = \frac{210(s+4)(s+6)(s+11)(s+13)}{s^3(s+7)(s+14)(s+19)}$$

Find the steady-state error in position. [Section: 7.3]

7. The velocity steady-state error of a system can be defined to be

$$\left(\frac{dr}{dt} - \frac{dc}{dt}\right)\Bigg|_{t\to\infty}$$

where $r(t)$ is the input, and $c(t)$ is the output. Find the velocity steady-state error for the configuration of Figure P7.1 when [Section: 7.2]

$$G(s) = \frac{200(s+2)(s+3)}{s^2(s+1)(s+15)}$$

and the input is $r(t) = t^3u(t)$.

8. For a system, the proportional error constant is $K_P = 2$. Indicate what will be the steady-state error if the inputs are $50u(t)$ and $50tu(t)$? [Section 7.3]

9. For the unity-feedback system shown in Figure P7.1, where [Section: 7.3] SS

$$G(s) = \frac{5000}{s(s+75)}$$

a. What is the expected percent overshoot for a unit step input?

b. What is the settling time for a unit step input?

c. What is the steady-state error for an input of $5u(t)$?

d. What is the steady-state error for an input of $5tu(t)$?

e. What is the steady-state error for an input of $5t^2u(t)$?

10. It is desired to achieve $K_v = 40{,}000$ for the system of Figure P7.1 when

$$G(s) = \frac{300{,}000(s+5)(s+10)(s+30)}{s(s+60)(s+\alpha)(s+90)}$$

Find the required value of α. [Section: 7.4]

SS **11.** For the unity-feedback system of Figure P7.1, where

$$G(s) = \frac{K(s+2)(s+4)(s+6)}{s^2(s+5)(s+7)}$$

find the value of K to yield a static error constant of 10,000. [Section: 7.4]

12. Refer to the system of Figure P7.4. [Section: 7.3]

a. Find the steady-state error for inputs $20u(t)$, $20tu(t)$, and $20t^2u(t)$.

b. Find the error constants K_p, K_v, and K_a.

c. Find the system type.

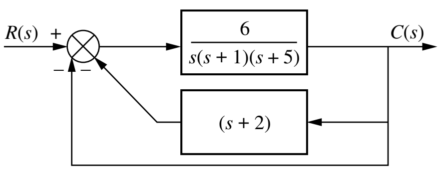


FIGURE P7.4

13. For the system of Figure P7.5, find the system type when [Section: 7.3]

a. $M(s) = 5$.

b. $M(s) = 5/s$.

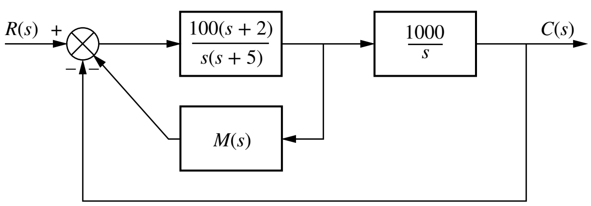


FIGURE P7.5

14. It is desired to obtain a zero steady-state error for step inputs in Figure P7.6. Find the restrictions on the feedforward transfer function $G_2(s)$ when: [Section: 7.3]

a. $G_1(s)$ is a Type 0 transfer function;

b. $G_1(s)$ is a Type 1 transfer function;

c. $G_1(s)$ is a Type 2 transfer function?

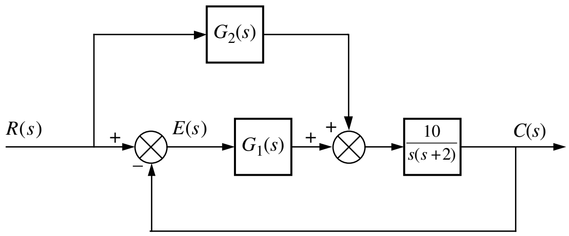


FIGURE P7.6

15. The steady-state error is defined to be the difference in position between input and output as time approaches infinity. Let us define a steady-state velocity error, which is the difference in velocity between input and output. Derive an expression for the error in velocity, $\dot{e}(\infty) = \dot{r}(\infty) - \dot{c}(\infty)$, and complete Table P7.1 for the error in velocity. [Sections: 7.2, 7.3] SS

TABLE P7.1

		Type		
		0	1	2
Input	Step			
	Ramp			
	Parabola			

16. Given the unity-feedback system of Figure P7.1, where

$$G(s) = \frac{K(s+a)}{s(s+2)(s+15)}$$

find the value of Ka so that a ramp input of slope 30 will yield an error of 0.005 in the steady state when compared to the output. [Section: 7.4]

17. For an input $50tu(t)$ to the system of Figure P7.7, find the value of K that will yield a steady-state error of 0.05. [Section: 7.4]

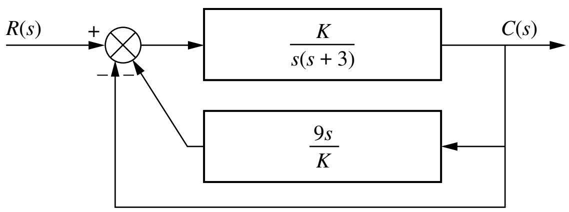


FIGURE P7.7

18. The unity-feedback system of Figure P7.1, where

$$G(s) = \frac{K(s^2+3s+30)}{s^n(s+5)}$$

is to have 1/6000 error between an input of $10tu(t)$ and the output in the steady state. [Section: 7.4]

a. Find K and n to meet the specification.

b. What are K_p, K_v, and K_a?

19. For the unity-feedback system of Figure P7.1, where [Section: 7.3]

$$G(s) = \frac{K(s^2+6s+6)}{(s+5)^2(s+3)}$$

a. Find the system type.

b. What error can be expected for an input of $12u(t)$?

c. What error can be expected for an input of $12tu(t)$?

20. Assume in the unity-feedback system of Figure P7.1 that

$$G(s) = \frac{K(s+4)}{(s+1)(s^2+7s+30)}$$

Find the value of K that will result in a 5% steady-state error. [Section: 7.4]

SS **21.** The unity-feedback system of Figure P7.1,where

$$G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$$

is to be designed to meet the following specifications: steady-state error for a unit step input = 0.1; damping ratio = 0.5; natural frequency = $\sqrt{10}$. Find K, α, and β. [Section: 7.4]

22. A second-order, unity-feedback system is to follow a ramp input with the following specifications: the steady-state output position shall differ from the input position by 0.01 of the input velocity; the natural frequency of the closed-loop system shall be 10 rad/s. Find the following:

a. The system type

b. The exact expression for the forward-path transfer function

c. The closed-loop system's damping ratio

23. The unity-feedback system of Figure P7.1 has a transfer function $G(s) = \dfrac{C(s)}{E(s)} = \dfrac{K}{s(s+\alpha)}$ and is to follow a ramp input, $r(t) = tu(t)$, so that the steady-state output position differs from the input position by 0.01 of the input velocity (e.g., $e(\infty) = \dfrac{1}{K_v} = 0.01$). The natural frequency of the closed-loop system will be $\omega_n = 5$ rad/s. [Section: 7.4]
Find the following:

a. The system type

b. The values of K and α

c. The closed-loop system's damping ratio, ζ

d. If K is reduced to 4 and $\alpha = 0.4$, find the corresponding new values of $e(\infty)$, ω_n, and ζ.

24. The unity-feedback system of Figure P7.1, where

$$G(s) = \frac{K(s+\alpha)}{s(s+\beta)}$$

is to be designed to meet the following requirements: The steady-state position error for a unit ramp input equals 1/10; the closed-loop poles will be located at $-1 \pm j1$. Find K, α, and β in order to meet the specifications. [Section: 7.4]

25. Given the unity-feedback control system of Figure P7.1 where SS

$$G(s) = \frac{K}{s(s+a)}$$

find the following: [Section: 7.4]

a. K and a to yield $K_v = 1000$ and a 20% overshoot

b. K and a to yield a 1% error in the steady state and a 10% overshoot.

26. Given the system in Figure P7.8, find the following: [Section: 7.3]

a. The closed-loop transfer function

b. The system type

c. The steady-state error for an input of $5u(t)$

d. The steady-state error for an input of $5tu(t)$

e. Discuss the validity of your answers to Parts **c** and **d**.

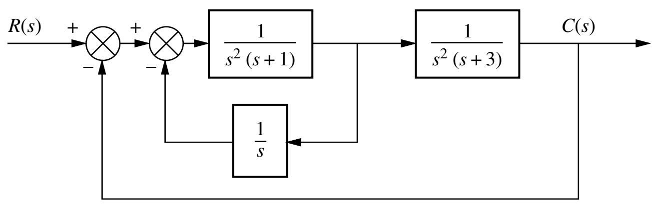


FIGURE P7.8

27. The system of Figure P7.9 is to have the following specifications: $K_v = 20$; $\zeta = 0.7$. Find the values of K_1 and K_f required for the specifications of the system to be met. [Section: 7.4] SS

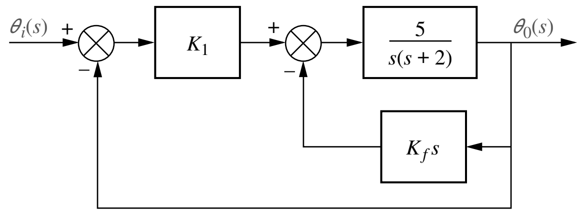


FIGURE P7.9

28. Design the values of K_1 and K_2 in the system of Figure P7.10 to meet the following specifications: Steady-state error component due to a unit step disturbance is -0.00001; steady-state error component due to a unit ramp input is 0.002. [Section: 7.5]

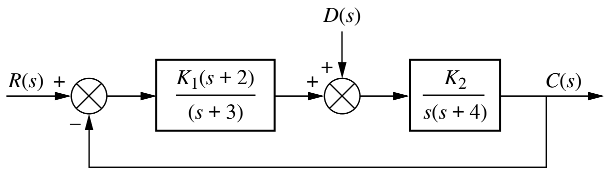


FIGURE P7.10

29. In Figure P7.11, let $G(s) = 5$ and $P(s) = \dfrac{7}{s+2}$.

SS

a. Calculate the steady-state error due to a command input $R(s) = \dfrac{3}{s}$ with $D(s) = 0$.

b. `Verify the result of Part` **a** `using Simulink.` Simulink **SL**

c. Calculate the steady-state error due to a disturbance input $D(s) = -\dfrac{1}{s}$ with $R(s) = 0$.

d. `Verify the result of Part` **c** `using Simulink.` Simulink **SL**

e. Calculate the total steady-state error due to a command input $R(s) = \dfrac{3}{s}$ and a disturbance $D(s) = -\dfrac{1}{s}$ applied simultaneously.

f. `Verify the result of Part` **e** `using Simulink.` Simulink **SL**

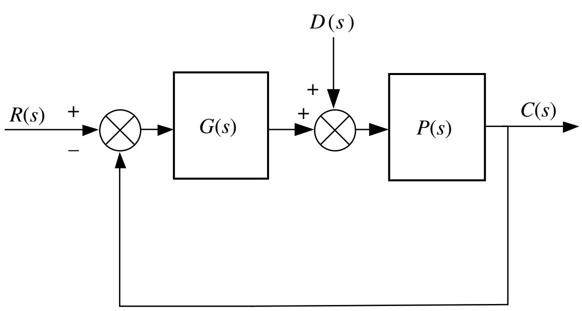


FIGURE P7.11

30. Derive Eq. (7.72) in the text, which is the final value of the actuating signal for nonunity-feedback systems. [Section: 7.6]

31. For each system shown in Figure P7.12, find the following: [Section: 7.6]

   a. The system type
   b. The appropriate static error constant
   c. The input waveform to yield a constant error
   d. The steady-state error for a unit input of the waveform found in Part **c**
   e. The steady-state value of the actuating signal.

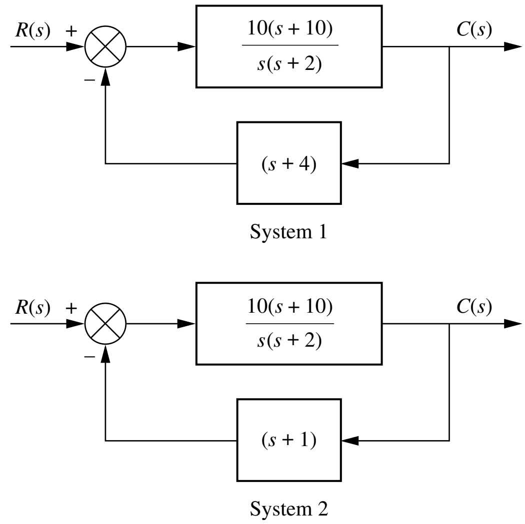


FIGURE P7.12 Closed-loop systems with nonunity feedback

32. For each system shown in Figure P7.13, find the appropriate static error constant as well as the steady-state error, $r(\infty) - c(\infty)$, for unit step, ramp, and parabolic inputs. [Section: 7.6]

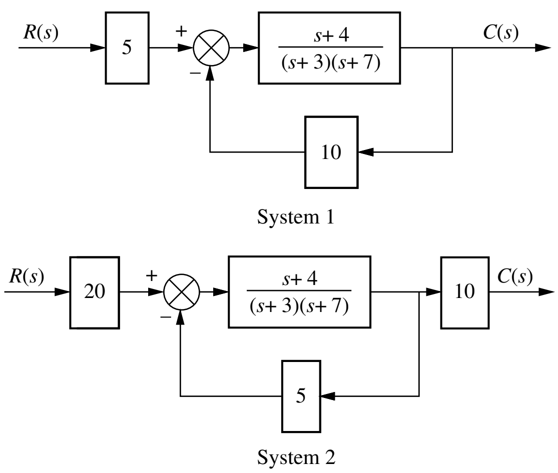


FIGURE P7.13

33. Given the system shown in Figure P7.14, find the following: [Section: 7.6]

a. The system type

b. The value of K to yield 0.1% error in the steady state.

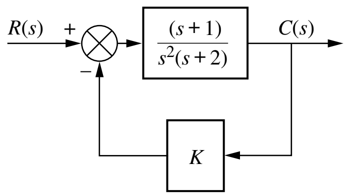


FIGURE P7.14

34. For the system shown in Figure P7.15, use MATLAB to find the following for $K = 10$, and $K = 10^6$: [Section: 7.6]

MATLAB **ML**

a. The system type

b. K_p, K_v, and K_a

c. The steady-state error for inputs of $30u(t)$, $30tu(t)$, and $30t^2u(t)$

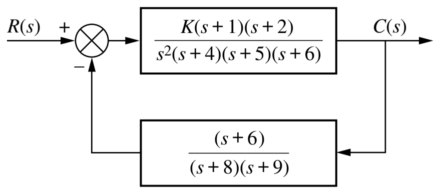


FIGURE P7.15

SS **35.** A dynamic voltage restorer (DVR) is a device that is connected in series to a power supply. It continuously monitors the voltage delivered to the load, and compensates voltage sags by applying the necessary extra voltage to maintain the load voltage constant.

In the model shown in Figure P7.16, u_r represents the desired reference voltage, u_o is the output voltage, and Z_L is the load impedance. All other parameters are internal to the DVR (*Lam, 2004*).

a. Assuming $Z_L = \dfrac{1}{sC_L}$, and $\beta \neq 1$, find the system's type.

b. Find the steady-state error to a unit step input as a function of β.

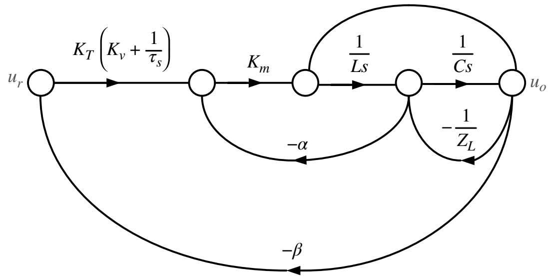


FIGURE P7.16 DVR Model

36. Derive Eq. (7.69) in the text. [Section: 7.6] **SS**

37. Given the system shown in Figure P7.17, do the following: [Section: 7.6]

a. Derive the expression for the error, $E(s) = R(s) - C(s)$, in terms of $R(s)$ and $D(s)$.

b. Derive the steady-state error, $e(\infty)$, if $R(s)$ and $D(s)$ are unit step functions.

c. Determine the attributes of $G_1(s)$, $G_2(s)$, and $H(s)$ necessary for the steady-state error to become zero.

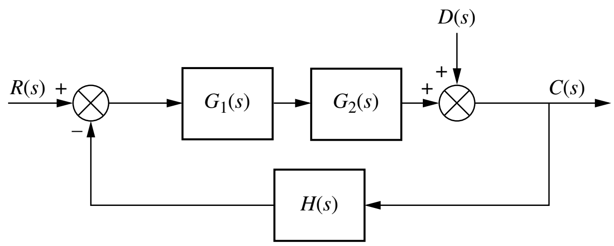


FIGURE P7.17 System with input and disturbance

38. Given the system shown in Figure P7.18, find the sensitivity of the steady-state error to parameter a. Assume a step input. Plot the sensitivity as a function of parameter a. [Section: 7.7]

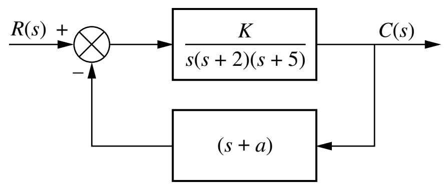


FIGURE P7.18

39. For the system shown in Figure P7.19, find the sensitivity of the steady-state error for changes in K_1 and in K_2, when $K_1 = 100$ and $K_2 = 0.1$. Assume step inputs for both the input and the disturbance. [Section: 7.7]

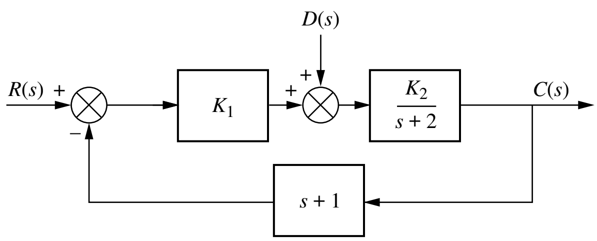


FIGURE P7.19 System with input and disturbance

40. Given the block diagram of the active suspension system shown in Figure P5.26 (*Lin, 1997*)

a. Find the transfer function from a road disturbance r to the error signal e.

b. Use the transfer function in Part **a** to find the steady-state value of e for a unit step road disturbance.

(*Continued*)

(*Continued*)

c. Use the transfer function in Part **a** to find the steady-state value of e for a unit ramp road disturbance.

d. From your results in Parts **b** and **c**, what is the system's type for e?

41. A simplified model of the steering of a four-wheel drive vehicle is shown in Figure P7.20.

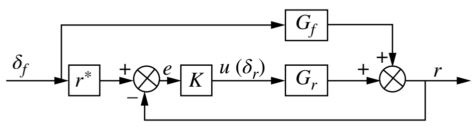


FIGURE P7.20 Steering model for a four-wheel drive vehicle[1]

In this block diagram, the output r is the vehicle's yaw rate, while δ_f and δ_r are the steering angles of the front and rear tires, respectively. In this model,

$$r^*(s) = \frac{\frac{s}{300} + 0.8}{\frac{s}{10} + 1}, G_f(s) = \frac{h_1 s + h_2}{s^2 + a_1 s + a_2}$$

$$G_r(s) = \frac{h_3 s + b_1}{s^2 + a_1 s + a_2}$$

and $K(s)$ is a controller to be designed. (*Yin*, *2007*).

a. Assuming a step input for δ_f, find the minimum system type of the controller $K(s)$ necessary so that in steady-state the error, as defined by the signal e in Figure P7.20, is zero if at all possible.

b. Assuming a step input for δ_f, find the system type of the controller $K(s)$ necessary so that in steady state the error as defined by $\delta_f(\infty) - r(\infty)$ is zero if at all possible.

42. As part of the development of a textile cross-lapper machine (*Kuo, 2010*), a torque input, $u(t) = \left\{ \begin{array}{cc} 1 & 0 \le t < 50 \\ -1 & 50 \le t < 100 \end{array} \right\}$, is applied to the motor of one of the movable racks embedded in a feedback loop. The corresponding velocity output response is shown in Figure P7.21.

a. What is the open-loop system's type?

b. What is the steady-state error?

c. What would be the steady-state error for a ramp input?

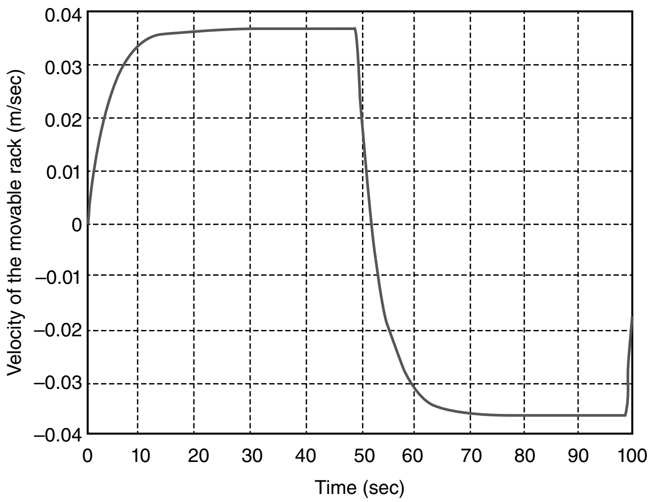


FIGURE P7.21 Velocity output response[2]

43. The block diagram in Figure P7.22 represents a motor driven by an amplifier with double-nested tachometer feedback loops (*Mitchell, 2010*).

a. Find the steady-state error of this system to a step input.

b. What is the system type?

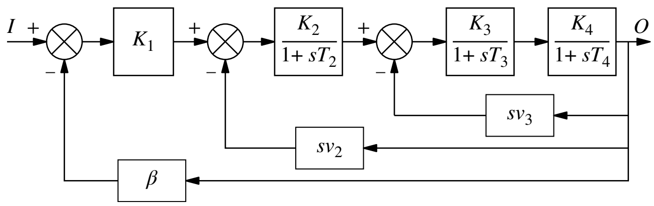


FIGURE P7.22[3]

44. PID control, which is discussed in Chapter 9, may be recommended for Type 3 systems when the output in a feedback system is required to perfectly track a parabolic as well as step and ramp reference signals (*Papadopoulos, 2013*). In the system of Figure P7.23, the transfer functions of the plant, $G_P(s)$, and the recommended controller, $G_C(s)$, are given by:

$$G_P(s) = \frac{127e^{-0.2s}}{s(s+1)(s+2)(s+5)^2(s+10)}$$

$$G_C(s) = \frac{(92.9s^2 + 13.63s + 1)}{97.6s^2(0.1s + 1)}$$

[1] Yin, G., Chen, N., and Li, P. Improving Handling Stability Performance of Four-Wheel Steering Vehicle via μ-Synthesis Robust Control. *IEEE Transactions on Vehicular Technology*, vol. 56, no. 5, 2007, pp. 2432–2439. Fig.2, p. 2434. IEEE transactions on vehicular technology by Vehicular Technology Society; Institute of Electrical and Electronics Engineers; IEEE Vehicular Technology Group Reproduced with permission of Institute of Electrical and Electronics Engineers, in the format Republish in a book via Copyright Clearance Center.

[2] Kuo, C-F. J., Tu, H-M., and Liu, C-H. Dynamic Modeling and Control of a Current New Horizontal Type Cross-Lapper Machine, *Textile Research Journal*, Vol. 80 (19), pp. 2016–2027, Figure 5. Copyright © 2010. Reprinted by Permission of SAGE.

[3] Mitchell, R. J. More Nested Velocity Feedback Control. *IEEE 9th International Conference on Cybernetic Intelligent Systems (CIS)*, 2010. Figure 5, page 3 of the paper.

Use Simulink to model this system and plot its response (from 0 to 300 seconds) to a unit-step reference input, $r(t)$, applied at $t = 0$, and (on the same graph) to a disturbance, $d(t) = 0.25\,r(t)$, applied at $t = 150$ seconds. What are the values of the steady-state error due to the reference input and due to the disturbance? What about the relative stability of this Type 3 system as evidenced by the percent overshoot in response to the unit-step reference input?

Simulink
SL

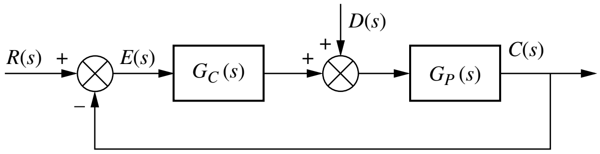


FIGURE P7.23

45. A Type 3 feedback control system (*Papadopoulos, 2013*) was presented in Problem 44. Modify the Simulink model you developed in that problem to plot its response (from 0 to 100 seconds) to a unit-ramp reference input, $r(t) = tu(t)$, applied at $t = 0$, and (on the same graph) to a disturbance, $d(t) = 0.25tu(t)$, applied at $t = 50$ seconds. What are the values of the steady-state position error due to the reference-input and disturbance ramps?

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Copy this model and paste it in the same file. Then, in that copy, change the reference input to a unit parabola, $r(t) = 0.5\,t^2\,u(t)$, applied at $t = 0$, and the disturbance to $d(t) = 0.125t^2u(t)$, applied at $t = 50$ seconds, and plot, on a new graph (Scope 1), the system's response to these parabolic signals.

DESIGN PROBLEMS

46. Motion control, which includes position or force control, is used in robotics and machining. Force control requires the designer to consider two phases: contact and noncontact motions. Figure P7.24(*a*) is a diagram of a mechanical system for force control under contact motion. A force command, $F_{cmd}(s)$, is the input to the system, while the output, $F(s)$, is the controlled contact force.

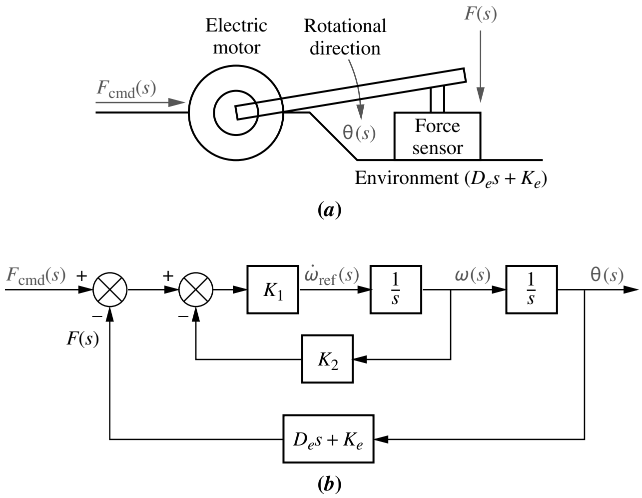


FIGURE P7.24 **a.** Force control mechanical loop under contact motion;[4] **b.** block diagram[4]

In the figure, a motor is used as the force actuator. The force output from the actuator is applied to the object through a force sensor. A block diagram representation of the system is shown in Figure P7.24(*b*). K_2 is velocity feedback used to improve the transient response. The loop is actually implemented by an electrical loop (not shown) that controls the armature current of the motor to yield the desired torque at the output. Recall that $T_m = K_t i_a$ (*Ohnishi, 1996*). Find an expression for the range of K_2 to keep the steady-state force error below 10% for ramp inputs of commanded force.

47. An open-loop swivel controller and plant for an industrial robot has the transfer function

$$G_e(s) = \frac{\omega_o(s)}{V_i(s)} = \frac{K}{(s+10)(s^2+4s+10)}$$

where $\omega_o(s)$ is the Laplace transform of the robot's angular swivel velocity and $V_i(s)$ is the input voltage to the controller. Assume $G_e(s)$ is the forward transfer function of a velocity control loop with an input transducer and sensor, each represented by a constant gain of 3 (*Schneider, 1992*).

a. Find the value of gain, K, to minimize the steady-state error between the input commanded angular

[4] Ohnishi, K.; Shibata, M.; and Murakami, T. Motion Control for Advanced Mechatronics, *IEEE/ASME Transactions on Mechatronics*, vol. 1, no. 1, March 1996, Figure 14 & 16, p. 62. IEEE/ASME transactions on mechatronics: a joint publication of the IEEE Industrial Electronics Society and the ASME Dynamic Systems and Control Division by Institute of Electrical and Electronics Engineers; IEEE Industrial Electronics Society; American Society of Mechanical Engineers.Dynamic Systems and Control Division Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

swivel velocity and the output actual angular swivel velocity.

b. What is the steady-state error for the value of K found in Part **a**?

c. For what kind of input does the design in Part **a** apply?

48. In Figure P7.11, the plant, $P(s)=\dfrac{48{,}500}{s^2+2.89s}$, represents the dynamics of a robotic manipulator joint. The system's output, $C(s)$, is the joint's angular position (*Low*, *2005*). The system is controlled in a closed-loop configuration as shown with $G(s) = K_P + \dfrac{K_I}{s}$, a proportional-plus-integral (PI) controller to be discussed in Chapter 9. $R(s)$ is the joint's desired angular position. $D(s)$ is an external disturbance, possibly caused by improper dynamics modeling, Coulomb friction, or other external forces acting on the joint.

a. Find the system's type.

b. Show that for a step disturbance input, $e_{ss} = 0$ when $K_I \neq 0$.

c. Find the value of K_I that will result in $e_{ss} = 5\%$ for a parabolic input.

d. Using the value of K_I found in Part **c**, find the range of K_P for closed-loop stability.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

49. Control of HIV/AIDS. Consider the HIV infection model of Problem 50 in Chapter 6 and its block diagram in Figure P6.15 (*Craig*, *2004*).

a. Find the system's type if $G(s)$ is a constant.

b. It was shown in Problem 50, Chapter 6, that when $G(s) = K$ the system will be stable when $K < 2.04 \times 10^{-4}$. What value of K will result in a unit step input steady-state error of 10%?

c. It is suggested that to reduce the steady-state error the system's type should be augmented by making $G(s) = \dfrac{K}{s}$. Is this a wise choice? What is the resulting stability range for K?

50. Hybrid vehicle. Figure P7.25 shows the block diagram of the speed control of an HEV taken from Figure P5.39, and rearranged as a unity-feedback system (*Preitl*, *2007*). Here the system output is $C(s) = K_{SS}V(s)$, the output voltage of the speed sensor/transducer.

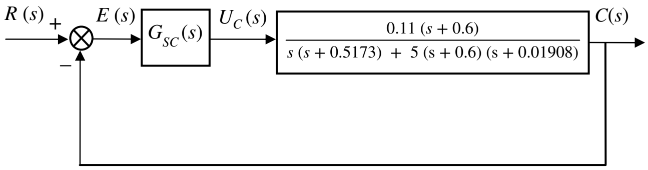


FIGURE P7.25

a. Assume the speed controller is given as $G_{SC}(s) = K_{P_{SC}}$. Find the gain, $K_{P_{SC}}$, that yields a steady-state error, $e_{\text{step}}(\infty) = 1\%$.

b. Now assume that in order to reduce the steady-state error for step inputs, integration is added to the controller yielding $G_{SC}(s) = K_{P_{SC}} + (K_{I_{SC}}/s) = 100 + (K_{I_{SC}}/s))$. Find the value of the integral gain, $K_{I_{SC}}$, that results in a steady-state error, $e_{\text{ramp}}(\infty) = 2.5\%$.

c. In Parts **a** and **b**, the HEV was assumed to be driven on level ground. Consider the case when, after reaching a steady-state speed with a controller given by $G_{SC}(s) = 100 + \dfrac{40}{s}$, the car starts climbing up a hill with a gradient angle, $\alpha = 5°$. For small angles $\sin\alpha = \alpha$ (in radians) and, hence, when reflected to the motor shaft the climbing torque is

$$T_{st} = \frac{F_{st}r}{i_{tot}} = \frac{mgr}{i_{tot}}\sin\alpha = \frac{mgr\alpha}{i_{tot}}$$

$$= \frac{1590 \times 9.8 \times 0.3 \times 5}{4.875 \times 57.3} = 83.7 \text{ Nm}.$$

The block diagram in Figure P7.26 represents the control system of the HEV rearranged for Part **c**.

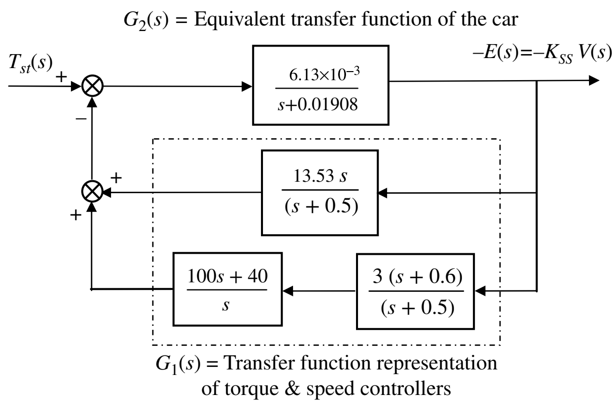


FIGURE P7.26

In this diagram, the input is $T_{st}(t) = 83.7\,u(t)$, corresponding to $\alpha = 5°$, and the output is the negative error,

$-e(t) = -c(t) = -K_{SS}v(t)$, proportional to the change in car speed, $v(t)$. Find the steady-state error $e(\infty)$ due to a step change in the disturbance; for example, the climbing torque, $T_{st}(t) = 83.7\,u(t)$.

51. Parabolic trough collector. The parabolic trough collector (*Camacho, 2012*) is embedded in a unit feedback configuration as shown in Figure P7.1, where $G(s) = G_C(s)P(s)$ and

$$P(s) = \frac{137.2 \times 10^{-6}}{S^2 + 0.0224s + 196 \times 10^{-6}} e^{-39s}$$

a. Assuming $G_C(s) = K$, find the value of K required for a unit-step input steady-state error of 3%. Use the result you obtained in Problem 52, Chapter 6, to verify that the system is closed-loop stable when that value of K is used.

b. What is the minimum unit-step input steady-state error achievable with $G_C(s) = K$?

c. What is the simplest compensator, $G_C(s)$, that can be used to achieve a steady-state error of 0%?

1. For each of the root loci shown in Figure P8.1, tell whether or not the sketch can be a root locus. If the sketch cannot be a root locus, explain why. Give *all* reasons. [Section: 8.4]

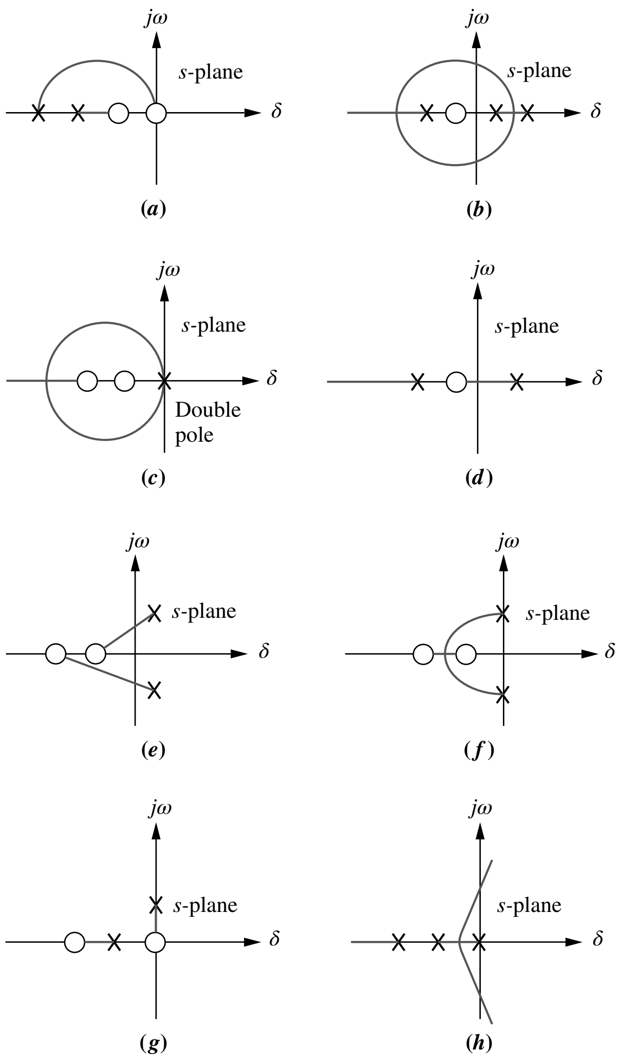


FIGURE P8.1

2. Sketch the general shape of the root locus for each of the open-loop pole-zero plots shown in Figure P8.2. [Section: 8.4]

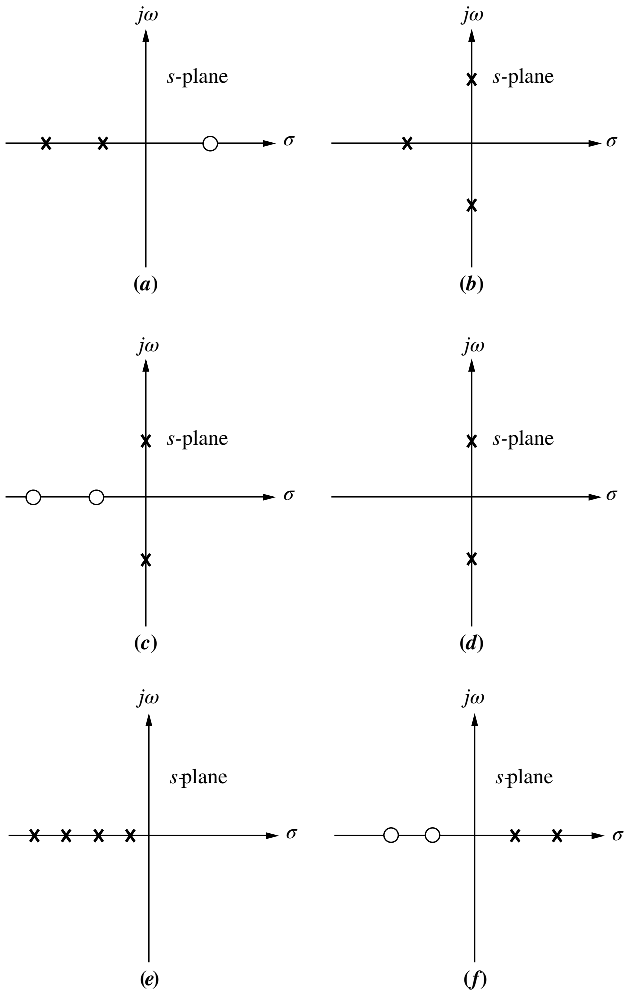


FIGURE P8.2

3. Let

$$G(s) = \frac{K\left(s + \frac{2}{3}\right)}{s^2(s+6)}$$

in Figure P8.3. [Section: 8.5]

FIGURE P8.3

a. Plot the root locus.

b. Write an expression for the closed-loop transfer function at the point where the three closed-loop poles meet.

4. For the open-loop pole-zero plot shown in Figure P8.4, sketch the root locus and find the break-in point. [Section: 8.5]

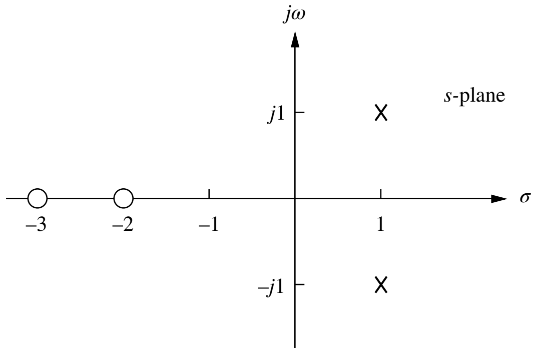


FIGURE P8.4

5. For Figure P8.3,

$$G(s) = \frac{K(s+1)(s+10)}{(s+4)(s-6)}$$

Sketch the root locus and find the value of K for which the system is closed-loop stable. Also find the break-in and breakaway points. [Section: 8.5]

6. The characteristic polynomial of a feedback control system, which is the denominator of the closed-loop transfer function, is given by $s^3 + 2s^2 + (20K + 7)s + 100K$. Sketch the root locus for this system. [Section: 8.8]

7. Plot the root locus for the unity-feedback system shown in Figure P8.3, where

$$G(s) = \frac{K(s+2)(s^2+4)}{(s+5)(s-3)}$$

For what range of K will the poles be in the right half-plane? [Section: 8.5]

8. Given the unity-feedback system of Figure P8.3, where

$$G(s) = \frac{K(s^2-16)}{(s^2+9)}$$

draw the root locus and indicate for what ranges of K the system is closed-loop stable. [Section: 8.5]

SS 9. For each system shown in Figure P8.5, make an accurate plot of the root locus and find the following: [Section: 8.5]

a. The breakaway and break-in points

b. The range of K to keep the system stable

c. The value of K that yields a stable system with critically damped second-order poles

d. The value of K that yields a stable system with a pair of second-order poles that have a damping ratio of 0.707

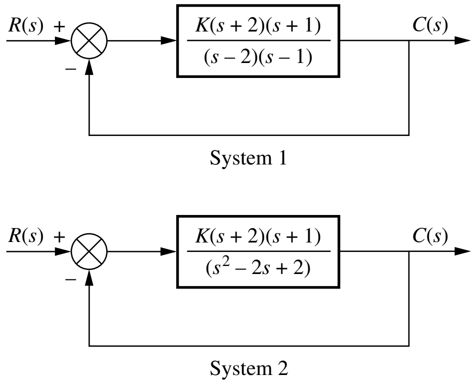


FIGURE P8.5

10. Sketch the root locus and find the range of K for stability for the unity-feedback system shown in Figure P8.3 for the following conditions: [Section: 8.5]

a. $G(s) = \dfrac{K(s^2+1)}{(s-1)(s+2)(s+3)}$

b. $G(s) = \dfrac{K(s^2-2s+2)}{s(s+1)(s+2)}$

11. Sketch the root locus and find the range of K for which the closed-loop system of Figure P8.3 will have only two right half-plane poles when [Section: 8.5]

$$G(s) = \frac{K(s+6)}{(s^2+1)(s-2)(s+4)}$$

12. For the unity-feedback system of Figure P8.3, where

$$G(s) = \frac{K}{s(s+5)(s+8)}$$

plot the root locus and calibrate your plot for gain. Find all the critical points, such as breakaways, asymptotes, $j\omega$-axis crossing, and so forth. [Section: 8.5]

13. Given the root locus shown in Figure P8.6. [Section: 8.5]

a. Find the value of gain that will make the system marginally stable.

b. Find the value of gain for which the closed-loop transfer function will have a pole on the real axis at -5.

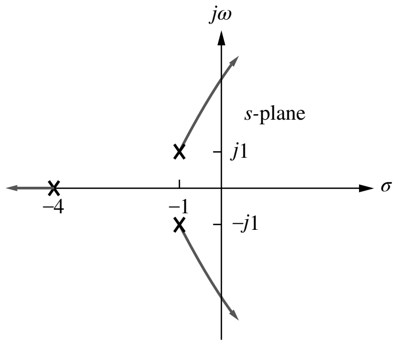


FIGURE P8.6

14. Let the unity-feedback system of Figure P8.3 be defined with

$$G(s) = \frac{K(s+3)}{s(s+1)(s+4)(s+6)}$$

Then do the following: [Section: 8.5]

a. Draw the root locus.

b. Obtain the asymptotes.

c. Obtain the value of gain that will make the system marginally stable.

d. Obtain the value of gain for which the closed-loop transfer function will have two identical real roots.

SS 15. For the unity-feedback system of Figure P8.3, where

$$G(s) = \frac{K(s+\alpha)}{s(s+3)(s+6)}$$

find the values of α and K that will yield a second-order closed-loop pair of poles at $-1 \pm j100$. [Section: 8.5]

16. Sketch the root locus for a unity-feedback system where

$$G(s) = \frac{K(s-2)(s-3)}{s(s+2)(s+3)}$$

Then find the following: [Section: 8.5]

a. The breakaway and break-in points

b. The crossing of the $j\omega$-axis

c. The range of K for closed-loop stability

d. The value of K that will result in a stable system with complex conjugate poles and damping factor of 0.5

SS 17. For the system of Figure P8.7(*a*), sketch the root locus and find the following: [Section: 8.7]

a. Asymptotes

b. Breakaway points

c. The range of K for stability

d. The value of K to yield a 0.7 damping ratio for the dominant second-order pair

To improve stability, we desire the root locus to cross the $j\omega$-axis at $j5.5$. To accomplish this, the open-loop function is cascaded with a zero, as shown in Figure P8.7(*b*).

e. Find the value of α and sketch the new root locus.

f. Repeat Part **c** for the new locus.

g. Compare the results of Part **c** and Part **f**. What improvement in transient response do you notice?

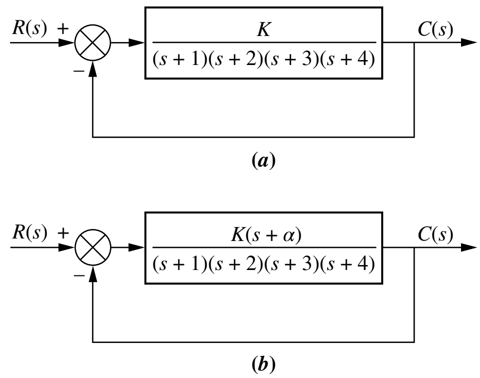


FIGURE P8.7

18. Sketch the root locus for the positive-feedback system shown in Figure P8.8. [Section: 8.9]

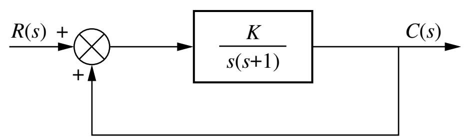


FIGURE P8.8

19. Given the unity-feedback system shown in Figure P8.3, where

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

do the following problem parts by first making a second-order approximation. After you are finished with all of the parts, justify your second-order approximation. [Section: 8.7]

a. Sketch the root locus.

b. Find K for 20% overshoot.

c. For K found in Part **b**, what is the settling time, and what is the peak time?

d. Find the locations of higher-order poles for K found in Part **b**.

e. Find the range of K for stability.

20. Assume for the unity-feedback system shown in Figure P8.3, that

$$G(s) = \frac{K(s^2 - 2s + 2)}{(s+1)(s+3)(s+4)(s+5)}$$

Then do the following: [Section: 8.7]

a. Make a sketch of the root locus.

b. Calculate the asymptotes.

c. Find the range of K for which the system is closed-loop stable.

d. Caculate the breakaway points.

e. Obtain the value of K that results in a step response with 20% overshoot.

f. Obtain the locations of all closed-loop poles when the system has 20% over shoot.

g. Discuss the validity of a second-order approximation for the given overshoot specification.

h. `Use MATLAB to verify or reject a second-order approximation for the closed-loop step response with the given percent overshoot.` MATLAB **ML**

SS **21.** The unity-feedback system shown in Figure P8.3, where

$$G(s) = \frac{K(s+2)(s+3)}{s(s+1)}$$

is to be designed for minimum damping ratio. Find the following: [Section: 8.7]

a. The value of K that will yield minimum damping ratio

b. The estimated percent overshoot for that case

c. The estimated settling time and peak time for that case

d. The justification of a second-order approximation (discuss)

e. The expected steady-state error for a unit ramp input for the case of minimum damping ratio

22. For the closed-loop system of Figure P8.3, it is specified to have a settling time of 1 second for large values of K when

$$G(s) = \frac{K(s+\alpha)}{s(s+5)(s+20)}$$

Find the appropriate value of α and sketch the resulting root locus. [Section: 8.8]

23. For the unity-feedback system shown in Figure 8.3, where SS

$$G(s) = \frac{K(s+5)}{(s^2+8s+25)(s+1)^2(s+\alpha)}$$

design K and α so that the dominant complex poles of the closed-loop function have a damping ratio of 0.5 and a natural frequency of 1.2 rad/s.

24. For the unity-feedback system shown in Figure 8.3, where

$$G(s) = \frac{K}{s(s+3)(s+4)(s+8)}$$

do the following: [Section: 8.7]

a. Sketch the root locus.

b. Find the value of K that will yield a 10% overshoot.

c. Locate all nondominant poles. What can you say about the second-order approximation that led to your answer in Part **b**?

d. Find the range of K that yields a stable system.

25. For the unity-feedback system shown in Figure 8.3, where SS

$$G(s) = \frac{K(s^2+4s+5)}{(s^2+2s+5)(s+3)(s+4)}$$

do the following: [Section: 8.7]

a. Find the gain, K, to yield a 1-second peak time if one assumes a second-order approximation.

b. `Check the accuracy of the second-order approximation using MATLAB to simulate the system.` MATLAB **ML**

26. For the unity-feedback system shown in Figure P8.3, where

$$G(s) = \frac{K(s+2)(s+3)}{(s^2+2s+2)(s+4)(s+5)(s+6)}$$

do the following: [Section: 8.7]

a. Sketch the root locus.

b. Find the $j\omega$-axis crossing and the gain, K, at the crossing.

c. Find all breakaway and break-in points.

d. Find angles of departure from the complex poles.

e. Find the gain, K, to yield a damping ratio of 0.3 for the closed-loop dominant poles.

27. Repeat Parts **a** through **c** and **e** of Problem 26 for [Section: 8.7]

$$G(s) = \frac{K(s+4)}{s(s+1)(s+2)(s+10)}$$

SS **28.** For the system shown in Figure P8.9, do the following: [Section: 8.7]

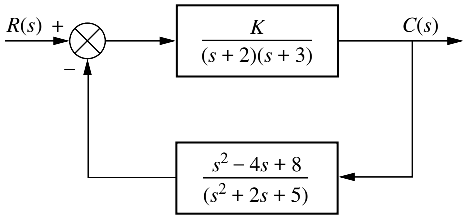


FIGURE P8.9

a. Sketch the root locus.

b. Find the $j\omega$-axis crossing and the gain, K, at the crossing.

c. Find the real-axis breakaway to two-decimal-place accuracy.

d. Find angles of arrival to the complex zeros.

e. Find the closed-loop zeros.

f. Find the gain, K, for a closed-loop step response with 30% overshoot.

g. Discuss the validity of your second-order approximation.

29. Sketch the root locus for the system of Figure P8.10 and find the following: [Section: 8.7]

a. The range of gain to yield stability

b. The value of gain that will yield a damping ratio of 0.707 for the system's dominant poles

c. The value of gain that will yield closed-loop poles that are critically damped

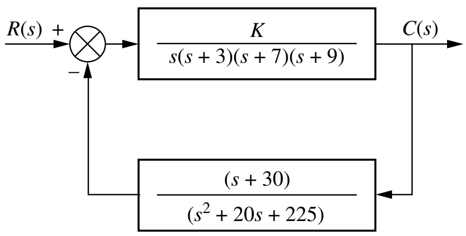


FIGURE P8.10

30. `Repeat Problem 29 using MATLAB. The program will do the following in one program:` MATLAB **ML**

a. `Display a root locus and pause.`

b. `Display a close-up of the root locus where the axes go from −2 to 0.5 on the real axis and −2 to 2 on the imaginary axis.`

c. `Overlay the 0.707 damping ratio line on the close-up root locus.`

d. `Allow you to select interactively the point where the root locus crosses the 0.707 damping ratio line, and respond by displaying the gain at that point as well as all of the closed-loop poles at that gain. The program will then allow you to select interactively the imaginary-axis crossing and respond with a display of the gain at that point as well as all of the closed-loop poles at that gain. Finally, the program will repeat the evaluation for critically damped dominant closed-loop poles.`

e. `Generate the step response for the critically damped case.`

31. Given the unity-feedback system shown in Figure P8.3, where

$$G(s) = \frac{K(s+z)}{s^2(s+10)}$$

do the following: [Section: 8.7]

a. If $z = 2$, find K so that the damped frequency of oscillation of the transient response is 5 rad/s.

b. For the system of Part **a**, what static error constant (finite) can be specified? What is its value?

c. The system is to be redesigned by changing the values of z and K. If the new specifications are $\%OS = 4.32\%$ and $T_s = 0.8$ s, find the new values of z and K.

32. Given the unity-feedback system shown in Figure P8.3, where

$$G(s) = \frac{K}{(s+1)(s+3)(s+6)^2}$$

find the following: [Section: 8.7]

a. The value of gain, K, that will yield a settling time of 4 seconds

b. The value of gain, K, that will yield a critically damped system

33. You are given the unity-feedback system of Figure P8.3, where MATLAB ML

$$G(s) = \frac{K(s+0.02)}{s^2(s+4)(s+10)(s+25)}$$

Use MATLAB to plot the root locus. Use a closeup of the locus (from -5 to 0 and -1 to 6) to find the gain, K, that yields a closed-loop unit-step response, $c(t)$, with 20.5% overshoot and a settling time of $T_s = 3$ seconds. Mark on the time response graph all other relevant characteristics, such as the peak time, rise time, and final steady-state value.

SS 34. Let

$$G(s) = \frac{K(s-1)}{(s+2)(s+3)}$$

in Figure P8.3. [Section: 8.7].

a. Find the range of K for closed-loop stability.

b. Plot the root locus for $K > 0$.

c. Plot the root locus for $K < 0$.

d. Assuming a step input, what value of K will result in the smallest attainable settling time?

e. Calculate the system's e_{ss} for a unit-step input assuming the value of K obtained in Part **d**.

f. Make an approximate hand sketch of the unit-step response of the system if K has the value obtained in Part **d**.

35. Figure P8.11 shows the block diagram of a closed-loop control of a linearized magnetic levitation system (*Galvão*, *2003*).

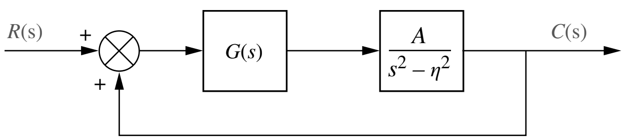


FIGURE P8.11 Linearized magnetic levitation system block diagram

Assuming $A = 1300$ and $\eta = 860$, draw the root locus and find the range of K for closed-loop stability when:

a. $G(s) = K$;

b. $G(s) = \dfrac{K(s+200)}{s+1000}$

36. The simplified transfer function model from steering angle $\delta(s)$ to tilt angle $\varphi(s)$ in a bicycle is given by

$$G(s) = \frac{\varphi(s)}{\delta(s)} = \frac{aV}{bh} \times \frac{s + \dfrac{V}{a}}{s^2 - \dfrac{g}{h}}$$

In this model, h represents the vertical distance from the center of mass to the floor, so it can be readily verified that the model is open-loop unstable. (*Åström*, *2005*). Assume that for a specific bicycle, $a = 0.6$ m, $b = 1.5$ m, $h = 0.8$ m, and $g = 9.8$ m/sec. In order to stabilize the bicycle, it is assumed that the bicycle is placed in the closed-loop configuration shown in Figure P8.3 and that the only available control variable is V, the rear wheel velocity.

a. Find the range of V for closed-loop stability.

b. Explain why the methods presented in this chapter cannot be used to obtain the root locus.

c. Use MATLAB to obtain the system's root locus. MATLAB ML

37. A technique to control the steering of a vehicle that follows a line located in the middle of a lane is to define a look-ahead point and measure vehicle deviations with respect to the point. A linearized model for such a vehicle is

$$\begin{bmatrix} \dot{V} \\ \dot{r} \\ \dot{\psi} \\ \dot{Y}_g \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & -b_1K & \dfrac{b_1K}{d} \\ a_{21} & a_{22} & -b_2K & \dfrac{b_2K}{d} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & U & 0 \end{bmatrix} \begin{bmatrix} V \\ r \\ \psi \\ Y_g \end{bmatrix}$$

where V = vehicle's lateral velocity, r = vehicle's yaw velocity, ψ = vehicle's yaw position, and Y_g = the y-axis coordinate of the vehicle's center of gravity. K is a parameter to be varied depending upon trajectory changes. In a specific vehicle traveling at a speed of $U = -10$ m/sec, the parameters are $a_{11} = -11.6842$, $a_{12} = 6.7632$, $b_1 = -61.5789$, $a_{21} = -3.5143$, $a_{22} = 24.0257$, and $b_2 = 66.8571$. $d = 5$ m is the look-ahead distance (*Ünyelioğlu*, *1997*). Assuming the vehicle will be controlled in closed loop:

a. Find the system's characteristic equation as a function of K.

b. Find the system's root locus as K is varied.

c. Using the root locus found in Part **b**, show that the system will be unstable for all values K.

38. For the dynamic voltage restorer (DVR) discussed in Problem 35, Chapter 7, do the following:

a. When $Z_L = \dfrac{1}{sC_L}$, a pure capacitance, the system is more inclined toward instability. Find the system's characteristic equation for this case.

b. Using the characteristic equation found in Part **a**, sketch the root locus of the system as a function of C_L. Let $L = 7.6$ mH, $C = 11\ \mu$F, $\alpha = 26.4$, $\beta = 1$, $K_m = 25$, $K_v = 15$, $K_T = 0.09565$, and $\tau = 2$ ms (*Lam, 2004*).

SS **39.** The closed-loop vehicle response in stopping a train depends on the train's dynamics and the driver, who is an integral part of the feedback loop. In Figure P8.3, let the input be $R(s) = v_r$ the reference velocity, and the output $C(s) = v$, the actual vehicle velocity. (*Yamazaki, 2008*) shows that such dynamics can be modeled by $G(s) = G_d(s)G_t(s)$ where

$$G_d(s) = h\left(1 + \frac{K}{s}\right)\frac{s - \frac{L}{2}}{s + \frac{L}{2}}$$

represents the driver dynamics with h, K, and L parameters particular to each individual driver. We assume here that $h = 0.003$ and $L = 1$. The train dynamics are given by

$$G_t(s) = \frac{k_b f K_p}{M(1 + k_e)s(\tau s + 1)}$$

where $M = 8000$ kg, the vehicle mass; $k_e = 0.1$ the inertial coefficient; $k_b = 142.5$, the brake gain; $K_p = 47.5$, the pressure gain; $\tau = 1.2$ seconds, the time constant; and $f = 0.24$, the normal friction coefficient.

a. Make a root locus plot of the system as a function of the driver parameter K.

b. Discuss why this model may not be an accurate description of a real driver-train situation.

40. Voltage droop control is a technique in which loads are driven at lower voltages than those provided by the source. In general, the voltage is decreased as current demand increases in the load. The advantage of voltage droop is that it results in lower sensitivity to load current variations.

Voltage droop can be applied to the power distribution of several generators and loads linked through a dc bus. In (*Karlsson, 2003*) generators and loads are driven by 3-phase ac power, so they are interfaced to the bus through ac/dc converters. Since each one of the loads works independently, a feedback system shown in Figure P8.12 is used in each to respond equally to bus voltage variations. Given that $C_s = C_r = 8{,}000\ \mu$F, $L_{cable} = 50\ \mu$H, $R_{cable} = 0.06\ \Omega$, $Z_r = R_r = 5\ \Omega$, $\omega_{lp} = 200$ rad/s, $G_{conv}(s) = 1$, $V_{dc\text{-}ref} = 750$ V, and $P_{ref\text{-}ext} = 0$, do the following:

a. If $Z_{r_{eq}}$ is the parallel combination of R_r and C_r, and $G_{conv}(s) = 1$, find

$$G(s) = \frac{V_s(s)}{I_s(s)} = \frac{V_s(s)}{I_{s-ref}(s)}$$

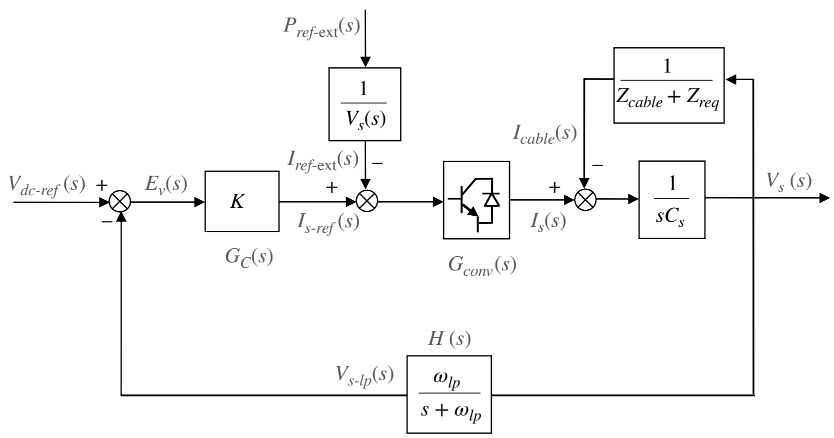


FIGURE P8.12[1]

[1] Karlsson, P., and Svensson, J. DC Bus Voltage Control for a Distributed Power System, *IEEE Trans. Power Electronics*, vol. 18, no. 6, 2003, pp. 1405–1412. Fig. 4, p. 1406. *IEEE Transactions on Power Electronics* by Institute of Electrical and Electronics Engineers; Power Electronics Council (Institute of Electrical and Electronic Engineers); IEEE Power Electronics Society. Reproduced with permission of Institute of Electrical and Electronics Engineers, in the format Republish in a book via Copyright Clearance Center.

b. Write a MATLAB M-file to plot and copy the full root locus for that system, then zoom-in the locus by setting the x-axis (real-axis) limits to −150 to 0 and the y-axis (imaginary-axis) limits to −150 to 150. Copy that plot, too, and find and record the following:

(1) The gain, K, at which the system would have complex-conjugate closed-loop dominant poles with a damping ratio $\zeta = 0.707$

(2) The coordinates of the corresponding point selected on the root-locus

(3) The values of all closed-loop poles at that gain

(4) The output voltage $v_s(t)$ for a step input voltage $v_{dc\text{-}ref}(t) = 750\,u(t)$ volts

c. Plot that step response and use the MATLAB **Characteristics** tool (in the graph window) to note on the curve the following parameters:

(1) The actual percent overshoot and the corresponding peak time, T_p

(2) The rise time, T_r, and the settling time, T_s

(3) The final steady-state value in volts

41. It has been suggested that the use of closed-loop feedback in ventilators can highly reduce mortality and health problems in patients in need of respiratory treatments (*Hahn, 2012*). A good knowledge of the transfer functions involved is necessary for the design of an appropriate controller. In a study with 18 patients it was found that the open-loop transfer function from minute ventilation (MV) to end-tidal carbon dioxide partial pressure ($P_{ET}CO_2$) can be nominally modeled as:

$$G(s) = \frac{0.415k_c(s+0.092)(s+0.25)}{s(s+0.007)(s+0.207)}$$

a. Make a sketch of the root locus of the system indicating the breakaway points and the value k_c takes in each of them.

b. In the design of ventilators it is very important to have negligible overshoot with the fastest possible settling time. It has been suggested that a value of $k_c = 5.35$ will achieve these specifications. Mark the position of the closed-loop poles for this value of k_c and explain why this is a reasonable gain choice.

42. Figure P8.13 shows a simplified drawing of a feedback system that includes the drive system

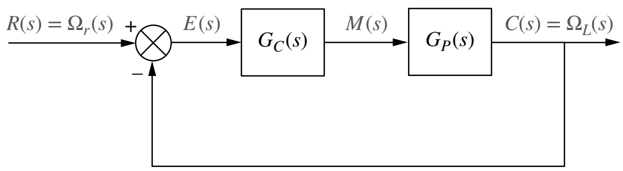


FIGURE P8.13

$$G(s) = \frac{25\,(s^2+1.2s+12500)}{s\,(s^2+5.6s+62000)}$$

presented in Problem 48, Chapter 5 (*Thomsen, 2011*). Referring to Figures P5.32 and P8.13, $G_p(s)$ in Figure P8.13 is given by:

$$G_p(s) = K_M \frac{G(s)}{1+0.1G(s)}$$

Given that the controller is proportional, that is, $G_C(s) = K_P$, use MATLAB and a procedure similar to that developed in Problem 30 in this chapter to plot the root locus[2] and obtain the output response, $c(t) = \omega_L(t)$, when a step input, $r(t) = \omega_r(t) = 260\,u(t)$ rad/s, is applied at $t = 0$. Mark on the time response graph, $c(t)$, all relevant characteristics, such as the percent overshoot (which should not exceed 16%), peak time, rise time, settling time, and final steady-state value.

DESIGN PROBLEMS

43. A simplified block diagram of a human pupil servomechanism is shown in Figure P8.14. The term $e^{-0.18s}$ represents a time delay. This function can be approximated by what is known as a Padé approximation. This approximation can take on many increasingly complicated forms, depending upon the degree of accuracy required. If we use the Padé approximation

$$e^{-x} = \frac{1}{1+x+\dfrac{x^2}{2!}}$$

then

$$e^{-0.18s} = \frac{61.73}{s^2+11.11s+61.73}$$

[2] Select a point on the closeup of the root locus that corresponds to a gain between 1 and 5.

Since the retinal light flux is a function of the opening of the iris, oscillations in the amount of retinal light flux imply oscillations of the iris (*Guy*, *1976*). Find the following:

a. The value of K that will yield oscillations

b. The frequency of these oscillations

c. The settling time for the iris if K is such that the eye is operating with 20% overshoot

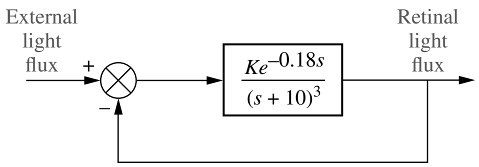


FIGURE P8.14 Simplified block diagram of pupil servomechanism

SS **44.** A hard disk drive (HDD) arm has an open-loop unstable transfer function,

$$P(s) = \frac{X(s)}{F(s)} = \frac{1}{I_b s^2}$$

where $X(s)$ is arm displacement and $F(s)$ is the applied force (*Yan*, *2003*). Assume the arm has an inertia of $I_b = 3 \times 10^{-5}$ kg-m^2 and that a lead controller, $G_c(s)$ (used to improve transient response and discussed in Chapter 9), is placed in cascade to yield

$$P(s)G_c(s) = G(s) = \frac{K}{I_b}\frac{(s+1)}{s^2(s+10)}$$

as in Figure P8.3.

a. Plot the root locus of the system as a function of K.

b. Find the value of K that will result in dominant complex conjugate poles with a $\zeta = 0.7$ damping factor.

45. Wind turbines, such as the one shown in Figure P8.15(*a*), are becoming popular as a way of generating electricity. Feedback control loops are designed to control the output power of the turbine, given an input power demand. Blade-pitch control may be used as part of the control loop for a constant-speed, pitch-controlled wind turbine, as shown in Figure P8.15(*b*). The drivetrain, consisting of the windmill rotor, gearbox, and electric generator (see Figure P8.15(*c*)), is part of the control loop. The torque created by the wind drives the rotor. The windmill rotor is connected to the generator through a gearbox.

The transfer function of the drivetrain is

$$\frac{P_o(s)}{T_R(s)} = G_{dt}(s) = \frac{3.92K_{LSS}K_{HSS}K_G N^2 s}{\{N^2K_{HSS}(J_R s^2 + K_{LSS})(J_G s^2[\tau_{el}s + 1] + K_G s) + J_R s^2 K_{LSS}[(J_G s^2 + K_{HSS})(\tau_{el}s + 1) + K_G s]\}}$$

Hammondovi/iStockphoto

(*a*)

FIGURE P8.15 (*continued*)

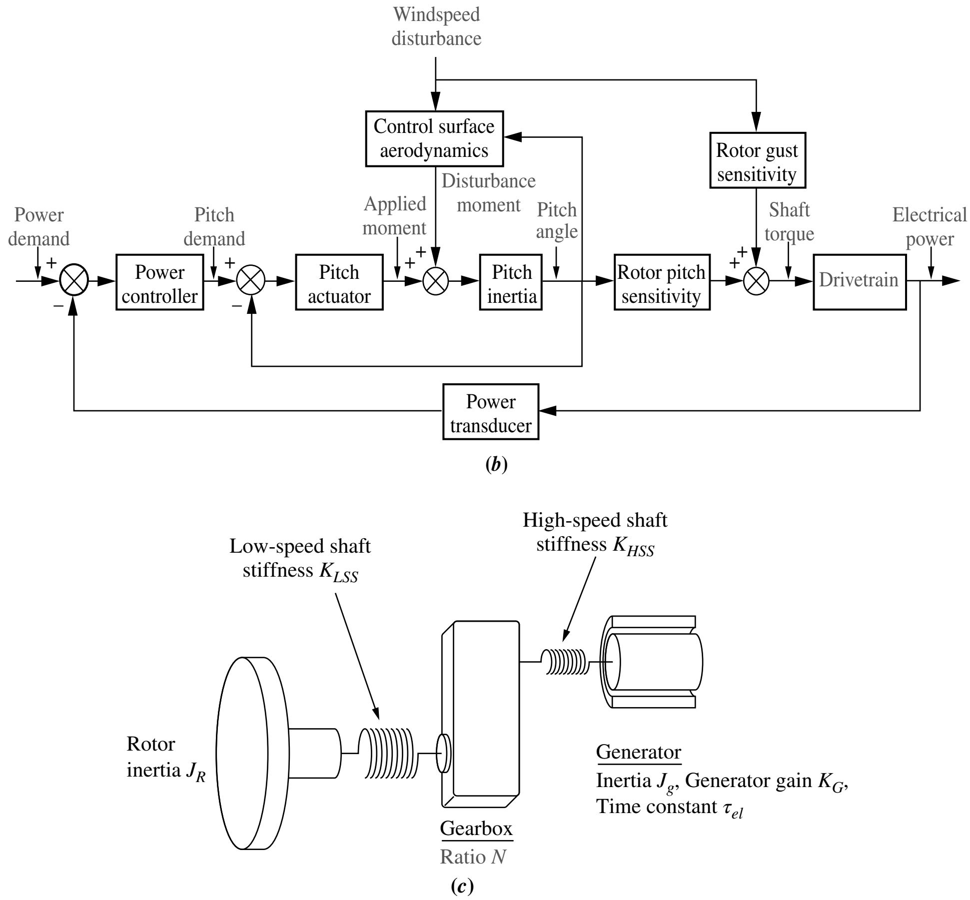


FIGURE P8.15 **a.** Wind turbines generating electricity near Palm Springs, California; **b.** control loop for a constant-speed pitch-controlled wind turbine;[3] **c.** drivetrain[3]

where $P_o(s)$ is the Laplace transform of the output power from the generator and $T_R(s)$ is the Laplace transform of the input torque on the rotor. Substituting typical numerical values into the transfer function yields

$$\frac{P_o(s)}{T_R(s)} = G_{dt}(s)$$
$$= \frac{(3.92)(12.6 \times 10^6)(301 \times 10^3)(688)N^2 s}{\{N^2(301 \times 10^3)(190,120s^2 + 12.6 \times 10^6)\times (3.8s^2[20 \times 10^{-3}s + 1] + 668s)+ 190,120s^2(12.6 \times 10^6)\times [(3.8s^2 + 301 \times 10^3)\times (20 \times 10^{-3}s + 1) + 668s]\}}$$

(*Anderson, 1998*). Do the following for the drivetrain dynamics, making use of any computational aids at your disposal:

a. Sketch a root locus that shows the pole locations of $G_{dt}(s)$ for different values of gear ratio, N.

b. Find the value of N that yields a pair of complex poles of $G_{dt}(s)$ with a damping ratio of 0.5.

46. Many implantable medical devices such as pacemakers, retinal implants, deep brain stimulators, and spinal cord stimulators are powered by an in-body battery that can be charged through a transcutaneous inductive device. Optimal battery charge can be obtained when the out-of-body charging circuit is in resonance with the implanted SS

[3] Adapted from Anderson, C. G.; Richon, J-B.; and Campbell, T. J. An Aerodynamic Moment-Controlled Surface for Gust Load Alleviation on Wind Turbine Rotors, *IEEE Transactions on Control System Technology*, vol. 6, no. 5, September 1998, pp. 577–595. © 1998 IEEE.

charging circuit (*Baker, 2007*). Under certain conditions, the coupling of both resonant circuits can be modeled by the feedback system in Figure P8.3 where

$$G(s) = \frac{Ks^4}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)^2}$$

The gain K is related to the magnetic coupling between the external and in-body circuits. K may vary due to positioning, skin conditions, and other variations. For this problem let $\zeta = 0.5$ and $\omega_n = 1$.

a. Find the range of K for closed-loop stability.

b. Draw the corresponding root locus.

47. MATLAB ML Harmonic drives are very popular for use in robotic manipulators due to their low backlash, high torque transmission, and compact size (*Spong, 2006*). The problem of joint flexibility is sometimes a limiting factor in achieving good performance. Consider that the idealized model representing joint flexibility is shown in Figure P8.16. The input to the drive is from an actuator and is applied at θ_m. The output is connected to a load at θ_l. The spring represents the joint flexibility and B_m and B_l represent the viscous damping of the actuator and load, respectively. Now we insert the device into the feedback loop shown in Figure P8.17. The first block in the forward path is a PD controller, which we will study in the next chapter. The PD controller is used to improve transient response performance.

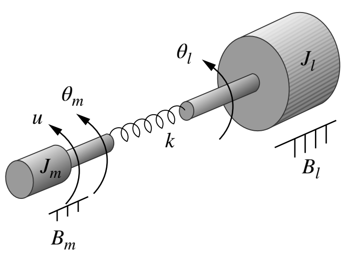


FIGURE P8.16 Idealized model representing joint flexibility[4]

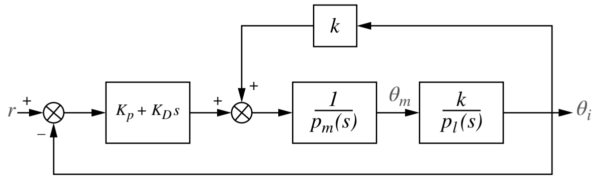


FIGURE P8.17 Joint flexibility model inserted in feedback loop[5]

Use MATLAB to find the gain K_D to yield an approximate 5% overshoot in the step response given the following parameters: $J_l = 10$; $B_l = 1$; $k = 100$; $J_m = 2$; $B_m = 0.5$; $\frac{K_P}{K_D} = 0.25$; $p_l(s) = J_l s^2 + B_l s + k$; and $p_m(s) = J_m s^2 + B_m s + k$

48. LabVIEW LV SS Using LabVIEW, the Control Design and Simulation Module, and the MathScript RT Module, open and customize the Interactive Root Locus VI from the Examples to implement the system of Problem 47. Select the parameter K_D to meet the requirement of Problem 47 by varying the location of the closed-loop poles on the root locus. Be sure your front panel shows the following: (1) open-loop transfer function, (2) closed-loop transfer function, (3) root locus, (4) list of closed-loop poles, and (5) step response.

49. MATLAB ML An automatic regulator is used to control the field current of a three-phase synchronous machine with identical symmetrical armature windings (*Stapleton, 1964*). The purpose of the regulator is to maintain the system voltage constant within certain limits. The transfer function of the synchronous machine is

$$G_{sm}(s) = \frac{\Delta\delta(s)}{\Delta P_m(s)} = \frac{M(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)(s - p_3)}$$

which relates the variation of rotor angle, $\Delta\delta(s)$, to the change in the synchronous machine's shaft power, $\Delta P_m(s)$. The closed-loop system is shown in Figure P8.3, where $G(s) = KG_C(s)G_{sm}(s)$ and K is a gain to be adjusted. The

[4] Spong, M., Hutchinson, S., and Vidyasagar, M.; *Robot Modeling and Control*. John Wiley & Sons, Hoboken, NJ, 2006. Figure 6.20, p. 221.

[5] Spong, M., Hutchinson, S., and Vidyasagar, M.; *Robot Modeling and Control*. John Wiley & Sons, Hoboken, NJ, 2006. Figure 6.24, p. 224.

regulator's transfer function, $G_c(s)$, is given by

$$G_c(s) = \frac{\mu / T_e}{s + \frac{1}{T_e}}$$

Assume the following parameter values:

$\mu = 4, M = 0.117, T_e = 0.5,$
$z_{1,2} = -0.071 \pm j6.25, p_1 = -0.047,$
and $p_{2,3} = -0.262 \pm j5.1,$

and do the following:

Write a MATLAB M-file to plot the root locus for the system and to find the following:

a. The gain K at which the system becomes marginally stable

b. The closed-loop poles, p, and transfer function, $T(s)$, corresponding to a 16% overshoot

c. The coordinates of the point selected on the root-locus corresponding to 16% overshoot

d. A simulation of the unit-step response of the closed-loop system corresponding to your 16% overshoot design. Note in your simulation the following values: (1) actual percent overshoot, (2) corresponding peak time, T_p, (3) rise time, T_r, (4) settling time, T_s, and (5) final steady-state value.

50. It is well known that when a person ingests a significant amount of water, the blood volume increases, causing an increase in arterial blood pressure until the kidneys are able to excrete the excess volume and the pressure returns to normal (*Shahin, 2010*). In order to describe this process mathematically, water-loading experiments are performed in various subjects while their mean arterial pressure is monitored. It was found that the open-loop transfer function of this process is

$$G(s) = \frac{b_p(1.759s^3 + 2.318s^2 + 2.173 \times 10^{-4})}{3.362s^3 + 11.34s^2 + 7.803s + 0.00293}$$

where b_p is an autonomous nervous activity parameter.

a. Make a sketch of the root locus of the system, indicating the breakaway points and the value of b_p for each point.

b. Indicate the range of b_p for which the system is overdamped.

c. Indicate the values of b_p for which the system is critically damped.

d. Indicate the range of b_p for which the system is underdamped.

e. Explain why the system will have a larger settling time for larger values of b_p.

51. One of the treatments for Parkinson's disease in some patients is Deep Brain Stimulation (DBS) (*Davidson, 2012*). In DBS a set of electrodes is surgically implanted and a vibrating current is applied to the subthalamic nucleus, also known as a brain pacemaker. Root locus has been used on a linearized model of the system to help explain the dynamics of DBS. The DBS model can be obtained by substituting $G(s) = \dfrac{ks}{(s-b)^2}$ $(b > 0)$ in the unity-feedback diagram of Figure P8.3.

a. Make a sketch of the resulting root locus as a function of k and find the break-in point and its corresponding value of gain.

b. Find the range of k for closed-loop stability in terms of b.

c. Find the frequency of oscillation when the system has closed-loop poles on the $j\omega$ axis.

52. A linear dynamic model of the α-subsystem of a grid-connected voltage-source converter (VSC) using a Y–Y transformer is shown in Figure P8.18(*a*) (*Mahmood, 2012*). Here, $C = 135\ \mu\text{F}$, $R_1 = 0.016\ \Omega$, $L_1 = 0.14$ mH, $R_2 = 0.014\ \Omega$, $L_2 = 10\ \mu$H, $R_g = 1.1\ \Omega$, and $L_g = 0.5$ mH.

a. Find the transfer function $G_P(s) = \dfrac{V_\alpha(s)}{M_\alpha(s)}$.

b. If $G_P(s)$ is the plant in Figure P8.18(*b*) and $G_C(s) = K$, use MATLAB to plot the root locus. On a closeup of the locus (from −300 to 0 on the real axis and from −50 to 5000 on the imaginary axis), find K and the coordinates of the dominant poles, which correspond to $\zeta = 0.012$. Plot the output response, $c(t) = v_\alpha(t)$, at that value of the gain when a step input, $r(t) = v_r(t) = 208\,u(t)$ volts, is applied at $t = 0$. Mark on the time response graph, $c(t)$, all relevant characteristics, such as the percent overshoot, peak time, rise time, settling time, and final steady-state value.

MATLAB
ML

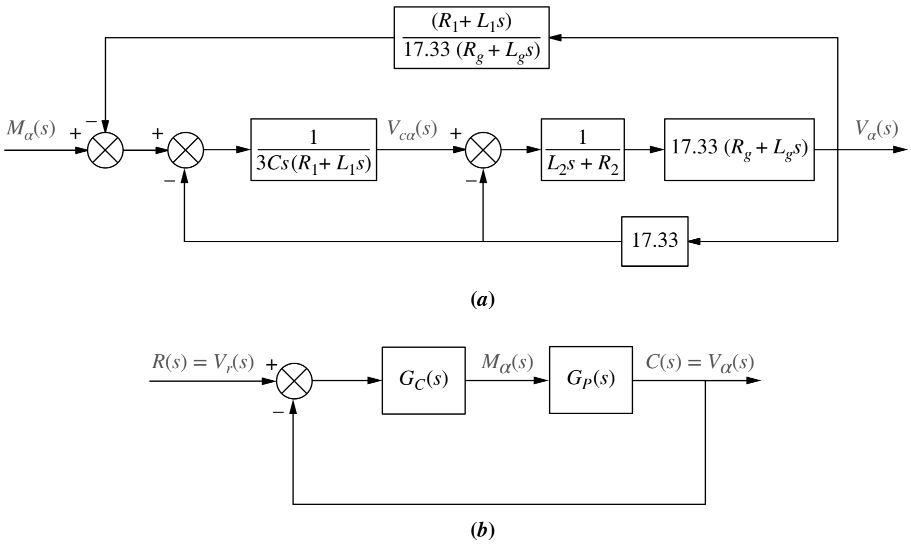


FIGURE P8.18

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

53. Control of HIV/AIDS. In the linearized model of Chapter 6, Problem 50, where virus levels are controlled by means of RTIs, the open-loop plant transfer function was shown to be

$$P(s)=\frac{Y(s)}{U_1(s)}=\frac{-520s-10.3844}{s^3+2.6817s^2+0.11s+0.0126}$$

The amount of RTIs delivered to the patient will automatically be calculated by embedding the patient in the control loop as $G(s)$ shown in Figure P6.15 (*Craig, 2004*).

a. In the simplest case, $G(s)=K$, with $K>0$. Note that this effectively creates a positive-feedback loop because the negative sign in the numerator of $P(s)$ cancels out with the negative-feedback sign in the summing junction. Use positive-feedback rules to plot the root locus of the system.

b. Now assume $G(s)=-K$ with $K>0$. The system is now a negative-feedback system. Use negative-feedback rules to draw the root locus. Show that in this case the system will be closed-loop stable for all $K>0$.

54. Hybrid vehicle. In Chapter 7, Figure P7.25 shows the block diagram of the speed control of an HEV rearranged as a unity-feedback system (*Preitl, 2007*).

MATLAB ML

Let the transfer function of the speed controller be

$$G_{SC}(s)=K_{P_{sc}}+\frac{K_{I_{sc}}}{s}=\frac{K_{P_{sc}}\left(s+\frac{K_{I_{sc}}}{K_{P_{sc}}}\right)}{s}$$

a. Assume first that the speed controller is configured as a proportional controller ($K_{I_{sc}}=0$ and $G_{SC}(s)=K_{P_{sc}}$). Calculate the forward-path open-loop poles. Now use MATLAB to plot the system's root locus and find the gain, $K_{P_{sc}}$ that yields a critically damped closed-loop response. Finally, plot the time-domain response, $c(t)$, for a unit-step input using MATLAB. Note on the curve the rise time, T_r, and settling time, T_s.

MATLAB ML

b. Now add an integral gain, $K_{I_{sc}}$, to the controller, such that $K_{I_{sc}}/K_{P_{sc}}=0.4$. Use MATLAB to plot the root locus and find the proportional gain, $K_{P_{sc}}$, that could lead to a closed-loop unit-step response with 10% overshoot. Plot $c(t)$ using MATLAB and note on the curve the peak time, T_p, and settling time, T_s. Does the response obtained resemble a second-order underdamped response?

MATLAB ML

55. **Parabolic trough collector.** Consider the fluid temperature control of a parabolic trough collector (*Camocho*, *2012*) embedded in the unity-feedback structure as shown in Figure P8.3, where the open-loop plant transfer function is given by

$$G(s) = \frac{137.2 \times 10^{-6} K}{s^2 + 0.0224s + 196 \times 10^{-6}} e^{-39s}$$

Approximating the time-delay term with $e^{-sT} \approx \dfrac{1 - \dfrac{T}{2}s}{1 + \dfrac{T}{2}s}$, make a sketch of the resulting root locus (Note: After substituting the approximation, $G(\infty) < 0$, the positive feedback rules of Section 8.9 must be used). Mark where appropriate in the plot and find:

a. The asymptotes and their intersection with the real axis;

b. The break-in and breakaway points. (The procedures presented in Section 8.5 are also valid for positive feedback systems);

c. The range of K for closed-loop stability;

d. The value of K that will make the system oscillate and the oscillation frequency.

Chapter 9 Problems

1. In the system of Figure P9.1, it is desired to have a step response with zero steady-state error, and a closed-loop damping ratio if 0.5. The open loop transfer function is

$$G(s) = \frac{K}{(s+2)^2(s+20)}$$

Design a PI controller for the given specifications. Compare the performance of the uncompensated and compensated systems. [Section: 9.2]

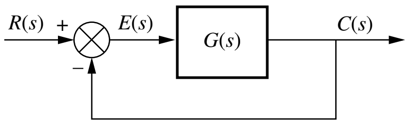


FIGURE P9.1

SS 2. Consider the unity-feedback system shown in Figure P9.1, where

$$G(s) = \frac{K}{s(s+3)(s+6)}$$

a. Design a PI controller to drive the ramp response error to zero for any K that yields stability. [Section: 9.2]

b. `Use MATLAB to simulate your design for K = 1. Show both the input ramp and the output response on the same plot.` MATLAB ML

3. Assume that for step inputs the system of Figure P9.1 exhibits 15% overshoot when

$$G(s) = \frac{K}{(s+1)(s+2)(s+5)}$$

[Section: 9.2]

a. What static error constant applies to this system, and what is its value?

b. Design a lag network so that the applicable static error constant has a value of 1 without significantly changing the position of the dominant poles of the system.

c. `Simulate the system to verify the effects of your compensator using MATLAB or any other computer program.` MATLAB ML

4. For the unity-feedback system of Figure P9.1, let

$$G(s) = \frac{K}{(s+2)(s+4)(s+6)}$$

a. Design a compensator that will not significantly change the position of the uncompensated dominant poles that result in 10% overshoot but yields $K_p = 20$. [Section: 9.2]

b. `Simulate the uncompensated and compensated systems using MATLAB or any other computer program.` MATLAB ML

c. `Find out how long will it take for the slow response of the lag compensator to reach 2% of the final value of the output. Use MATLAB or any other computer program.` MATLAB ML

5. The unity-feedback system shown in Figure P9.1 with SS

$$G(s) = \frac{K(s+6)}{(s+2)(s+3)(s+5)}$$

is operating with a dominant-pole damping ratio of 0.707. Design a PD controller so that the settling time is reduced by a factor of 2. Compare the transient and steady-state performance of the uncompensated and compensated systems. Describe any problems with your design. [Section: 9.3]

6. Let $G(s)$ in the unity-feedback system shown in Figure P9.1 be [Section: 9.3]

$$G(s) = \frac{K}{(s+4)^3}$$

a. Find the dominant poles' location to yield a 1.2 second settling time and an overshoot of 15%.

b. Assuming that a compensator is designed with a zero at -1 to achieve the conditions of Part **a**, find the angular contribution of the compensator pole.

c. Where is the compensator pole located?

d. Find the gain required to meet the requirements of Part **a**.

e. Find the location of other closed-loop poles for the compensated system.

f. Make an argument for the validity of your second-order approximation.

g. `Check your design by simulating your system using MATLAB or any other computer program.` MATLAB ML

7. The unity-feedback system shown in Figure P9.1 with

$$G(s) = \frac{K}{s^2}$$

is to be designed for a settling time of 1.667 seconds and a 16.3% overshoot. If the compensator zero is placed at -1, do the following: [Section: 9.3]

a. Find the coordinates of the dominant poles.

b. Find the compensator pole.

c. Find the system gain.

d. Find the location of all nondominant poles.

(*problem continues*)

(*continued*)

e. Estimate the accuracy of your second-order approximation.

f. Evaluate the steady-state error characteristics.

g. `Use MATLAB or any other computer program to simulate the system and evaluate the actual transient response characteristics for a step input.` MATLAB **ML**

8. Consider the unity-feedback system of Figure P9.1, with

$$G(s) = \frac{K(s+5)}{(s+2)(s+3)(s+7)(s+10)}$$

do the following: [Section: 9.3]

a. Draw the root locus.

b. Find the location of the dominant poles when $\zeta = 0.8$.

c. Find the gain at which $\zeta = 0.8$.

d. If the system is to be cascade-compensated to attain $T_s = 1$ second and $\zeta = 0.8$, find the compensator pole if the compensator zero is at -4.

e. Make an argument for the validity of your second-order approximation.

f. `Verify the validity of your design by simulating your system using MATLAB or any other computer program.` MATLAB **ML**

9. `Redo Problem 8 using MATLAB in the following way:` MATLAB **ML**

a. `MATLAB will generate the root locus for the uncompensated system along with the 0.8 damping ratio line. You will interactively select the operating point. MATLAB will then inform you of the coordinates of the operating point, the gain at the operating point, as well as the estimated` $\%OS$, T_s, T_p, ζ, ω_n, `and` K_p `represented by a second-order approximation at the operating point.`

b. `MATLAB will display the step response of the uncompensated system.`

c. `Without further input, MATLAB will calculate the compensated design point and will then ask you to input a value for the lead compensator pole from the keyboard. MATLAB will respond with a plot of the root locus showing the compensated design point. MATLAB will then allow you to keep changing the lead compensator pole value from the keyboard until a root locus is plotted that goes through the design point.`

d. `For the compensated system, MATLAB will inform you of the coordinates of the operating point, the gain at the operating point, as well as the estimated` $\%OS$, T_s, T_p, ζ, ω_n, `and` K_p `represented by a second-order approximation at the operating point.`

e. `MATLAB will then display the step response of the compensated system.`

f. `Change the compensator's zero location a few times and collect data on the compensated system to see if any other choices of compensator zero yield advantages over the original design.`

10. Consider the unity-feedback system of Figure P9.1 with **SS**

$$G(s) = \frac{K}{s(s+20)(s+40)}$$

The system is operating at 20% overshoot. Design a compensator to decrease the settling time by a factor of 2 without affecting the percent overshoot and do the following: [Section: 9.3]

a. Evaluate the uncompensated system's dominant poles, gain, and settling time.

b. Evaluate the compensated system's dominant poles and settling time.

c. Evaluate the compensator's pole and zero. Find the required gain.

d. `Use MATLAB or any other computer program to simulate the compensated and uncompensated systems' step response.` MATLAB **ML**

11. The unity-feedback system shown in Figure P9.1 with **SS**

$$G(s) = \frac{K}{(s+15)(s^2+6s+13)}$$

is operating with 30% overshoot. [Section: 9.3]

a. Find the transfer function of a cascade compensator, the system gain, and the dominant pole location that will cut the settling time in half if the compensator zero is at -7.

b. Find other poles and zeros and discuss your second-order approximation.

c. `Use MATLAB or any other computer program to simulate both the uncompensated and compensated systems to see the effect of your compensator.` MATLAB **ML**

12. A unity-feedback control system has the following forward transfer function: [Section: 9.3]

$$G(s) = \frac{K}{s^2(s+4)(s+12)}$$

   a. Design a lead compensator to yield a closed-loop step response with 20.5% overshoot and a settling time of 3 seconds. Be sure to specify the value of K.
   b. Is your second-order approximation valid?
   c. `Use MATLAB or any other computer program to simulate and compare the transient response of the compensated system to the predicted transient response.` MATLAB ML

13. For the unity-feedback system of Figure P9.1, with

$$G(s) = \frac{K}{(s^2+20s+101)(s+20)}$$

the damping ratio for the dominant poles is to be 0.4, and the settling time is to be 0.5 second. [Section: 9.3]

   a. Find the coordinates of the dominant poles.
   b. Find the location of the compensator zero if the compensator pole is at -15.
   c. Find the required system gain.
   d. Compare the performance of the uncompensated and compensated systems.
   e. `Use MATLAB or any other computer program to simulate the system to check your design. Redesign if necessary.` MATLAB ML

14. Consider the unity-feedback system of Figure P9.1, with

$$G(s) = \frac{K}{(s+3)(s+5)}$$

   a. Show that the system cannot operate with a settling time of 2/3 second and a percent overshoot of 1.5% with a simple gain adjustment.
   b. Design a lead compensator so that the system meets the transient response characteristics of Part **a.** Specify the compensator's pole, zero, and the required gain.

SS 15. Given the unity-feedback system of Figure P9.1, with

$$G(s) = \frac{K}{(s+2)(s+4)(s+6)(s+8)}$$

Find the transfer function of a lag–lead compensator that will yield a settling time 0.5 second shorter than that of the uncompensated system. The compensated system also will have a damping ratio of 0.5, and improve the steady-state error by a factor of 30. The compensator zero is at -5. Also, find the compensated system's gain. Justify any second-order approximations or verify the design through simulation. [Section: 9.4]

16. Given the uncompensated unity-feedback system of Figure P9.1, with SS

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

do the following: [Section: 9.4]

   a. Design a compensator to yield the following specifications: settling time = 2.86 seconds; percent overshoot = 4.32%; the steady-state error is to be improved by a factor of 2 over the uncompensated system.
   b. Compare the transient and steady-state error specifications of the uncompensated and compensated systems.
   c. Compare the gains of the uncompensated and compensated systems.
   d. Discuss the validity of your second-order approximation.
   e. `Use MATLAB or any other computer program to simulate the uncompensated and compensated systems and verify the specifications.` MATLAB ML

17. For the unity-feedback system given in Figure P9.1 with

$$G(s) = \frac{K}{s(s+5)(s+11)}$$

do the following: [Section: 9.4]

   a. Find the gain, K, for the uncompensated system to operate with 30% overshoot.
   b. Find the peak time and K_v for the uncompensated system.
   c. Design a lag–lead compensator to decrease the peak time by a factor of 2, decrease the percent overshoot by a factor of 2, and improve the steady-state error by a factor of 30. Specify all poles, zeros, and gains.

18. Consider the unity-feedback system in Figure P9.1, with

$$G(s) = \frac{K}{(s+2)(s+4)}$$

The system is operated with 4.32% overshoot. In order to improve the steady-state error, K_p is to be increased by at least a factor of 5. A lag compensator of the form

$$G_c(s) = \frac{(s+0.5)}{(s+0.1)}$$

is to be used. [Section: 9.4]

   a. Find the gain required for both the compensated and the uncompensated systems.
   b. Find the value of K_p for both the compensated and the uncompensated systems.

(problem continues)

(*continued*)

c. Estimate the percent overshoot and settling time for both the compensated and the uncompensated systems.

d. Discuss the validity of the second-order approximation used for your results in Part **c**.

e. `Use MATLAB or any other computer program to simulate the step response for the uncompensated and compensated systems. What do you notice about the compensated system's response?` MATLAB ML

f. Design a lead compensator that will correct the objection you notice in Part **e**.

19. For the unity-feedback system in Figure P9.1, with

$$G(s) = \frac{K}{(s+1)(s+3)}$$

design a PID controller that will yield a peak time of 1.122 seconds and a damping ratio of 0.707, with zero error for a step input. [Section: 9.4]

SS **20.** For the unity-feedback system in Figure P9.1, with

$$G(s) = \frac{K}{(s+4)(s+6)(s+10)}$$

do the following:

a. Design a controller that will yield no more than 25% overshoot and no more than a 2-second settling time for a step input and zero steady-state error for step and ramp inputs.

b. `Use MATLAB and verify your design.` MATLAB ML

SS **21.** `Redo Problem 20 using MATLAB in the following way:` MATLAB ML

a. `MATLAB will ask for the desired percent overshoot, settling time, and PI compensator zero.`

b. `MATLAB will design the PD controller's zero.`

c. `MATLAB will display the root locus of the PID-compensated system with the desired percent overshoot line.`

d. `The user will interactively select the intersection of the root locus and the desired percent overshoot line.`

e. `MATLAB will display the gain and transient response characteristics of the PID-compensated system.`

f. `MATLAB will display the step response of the PID-compensated system.`

g. `MATLAB will display the ramp response of the PID-compensated system.`

22. If the system of Figure P9.2 operates with a damping ratio of 0.456 for the dominant second-order poles, find the location of all closed-loop poles and zeros.

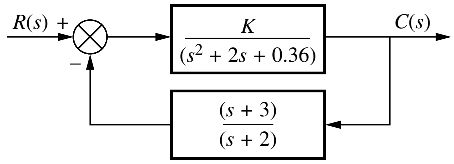


FIGURE P9.2

23. For the unity-feedback system in Figure P9.1, with SS

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)}$$

do the following: [Section: 9.5]

a. Design rate feedback to yield a step response with no more than 15% overshoot and no more than 3 seconds settling time. Use Approach 1.

b. `Use MATLAB and simulate your compensated system.` MATLAB ML

24. Given the system of Figure P9.3: [Section: 9.5]

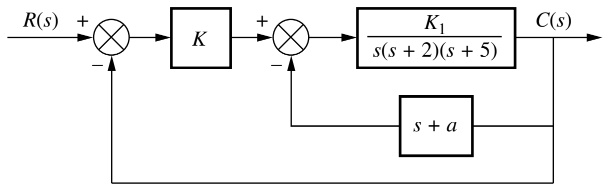


FIGURE P9.3

a. Design the value of K_1, as well as a in the feedback path of the minor loop, to yield a settling time of 4 seconds with 5% overshoot for the step response.

b. Design the value of K to yield a major-loop response with 10% overshoot for a step input.

c. `Use MATLAB or any other computer program to simulate the step response to the entire closed-loop system.` MATLAB ML

d. `Add a PI compensator to reduce the major-loop steady-state error to zero and simulate the step response using MATLAB or any other computer program.` MATLAB ML

25. Design a PI controller to drive the step-response error to zero for the unity-feedback system shown in Figure P9.1, where SS

$$G(s) = \frac{K}{(s+1)(s+3)(s+10)}$$

The system operates with a damping factor of 0.4. Design for each of the following two cases:

(1) compensator zero at -0.1 and (2) compensator zero at -0.7.

Compare the specifications of the uncompensated and each one of the compensated systems. Simulate each one of the systems using any software program.

26. An inverted pendulum mounted on a motor-driven cart was introduced in Problem 25 in Chapter 3. Its state-space model was linearized around a stationary point, $\mathbf{x}_0 = \mathbf{0}$ (*Prasad, 2012*). At the stationary point, the pendulum point-mass, m, is in the upright position at $t = 0$, and the force applied to the cart, u_0, is 0. Its model was then modified in Problem 37 in Chapter 6 to have two output variables: the pendulum angle relative to the y-axis, $\theta(t)$, and the horizontal position of the cart, $x(t)$. MATLAB was then used to find its eigenvalues. Noting that only one pole (out of four) is located in the left half of the s-plane, we concluded that this unit requires stabilization.

To accomplish stability and design an appropriate control system, do the following:

a. Draw a signal-flow diagram for that unit and use it to develop Simulink models for two feedback systems: one to control the cart position, $x(t)$, and the other to control the pendulum angle, $\theta(t)$. Set the upper and lower saturation limits of the second integrator in the angle loop to 100 and -100, respectively, and those limits in the position loop to 10 and -10. Simulink **SL**

b. Use rate feedback with a gain of 12.5 to stabilize the pendulum angle control system. The forward path of each of these systems should include a PD (proportional-plus-derivative) controller that adjusts the force applied to the cart, $u(t)$. These controllers[1] may be configured with the following settings:[2]

$P = 5$, $I = 0$, and $D = 5$ for the cart position controller

$P = 2$, $I = 0$, and $D = 10$ for the pendulum angle controller

c. Utilizing scopes to capture the two output responses, use a unit-impulse[3] as the reference input in the angle control loop and a unit-step source (configured to start at $t = 0$ with a final value of 1) as the input for the cart position control loop.

d. If either of the responses has a steady-state error, $e(\infty) > 2\%$, a peak time, $T_P > 1.2$ seconds, or a percent overshoot, $\%OS > 20.5\%$, suggest appropriate changes to its controller settings.

27. Identify and realize the following controllers with operational amplifiers. [Section: 9.6]

a. $\dfrac{s + 0.01}{s}$

b. $s + 2$

28. Identify and realize the following compensators with passive networks. [Section: 9.6]

a. $\dfrac{s + 0.1}{s + 0.01}$

b. $\dfrac{s + 2}{s + 5}$

c. $\left(\dfrac{s + 0.1}{s + 0.01}\right)\left(\dfrac{s + 1}{s + 10}\right)$

29. Repeat Problem 28 using operational amplifiers. [Section: 9.6]

DESIGN PROBLEMS

30. Figure P9.4(a) shows a heat-exchanger process whose purpose is to maintain the temperature of a liquid at a prescribed temperature. **SS**

The temperature is measured using a sensor and a transmitter, TT 22, that sends the measurement to a corresponding controller, TC 22, that compares the actual temperature with a desired temperature set point, SP. The controller automatically opens or closes a valve to allow or prevent the flow of steam to change the temperature in the tank. The corresponding block diagram for this system is shown in Figure P9.4(b) (*Smith, 2002*). Assume the following transfer functions:

$$G_v(s) = \frac{0.02}{4s + 1}; \quad G_1(s) = \frac{70}{50s + 1}; \quad H(s) = \frac{1}{12s + 1}$$

a. Assuming $G_c(s) = K$, find the value of K that will result in a dominant pole with $\zeta = 0.7$. Obtain the corresponding T_s.

b. Design a PD controller to obtain the same damping factor as Part **a** but with a settling time 20% smaller.

c. Verify your results through MATLAB simulation. MATLAB **ML**

31. Repeat Problem 30, Parts **b** and **c**, using a lead compensator.

---

[1] These are PID controllers, in which the integral actions are set to zero to avoid any negative effect on stability.

[2] Note that a high value for the derivative action of the angle controller and a low value for its proportional gain have been selected to further stabilize the pendulum.

[3] To create a unit-impulse, use a unit-step source followed by a derivative block.

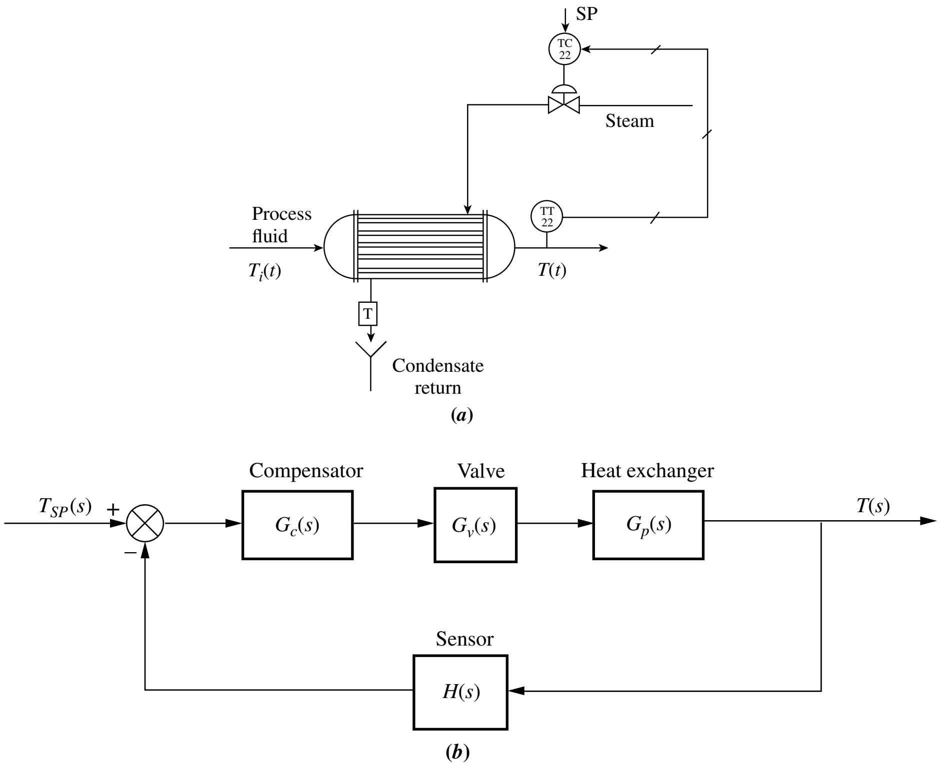


FIGURE P9.4 **a.** Heat-exchanger process;[4] **b.** block diagram

32. **a.** Find the transfer function of a motor whose torque–speed curve and load are given in Figure P9.5.
    b. Design a tachometer compensator to yield a damping ratio of 0.5 for a position control employing a power amplifier of gain 1 and a preamplifier of gain 5000.
    c. Compare the transient and steady-state characteristics of the uncompensated system and the compensated system.

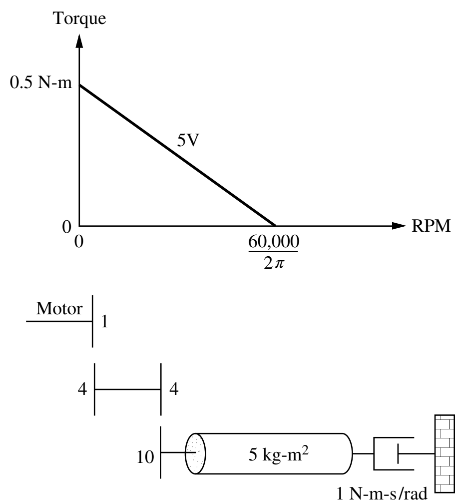


FIGURE P9.5

33. A position control is to be designed with a 20% overshoot and a settling time of 2 seconds. You have on hand an amplifier and a power amplifier whose cascaded transfer function is $K_1/(s+20)$ with which to drive the motor. Two 10-turn pots are available to convert shaft position into voltage. A voltage of $\pm 5\pi$ volts is placed across the pots. A dc motor whose transfer function is of the form

$$\frac{\theta_o(s)}{E_a(s)} = \frac{K}{s(s+a)}$$

is also available. The transfer function of the motor is found experimentally as follows: The motor and geared load are driven open-loop by applying a large, short, rectangular pulse to the armature. An oscillogram of the response shows that the motor reached 63% of its final output value at 1/2 second after the application of the pulse. Further, with a constant 10 volts dc applied to the armature, the constant output speed was 100 rad/s.
    a. Draw a complete block diagram of the system, specifying the transfer function of each component when the system is operating with 20% overshoot.
    b. What will the steady-state error be for a unit ramp input?
    c. Determine the transient response characteristics.

(*problem continues*)

[4] Smith, C.A. Automated Continuous Process Control. John Wiley & Sons, New York, NY, 2002. p. 128, Figure 6-1.1.

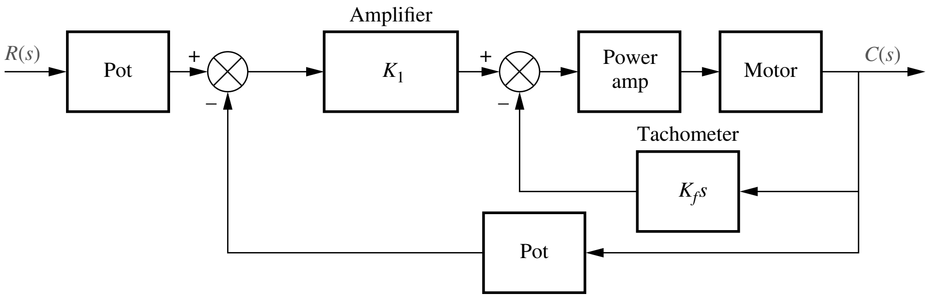


FIGURE P9.6

(*continued*)

d. If tachometer feedback is used around the motor, as shown in Figure P9.6, find the tachometer and the amplifier gain to meet the original specifications. Summarize the transient and steady-state characteristics.

34. A position control is to be designed with a 10% overshoot, a settling time of 1 second, and $K_v = 1000$. You have on hand an amplifier and a power amplifier whose cascaded transfer function is $K_1/(s+40)$ with which to drive the motor. Two 10-turn pots are available to convert shaft position into voltage. A voltage of $\pm 20\pi$ volts is placed across the pots. A dc motor whose transfer function is of the form

$$\frac{\theta_o(s)}{E_a(s)} = \frac{K}{s(s+a)}$$

is also available. The following data are observed from a dynamometer test at 50 V. At 25 N-m of torque, the motor turns at 1433 rpm. At 75 N-m of torque, the motor turns at 478 rpm. The speed measured at the load is 0.1 that of the motor. The equivalent inertia, including the load, at the motor armature is 100 kg-m^2, and the equivalent viscous damping, including the load, at the motor armature is 50 N-m-s/rad.

a. Draw a complete block diagram of the system, specifying the transfer function of each component.

b. Design a passive compensator to meet the requirements in the problem statement.

c. Draw the schematic of the compensator showing all component values. Use an operational amplifier for isolation where necessary.

d. Use MATLAB or any other computer program to simulate your system and show that all requirements have been met. MATLAB **ML**

35. Given the system shown in Figure P9.7, find the values of K and K_f so that the closed-loop dominant poles will have a damping ratio of 0.5 and the under-damped poles of the minor loop will have a damping ratio of 0.8.

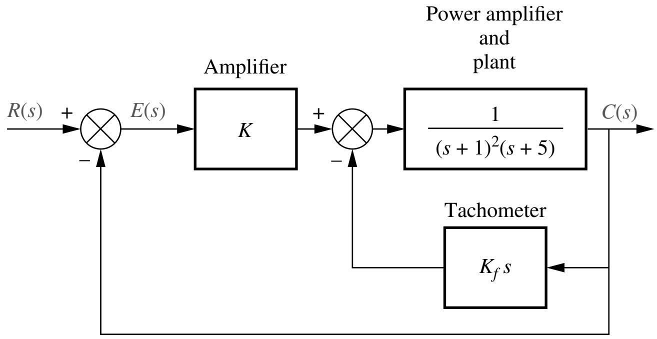


FIGURE P9.7

36. Steam-driven power generators rotate at a constant speed via a governor that maintains constant steam pressure in the turbine. In addition, automatic generation control (AGC) or load frequency control (LFC) is added to ensure reliability and consistency despite load variations or other disturbances that can affect the distribution line frequency output. A specific turbine-governor system can be described only using the block diagram of Figure P9.1 in which $G(s) = G_c(s)G_g(s)G_t(s)G_m(s)$, where (*Khodabakhshian, 2005*)

$G_g(s) = \dfrac{1}{0.2s+1}$ is the governor's transfer function

$G_t(s) = \dfrac{1}{0.5s+1}$ is the turbine transfer function

$G_m(s) = \dfrac{1}{10s+0.8}$ represents the machine and load transfer functions

$G_c(s)$ is the LFC compensation to be designed

a. Assuming $G_c(s) = K$, find the value of K that will result in a dominant pole with $\zeta = 0.7$. Obtain the corresponding T_s.

b. Design a PID controller to obtain the same damping factor as in Part **a**, but with a settling time of 2 seconds and zero steady-state error to step input commands.

c. Verify your results using a MATLAB simuation. MATLAB **ML**

37. Repeat Problem 36 using a lag–lead compensator instead of a PID controller. Design for a steady-state error of 1% for a step input command.

38. Digital versatile disc (DVD) players incorporate several control systems for their operations. The control tasks include (1) keeping the laser beam focused on the disc surface, (2) fast track selection, (3) disc rotation speed control, and (4) following a track accurately. In order to follow a track, the pickup-head radial position is controlled via a voltage that operates a voice coil embedded in a magnet configuration. For a specific DVD player, the transfer function is given by

$$P(s) = \frac{X(s)}{V(s)} = \frac{0.63}{\left(1 + \frac{0.36}{305.4}s + \frac{s^2}{305.4^2}\right)\left(1 + \frac{0.04}{248.2}s + \frac{s^2}{248.2^2}\right)}$$

where $x(t)$ = radial pickup position and $v(t)$ = the coil input voltage (*Bittanti, 2002*).

a. Assume that the system will be controlled in a closed-loop configuration, such as the one shown in Figure P9.1. Assuming that the plant, $P(s)$, is cascaded with a proportional compensator, $G_c(s) = K$, plot the root locus of the system.

b. `Repeat Part` **a** `using MATLAB if your root locus plot was created by any other tool.` MATLAB **ML**

c. Find the range of K for closed-loop stability, the resulting damping factor range, and the smallest settling time.

d. Design a notch filter compensator so that the system's dominant poles have a damping factor of $\zeta = 0.7$ with a closed-loop settling time of 0.1 second.

e. `Simulate the system's step response for Part` **c** `using MATLAB.` MATLAB **ML**

f. Add a PI compensator to the system to achieve zero steady-state error for a step input without appreciably affecting the transient response achieved in Part **b**.

g. `Simulate the system's step response for Part` **e** `using MATLAB.` MATLAB **ML**

39. Problem 8.40 described an ac/dc conversion and power distribution system for which droop control is implemented through the use of a proportional controller to stabilize the dc-bus voltage. For simplification, a system with only one source converter and one load converter was considered. The parameters and design considerations presented in that problem, along with some solution results, allow us to represent the block-diagram of that system as shown in the Figure P9.8.

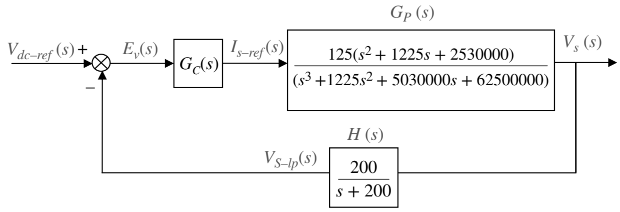


FIGURE P9.8

Here $G_c(s)$ is the transfer function of the controller, $G_p(s)$ represents the forward path of the controlled plant (a conversion and power distribution unit), and $H(s)$ is the transfer function of the feedback low-pass filter (*Karlsson, 2003*).

Prepare a table, such as Table 9.5, where the first column, headed *Uncompensated*, is filled in with your results from the proportional design of Problem 8.40, assuming an input step, $v_{dc-ref}(t) = 750\,u(t)$.

Follow Steps 2–8 as described in Section 9.4 (Example 9.5), to design a proportional-plus-integral-plus-derivative (PID) controller so that the system can operate with a percent overshoot ≤ 4.4 %, a peak time 20% smaller than that of the uncompensated system, and zero steady-state error, $e_{V\text{step}}(\infty) = 0$. Fill in the remaining two columns of your table, *PD-compensated* and *PID-compensated*.

40. Testing of hypersonic flight is performed in wind tunnels where maintaining a constant air pressure is important. Air pressure control is accomplished in several stages. For a specific setup, a simplified transfer function has been found to be (*Varghese, 2009*)

$$\frac{P(s)}{M(s)} = \frac{-2.369 \times 10^6 s^2 + 7.897 \times 10^7 s + 4.21 \times 10^5}{0.015s^5 + 0.7802s^4 + 9.89s^3 + 18.46s^2 + 3.377s + 0.01937}$$

where $M(s)$ is the stem movement of a valve feeding compressed air into the storage tank, and $P(s)$ is the settling chamber pressure.

In order to achieve steady-state error, design a PI controller that operates with a damping factor of 0.4. Compare the characteristics of the uncompensated and compensated systems, and use a computer program to simulate the step response to the compensated system.

41. A linear model of the α-subsystem of a grid-connected converter (*Mahmood, 2012*) with a Y–Y transformer was presented as the plant in Problem 52 in Chapter 8. You were asked to find the transfer function of that plant, $G_P(s) = \frac{V_\alpha(s)}{M_\alpha(s)}$ (see Figure P8.18(*b*)).

a. Use the results of your solution to Problem 52, Chapter 8, to write the open-loop transfer function in pole-zero form with a unity gain. Then design a PID controller to yield a zero steady-state error for a step input with an overshoot of less than 10% and a natural frequency of 135.3.

b. Plot the time response, $c(t)$, marking on it all relevant characteristics, such as the percent overshoot (if any), rise time, settling time, and final steady-state value. Also find all closed-loop poles of this system and the velocity error constant, K_v. Do you have any observations about the time response and/or the poles?

42. Design a PID controller for the drive system of Problem 42, Chapter 8, and shown in Figure P8.13 (*Thomsen*, *2011*). Obtain an output response, $\omega_L(t)$, with an overshoot $\leq 15\%$, a settling time of preferably 0.2 second, but not more than 5 seconds, a zero steady-state position error, and a velocity error of $< 2\%$, for a step input, $\omega_r(t) = 260\ u(t)$ rad/s, applied at $t = 0$.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

43. Control of HIV/AIDS. It was shown in Chapter 6, Problem 50, that when the virus levels in an HIV/AIDS patient are controlled using RTIs the linearized plant model is

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$

Assume that the system is embedded in a configuration, such as the one shown in Figure P9.1, where $G(s) = G_c(s)\,P(s)$. Here, $G_c(s)$ is a cascade compensator. For simplicity in this problem, choose the dc gain of $G_c(s)$ less than zero to obtain a negative-feedback system (the negative signs of $G_c(s)$ and $P(s)$ cancel out) (*Craig, I. K., 2004*).

a. Consider the uncompensated system with $G_c(s) = -K$. Find the value of K that will place all closed-loop poles on the real axis.

b. Use MATLAB to simulate the unit-step response of the gain-compensated system. Note the *%OS* and the T_s from the simulation.

MATLAB **ML**

c. Design a PI compensator so that the steady-state error for step inputs is zero. Choose a gain value to make all poles real.

d. Use MATLAB to simulate the design in Part c for a unit-step input. Compare the simulation to Part b.

MATLAB **ML**

44. Hybrid vehicle. In the previous chapter, we used the root locus to design a proportional controller for the speed control of an HEV. We rearranged the block diagram to be a unity-feedback system, as shown in the block diagram of Figure P7.25 (*Preitl, 2007*). The plant and compensator resulted in

$$G(s) = \frac{K(s + 0.60)}{(s + 0.5858)(s + 0.0163)}$$

and we found that $K = 0.78$ resulted in a critically damped system.

a. Use this design to itemize the performance specifications by filling in a table, similar to Table 9.5, under the column *Uncompensated*. Take advantage of the results from Chapter 8 or use MATLAB to find the entries. Plot $c(t)$ for $r(t) = 4\ u(t)$ volts.

b. Now assume that the system specifications require zero steady-state error for step inputs, a steady-state error for ramp inputs $\leq 2\%$, a $\%OS \leq 4.32\%$, and a settling time ≤ 4 seconds. It should be evident that this is not accomplished with a proportional controller. Thus, start by designing a PI controller to meet the requirements. If necessary, add a PD mode to get a PID controller. Simulate your final design using MATLAB. Fill in the results of this design in the second column of your table with the heading *Compensated*.

MATLAB **ML**

c. Now note the following limitations of linear control system modeling:

1. No limit is set on system variables. For example, vehicle acceleration as well as motor and power amplifier current, torque or power do not have upper limits.
2. It is assumed that to improve the speed of response in Part **b**, we could place the PI controller's zero on top of the pole closest to the origin. Realistically, such pole-zero cancellation is not always possible to maintain.

If you do not expand your model beyond the described limitations if required for accuracy, unrealistic response characteristics, such as rise and settling times could result. Look at your design results including

Simulink **SL**

response curves. Are they realistic? If not, revise your Simulink model, which you developed for Problem 5.57, according to the following 4 steps:

i. Represent the motor armature as a first-order system with a unity steady-state gain and a time constant of 50 ms, which avoids the creation of internal algebraic closed-loops and should have negligible effect on system response;

ii. Add a saturation element at the output of the motor armature and set it to an upper limit of 250 A;

iii. Use the following PI settings. The PI settings of the speed controller are P = 61 and I = 0.795. The PI settings of the torque controller are P = 10 and I = 6;

iv. Run the modified model and comment on the graphs obtained for motor current, car acceleration, and speed.

45. Parabolic trough collector. The parabolic trough collector (*Camacho*, *2012*) is a Type 0 system as can be seen from its transfer function,

$$G(s) = \frac{137.2 \times 10^{-6}\,K}{s^2 + 0.0224s + 196 \times 10^{-6}}\,e^{-39s}$$

We want the system operating in the critically damped mode, but with reduced steady-state error. Using the root locus and a Padé approximation, $e^{-sT} \approx \dfrac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$, do the following:

a. Substitute the Padé approximation for the delay and find the gain necessary to have the system operating with a damping factor, $\zeta = 0.5$. Also, find the corresponding steady-state error.

b. Design a PI compensator to obtain a zero steady-state error while maintaining $\zeta = 0.5$.

c. Simulate the resulting design using MATLAB to verify your design.

MATLAB
ML

Chapter 10 Problems

1. For each of the following $G(s)$, find analytical expressions for the magnitude and phase response. [Section: 10.1]

   a. $G(s) = \dfrac{1}{s(s+1)(s+3)}$

   b. $G(s) = \dfrac{(s+2)}{(s+1)(s+3)}$

   c. $G(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+3)}$

2. For each function in Problem 1, make a plot of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate. Do not use asymptotic approximations. [Section: 10.1]

SS 3. For each function in Problem 1 in the text problems, make a polar plot of the frequency response. [Section: 10.1]

4. Sketch the Nyquist diagram for each of the systems in Figure P10.1. [Section: 10.4]

5. Draw the polar plot from the separate magnitude and phase curves shown in Figure P10.2. [Section: 10.1]

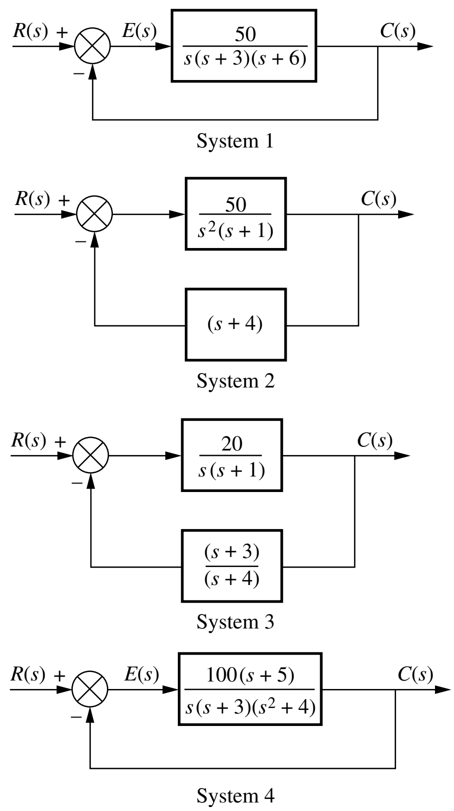


FIGURE P10.1

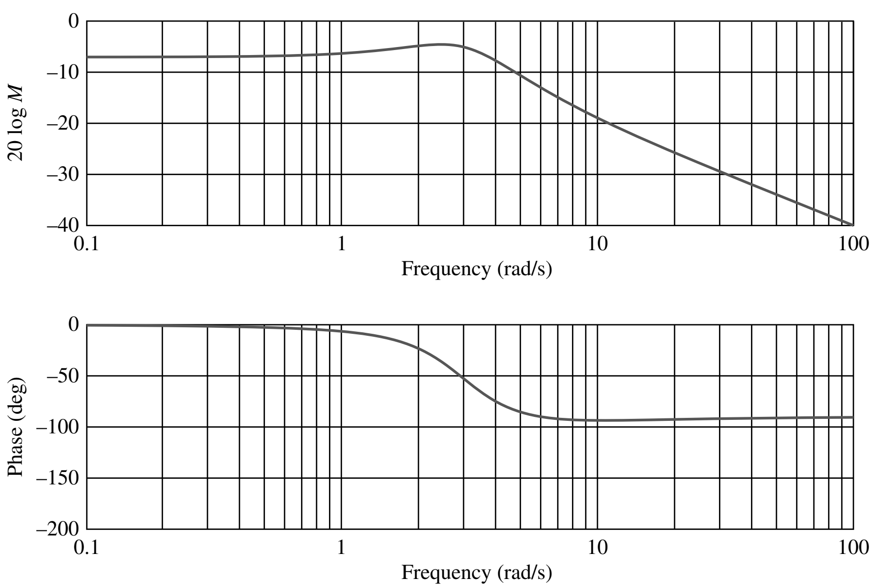


FIGURE P10.2

SS 6. Draw the separate magnitude and phase curves from the polar plot shown in Figure P10.3. [Section: 10.1]

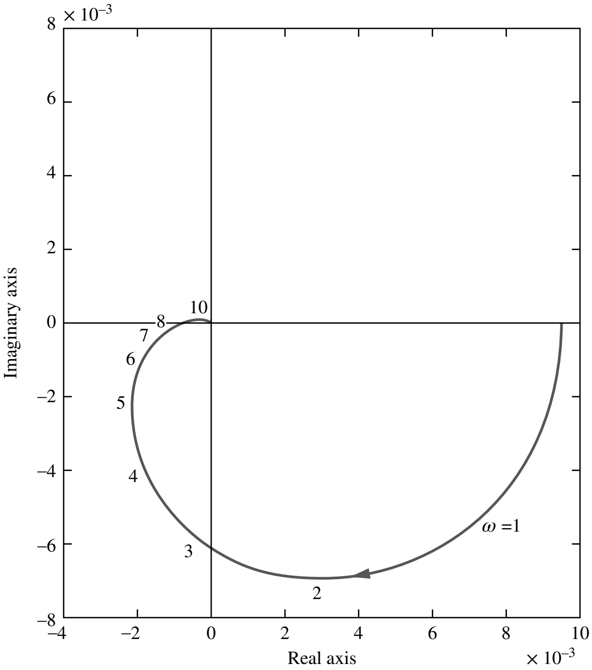


FIGURE P10.3

7. Using the Nyquist criterion, find out whether each system of Problem 4 is stable. [Section: 10.3]

SS 8. Using the Nyquist criterion, find the range of K for stability for each of the systems in Figure P10.4. [Section: 10.3]

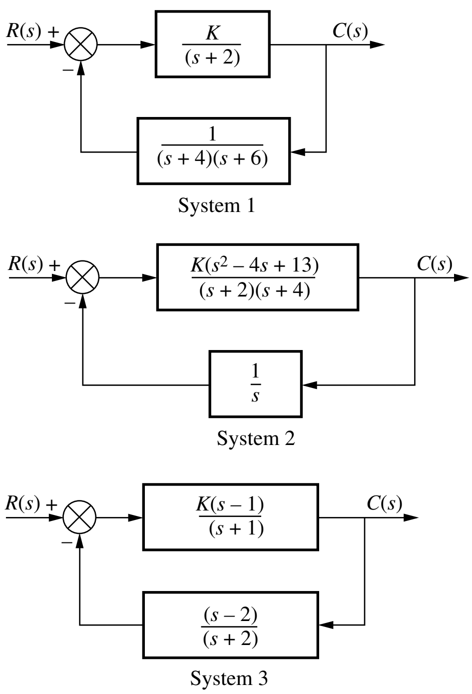


FIGURE P10.4

9. Find the gain margin and the phase margin for each one of the systems of Problem 8 assuming that in each part: [Section: 10.6]

   a. $K = 500$
   b. $K = 50$
   c. $K = 0.5$

10. `Write a program in MATLAB that will do the following:` MATLAB ML

    a. `Allow a value of gain, K, to be entered from the keyboard`
    b. `Display the Bode plots of a system for the entered value of K`
    c. `Calculate and display the gain and phase margin for the entered value of K`

    `Test your program on a unity-feedback system with` $G(s) = K/[s(s+3)(s+12)]$.

11. Derive Eq. (10.54), the closed-loop bandwidth in terms of ζ and ω_n of a two-pole system. [Section: 10.8] SS

12. Find the closed-loop bandwidth that corresponds to each system with the following characteristics. [Section: 10.8]

    a. $\zeta = 0.3,\ T_s = 1.5$ seconds
    b. $\zeta = 0.3,\ T_p = 1.5$ seconds
    c. $T_s = 5$ seconds, $T_p = 3$ seconds
    d. $\zeta = 0.2,\ T_r = 0.5$ seconds.

13. Consider the unity-feedback system of Figure 10.10. For each $G(s)$ that follows, use the M and N circles to make a plot of the closed-loop frequency response: [Section: 10.9]

    a. $G(s) = \dfrac{10}{s(s+1)(s+2)}$

    b. $G(s) = \dfrac{1000}{(s+3)(s+4)(s+5)(s+6)}$

    c. $G(s) = \dfrac{50(s+3)}{s(s+2)(s+4)}$

14. Repeat Problem 13, using the Nichols chart in place of the M and N circles. [Section: 10.9] SS

15. Using the results of Problem 13, estimate the percent overshoot that can be expected in the step response for each system shown. [Section: 10.10]

16. Use the results of Problem 14 to estimate the percent overshoot if the gain term in the numerator of the forward path of each part of the problem is respectively changed as follows: [Section: 10.10]

    a. From 10 to 30
    b. From 1000 to 2500
    c. From 50 to 75

17. Write a program in MATLAB that will do the following:

MATLAB
ML

a. Allow a value of gain, K, to be entered from the keyboard

b. Display the closed-loop magnitude and phase frequency response plots of a unity-feedback system with an open-loop transfer function, $KG(s)$

c. Calculate and display the peak magnitude, frequency of the peak magnitude, and bandwidth for the closed-loop frequency response and the entered value of K

Test your program on the system of Figure P10.5 for $K = 50$.

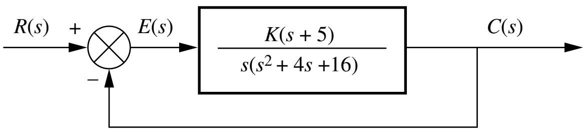


FIGURE P10.5

18. For a unity-feedback system with a forward-path transfer function

GUI Tool
GUIT

$$G(s) = \frac{7(s+5)}{s(s^2+5s+20)}$$

use MATLAB's Linear System Analyzer Nichols plot to find the gain margin, dB frequency, and the $-180°$ frequency.

19. For each one of the system in Figure P10.6, estimate the transient response using Bode P1ots. [Section: 10.10]

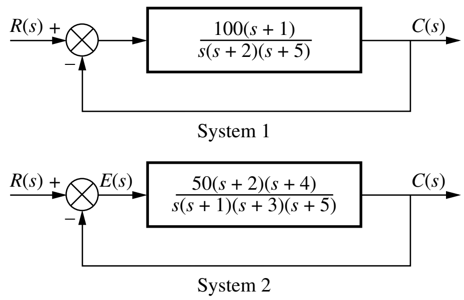


FIGURE P10.6

20. For the system of Figure P10.5, do the following: [Section: 10.10]

SS

a. Plot the Bode magnitude and phase plots.

b. Assuming a second-order approximation, estimate the transient response of the system if $K = 2$.

(*problem continues*)

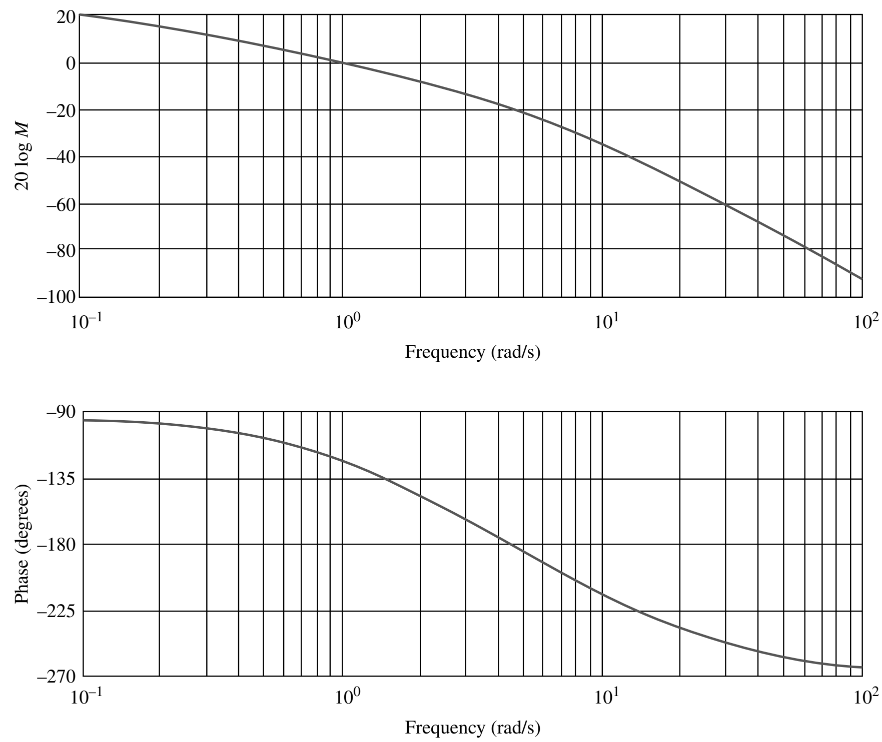


FIGURE P10.7

(*continued*)

c. Use MATLAB or any other program to check your assumptions by simulating the step response of the system.

21. Write a program in MATLAB that will use an open-loop transfer function, $G(s)$, to do the following:

a. Make a Bode plot

b. Use frequency response methods to estimate the percent overshoot, settling time, and peak time

c. Plot the closed-loop step response

Test your program by comparing the results to those obtained for the systems of Problem 19.

22. The Bode plots for a plant, $G(s)$, used in a unity-feedback system are shown in Figure P10.7. Do the following:

a. Find the gain margin, phase margin, zero dB frequency, 180° frequency, and the closed-loop bandwidth.

b. Use your results in Part **a** to estimate the damping ratio, percent overshoot, settling time, and peak time.

23. For the system in Figure P10.8. [Section: 10.12]

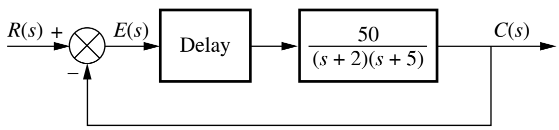


FIGURE P10.8

a. Caculate the phase margin if the system is stable for time delays of 0, 0.1, 0.2, 0.5, and 1 second.

b. Caculate the gain margin if the system is stable for each one of the time delays in Part **a**.

c. Find out for which of the time delays in Part a the system is closed-loop stable.

d. Find out by what amount the gain should be reduced to obtain a stable closed-loop system for those time delays for which the system was closed-loop unstable.

24. Given a unity-feedback system with the forward-path transfer function

$$G(s) = \frac{K}{s(s+1)(s+15)}$$

and a delay of 0.2 second, make a second-order approximation and estimate the percent overshoot if $K = 30$. Use Bode plots and frequency response techniques. [Section: 10.12]

25. Use the MATLAB function pade(T,n) to model the delay in Problem 24. Obtain the unit-step response and evaluate your second-order approximation in Problem 24.

26. For the Bode plots shown in Figure P10.9 determine the transfer function by hand or via MATLAB. [Section: 10.13]

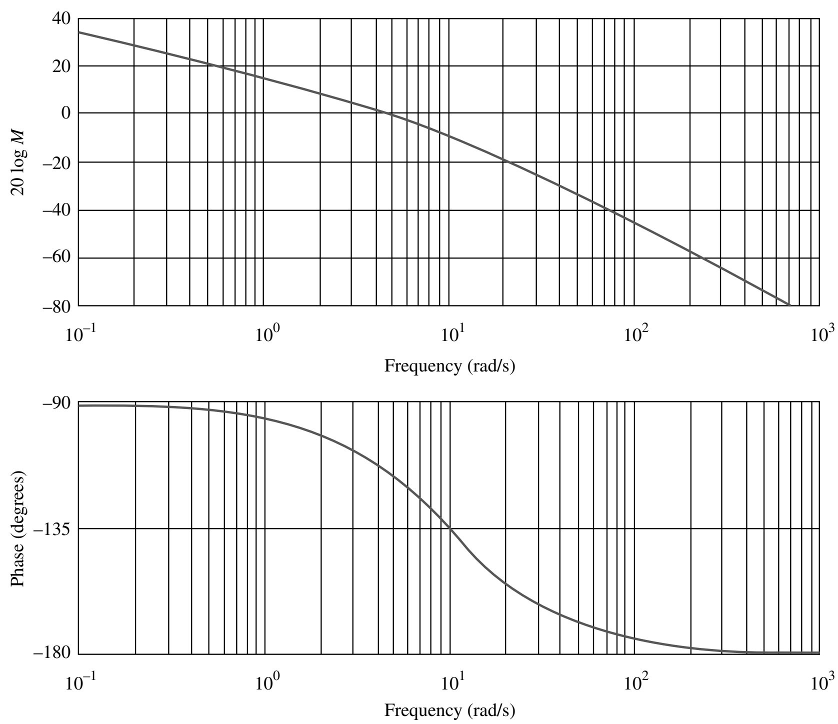


FIGURE P10.9

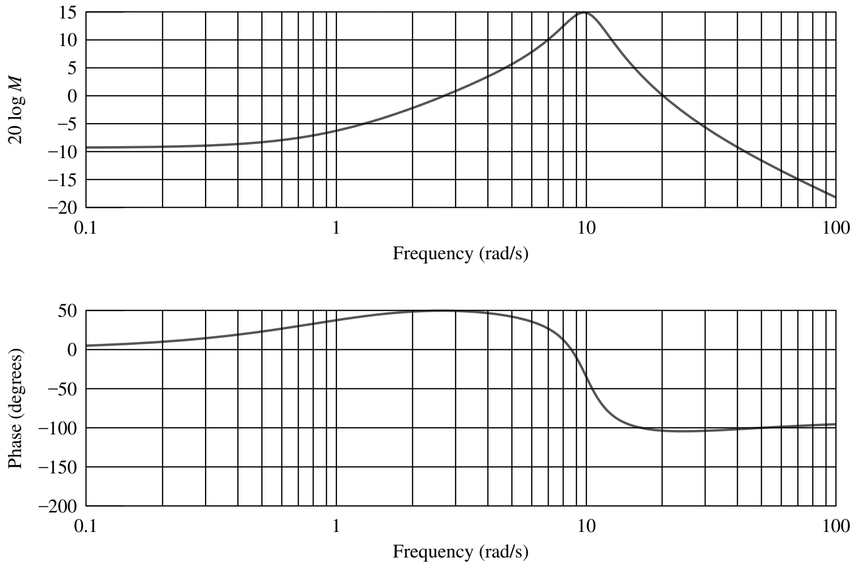


FIGURE P10.10

27. Repeat Problem 26 for the Bode plots shown in Figure P10.10. [Section: 10.13]

28. A room's temperature can be controlled by varying the radiator power. In a specific room, the transfer function from indoor radiator power, $\dot{Q}$, to room temperature, T in °C is (*Thomas, 2005*)

$$P(s) = \frac{T(s)}{\dot{Q}(s)} = \frac{(1\times10^{-6})s^2 + (1.314\times10^{-9})s + (2.66\times10^{-13})}{s^3 + 0.00163\,s^2 + (5.272\times10^{-7})s + (3.538\times10^{-11})}$$

The system is controlled in the closed-loop configuration shown in Figure 10.20 with $G(s) = KP(s)$, $H = 1$.

a. Draw the corresponding Nyquist diagram for $K = 1$.

b. Obtain the gain and phase margins.

c. Find the range of K for the closed-loop stability. Compare your result with that of Problem 40, Chapter 6.

29. Problem 35, Chapter 8 discusses a magnetic levitation system with a plant transfer function $P(s) = -\dfrac{1300}{s^2 - 860^2}$ (*Galvão, 2003*). Assume that the plant is in cascade with an $M(s)$ and that the system will be controlled by the loop shown in Figure 10.20, where $G(s) = M(s)P(s)$ and $H = 1$. For each $M(s)$ that follows, draw the Nyquist diagram when $K = 1$, and find the range of closed-loop stability for $K > 0$.

a. $M(s) = -K$

b. $M(s) = -\dfrac{K(s+200)}{s+1000}$

c. Compare your results with those obtained in Problem 35, Chapter 8.

30. A simple modified and linearized model for the transfer function of a certain bicycle from steer angle (δ) to roll angle (φ) is given by (*Åstrom, 2005*) **SS**

$$P(s) = \frac{\varphi(s)}{\delta(s)} = \frac{12(s+20)}{s^2+25}$$

Assume the rider can be represented by a gain K, and that the closed-loop system is shown in Figure 10.20 with $G(s) = KP(s)$ and $H = 1$.

Use MATLAB and the Nyquist stability criterion to find the range of K for closed-loop stability. MATLAB **ML**

31. A ship's roll can be stabilized with a control system. A voltage applied to the fins' actuators creates a roll torque that is applied to the ship. The ship, in response to the roll torque, yields a roll angle. Assuming the block diagram for the roll control system shown in Figure P10.11, determine the gain and phase margins for the system.

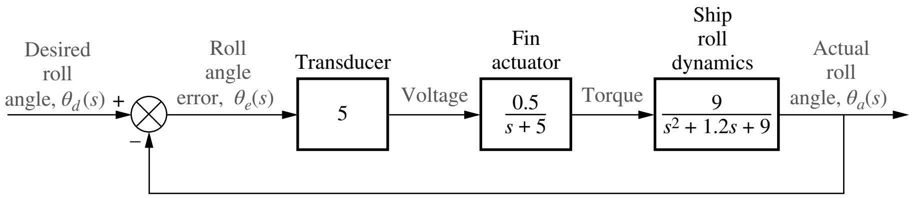


FIGURE P10.11 Block diagram of a ship's roll-stabilizing system

32. The linearized model of a particular network link working under TCP/IP and controlled using a random early detection (RED) algorithm can be described by Figure 10.20 where $G(s) = M(s)P(s)$, $H = 1$, and (*Hollot, 2001*)

$$M(s) = \frac{0.005L}{s + 0.005};\ P(s) = \frac{140625e^{-0.1s}}{(s + 2.67)(s + 10)}$$

a. Plot the Nichols chart for $L = 1$. Is the system closed-loop stable?

b. Find the range of L for closed-loop stability.

c. Use the Nichols chart to predict $\%OS$ and T_s for $L = 0.95$. Make a hand sketch of the expected unit-step response.

d. Verify Part **c** with a Simulink unit-step response simulation. Simulink **SL**

SS **33.** In the TCP/IP network link of Problem 32, let $L = 0.8$, but assume that the amount of delay is an unknown variable.

a. Plot the Nyquist diagram of the system for zero delay, and obtain the phase margin.

b. Find the maximum delay allowed for closed-loop stability.

SS **34.** An experimental holographic media storage system uses a flexible photopolymer disk. During rotation, the disk tilts, making information retrieval difficult. A system that compensates for the tilt has been developed. For this, a laser beam is focused on the disk surface and disk variations are measured through reflection. A mirror is in turn adjusted to align with the disk and makes information retrieval possible. The system can be represented by a unity-feedback system in which a controller with transfer function

$$G_C(s) = \frac{78.575(s + 436)^2}{(s + 132)(s + 8030)}$$

and a plant

$$P(s) = \frac{1.163 \times 10^8}{s^3 + 962.5s^2 + 5.958 \times 10^5 s + 1.16 \times 10^8}$$

form an open loop transmission $L(s) = G_c(s)P(s)$ (*Kim, 2009*).

a. Use MATLAB to obtain the system's Nyquist diagram. Find out if the system is stable. MATLAB **ML**

b. Find the system's phase margin.

c. Use the value of phase margin obtained in **b** to calculate the expected system's overshoot to a step input.

d. Simulate the system's response to a unit-step input and verify the %OS calculated in **c**.

35. Use LabVIEW with the Control Design and Simulation Module and MathScript RT Module to do the following: Modify the CDEx Nyquist Analysis.vi to obtain the range of K for stability using the Nyquist plot for any system you enter. In addition, design a LabVIEW VI that will accept as an input the polynomial numerator and polynomial denominator of an open-loop transfer function and obtain a Nyquist plot for a value of $K = 10{,}000$. Your VI will also display the following as generated from the Nyquist plot: (1) gain margin, (2) phase margin, (3) zero dB frequency, and (4) 180 degrees frequency. Use the system and results of Skill-Assessment Exercise 10.6 to test your VIs. LabVIEW **LV** MATLAB **ML**

36. Use LabVIEW with the Control Design and Simulation Module, and MathScript RT Module to build a VI that will accept an open-loop transfer 51 function, plot the Bode diagram, and plot the closed-loop step response. Your VI will also use the CD Parametric Time Response VI to display (1) rise time, (2) peak time, (3) settling time, (4) percent overshoot, (5) steady-state value, and (6) peak value. Use the system in Skill-Assessment Exercise 10.9 to test your VI. Compare the results obtained from LabVIEW **LV** MATLAB **ML**

your VI with those obtained in Skill-Assessment Exercise 10.9.

37. The block diagram of a cascade system used to control water level in a steam generator of a nuclear power plant (*Wang, 2009*) was presented in Figure P6.12. In that system, the level controller, $G_{LC}(s)$, is the *master* controller and the feed-water flow controller, $G_{FC}(s)$, is the slave controller. Consider that the inner feedback loop is replaced by its equivalent transfer function, $G_{WX}(s)$.

MATLAB ML

Using numerical values (*Wang, 2009; Bhambhani, 2008*), the transfer functions with a 1-second pure delay are:

$$G_{fw}(s) = \frac{2 \cdot e^{-\tau s}}{s(T_1 s + 1)} = \frac{2 \cdot e^{-s}}{s(25s + 1)};$$

$$G_{WX}(s) = \frac{(4s + 1)}{3(3.333s + 1)};$$

$$G_{LC}(s) = K_{P_{LC}} + K_{D_{LC}} s = 1.5(10s + 1)$$

Use MATLAB or any other program to:

a. Obtain Bode magnitude and phase plots for this system using a fifth-order Padé approximation (available in MATLAB). Note on these plots, if applicable, the gain and phase margins.

b. Plot the response of the system, $c(t)$, to a unit-step input, $r(t) = u(t)$. Note on the $c(t)$ curve, the rise time, T_r, the settling time, T_s, the final value of the output, and, if applicable, the percent overshoot, $\%OS$, and mid peak time, T_p.

c. Repeat the above two steps for a pure delay of 1.5 seconds.

38. In order to self-balance a bicycle, its open-loop transfer function is found to be (*Lam, 2011*):

$$G(s) = \frac{\theta(s)}{U(s)} = \frac{334019}{s^4 + 5126.16s^3 + 2470.7s^2 + 428419s - 34040}$$

where $\theta(s)$ is the angle of the bicycle with respect to the vertical, and $U(s)$ is the voltage applied to the motor that drives a flywheel used to stabilize the bicycle. Note that the bicycle is open-loop unstable with one open-loop pole in the right half-plane.

a. Draw the Nyquist diagram of the system.

b. Find the system's gain and phase margins.

c. Assuming a unit feedback system, find the range of K for closed-loop stability if the forward path transfer function is $KG(s)$.

d. Assuming a second-order approximation, what is the expected $\%OS$ if $K = 0.141$?

e. Use a computer program to simulate your system for a unit-step response using the value of K in Part **d**.

39. Modify the MATLAB program you developed in Problem 10.17 to do the following:

MATLAB ML

a. Display the closed-loop magnitude and phase frequency response plots for the drive system (*Thomsen, 2011*) presented in Problem 42, Chapter 8. Using the graph properties, specify the value of K in the Bode plot title.

b. Calculate and display the closed-loop transfer function, $T(s)$, the peak magnitude, frequency of the peak magnitude, and bandwidth for the closed-loop frequency response at the following two values of the proportional controller's gain, $K = K_P = 3.2$ and 10.

40. A linear model of the α-subsystem of a grid-connected voltage-source converter (VSC) with a Y-Y transformer (*Mahmood, 2012*) was presented in Problem 52, Chapter 8. In Figure P8.18(*b*), $G_C(s) = K$ and $G_P(s)$ is given in a pole zero form (with a unity gain and slightly modified parameters) as follows:

$$G_P(s) = \frac{V_\alpha(s)}{M_\alpha(s)} = \frac{(s + 2200)}{(s + 220)(s^2 + 120s + 16 \times 10^{6)}}$$

Use MATLAB and frequency response techniques to obtain the Bode plots for this system and find the following:

MATLAB ML

a. The range of K for system stability

b. The gain margin, phase margin, zero dB frequency, and 180° frequency, if $K = 5 \times 10^5$.

41. A new measurement-based technique to design fixed-structure controllers for unknown SISO systems, which does not require system identification, has been proposed. The fourth-order transfer function shown below and modified to have a unity steady-state gain is used as an example (*Khadraoui, 2013*).

$$G(s) = \frac{0.1111(4s^2 + 5s + 1)}{s^4 + 3.1s^3 + 0.85s^2 + 0.87s + 0.1111}$$

The interested reader is referred to the reference to explore this new technique. In this problem and its companion design problem in Chapter 11, however, we take a standard approach as covered in Chapters 10 and 11.

MATLAB **ML**

Assuming that a cascade-connected proportional controller, $G_C(s) = K$, is used, utilize MATLAB and frequency response techniques to obtain the Bode plots for this system and find:

a. The range of K for system stability

b. The gain margin, phase margin, zero dB frequency, and 180° frequency, if $K = 0.3$.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

42. Control of HIV/AIDS. The linearized model for an HIV/AIDS patient treated with RTIs was obtained in Chapter 6 as (*Craig, 2004*);

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$

a. Consider this plant in the feedback configuration in Figure 10.20 with $G(s) = P(s)$ and $H(s) = 1$. Obtain the Nyquist diagram. Evaluate the system for closed-loop stability.

b. Consider this plant in the feedback configuration in Figure 10.20 with $G(s) = -P(s)$ and $H(s) = 1$. Obtain the Nyquist diagram. Evaluate the system for closed-loop stability. Obtain the gain and phase margins.

43. Hybrid vehicle. In Problem 54, Chapter 8, we used MATLAB to plot the root locus for the speed control of an HEV rearranged as a unity-feedback system, as shown in Figure P7.25 (*Preitl, 2007*). The plant and compensator were given by

MATLAB **ML**

$$G(s) = \frac{K(s + 0.6)}{(s + 0.5858)(s + 0.0163)}$$

and we found that $K = 0.78$, resulted in a critically damped system.

a. Use MATLAB or any other program to plot the following:

i. The Bode magnitude and phase plots for that system, and

ii. The response of the system, $c(t)$, to a step input, $r(t) = 4\,u(t)$. Note on the $c(t)$ curve the rise time, T_r, and settling time, T_s, as well as the final value of the output.

b. Now add an integral gain to the controller, such that the plant and compensator transfer function becomes

$$G(s) = \frac{K_1(s + Z_c)(s + 0.6)}{s(s + 0.5858)(s + 0.0163)}$$

where K_1=0.78 and $Z_c = \frac{K_2}{K_1} = 0.4$. Use MATLAB or any other program to do the following:

i. Plot the Bode magnitude and phase plots for this case.

ii. Obtain the response of the system to a step input, $r(t) = 4\,u(t)$. Plot $c(t)$ and note on it the rise time, T_r, percent overshoot, $\%OS$, peak time, T_p, and settling time, T_s.

c. Does the response obtained in Parts **a** or **b** resemble a second-order overdamped, critically damped, or underdamped response? Explain.

44. Parabolic trough collector. As discussed in Section 10.12, the Nyquist stability criterion can be applied to systems with pure time delay without the need for rational approximations as required in Problems 8.55 and 9.44. You will verify this by applying the Nyquist stability criterion to the parabolic trough collector by assuming a unity-feedback system and a forward-path transfer function (*Camacho, 2012*),

$$G(s) = \frac{137.2 \times 10^{-6} K}{s^2 + 0.0224s + 196 \times 10^{-6}} e^{-39s}$$

a. Draw the corresponding Nyquist diagram for $K = 1$.

b. Use the Nyquist diagram to find the range of K for which the system is closed-loop stable.

c. Find the value of K that will make the system marginally stable and the associated frequency of oscillation.

Chapter 11 Problems

1. For the unity-feedback system of Figure P11.1, find the value of K required to obtain a gain margin of 10 dB when: [Section: 11.2]

   a. $G(s) = \dfrac{K}{(s+5)(s+15)(s+20)}$

   b. $G(s) = \dfrac{K}{s(s+5)(s+15)}$

   c. $G(s) = \dfrac{K(s+1)}{s(s+3)(s+5)(s+10)}$

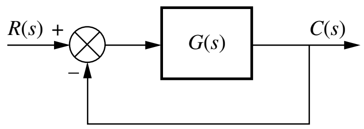


FIGURE P11.1

2. For each of the systems in Problem 1, design the gain, K, for a phase margin of 40°. [Section: 11.2]

3. Use frequency response methods to find the value of K necessary to achieve a step response with a 10% overshoot for the unity-feedback system of Figure P11.1 when: [Section: 11.2]

   a. $G(s) = \dfrac{K}{s(s+5)(s+10)}$

   b. $G(s) = \dfrac{K(s+2)}{s(s+4)(s+6)(s+10)}$

   c. $G(s) = \dfrac{K(s+1)(s+5)}{s(s+3)(s+6)(s+10)(s+15)}$

4. The system of Figure P11.1 is operating with 10% overshoot when

$$G(s) = \frac{K}{s(s+5)}$$

   Design a compensator using frequency response techniques to yield $K_v = 50$ without significantly changing the uncompensated system's phase-margin frequency and phase margin. [Section: 11.3]

5. The system of Figure P11.1 is operating with 10% overshoot when

$$G(s) = \frac{K}{(s+2)(s+8)(s+15)}$$

   Design a compensator using frequency response techniques to give a fivefold improvement in steady-state error without significantly changing the transient response. [Section: 11.3]

6. It is desired to have zero steady-state error for ramp inputs and a 15% overshoot in the system of Figure 11.2. Design a PI controller to achieve the specifications. [Section: 11.3]

7. MATLAB ML Write a MATLAB program that will design a PI controller assuming a second-order approximation as follows:

   a. Allow the user to input from the keyboard the desired percent overshoot

   b. Design a PI controller and gain to yield zero steady-state error for a closed-loop step response as well as meet the percent overshoot specification

   c. Display the compensated closed-loop step response

   Test your program on

$$G(s) = \frac{K}{(s+5)(s+10)}$$

   and 25% overshoot.

8. Design a compensator for the unity-feedback system of Figure P11.1 with

$$G(s) = \frac{K}{s(s+2)(s+10)(s+25)}$$

   to yield a $K_v = 4$ and a phase margin of 45°. [Section: 11.4]

9. SS Consider the unity-feedback system of Figure P11.1 with

$$G(s) = \frac{K}{s(s+5)(s+20)}$$

   The uncompensated system has about 55% overshoot and a peak time of 0.5 second when $K_v = 10$. Do the following: [Section: 11.4]

   a. Use frequency response methods to design a lead compensator to reduce the percent overshoot to 10%, while keeping the peak time and steady-state error about the same or less. Make any required second-order approximations.

   b. MATLAB ML Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K.

10. SS The unity-feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+4)}{(s+2)(s+5)(s+12)}$$

   is operating with 20% overshoot. [Section: 11.4]

   a. Find the settling time.

   b. Find K_p.

   c. Find the phase margin and the phase-margin frequency.

   d. Using frequency response techniques, design a compensator that will yield a threefold improvement in K_p and a twofold reduction in settling time while keeping the overshoot at 20%.

11. Repeat Problem 9 using a PD compensator. [Section: 11.4]

12. Write a MATLAB program that will design a lead compensator assuming second-order approximations as follows: MATLAB **ML**
    a. Allow the user to input from the keyboard the desired percent overshoot, peak time, and gain required to meet a steady-state error specification
    b. Display the gain-compensated Bode plot
    c. Calculate the required phase margin and bandwidth
    d. Display the pole, zero, and gain of the lead compensator
    e. Display the compensated Bode plot
    f. Output the step response of the lead-compensated system to test your second-order approximation

    Test your program on a unity-feedback system where

$$G(s) = \frac{K(s+1)}{s(s+2)(s+6)}$$

and the following specifications are to be met: percent overshoot = 10%, peak time = 0.1 second, and $K_v = 30$.

SS 13. Use frequency response methods to design a lag-lead compensator for a unity-feedback system where

$$G(s) = \frac{K(s+5)}{s(s+2)(s+10)}$$

and the following specifications are to be met: percent overshoot = 10%, settling time = 0.2 second, and $K_v = 1000$. [Section: 11.4]

14. Write a MATLAB program that will design a lag-lead compensator assuming second-order approximations as follows: [Section: 11.5] MATLAB **ML**
    a. Allow the user to input from the keyboard the desired percent overshoot, settling time, and gain required to meet a steady-state error specification
    b. Display the gain-compensated Bode plot
    c. Calculate the required phase margin and bandwidth
    d. Display the poles, zeros, and the gain of the lag-lead compensator
    e. Display the lag-lead-compensated Bode plot
    f. Display the step response of the lag-lead compensated system to test your second-order approximation

    Use your program to do Problem 13.

15. Given a unity-feedback system with SS

$$G(s) = \frac{K}{s(s+1.75)(s+6)}$$

design a PID controller to yield zero steady-state error for a ramp input, as well as a 20% overshoot, and a peak time less than 1.8 seconds for a step input. Use only frequency response methods. [Section: 11.5]

16. A unity-feedback system has MATLAB **ML**

$$G(s) = \frac{K}{s(s+3)(s+6)}$$

If this system has an associated 0.5 second delay, use MATLAB to design the value of K for 20% overshoot. Make any necessary second-order approximations, but test your assumptions by simulating your design. The delay can be represented by cascading the MATLAB function padé (T, n) with $G(s)$, where T is the delay in seconds and n is the order of the Pade approximation (use 5). Write the program to do the following:
    a. Accept your value of percent overshoot from the keyboard
    b. Display the Bode plot for $K = 1$
    c. Calculate the required phase margin and find the phase-margin frequency and the magnitude at the phase-margin frequency
    d. Calculate and display the value of K

DESIGN PROBLEMS

17. An electric ventricular assist device (EVAD) that helps pump blood concurrently to a defective natural heart in sick patients can be shown to have a transfer function SS

$$G(s) = \frac{P_{ao}(s)}{E_m(s)} = \frac{1361}{s^2 + 69s + 70.85}$$

The input, $E_m(s)$, is the motor's armature voltage, and the output is $P_{ao}(s)$, the aortic blood pressure (*Tasch*, *1990*).

The EVAD will be controlled in the closed-loop configuration shown in Figure P11.1.

a. Design a phase lag compensator to achieve a tenfold improvement in the steady-state error to step inputs without appreciably affecting the transient response of the uncompensated system.

b. Use MATLAB to simulate the uncompensated and compensated systems for a unit-step input. MATLAB ML

18. A Tower Trainer 60 Unmanned Aerial Vehicle has a transfer function

$$P(s) = \frac{h(s)}{\delta_e(s)} = \frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s}$$

where $\delta_e(s)$ is the elevator angle and $h(s)$ is the change in altitude (*Barkana, 2005*).

a. Assuming the airplane is controlled in the closed-loop configuration of Figure P11.1 with $G(s) = KP(s)$, find the value of K that will result in a 30° phase margin.

b. For the value of K calculated in Part **a**, obtain the corresponding gain margin.

c. Obtain estimates for the system's $\%OS$ and settling times T_s for step inputs.

d. Simulate the step response of the system using MATLAB. MATLAB ML

e. Explain the simulation results and discuss any inaccuracies in the estimates obtained in Part **c**.

19. The transfer function from applied force to arm displacement for the arm of a hard disk drive has been identified as

$$G(s) = \frac{X(s)}{F(s)} = \frac{3.3333 \times 10^4}{s^2}$$

The position of the arm will be controlled using the feedback loop shown in Figure P11.1 (*Yan, 2003*).

a. Design a lead compensator to achieve closed-loop stability with a transient response of 16% overshoot and a settling time of 2 msec for a step input.

b. Verify your design through MATLAB simulations. MATLAB ML

20. For the heat exchange system described in Problem 30, Chapter 9 (*Smith, 2002*):

a. Design a passive lag-lead compensator to achieve 5% steady-state error with a transient response of 10% overshoot and a settling time of 60 seconds for step inputs.

b. Use MATLAB to simulate and verify your design. MATLAB ML

21. Figure P11.2 illustrates a set of booms used for the delivery of chemicals in agriculture (*Sun, 2011*). Each of the booms has equally spaced nozzles, the purpose of which is to maintain a constant gap between the nozzles and the soil despite car movements due to road unevenness. The booms are tethered to a vehicle (not shown in figure), and the gap is measured using an infrared sensor. This measurement is fed to a controller that drives two hydraulic cylinders to adjust the boom's positions. Under certain operating conditions, it was found that the system can be described by the unity-feedback configuration of Figure P11.1 where SS

$$G(s) = \frac{K}{s}\,\frac{2.78 \times 10^{-4}}{\left(\frac{s^2}{60^2} + \frac{s}{60} + 1\right)}\,\frac{509.3}{\left(\frac{s^2}{213^2} + \frac{3s}{213} + 1\right)}$$

a. Design a lag compensator to achieve $K_v = 30$, and $\%OS = 10\%$.

b. Use a computer program to obtain the step response of the closed-loop system and verify its performance.

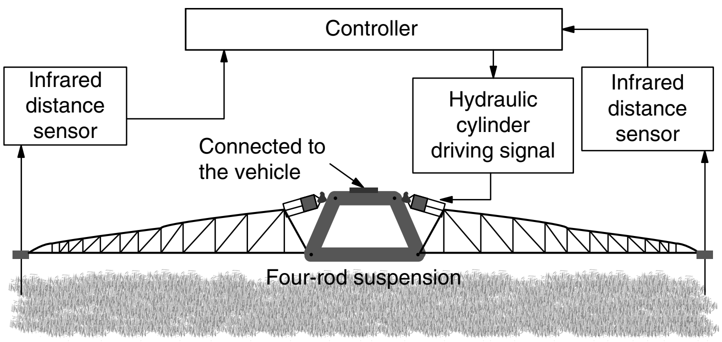


FIGURE P11.2[1]

[1] Sun, J., and Miao, Y. Modeling and simulation of the agricultural sprayer boom leveling system. *IEEE Third International Conf. on Measuring Tech. and Mechatronics Automation, 2011*, pp. 613–618. Figure 2, p. 613. 2011 Third International Conference on Measuring Technology and Mechatronics Automation by IEEE. Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

22. Problem 41 in Chapter 10 mentioned a measurement-based technique to design fixed-structure controllers, which does not require system identification. In that problem, we assumed a plant transfer function of (*Khadraoui*, *2013*)

$$G(s) = \frac{0.1111(4s^2 + 5s + 1)}{s^4 + 3.1s^3 + 0.85s^2 + 0.87s + 0.1111}$$

Again, the interested reader is directed to the reference for further study. In this problem, however, we use the design and analysis techniques developed in this and the previous chapters.

MATLAB **ML**

Use MATLAB and Bode plots to design a PID controller, $G_c(s)$, to yield zero steady-state error for a step input, an overshoot of 10–20%, and a settling time of 20–50 seconds. Start your design assuming an overshoot of 10% and a settling time of 50 seconds. Consider the design acceptable if the PID-controlled response satisfies the above requirements.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

23. **Control of HIV/AIDS.** In Chapter 6, the model for an HIV/AIDS patient treated with RTIs was linearized and shown to be

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$
$$= \frac{-520(s + 0.02)}{(s + 2.2644)(s^2 + 0.04s + 0.0048)}$$

It is assumed here that the patient will be treated and monitored using the closed-loop configuration shown in Figure P11.1 Since the plant has a negative dc gain, assume for simplicity that $G(s) = G_c(s)\,P(s)$ and $G_c(0) < 0$. Assume also that the specifications for the design are (1) zero steady-state error for step inputs, (2) overdamped time-domain response, and (3) settling time $T_s \approx 100$ days (*Craig*, *2004*).

a. The overdamped specification requires a $\Phi_M \approx 90°$. Find the corresponding bandwidth required to satisfy the settling time requirement.

b. The zero steady-state error specification implies that the open-loop transfer function must be augmented to Type 1. The -0.02 zero of the plant adds too much phase lead at low frequencies, and the complex conjugate poles, if left uncompensated within the loop, result in undesired oscillations in the time domain. Thus, as an initial approach to compensation for this system we can try

$$G_c(s) = \frac{-K(s^2 + 0.04s + 0.0048)}{s(s + 0.02)}$$

For $K = 1$, make a Bode plot of the resulting system. Obtain the value of K necessary to achieve the design demands. Check for closed-loop stability.

c. Simulate the unit-step response of the system using MATLAB. Adjust K to achieve the desired response.

MATLAB **ML**

24. **Hybrid vehicle.** In Part **b** of Problem 43 in Chapter 10, we used a proportional-plus-integral (PI) speed controller that resulted in an overshoot of 20% and a settling time, $T_s = 3.92$ seconds (*Preitl*, *2007*).

a. Now assume that the system specifications require a steady-state error of zero for a step input, a ramp input steady-state error $\leq 2\%$, a $\%OS \leq 4.32\%$, and a settling time ≤ 4 seconds. One way to achieve these requirements is to cancel the PI-controller's zero, Z_I, with the real pole of the uncompensated system closest to the origin (located at -0.0163). Assuming exact cancellation is possible, the plant and controller transfer function becomes

$$G(s) = \frac{K(s + 0.6)}{s(s + 0.5858)}$$

Design the system to meet the requirements. You may use the following steps:

i. Set the gain, K, to the value required by the steady-state error specifications. Plot the Bode magnitude and phase diagrams.

ii. Calculate the required phase margin to meet the damping ratio or equivalently the $\%OS$ requirement, using Eq. (10.73). If the phase margin found from the Bode plot obtained in Step **i** is greater than the required value, simulate the system to check whether the settling time is less than 4 seconds and whether the requirement of a $\%OS \leq 4.32\%$ has been met. Redesign if the simulation shows that the $\%OS$ and/or the steady-state error requirements have not been met. If all requirements are met, you have completed the design.

b. In most cases, perfect pole-zero cancellation is not possible. Assume that you want to check what happens if the PI-controller's zero changes by ± 20%, for example, if Z_I moves to:

Case 1: -0.01304

or to

Case 2: -0.01956.

The plant and controller transfer function in these cases will be, respectively:

$$\text{Case 1: } G(s) = \frac{K(s+0.6)(s+0.01304)}{s(s+0.0163)(s+0.5858)}$$

$$\text{Case 2: } G(s) = \frac{K(s+0.6)(s+0.01956)}{s(s+0.0163)(s+0.5858)}$$

Set K in each case to the value required by the steady-state error specifications and plot the Bode magnitude and phase diagrams. Simulate the closed-loop step response for each of the three locations of Z_I: pole/zero cancellation, Case 1, and Case 2, given in the problem.

Do the responses obtained resemble a second-order overdamped, critically damped, or underdamped response? Is there a need to add a derivative mode?

25. Parabolic trough collector. In order to reduce the steady-state error of the parabolic trough collector system, a PI controller is added to the open-loop transfer function so that (*Camacho*, *2012*)

$$G(s) = \frac{137.2 \times 10^{-6} K(s+0.01)}{s(s^2+0.0224s+196 \times 10^{-6})} e^{-39s}$$

a. Draw the new resulting Nyquist diagram when $K = 1$.

b. Find the range of K for closed-loop stability.

c. Use a phase margin argument to find the value of K that will yield $\zeta = 0.5$ damping factor.

d. Using the value found in Part **a**, simulate the system for a unit-step response using a computer program.

Chapter 12 Problems

1. Consider the following open-loop transfer functions, where $G(s) = Y(s)/U(s)$. $Y(s)$ is the Laplace transform of the output, and $U(s)$ is the Laplace transform of the input control signal:

   i. $G(s) = \dfrac{(s+3)}{(s+4)^2}$

   SS ii. $G(s) = \dfrac{s}{(s+5)(s+7)}$

   iii. $G(s) = \dfrac{20s(s+7)}{(s+3)(s+7)(s+9)}$

   iv. $G(s) = \dfrac{30(s+2)(s+3)}{(s+4)(s+5)(s+6)}$

   v. $G(s) = \dfrac{s^2+8s+15}{(s^2+4s+10)(s^2+3s+12)}$

   For each of these transfer functions, do the following: [Section: 12.2]

   a. Draw the signal-flow graph in phase-variable form.
   b. Add state-variable feedback to the signal-flow graph.
   c. For each closed-loop signal-flow graph, write the state equations.
   d. Write, *by inspection*, the closed-loop transfer function, $T(s)$, for your closed-loop signal-flow graphs.
   e. Verify your answers for $T(s)$ by finding the closed-loop transfer functions from the state equations and Eq. (3.73).

2. Assume that each of the following open-loop transfer functions is represented in signal-flow cascade form.

   i. $G(s) = \dfrac{40(s+1)(s+5)}{s(s+2)(s+10)}$

   ii. $G(s) = \dfrac{70(s^2+2s+9)}{(s+3)(s^2+s+10)}$

   For each, do the following: [Section: 12.4]

   a. Sketch the signal-flow graph showing state-variable feedback.
   b. Obtain the closed-loop transfer function with state-variable feedback.

3. For each of the following open-loop transfer functions represented in signal-flow parallel form: [Section: 12.4]

   a. Sketch the signal-flow graph showing state-variable feedback.
   b. Obtain the closed-loop transfer function with state-variable feedback.

   i. $G(s) = \dfrac{70(s^2+5s+80)}{s(s+20)(s+30)}$

   ii. $G(s) = \dfrac{40(s+4)(s+10)}{(s+1)(s+2)(s+3)}$

4. Given the following open-loop plant: [Section: 12.2] SS

$$G(s) = \frac{100(s+2)(s+25)}{(s+1)(s+3)(s+5)}$$

   design a controller to yield 10% overshoot with a peak time of 0.5 second. Use the controller canonical form for state-variable feedback.

5. Design a controller using phase variables for state-variable feedback that will result in 15% overshoot and a settling time of 1 second when the plant is: [Section: 12.2]

$$G(s) = \frac{30(s+1)}{s(s+3)(s+5)}$$

   Place the third pole 10 times farther from the imaginary axis as the dominant pole pair.

6. Repeat Problem 5 but represent the plant in parallel form without converting to phase-variable form. [Section: 12.4]

7. a. Given the plant shown in Figure P12.1, what relationship exists between b_1 and b_2 to make the system uncontrollable?
   b. What values of b_2 will make the system uncontrollable if $b_1 = 1$? [Section: 12.3]

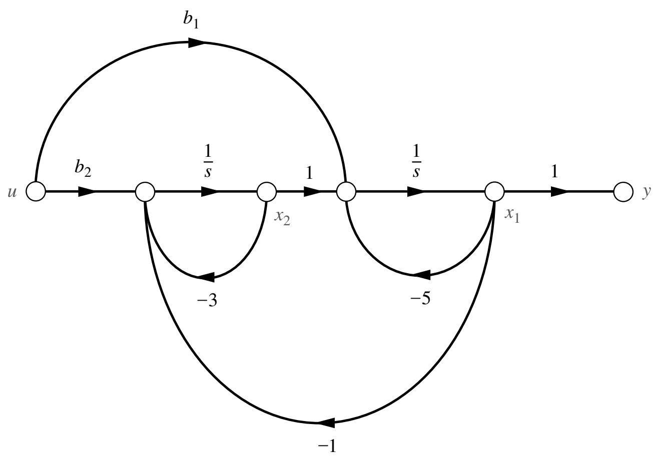


FIGURE P12.1

8. For each of the plants represented by signal-flow graphs in Figure P12.2, determine the controllability. If the controllability can be determined by inspection, state that it can and then verify your conclusions using the controllability matrix. [Section: 12.3] SS

9. `Use MATLAB to determine the controllability of the systems of Figure P12.2(d) and (f).` MATLAB ML

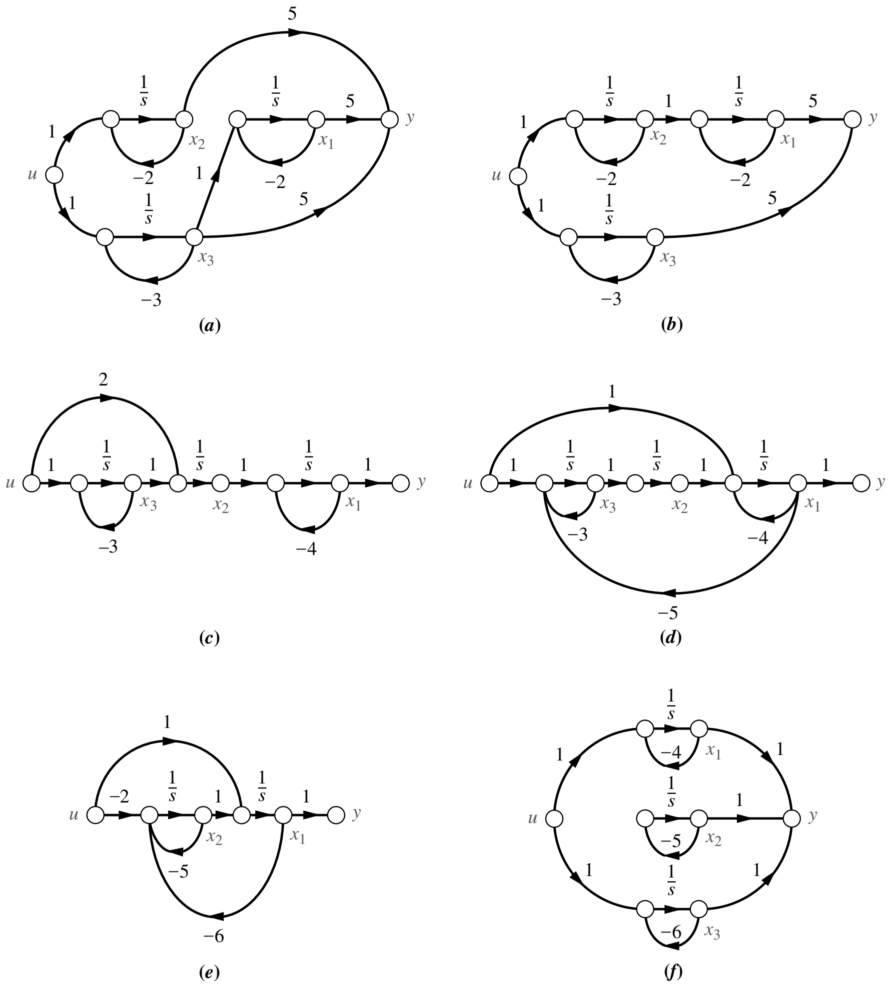


FIGURE P12.2

10. Consider the following transfer function:

$$G(s) = \frac{(s+6)}{(s+3)(s+8)(s+10)}$$

If the system is represented in cascade form, as shown in Figure P12.3, design a controller to yield a closed-loop response of 10% overshoot with a settling time of 1 second. Design the controller by first transforming the plant to phase variables. [Section: 12.4]

11. `Use MATLAB to design the controller gains for the system given in Problem 10.`

MATLAB **ML**

12. If an open-loop plant

$$G(s) = \frac{100}{s(s+4)(s+10)}$$

is represented in parallel form, design a controller to yield a closed-loop response of 20% overshoot and a peak time of 0.2 second. Design the controller by first transforming the plant to controller canonical form. [Section: 12.4]

13. Assume for the plant

SS

$$G(s) = \frac{1}{s(s+3)(s+6)}$$

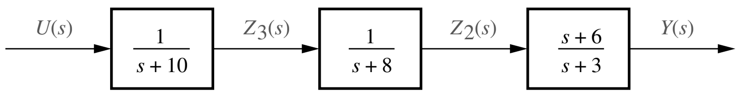


FIGURE P12.3

that the state variables are not accessible for measurement. Using observer canonical variables, design an observer that will have a $\zeta = 0.5$ and $\omega_n = 40$. Choose the third pole 10 times farther to the left than the dominant poles. [Section: 12.5]

SS **14.** Design an observer for the plant

$$G(s) = \frac{10}{(s+3)(s+7)(s+15)}$$

operating with 10% overshoot and 2 seconds peak time. Design the observer to respond 10 times as fast as the plant. Place the observer third pole 20 times as far from the imaginary axis as the observer dominant poles. Assume the plant is represented in observer canonical form. [Section: 12.5]

15. Consider the plant

$$G(s) = \frac{(s+2)}{(s+5)(s+9)}$$

whose phase variables are not available. Design an observer for the phase variables with a transient response described by $\zeta = 0.6$ and $\omega_n = 120$. Do not convert to observer canonical form. [Section: 12.7]

16. Determine whether or not each of the systems shown in Figure P12.2 is observable. [Section: 12.6]

17. `Use MATLAB to determine the observability of the systems of Figure P12.2(a) and (f).` MATLAB ML

18. Design an observer for the plant

$$G(s) = \frac{1}{(s+5)(s+13)(s+20)}$$

represented in cascade form. Transform the plant to observer canonical form for the design. Then transform the design back to cascade form. The characteristic polynomial for the observer is to be $s^3 + 600s^2 + 40{,}000s + 1{,}500{,}000$.

19. `Use MATLAB to design the observer gains for the system given in Problem 19.` MATLAB ML

20. Design an observer for

$$G(s) = \frac{45}{(s+3)(s+5)(s+10)}$$

represented in phase-variable form with a desired performance of 10% overshoot and a settling time of 0.5 second. The observer will be 10 times as fast as the plant, and the observer's nondominant pole will be 10 times as far from the imaginary axis as the observer's dominant poles. Design the observer by first converting to observer canonical form. [Section: 12.7]

21. Observability and controllability properties depend on the state-space representation chosen for a given system. In general, observability and controllability are affected when pole-zero cancellations are present in the transfer function. Consider the following two systems with representations: SS

$$\dot{x}_i = \mathbf{A}_i\mathbf{x}_i = \mathbf{B}_i r$$
$$y = \mathbf{C}_i\mathbf{x}_i;$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix};\ \mathbf{B}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ \mathbf{C}_1 = [2 \quad 0]$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix};\ \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};\ \mathbf{C}_2 = [6 \quad 2 \quad 0]$$

a. Show that both systems have the same transfer function $G_i(s) = \dfrac{Y(s)}{R(s)}$ after pole-zero cancellations.

b. Evaluate the observability of both systems.

22. Given the plant SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathrm{u};\quad y = [1 \quad 1]\mathbf{x}$$

design an integral controller to yield a 15% overshoot, 0.6-second settling time, and zero steady-state error for a step input. [Section: 12.8]

DESIGN PROBLEMS

23. Figure P12.4 shows a continuous stirred tank reactor in which an aqueous solution of sodium acetate (CH_3COONa) is neutralized in the mixing tank with hydrochloric acid (HCl) to maintain a particular pH in the mixing tank.

The amount of acid in the mix is controlled by varying the rotational speed of a feeding peristaltic pump. A nominal linearized transfer function from HCl flowrate to pH has been shown to be (*Tadeo*, *2000*)

$$G(s) = \frac{-0.9580 \times 10^{-4}s - 0.01197 \times 10^{-4}}{s^3 + 0.5250s^2 + 0.01265s + 0.000078}$$

a. Write the system in state-space phase-variable form.

b. Use state-feedback methods to design a matrix **K** that will yield an overdamped output pH response with a settling time of $T_s \approx 5$ min for a step input change in pH.

c. `Simulate the step response of the resulting closed-loop system using MATLAB.` MATLAB ML

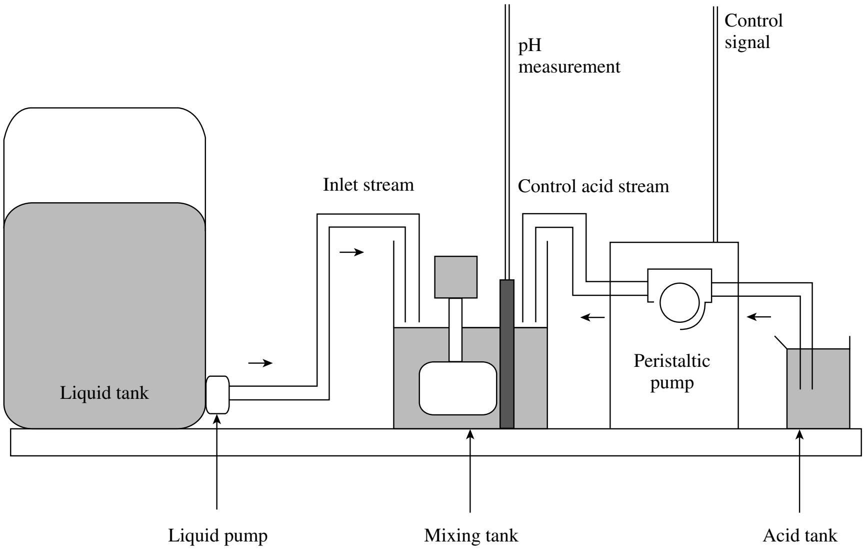


FIGURE P12.4[1]

24. a. Design an observer for the neutralization system using the continuous stirred tank reactor of Problem 24. The observer should have time constants 10 times smaller than those of the original system. Assume that the original state variables are those obtained in the phase-variable representation.

b. Simulate your system and observer for a unit-step input using Simulink. Assume that the initial conditions for the original system are $\mathbf{x}(0)=\begin{bmatrix}-1\\-10\\3\end{bmatrix}$. The observer should have initial conditions $\hat{\mathbf{x}}(0)=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

Simulink **SL**

25. The use of feedback control to vary the pitch angle in the blades of a variable speed wind turbine allows power generation optimization under variable wind conditions (*Liu, 2008*). At a specific operating point, it is possible to linearize turbine models. For example, the model of a three-blade turbine with a 15 m radius working in 12 m/s wind-speed and generating 220 V can be expressed as

$$\dot{\mathbf{x}}=\begin{bmatrix}-5 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ -10.5229 & -1066.67 & -3.38028 & 23.5107 & 0\\ 0 & 993.804 & 3.125 & -23.5107 & 0\\ 0 & 0 & 0 & 10 & -10\end{bmatrix}\mathbf{x}+\begin{bmatrix}5\\0\\0\\0\\0\end{bmatrix}u$$

$$y=\begin{bmatrix}0 & 0 & 0 & 1.223x10^5 & 0\end{bmatrix}x$$

where the state variable vector is given by

$$\mathbf{x}=\begin{bmatrix}\beta & \xi & \dot{\xi} & \omega_g & \omega_{gm}\end{bmatrix}$$

Here, β = pitch angle of the wind turbine blades, ξ = relative angle of the secondary shaft, ω_g = generator speed, and ω_{gm} = generator measurement speed. The

[1] Tadeo F., Perez, Loepez O., and Alvarez T. Control of Neutralization Processes by Robust Loopsharing. *IEEE Trans. on Cont. Syst. Tech.*, vol. 8, no. 2, 2000. Fig. 2, p. 239. IEEE Transactions on Control Systems Technology by Institute of Electrical and Electronics Engineers; IEEE Control Systems Society. Reproduced with permission of Institute of Electrical and Electronics Engineers, in the format Republish in a book via Copyright Clearance Center.

system input is u, the pitch angle reference, and the output is y, the active power generated.

a. Find a state feedback vector gain such that the system responds with a 10% overshoot and a settling time of 2 seconds for a step input.

b. `Use MATLAB to verify the operation of the system under state feedback.` MATLAB ML

26. The study of the flexible links, such as the one shown in Figure P12.5, is important because of their application to the control of flexible lightweight robots (*Saini*, *2012*). The flexible link angle is deflected by a servomotor. It is assumed that the base angle, $\theta(t)$, and the tip angular deflection relative to the undeformed link, $\alpha(t)$, can be measured. For a specific setup, a state-space model of the system was developed. The state vector is $\mathbf{x} = [\theta \quad \alpha \quad \omega \quad \dot{\alpha}]^T$, where $\omega(t) = \dot{\theta}(t)$ and input $u(t)$ is the voltage applied to the servomotor. Thus the system is represented as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$, $y = \mathbf{C}\mathbf{x}$ where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 673.07 & -35.1667 & 0 \\ 0 & -1023.07 & 35.1667 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 61.7325 \\ -61.7325 \end{bmatrix}$$

$$\mathbf{C} = [1 \quad 1 \quad 0 \quad 0]$$

It is desired to build state-feedback compensation around this system so that the system's characteristic equation becomes $D(s) = (s+10)^4$. In order to do this:

a. Find the system's controllability matrix $\mathbf{C_{M_O}}$ and show that the system is controllable.

b. Find the original system's characteristic equation and use it to find a phase-variable representation of the system.

c. Find the phase-variable system's controllability matrix $\mathbf{C_{M_P}}$ and then find the transformation matrix $\mathbf{P} = \mathbf{C_{M_O}}\mathbf{C_{M_P}^{-1}}$.

d. Use the phase-variable representation to find a feedback gain matrix $\mathbf{K_P} = [k_{1p} \quad k_{2p} \quad k_{3p} \quad k_{4p}]$ that will place the closed-loop poles in the desired positions.

e. Find the corresponding feedback gain matrix $\mathbf{K_O} = \mathbf{K_P}\,\mathbf{P}^{-1}$.

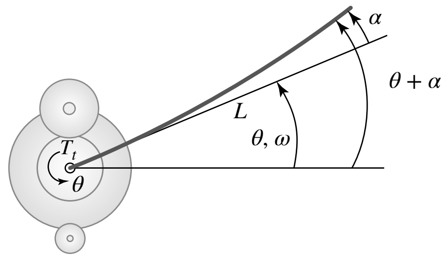


FIGURE P12.5[2]

27. We want to use an observer in a textile machine to estimate the state variables. The 2-input, 1-output system's model is $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$; $y = \mathbf{C}\mathbf{x}$, where (*Cardona*, *2010*)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -52.6532 & -4.9353 & -2768.1557 \\ -0.001213 & 0 & -0.06106 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.001613 & -0.001812 \end{bmatrix}$$

$$\mathbf{C} = [1 \quad 0 \quad 0]$$

a. Find the system's observability matrix $\mathbf{O_{Mz}}$ and show that the system is observable.

b. Find the original system's characteristic equation and use it to find an observable canonical representation of the system.

c. Find the observable canonical system's observability matrix $\mathbf{O_{Mx}}$ and then find the transformation matrix $\mathbf{P} = \mathbf{O_{Mz}^{-1}}\mathbf{O_{Mx}}$.

d. Use the observable canonical representation to find an observer gain matrix $\mathbf{L_x} = [l_{1x} \quad l_{2x} \quad l_{3x} \quad l_{4x}]^T$ so that the observer characteristic polynomial is $D(s) = s^3 + 30s^2 + 316s + 1160$.

e. Find the corresponding observer gain matrix $\mathbf{L_z} = \mathbf{P}\mathbf{L_x}$.

28. An inverted pendulum mounted on a motor-driven cart was presented in Problem 25, Chapter 3. Its state-space model was linearized (*Prasad*, *2012*) around

[2] Saini, S. C., Sharma, Y., Bhandari, M., and Satija, U. Comparison of Pole Placement and LQR Applied to Single Link Flexible Manipulator, *International Conference on Communication Systems and Network Technologies*, IEEE Computer Society, 2012, pp. 843–847, Figure 3, p. 844.

a stationary point, ($\mathbf{x_0} = 0$), corresponding to the pendulum point-mass, m, being in the upright position at $t = 0$. Its model was then modified in Problem 37, Chapter 6, to have two output variables: the pendulum angle relative to the y-axis, $\theta(t)$, and the horizontal position of the cart, $x(t)$. Noting that the unit requires stabilization, you were asked in Problem 26, Chapter 9, to develop Simulink models for two feedback systems: one to control the cart position, $x(t)$, and the second to control the pendulum angle, $\theta(t)$.

Modify the second model, using state-feedback amplifiers with appropriate gains (in addition to the rate feedback amplifier and the PD controller), to improve the unit-impulse response of the angle control loop. Compare the response you get here with that obtained for Problem 26, Chapter 9.

29. Let the plant in the drive system with an elastically coupled load (*Thomsen, 2011*) shown in Figure P8.13 be

$$G_p(s) = \frac{Y(s)}{M(s)} = \frac{250\,(s^2 + 1.2s + 12500)}{s^3 + 8.1s^2 + 62003s + 31250}$$

where $Y(s) = \Omega_L(s)$, the load speed. Represent $G_p(s)$ in observer canonical form. Then design an observer for it, such that it responds 10 times faster than the output, $y(t)$, if $G_C(s) = K_P$. [Section: 12.5]

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

30. Control of HIV/AIDS. The linearized model of HIV infection when RTIs are used for treatment was introduced in Chapter 4 and repeated here for convenience (*Craig, 2004*):

$$\begin{bmatrix} \dot{T} \\ \dot{T}^* \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -0.04167 & 0 & -0.0058 \\ 0.0217 & -0.24 & 0.0058 \\ 0 & 100 & -2.4 \end{bmatrix} \begin{bmatrix} T \\ T^* \\ v \end{bmatrix} + \begin{bmatrix} 5.2 \\ -5.2 \\ 0 \end{bmatrix} u_1$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ T^* \\ v \end{bmatrix}$$

T represents the number of healthy T-cells, T^* the number of infected cells, and v the number of free viruses.

a. Design a state-feedback scheme to obtain

(1) zero steady-state error for step inputs

(2) 10% overshoot

(3) a settling time of approximately 100 days

(Hint: the system's transfer function has an open-loop zero at approximately -0.02. Use one of the poles in the desired closed-loop-pole polynomial to eliminate this zero. Place the higher-order pole 6.25 times farther than the dominant pair.)

b. `Simulate the unit-step response of your design using Simulink.`

Simulink
SL

31. Hybrid vehicle. In Problem 27, Chapter 3, we introduced the idea that when an electric motor is the sole motive force provider for a hybrid electric vehicle (HEV), the forward paths of all HEV topologies are similar. It was

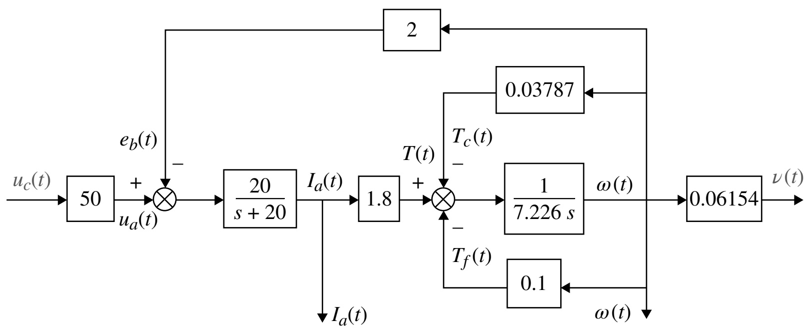


FIGURE P12.6

noted that, in general, the forward path of an HEV cruise control system can be represented by a block diagram similar to that of Figure P3.16 (*Preitl, 2007*). The diagram is shown in Figure P12.6, with the parameters substituted by their numerical values from Problem 51, Chapter 6; the motor armature represented as a first-order system with a unity steady-state gain and a time constant of 50 ms; and the power amplifier gain set to 50. Whereas the state variables remain as the motor angular speed, $\omega(t)$, and armature current, $I_a(t)$, we assume now that we have only one input variable, $u_c(t)$, the command voltage from the electronic control unit, and one output variable, car speed, $v = r\omega/i_{tot} = 0.06154\omega$. The change in the load torque, $T_c(t)$, is represented as an internal feedback proportional to $\omega(t)$.

Looking at the diagram, the state equations may be written as

$$\begin{bmatrix} \dot{I}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -20 & -40 \\ 0.2491 & -0.0191 \end{bmatrix} \begin{bmatrix} I_a \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 1000 \end{bmatrix} u_c(t)$$

$$y(t) = v(t) = [0 \quad 0.05154] \begin{bmatrix} I_a \\ \omega \end{bmatrix}$$

a. Design an integral controller for $\%OS \leq 4.32\%$, a settling time, $T_s \leq 4.4$ sec, and a zero steady-state error for a step input (Hint: To account for the effect of the integral controller on the transient response, use $T_s = 4$ seconds in your calculation of the value of the natural frequency, ω_n, of the required dominant poles).

b. `Use MATLAB to verify that the design requirements are met.` MATLAB **ML**

32. Parabolic trough collector. A parabolic trough collector can be designed using state-space techniques. For simplicity, pure time delay will be ignored here, although it could be handled in several different ways. Consider the open-loop transfer function (*Camacho, 2012*):

$$G(s) = \frac{137.2 \times 10^{-6} K}{s^2 + 0.0224s + 196 \times 10^{-6}}$$

Design a state feedback controller with integral control to yield zero steady-state error, such that the system transient response results in a damping factor of $\zeta = 0.5$ with a settling time $T_s = 200$ sec. Simulate the step response of your designed system using a computer program.

Chapter 13 Problems

1. Derive the z-transforms for the time functions listed below. Do not use any z-transform tables. Use the plan $f(t) \to f^*(t) \to F^*(s) \to F(z)$, followed by converting $F(z)$ into closed form making use of the fact that $1/(1-z^{-1}) = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$. Assume ideal sampling. [Section: 13.3]

   a. $e^{-at}u(t)$
   b. $u(t)$
   c. $t^2e^{-at}u(t)$
   d. $\cos \omega t\, u(t)$

2. Repeat all parts of Problem 1 using MATLAB and MATLAB's Symbolic Math Toolbox. Symbolic Math **SM**

3. Use partial-fraction expansions to find $f(kT)$ for each of the following transfer functions: [Section: 13.3]

   a. $F(z) = \dfrac{z(z+2)(z+4)}{(z-0.3)(z-0.5)(z-0.7)}$

   b. $F(z) = \dfrac{(z+0.3)(z+0.5)}{(z-0.2)(z-0.6)(z-0.8)}$

   c. $F(z) = \dfrac{(z+1)(z+0.2)(z+0.5)}{z(z-0.1)(z-0.6)(z-0.9)}$

4. Use MATLAB's Symbolic Math Toolbox to solve all parts of Problem 3. Symbolic Math **SM**

5. Using partial-fraction expansion and Table 13.1, find the z-transform for each $G(s)$ shown below if $T = 0.5$ second. [Section: 13.3]

   a. $G(s) = \dfrac{(s+4)}{(s+2)(s+5)}$

   b. $G(s) = \dfrac{(s+1)(s+2)}{s(s+3)(s+4)}$ **SS**

   c. $G(s) = \dfrac{20}{(s+3)(s^2+6s+25)}$

   d. $G(s) = \dfrac{15}{s(s+1)(s^2+10s+81)}$

6. Repeat all parts of Problem 5 using MATLAB and MATLAB's Symbolic Math Toolbox. Symbolic Math **SM**

7. Find $G(z) = C(z)/R(z)$ for each of the block diagrams shown in Figure P13.1 if $T = 0.3$ second. [Section: 13.4] **SS**

8. Find $T(z) = C(z)/R(z)$ for each of the systems shown in Figure P13.2. [Section: 13.5] **SS**

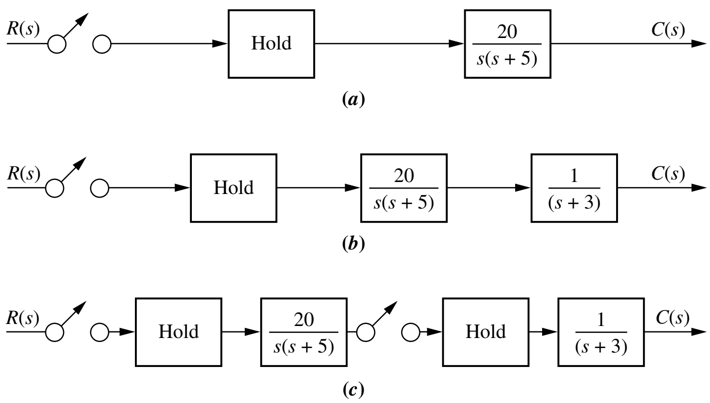


FIGURE P13.1

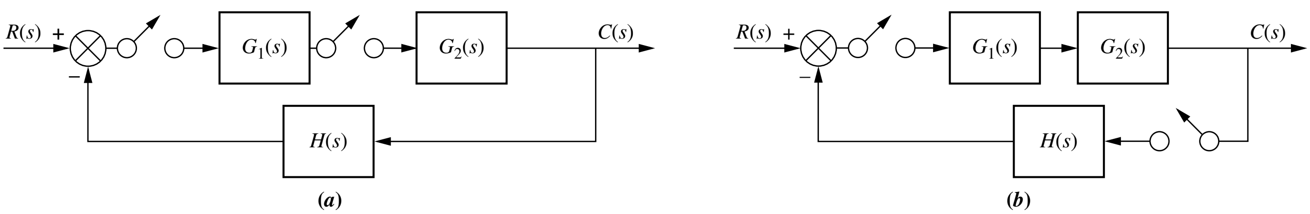


FIGURE P13.2

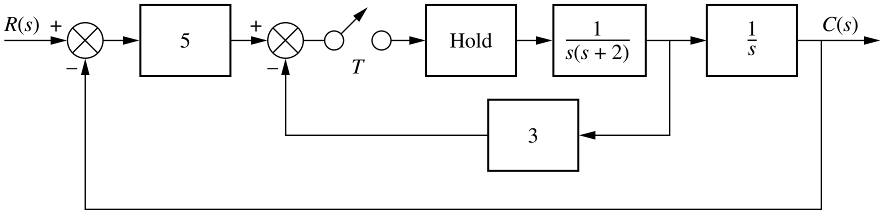


FIGURE P13.3

9. Find the closed-loop transfer function, $T(z) = C(z)/R(z)$, for the system shown in Figure P13.3. [Section: 13.5]

10. `Write a MATLAB program that can be used to find the range of sampling time, T, for stability. The program will be used for systems of the type represented in Figure P13.4 and should meet the following requirements:`

MATLAB **ML**

a. `MATLAB will convert` $G_1(s)$ `cascaded with a sample-and-hold to` $G(z)$.

b. `The program will calculate the` z`-plane roots of the closed-loop system for a range of T and determine the value of T, if any, below which the system will be stable. MATLAB will display this value of T along with the z-plane poles of the closed-loop transfer function.`

`Test the program on`

$$G_1(s) = \frac{20(s+6)}{(s+1)(s+3)(s+4)(s+8)}$$

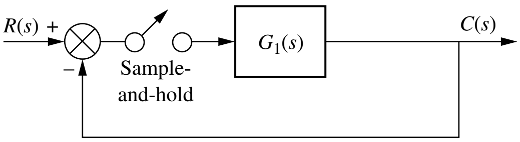


FIGURE P13.4

11. Find the range of sampling interval, T, for which the system in Figure P13.5 is closed-loop stable. [Section: 13.6]

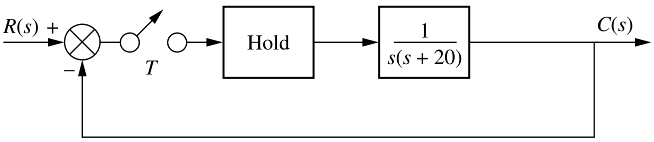


FIGURE P13.5

12. Find the range of gain, K, to make the system shown in Figure P13.6 stable. [Section: 13.6]

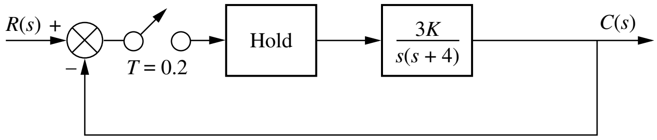


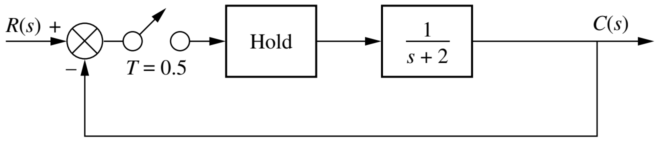
FIGURE P13.6

13. Find the static error constants and the steady-state error for each of the digital systems shown in Figure P13.7 if the inputs are [Section: 13.7] **SS**

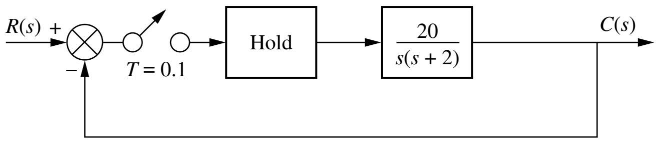
a. $u(t)$

b. $tu(t)$

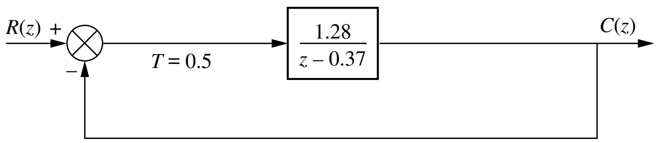
c. $\frac{1}{2}t^2u(t)$



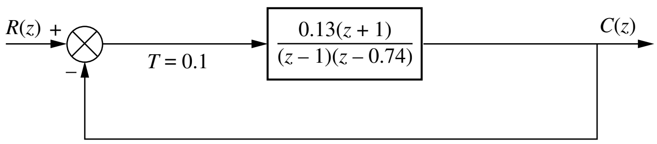
System 1



System 2



System 3



System 4

FIGURE P13.7

14. `Write a MATLAB program that can be used to find` K_p, K_v, `and` K_a `for digital systems. The program will be used for systems of the type represented in Figure P13.4. Test your program for`

MATLAB ML

$$G(z) = \frac{0.04406z^3 - 0.03624z^2 - 0.03284z + 0.02857}{z^4 - 3.394z^3 + 4.29z^2 - 2.393z + 0.4966}$$

`where` $G(z)$ `is the pulse transfer function for` $G_1(s)$ `in cascade with the z.o.h. and` $T = 0.1$ `second.`

15. For the digital system shown in Figure P13.4, where $G_1(s) = K/[(s+1)(s+5)]$, find the value of K to yield a 15% overshoot. Also find the range of K for stability. Let $T = 0.1$ second. [Section: 13.9]

16. `Use Simulink to simulate the step response for the system of Problem 15. Set the value of gain,` K, `to that designed in Problem 15 for 15% overshoot.`

Simulink SL

17. `Write a MATLAB program that can be used to design the gain of a digital control system to meet a percent overshoot requirement. The program will be used for systems of the type represented in Figure P13.4 and meet the following requirements:`

MATLAB ML

a. `The user will input the desired percent overshoot.`

b. `MATLAB will convert` $G_1(s)$ `cascaded with the sample-and-hold to` $G(z)$.

c. `MATLAB will display the root locus on the` z`-plane along with an overlay of the percent overshoot curve.`

d. `The user will click with the mouse at the intersection of the root locus and percent overshoot overlay and MATLAB will respond with the value of gain followed by a display of the step response of the closed-loop system.`

`Apply your program to Problem 15 and compare results.`

18. Let $G_1 = K/\big(s(s+1)\big)$ in Figure P13.4. Find the range of K for closed-loop stability. Also, find the value of K that will result in a peak time of 1.5 seconds if the sampling interval is $T = 0.1$ second. [Section: 13.9]

19. In Figure P13.4 assume $G_1(s) = \big(K(s+3)\big)/\big(s(s+1)(s+4)\big)$. It is desired to have a settling time of 10 seconds when the sampling interval, T, is 0.5 second. Find the required value of K as well as the range of K for closed-loop stability. [Section: 13.9]

20. A PID controller was designed in Example 9.5 for a continuous system with unity feedback. The system's plant was

$$G(s) = \frac{(s+8)}{(s+3)(s+6)(s+10)}$$

The designed PID controller was

$$G_c(s) = 4.6\frac{(s+55.92)(s+0.5)}{s}$$

Find the digital transfer function, $G_c(z)$, of the PID controller in order for the system to be computer controlled if the sampling interval, T, is 0.005 second. [Section: 13.10]

21. A unity-feedback system has a continuous transfer function

$$G(s) = \frac{1}{s(s+4)(s+10)}$$

Design a lead compensator so the system is computer controlled in closed loop with the following specifications: [Section: 13.10]

Percent overshoot: 10%
Settling time: 2 seconds
Sampling interval: 0.05 second

22. `Solve Problem 21 using MATLAB.`

MATLAB ML

DESIGN PROBLEMS

23. **a.** Convert the heading control for the UFSS vehicle shown in Appendix A3 (*Johnson, 1980*) into a digitally controlled system. SS

b. Find the closed-loop pulse transfer function, $T(z)$, if $T = 0.1$ second.

c. Find the range of heading gain to keep the digital system stable.

24. In Problem 37, Chapter 9, a steam-driven turbine-governor system was implemented by a unity-feedback system with a forward-path transfer function (*Khodabakhshian, 2005*)

$$G(s) = \frac{K}{(s+0.08)(s+2)(s+5)}$$

a. Use a sampling period of $T = 0.5$ s and find a discrete equivalent for this system.

b. Use MATLAB to draw the root locus. MATLAB **ML**

c. Find the value of K that will result in a stable system with a damping factor of $\zeta = 0.7$.

d. Use the root locus found in Part **a** to predict the step-response settling time, T_s, and peak time, T_p.

e. Calculate the final value of the closed-loop system unit step response.

f. Obtain the step response of the system using Simulink. Verify the predictions you made in Parts **c** and **d**. Simulink **SL**

25. Given LabVIEW **LV**

$$G(s) = \frac{8}{s+4}$$

Use the LabVIEW Control Design and Simulation Module to (1) convert $G(s)$ to a digital transfer function using a sampling rate of 0.25 second; and (2) plot the step responses of the discrete and the continuous transfer functions.

26. Given LabVIEW **LV**

$$G(z) = \frac{K(z+0.5)}{(z-0.25)(z-0.75)}$$

Use the LabVIEW Control Design and Simulation Module and the MathScript RT Module to (1) obtain the value of K that will yield a damping ratio of 0.5 for the closed-loop system in Figure 13.20, where $H(z)=1$; and (2) display the step response of the closed-loop system in Figure 13.20 where $H(z) = 1$. Compare your results with those of Skill-Assessment Exercise 13.8. MATLAB **ML**

27. The purpose of an artificial pacemaker is to regulate heart rate in those patients in which the natural feedback system malfunctions. Assume a unity-feedback system with a forward path,

$$G(s) = \frac{1352k}{s(s+8)(s+20)}$$

as a simplified model of a pacemaker (*Neogi, 2010*).

a. Convert the pacemaker model to a discrete system with a sampling rate of 0.01 second.

b. Draw the root locus of the system using a computer program.

c. Use the root locus in Part **b** to find the range of k for which the system is closed-loop stable.

d. Use the root locus from Part **b** to find the value of k that will yield a 5% overshoot for a step input.

e. Simulate the unit step input of your discretized system to verify your design.

28. A linear model of the α-subsystem of a grid-connected voltage-source converter with a Y-Y transformer (*Mahmood, 2012*) was presented in Problem 52, Chapter 8, and Problem 40, Chapter 10. The system was represented with unity-feedback and a forward path consisting of the cascading of a compensator and a plant. The plant is given by

$$G_P(s) = \frac{V_\alpha(s)}{M_\alpha(s)} = \frac{(s+2200)}{(s+220)(s^2+120s+16\times10^6)}$$

This system is now to be digitally controlled with the following specifications: percent overshoot, $\%OS = 10\%$; settling time, $T_S = 0.1$ second; and sampling interval, $T = 0.001$ second. Design a lead compensator for that system to meet these specifications. [Section: 13.10]

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

29. Control of HIV/AIDS. In Chapter 11, a continuous cascaded compensator for a unity-feedback system was designed for the treatment of the HIV-infected patient treated with RTIs (*Craig, 2004*). The transfer function of the designed compensator was

$$G_c(s) = \frac{-2\times10^{-4}(s^2+0.04s+0.0048)}{s(s+0.02)}$$

The linearized plant was given by

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s-10.3844}{s^3+2.6817s^2+0.11s+0.0126}$$

The compensated system is overdamped with an approximate settling time of 100 seconds. This system must be discretized for practical reasons: (1) HIV patient cannot be monitored continuously and (2) medicine dosage cannot be adjusted continuously.

a. Show that a reasonable sampling period for this system is $T = 8$ days (medicine dosage will be updated on a weekly basis).

b. Use Tustin's method and $T = 8$ days to find a discrete equivalent to $G_c(s)$.

c. Use Simulink to simulate the continuous and discrete compensated systems for a unit step input. Plot both responses on the same graph.

Simulink SL

30. **Hybrid vehicle.** In Problem 50, Chapter 7 (Figure P7.25), the block diagram of a cascade scheme for the speed control of an HEV (*Preitl, 2007*) was represented as a unity-feedback system. In that diagram the output of the system is the speed transducer's output voltage, $C(s) = K_{ss}V(s)$. In Part **b** of Problem 24, Chapter 11, where a compensator was designed for this problem, we discussed the feasibility of achieving full pole–zero cancellation when we place a PI speed controller's zero, Z_I, on top of the uncompensated system's real pole, closest to the origin (located at -0.0163). Noting that perfect pole–zero cancellation may not be maintained, we studied a case where the PI-controller's zero changed by +20%, moving to -0.01304. In that case, the transfer function of the plant with a PI speed controller, which has a proportional gain = K, was given by

MATLAB ML

$$G(s) = \frac{K(s+0.6)(s+0.01304)}{s(s+0.0163)(s+0.5858)}$$

Assuming that $G_1(s)$ in Figure P13.4 equals the transfer function, $G(s)$, given above for the vehicle with the speed controller:

a. Develop a MATLAB M-file that would allow you to do the following: (Hint: Refer to the M-files you developed for Problems 10 and 17 of this chapter)

(1) Convert $G_1(s)$ cascaded with a sample-and-hold to $G(z)$.

(2) Search over the range $0 < T < 5$ seconds for the largest sampling period T_{max} below which the system is stable. Calculate the z-plane roots of the closed-loop system for the whole range of the sampling time, T. Subsequently set $T = 0.75T_{\max}$.

(3) Design the gain of a digital control system to meet a percent overshoot requirement, $\%OS$, allowing the user to input the value of the desired $\%OS$ and the value of the PI speed controller's proportional gain, K.

(4) Plot the step response of that digital system (in per unit, p. u., vs. time in seconds).

b. Run the M-file you developed in Part **a** and enter the values of the desired percent overshoot, $\%OS = 0$, and the PI speed controller's proportional gain, $K = 61$.

c. Select a point in the graphics window displaying the root locus, such that all poles of the closed-loop transfer function, T_z, are inside the unit circle.

d. Write the sampled-data transfer functions obtained, G_z and T_z, indicating the corresponding value of the sampling time, T, and all poles, r, of the closed-loop transfer function, T_z.

e. Plot the step response of that digital system (in per unit, p. u., vs. time in seconds) noting the following characteristics: final value, rise time, and settling time.

31. **Parabolic trough collector.** In Problem 25, Chapter 11, a zero steady-state error for a unit step input was achieved through the design of a lag compensator with integral control. In that problem, the open-loop transmission can be written as $L(s) = G_c(s)G(s)$, where the parabolic trough plant is given by (*Camacho, 2012*)

$$G(s) = \frac{137.2 \times 10^{-6}}{s^2 + 0.0224s + 196 \times 10^{-6}} e^{-39s}$$

and the lag compensator is given by

$$G_c(s) = 1.12\frac{(s+0.01)}{s}$$

We want to substitute for the continuous compensator with a digital one.

a. Find a suitable sampling period for the system.

b. Find the equivalent compensator's transfer function in z-domain.

c. Use Simulink to simulate the digital compensator with the continuous plant. Compare the resulting response with that of the original system using the continuous compensator on the same graph.

Simulink SL