Instructor’s Supplement Problems

Chapter 02

1. Use MATLAB and the Symbolic Math Toolbox to find the Laplace transform of the following time functions: [Section: 2.2]

Symbolic Math **SM**

a. $f(t) = 8t^2 \cos(3t + 45°)$

b. $f(t) = 3te^{-2t} \sin(4t + 60°)$

2. Repeat Problem 12 in the text problems for the following transfer function: [Section: 2.3]

MATLAB **ML**

$$G(s) = \frac{s^4 + 25s^3 + 20s^2 + 15s + 42}{s^5 + 13s^4 + 9s^3 + 37s^2 + 35s + 50}$$

3. Use MATLAB and the Symbolic Math Toolbox to input and form LTI objects in polynomial and factored form for the following frequency functions: [Section: 2.3]

Symbolic Math **SM**

a. $G(s) = \dfrac{45(s^2 + 37s + 74)(s^3 + 28s^2 + 32s + 16)}{(s + 39)(s + 47)(s^2 + 2s + 100)(s^3 + 27s^2 + 18s + 15)}$

b. $G(s) = \dfrac{56(s + 14)(s^3 + 49s^2 + 62s + 53)}{(s^3 + 81s^2 + 76s + 65)(s^2 + 88s + 33)(s^2 + 56s + 77)}$

4. Find the transfer function, $G(s) = V_L(s)/V(s)$, for each network shown in Figure I-2.1. [Section: 2.4]

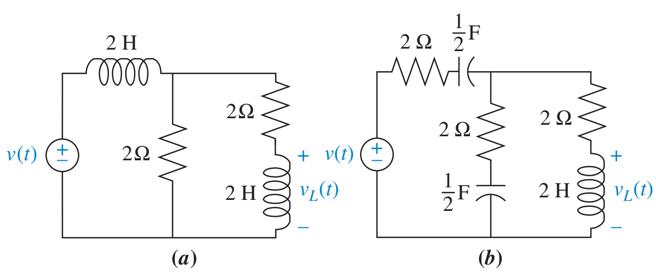

FIGURE I-2.1

5. **a.** Write, but do not solve, the mesh and nodal equations for the network of Figure I-2.2. [Section: 2.4]

b. Use MATLAB, the Symbolic Math Toolbox, and the equations found in part a to solve for the transfer function, $G(s) = V_o(s)/V(s)$. Use both the mesh and nodal equations and show that either set yields the same transfer function. [Section: 2.4]

Symbolic Math **SM**

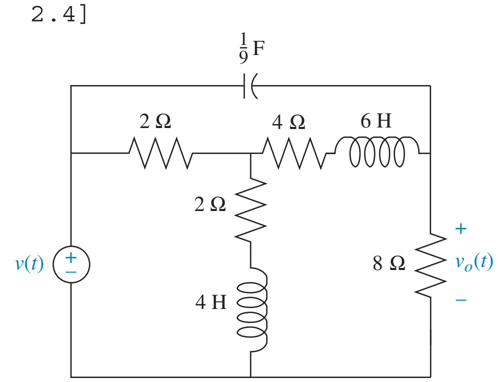

FIGURE I-2.2

6. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure I-2.3. (Hint: place a zero mass at $x_2(t)$.) [Section: 2.5]

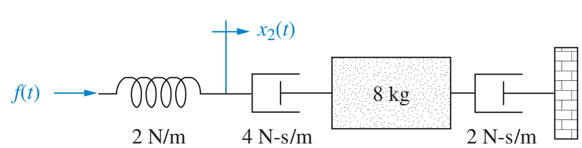

FIGURE I-2.3

7. Find the transfer function, $G(s) = X_3(s)/F(s)$, for the translational mechanical system shown in Figure I-2.4. [Section: 2.5]

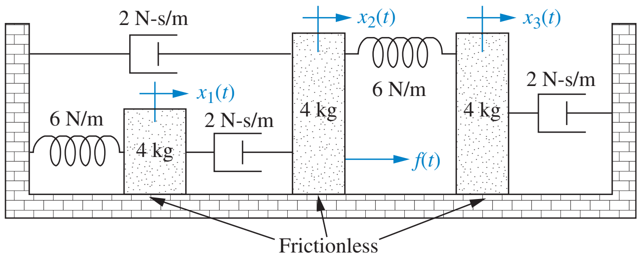

FIGURE I-2.4

8. Find the transfer function, $\dfrac{\theta_1(s)}{T(s)}$, for the system shown in Figure I-2.5.

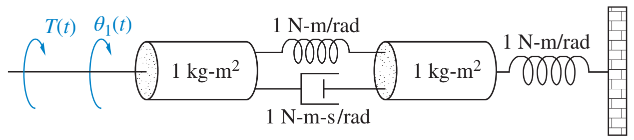

FIGURE I-2.5

9. For the rotational system shown in Figure I-2.6, write the equations of motion from which the transfer function, $G(s) = \theta_1(s)/T(s)$, can be found. [Section: 2.7]

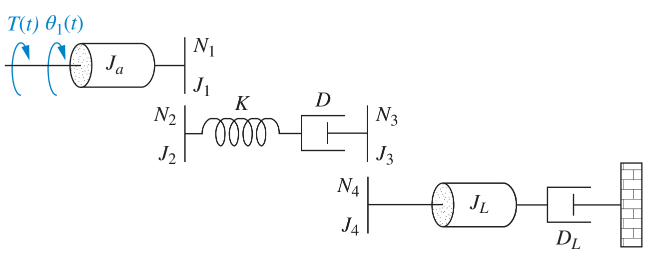

FIGURE I-2.6

10. In the system shown in Figure I-2.7, the inertia, J, of radius, r, is constrained to move only about the stationary axis A. A viscous damping force of translational value f_v exists between the bodies J and M. If an external force, $f(t)$, is applied to the mass, find the transfer function, $G(s) = \theta(s)/F(s)$. [Sections: 2.5; 2.6]

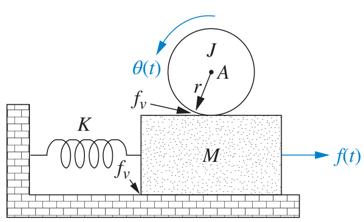

FIGURE I-2.7

11. Given the combined translational and rotational system shown in Figure I-2.8, find the transfer function, $G(s) = X(s)/T(s)$. [Sections: 2.5; 2.6]

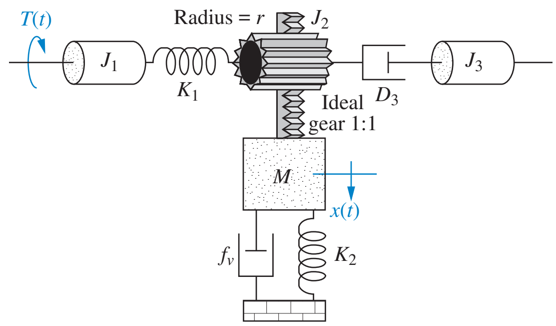

FIGURE I-2.8

12. A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m^2 and 3 N-m-s/rad, respectively, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$, of this motor if it drives an inertia load of 105 kg-m^2 through a gear train, as shown in Figure I-2.9. [Section: 2.8]

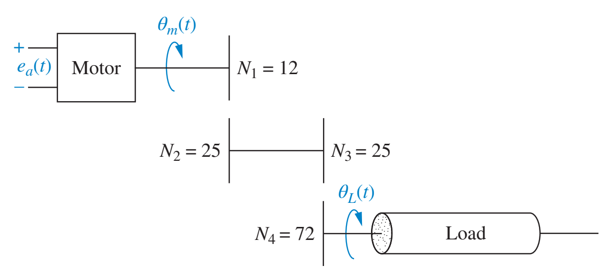

FIGURE I-2.9

13. Consider the differential equation

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = f(x)$$

where $f(x)$ is the input and is a function of the output, x. If $f(x) = \sin x$, linearize the differential equation for small excursions. [Section: 2.10]

a. $x = 0$

b. $x = \pi$

14. Many systems are *piecewise* linear. That is, over a *large* range of variable values, the system can be described linearly. A system with amplifier saturation is one such example. Given the differential equation

$$\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x = f(x)$$

assume that $f(x)$ is as shown in Figure I-2.10. Write the differential equation for each of the following ranges of x: [Section: 2.10]

a. $-\infty < x < -3$

b. $-3 < x < 3$

c. $3 < x < \infty$

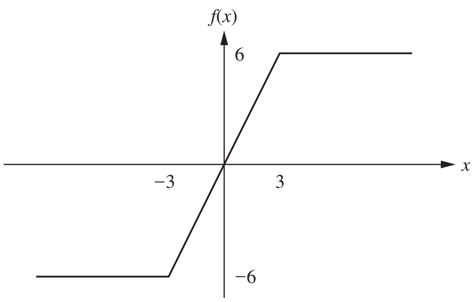

FIGURE I-2.10

15. Figure I-2.11 shows a crane hoisting a load. Although the actual system's model is highly nonlinear, if the rope is considered to be stiff with a fixed length L, the system can be modeled using the following equations:

$$\begin{aligned} m_L \ddot{x}_{La} &= m_L g \phi \\ m_T \ddot{x}_T &= f_T - m_L g \phi \\ x_{La} &= x_T - x_L \\ x_L &= L\phi \end{aligned}$$

where m_L is the mass of the load, m_T is the mass of the cart, x_T and x_L are displacements as defined in the figure, ϕ is the rope angle with respect to the vertical, and f_T is the force applied to the cart (*Marttinen, 1990*).

a. Obtain the transfer function from cart velocity to rope angle $\dfrac{\Phi(s)}{V_T(s)}$.

b. Assume that the cart is driven at a constant velocity V_0 and obtain an expression for the resulting $\phi(t)$. Show that under this condition, the load will sway with a frequency $\omega_0 = \sqrt{\dfrac{g}{L}}$.

c. Find the transfer function from the applied force to the cart's position, $\dfrac{X_T(s)}{F_T(s)}$.

d. Show that if a constant force is applied to the cart, its velocity will increase without bound as $t \to \infty$.

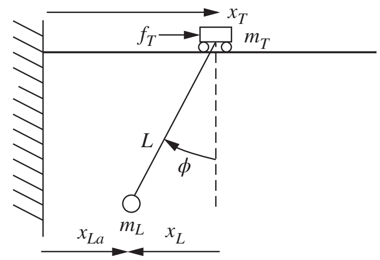

FIGURE I-2.11[1]

16. In 1978, Malthus developed a model for human growth population that is also commonly used to model bacterial growth as follows. Let $N(t)$ be the population density observed at time t. Let K be the rate of reproduction per unit time. Neglecting population deaths, the population density at a time $t + \Delta t$ (with small Δt) is given by

$$N(t + \Delta t) \approx N(t) + KN(t)\Delta t$$

which also can be written as

$$\frac{N(t + \Delta t) - N(t)}{\Delta t} = KN(t)$$

Since $N(t)$ can be considered to be a very large number, letting $\Delta t \to 0$ gives the following differential equation (*Edelstein-Keshet, 2005*):

$$\frac{dN(t)}{dt} = KN(t)$$

a. Assuming an initial population $N(0) = N_0$, solve the differential equation by finding $N(t)$.

b. Find the time at which the population is double the initial population.

[1] Marttinen A., Virkkunen J., Salminen R.T. Control Study with Pilot Crane. *IEEE Transactions on Education*, Vol. 33, No.3, August 1990. Fig. 2. p. 300. IEEE Transactions on Education by Institute of Electrical and Electronics Engineers; IEEE Education Group; IEEE Education Society. Reproduced with permission of Institute of Electrical and Electronics Engineers, in the format Republish in a book via Copyright Clearance Center.

Bibliography

Edelstein-Keshet, L. *Mathematical Models in Biology*. Society for Industrial and Applied Mathematics, Philadelphia, PA, 2005.

Marttinen, A., Virkkunen, J., and Salminen, R. T. Control Study with Pilot Crane. *IEEE Transactions on Education*, vol. 33, no. 3, August 1990, pp. 298–305.