Instructor's Supplement Problems

Chapter 13

1. For each $F(z)$ in Problem 3, in the text problems do the following: [Section: 13.3]

a. Find $f(kT)$ using the power series expansion.

b. Check your results against answers from Problem 3 in the text problems.

2. Find $C(z)$ in general terms for the digital system shown in Figure I-13.1. [Section: 13.5]

3. GUI Tool **GUIT** `Use MATLAB's Linear System Analyzer to determine the peak time and settling time of the closed-loop step response for System 4 in Figure P13.7 in the text problems.`

4. For the digital system shown in Figure P13.4 in the text problems, where $G_1(s) = K/[s(s+1)(s+3)]$, find the value of K to yield a 20% overshoot if the sampling interval, T, is 0.1 second. Also, find the range of K for stability. [Section: 13.9]

5. Simulink **SL** An inverted pendulum mounted on a motor-driven cart (*Prasad, 2012*) was the subject of Problem 26, Chapter 9 in the text problems. In that problem you were asked to develop Simulink models for two feedback systems, one of which was to control the cart position, $x(t)$. At the recommended settings, the step response of that system was expected to satisfy the following requirements: a steady-state error, $e(\infty) < 2\%$, a peak time, $T_P < 1.2$ seconds, and a percent overshoot, $\%OS < 20.5\%$. Having concluded that the steady-state error was unacceptable, you designed a PID controller and found its recommended settings.

`Digitize the Simulink model developed for PID control of the cart position in Part` **`d`** `of Problem 26, Chapter 9 in the text problems, by adding a zero-order-hold set to 0.01 second. Then, run a simulation to evaluate performance.`

DESIGN PROBLEMS

6. The analog system of a disk drive is shown in Figure I-13.2. Do the following:

a. Convert the disk drive to a digital system. Use a sampling time of 0.01 second.

b. Find the range of digital controller gain to keep the system stable.

c. Find the value of digital controller gain to yield 15% overshoot for a digital step response.

d. MATLAB **ML** `Repeat all previous parts and obtain the step response for Part` **`c`** `using MATLAB.`

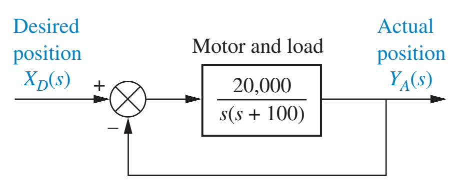

FIGURE I-13.2 Simplified block diagram of a floppy disk drive

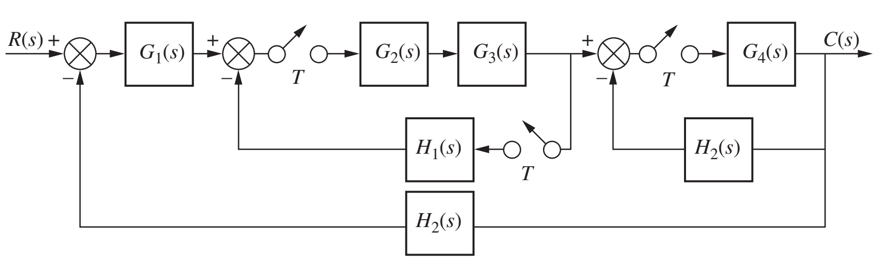

FIGURE I-13.1

7. In Problem 38, Chapter 9 in the text problems, and Problem 9, Chapter 10 in this supplement, we considered the radial pickup position control of a DVD player. A controller was designed and placed in cascade with the plant in a unit feedback configuration to stabilize the system. The controller was given by

$$M(s) = \frac{0.5(s+1.63)}{s(s+0.27)}$$

and the plant by (*Bittanti, 2002*)

$$P(s) = \frac{0.63}{\left(1 + \frac{0.36}{305.4}s + \frac{s^2}{305.4^2}\right)\left(1 + \frac{0.04}{248.2}s + \frac{s^2}{248.2^2}\right)}$$

It is desired to replace the continuous system by an equivalent discrete system without appreciably affecting the system performance.

a. Find an appropriate sampling frequency for the discretization.

b. Using the chosen sampling frequency, translate the continuous compensator into a discrete compensator.

c. Use Simulink to simulate the continuous and discrete systems on the same graph.

Simulink SL

Assume a unit step input. Are there significant differences in the system's performance?

8. In Problem 17, Chapter 11 in the text problems, we discussed an EVAD, a device that works in parallel with the human heart to help pump blood in patients with cardiac conditions. The device has a transfer function

$$G(s) = \frac{P_{ao}(s)}{E_m(s)} = \frac{1361}{s^2 + 69s + 70.85}$$

where $E_m(s)$ is the motor's armature voltage, and $P_{ao}(s)$ is the aortic blood pressure (*Tasch, 1990*). Using continuous techniques, a cascaded compensator is designed in a unity-feedback configuration with a transfer function

$$G_c(s) = \frac{0.5(s+1)}{s+0.05}$$

Selecting to control the device using a microcontroller, a discrete equivalent has to be found for $G_c(s)$. Do the following:

a. Find an appropriate sampling frequency for the discretization.

b. Translate the continuous compensator into a discrete compensator using the sampling frequency found in Part **a**.

c. Use Simulink to simulate the continuous and discrete systems on the same graph for a unit step input. There should be little difference between the compensated continuous and discrete systems.

Simulink SL

9. Discrete-time control systems can exhibit unique characteristics not available in continuous controllers. For example, assuming a specific input and some conditions, it is possible to design a system to achieve steady state within one single time sample without overshoot. This scheme is well known and referred to as *deadbeat control*. We illustrate deadbeat control design with a simple example. For a more comprehensive treatment see (*Ogata, 1987*). Assume in Figure 13.25(a) in the text that $G_p(s) = \frac{1}{s+1}$. The purpose of the design will be to find a compensator, $G_c(z)$, such that for a step input the system achieves steady state within one sample. We start by translating the system into the discrete domain to obtain the equivalent of Figure 13.25(c) in the text. The pulse transfer function, $G_p(z) = \frac{(1-e^{-T})z^{-1}}{1-e^{-T}z^{-1}}$, is found using Eq. (13.40) in the text, since it is assumed that the compensator will be followed by a zero-order hold. In Figure 13.25(c) in the text, the closed-loop transfer function is given by $\frac{C(z)}{R(z)} = T(z) = \frac{G_c(z)G_p(z)}{1+G_c(z)G_p(z)}$, or, solving for the compensator, we get $G_c(z) = \frac{1}{G_p(z)}\frac{T(z)}{1-T(z)}$. The desired system output is a unit step delayed by one unit sample. Thus, $C(z) = \frac{z}{z-1}z^{-1} = \frac{1}{z-1}$. Since the input is a unit step, $R(z) = \frac{z}{z-1}$; the desired closed-loop transfer function is $\frac{C(z)}{R(z)} = T(z) = z^{-1}$, and the resulting compensator, found by direct substitution, is given by $G_c(z) = \frac{1}{1-e^{-T}}\frac{(z-e^{-T})}{z-1}$.

Assume now that the plant is given by $G_p(s) = \frac{1}{s}$, and a sampling period of $T = 0.05$ second is used.

a. Design a deadbeat compensator to reach steady state within one time sample for a step input.

b. Calculate the resulting steady-state error for a unit-slope ramp input.

c. `Simulate your system using SIMULINK. (Hint: Following Figure 13.25 in the text, the forward path will consist of the cascading of` $G_C(z)$`, a zero-order hold, and` $G_p(s)$`.) Show that the system reaches steady state after one sample. Also verify your steady-state error ramp result.`

Simulink SL

10. Obtaining an exact shape in metal forming can be tricky because of material springback. A feedback system has been devised in which critical deviations from specifications are measured as soon as a part is formed and automatic incremental corrections to the forming tools are made before the next part is formed. Eventually, the system compensates for material springback and results in parts compliant with specifications. A unity-feedback digital system with a forward path

$$G(z) = \frac{k}{z-1}$$

can be used as a simplified representation of the system (*Fu, 2013*).

a. Make a sketch of the system's root locus.

b. Find the range of k for which the system is closed-loop stable.

c. Find the system's steady-state error for a step input.

d. Find the value of k that will result in the fastest possible response.

Bibliography

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