

Topic: Solving $Ax=b$ **Question:** Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 7 & -9 \\ -6 & 8 & 0 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

Answer choices:

A $\vec{x} = \begin{bmatrix} \frac{5}{3} \\ 3 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$

B $\vec{x} = \begin{bmatrix} \frac{1}{3} \\ 3 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$

C $\vec{x} = \begin{bmatrix} \frac{5}{3} \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$

D $\vec{x} = \begin{bmatrix} \frac{1}{3} \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$



Solution: D

To find the general solution to $A\vec{x} = \vec{b}$, we need to find the solutions in the null space, and then the particular solution. Let's start with the solutions in the null space, which we'll find by solving $A\vec{x} = \vec{0}$. To get those null space solutions, we'll augment the matrix.

$$\left[\begin{array}{ccc|c} 3 & -6 & 6 & 0 \\ -3 & 7 & -9 & 0 \\ -6 & 8 & 0 & 0 \end{array} \right]$$

We want to put the augmented matrix into reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ -3 & 7 & -9 & 0 \\ -6 & 8 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 1 & -3 & 0 \\ -6 & 8 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -4 & 12 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Let's parse out a system of equations.

$$x_1 - 4x_3 = 0$$

$$x_2 - 3x_3 = 0$$

Solve for the pivot variables in terms of the free variable.

$$x_1 = 4x_3$$

$$x_2 = 3x_3$$



The vector that satisfies the null space is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

We could therefore write the complementary solution as

$$\vec{x}_n = c_1 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

Now we need to find the particular solution that satisfies $A\vec{x}_p = \vec{b}$. So instead of augmenting the matrix with the zero vector, we augment the matrix with $\vec{b} = (b_1, b_2, b_3)$.

$$\left[\begin{array}{ccc|c} 3 & -6 & 6 & b_1 \\ -3 & 7 & -9 & b_2 \\ -6 & 8 & 0 & b_3 \end{array} \right]$$

Now we'll put the matrix into reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & \frac{1}{3}b_1 \\ -3 & 7 & -9 & b_2 \\ -6 & 8 & 0 & b_3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 2 & \frac{1}{3}b_1 \\ 0 & 1 & -3 & 3(\frac{1}{3}b_1) + b_2 \\ -6 & 8 & 0 & b_3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & \frac{1}{3}b_1 \\ 0 & 1 & -3 & b_1 + b_2 \\ 0 & -4 & 12 & 6(\frac{1}{3}b_1) + b_3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 2 & \frac{1}{3}b_1 \\ 0 & 1 & -3 & b_1 + b_2 \\ 0 & 0 & 0 & 4(b_1 + b_2) + 2b_1 + b_3 \end{array} \right]$$



$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 2(b_1 + b_2) + \frac{1}{3}b_1 \\ 0 & 1 & -3 & b_1 + b_2 \\ 0 & 0 & 0 & 6b_1 + 4b_2 + b_3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & \frac{7}{3}b_1 + 2b_2 \\ 0 & 1 & -3 & b_1 + b_2 \\ 0 & 0 & 0 & 6b_1 + 4b_2 + b_3 \end{array} \right]$$

Substitute the values from $\vec{b} = (1, -1, -2)$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & \frac{7}{3}(1) + 2(-1) \\ 0 & 1 & -3 & 1 + (-1) \\ 0 & 0 & 0 & 6(1) + 4(-1) + (-2) \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & \frac{1}{3} \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Rewrite the matrix as a system of equations.

$$x_1 - 4x_3 = \frac{1}{3}$$

$$x_2 - 3x_3 = 0$$

Now, because x_3 is a free variable, set $x_3 = 0$.

$$x_1 - 4(0) = \frac{1}{3}$$

$$x_2 - 3(0) = 0$$

The system becomes

$$x_1 = \frac{1}{3}$$

$$x_2 = 0$$

So the particular solution is



$$\vec{x}_p = \begin{bmatrix} \frac{1}{3} \\ 0 \\ 0 \end{bmatrix}$$

We'll get the general solution by adding the particular and complementary solutions.

$$\vec{x} = \vec{x}_p + \vec{x}_n$$

$$\vec{x} = \begin{bmatrix} \frac{1}{3} \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$



Topic: Solving $Ax=b$ **Question:** Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 1 & -1 & 3 & -1 & 3 \\ -1 & 0 & 0 & 2 & 1 \\ 0 & 1 & -3 & -1 & -4 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Answer choices:

$$\text{A} \quad \vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{B} \quad \vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \\ 4 \end{bmatrix}$$

$$\text{C} \quad \vec{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



$$\text{D} \quad \vec{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \\ 4 \end{bmatrix}$$

Solution: A

To find the general solution to $A\vec{x} = \vec{b}$, we need to find the solutions in the null space, and then the particular solution. Let's start with the solutions in the null space, which we'll find by solving $A\vec{x} = \vec{0}$. To get those null space solutions, we'll augment the matrix.

$$\left[\begin{array}{ccccc|c} 1 & -1 & 3 & -1 & 3 & 0 \\ -1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & -3 & -1 & -4 & 0 \end{array} \right]$$

We want to put the augmented matrix into reduced row-echelon form.

$$\left[\begin{array}{ccccc|c} 1 & -1 & 3 & -1 & 3 & 0 \\ 0 & -1 & 3 & 1 & 4 & 0 \\ 0 & 1 & -3 & -1 & -4 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} 1 & -1 & 3 & -1 & 3 & 0 \\ 0 & 1 & -3 & -1 & -4 & 0 \\ 0 & 1 & -3 & -1 & -4 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 3 & -1 & 3 & 0 \\ 0 & 1 & -3 & -1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & -1 & 0 \\ 0 & 1 & -3 & -1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Let's parse out a system of equations.



$$x_1 - 2x_4 - x_5 = 0$$

$$x_2 - 3x_3 - x_4 - 4x_5 = 0$$

Solve for the pivot variables in terms of the free variables.

$$x_1 = 2x_4 + x_5$$

$$x_2 = 3x_3 + x_4 + 4x_5$$

The vectors that satisfy the null space are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We could therefore write the complementary solution as

$$\vec{x}_n = c_1 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Now we need to find the particular solution that satisfies $A\vec{x}_p = \vec{b}$. So instead of augmenting the matrix with the zero vector, we augment the matrix with $\vec{b} = (b_1, b_2, b_3)$.

$$\left[\begin{array}{ccccc|c} 1 & -1 & 3 & -1 & 3 & b_1 \\ -1 & 0 & 0 & 2 & 1 & b_2 \\ 0 & 1 & -3 & -1 & -4 & b_3 \end{array} \right]$$



Now we'll put the matrix into reduced row-echelon form.

$$\begin{bmatrix} 1 & -1 & 3 & -1 & 3 & | & b_1 \\ 0 & -1 & 3 & 1 & 4 & | & b_2 + b_1 \\ 0 & 1 & -3 & -1 & -4 & | & b_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 & -1 & 3 & | & b_1 \\ 0 & 1 & -3 & -1 & -4 & | & -b_2 - b_1 \\ 0 & 1 & -3 & -1 & -4 & | & b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 3 & -1 & 3 & | & b_1 \\ 0 & 1 & -3 & -1 & -4 & | & -b_2 - b_1 \\ 0 & 0 & 0 & 0 & 0 & | & b_3 + b_2 + b_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -2 & -1 & | & -b_2 \\ 0 & 1 & -3 & -1 & -4 & | & -b_2 - b_1 \\ 0 & 0 & 0 & 0 & 0 & | & b_3 + b_2 + b_1 \end{bmatrix}$$

Substitute the values from $\vec{b} = (1, -2, 1)$.

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -1 & | & -(-2) \\ 0 & 1 & -3 & -1 & -4 & | & -(-2) - 1 \\ 0 & 0 & 0 & 0 & 0 & | & 1 - 2 + 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -2 & -1 & | & 2 \\ 0 & 1 & -3 & -1 & -4 & | & 1 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

Rewrite the matrix as a system of equations.

$$x_1 - 2x_4 - x_5 = 2$$

$$x_2 - 3x_3 - x_4 - 4x_5 = 1$$

Now, because x_3 , x_4 , and x_5 are free variables, set $x_3 = 0$, $x_4 = 0$, and $x_5 = 0$.

$$x_1 - 2(0) - 0 = 2$$

$$x_2 - 3(0) - 0 - 4(0) = 1$$

The system becomes

$$x_1 = 2$$



$$x_2 = 1$$

So the particular solution is

$$\vec{x}_p = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We'll get the general solution by adding the particular and complementary solutions.

$$\vec{x} = \vec{x}_p + \vec{x}_n$$

$$\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



Topic: Solving $Ax=b$ **Question:** Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 2 & 3 & 4 & -4 \\ 2 & 3 & 8 & -10 \\ 6 & 9 & 16 & -18 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Answer choices:

A
$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -\frac{1}{4} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ \frac{3}{2} \end{bmatrix}$$

B
$$\vec{x} = \begin{bmatrix} \frac{3}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ \frac{3}{2} \\ 1 \end{bmatrix}$$

C
$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -\frac{1}{4} \\ 0 \end{bmatrix}$$



D $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{4} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 1 \\ \frac{3}{2} \end{bmatrix}$

Solution: B

To find the general solution to $A\vec{x} = \vec{b}$, we need to find the solutions in the null space, and then the particular solution. Let's start with the solutions in the null space, which we'll find by solving $A\vec{x} = \vec{0}$. To get those null space solutions, we'll augment the matrix.

$$\left[\begin{array}{cccc|c} 2 & 3 & 4 & -4 & 0 \\ 2 & 3 & 8 & -10 & 0 \\ 6 & 9 & 16 & -18 & 0 \end{array} \right]$$

We want to put the augmented matrix into reduced row-echelon form.

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & 0 \\ 2 & 3 & 8 & -10 & 0 \\ 6 & 9 & 16 & -18 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & 0 \\ 0 & 0 & 4 & -6 & 0 \\ 6 & 9 & 16 & -18 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & 0 \\ 0 & 0 & 4 & -6 & 0 \\ 0 & 0 & 4 & -6 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 4 & -6 & 0 \end{array} \right]$$



$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Let's parse out a system of equations.

$$x_1 + \frac{3}{2}x_2 + x_4 = 0$$

$$x_3 - \frac{3}{2}x_4 = 0$$

Solve for the pivot variables in terms of the free variables.

$$x_1 = -\frac{3}{2}x_2 - x_4$$

$$x_3 = \frac{3}{2}x_4$$

The vectors that satisfy the null space are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ \frac{3}{2} \\ 1 \end{bmatrix}$$

We could therefore write the complementary solution as



$$\vec{x}_n = c_1 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ \frac{3}{2} \\ 1 \end{bmatrix}$$

Now we need to find the particular solution that satisfies $A\vec{x}_p = \vec{b}$. So instead of augmenting the matrix with the zero vector, we augment the matrix with $\vec{b} = (b_1, b_2, b_3)$.

$$\left[\begin{array}{cccc|c} 2 & 3 & 4 & -4 & b_1 \\ 2 & 3 & 8 & -10 & b_2 \\ 6 & 9 & 16 & -18 & b_3 \end{array} \right]$$

Now we'll again put the matrix into reduced row-echelon form.

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & \frac{1}{2}b_1 \\ 2 & 3 & 8 & -10 & b_2 \\ 6 & 9 & 16 & -18 & b_3 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & \frac{1}{2}b_1 \\ 0 & 0 & 4 & -6 & -2(\frac{1}{2}b_1) + b_2 \\ 6 & 9 & 16 & -18 & b_3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & \frac{1}{2}b_1 \\ 0 & 0 & 4 & -6 & -b_1 + b_2 \\ 0 & 0 & 4 & -6 & -6(\frac{1}{2}b_1) + b_3 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & \frac{1}{2}b_1 \\ 0 & 0 & 4 & -6 & -b_1 + b_2 \\ 0 & 0 & 0 & 0 & -3b_1 + b_3 - (-b_1 + b_2) \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 2 & -2 & \frac{1}{2}b_1 \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{4}(-b_1 + b_2) \\ 0 & 0 & 0 & 0 & -2b_1 + b_3 - b_2 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 0 & 1 & -2 \cdot \frac{1}{4}(-b_1 + b_2) + \frac{1}{2}b_1 \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{4}(-b_1 + b_2) \\ 0 & 0 & 0 & 0 & -2b_1 + b_3 - b_2 \end{array} \right]$$



$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 0 & 1 & b_1 - \frac{1}{2}b_2 \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{4}(-b_1 + b_2) \\ 0 & 0 & 0 & 0 & -2b_1 + b_3 - b_2 \end{array} \right]$$

Substitute the values from $\vec{b} = (1, -1, 1)$.

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 0 & 1 & 1 - \frac{1}{2}(-1) \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{4}(-1 + (-1)) \\ 0 & 0 & 0 & 0 & -2(1) + 1 - (-1) \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{3}{2} & 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Rewrite the matrix as a system of equations.

$$x_1 + \frac{3}{2}x_2 + x_4 = \frac{3}{2}$$

$$x_3 - \frac{3}{2}x_4 = -\frac{1}{2}$$

Now, because x_2 and x_4 are free variables, set $x_2 = 0$ and $x_4 = 0$.

$$x_1 + \frac{3}{2}(0) + 0 = \frac{3}{2}$$

$$x_3 - \frac{3}{2}(0) = -\frac{1}{2}$$

The system becomes

$$x_1 = \frac{3}{2}$$



$$x_3 = -\frac{1}{2}$$

So the particular solution is

$$\vec{x}_p = \begin{bmatrix} \frac{3}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$

We'll get the general solution by adding the particular and complementary solutions.

$$\vec{x} = \vec{x}_p + \vec{x}_n$$

$$\vec{x} = \begin{bmatrix} \frac{3}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ \frac{3}{2} \\ 1 \end{bmatrix}$$

