Instructor's Supplement Problems

Chapter 12

1. Given the following open-loop plant, [Section: 12.2]

$$G(s) = \frac{50}{(s+1)(s+3)(s+10)}$$

design a controller to yield a 10% overshoot and a settling time of 0.5 second. Place the third pole 10 times as far from the imaginary axis as the dominant pole pair. Use the phase variables for state-variable feedback.

2. Section 12.2 showed that controller design is easier to implement if the uncompensated system is represented in phase-variable form with its typical lower companion matrix. We alluded to the fact that the design can just as easily progress using the controller canonical form with its upper companion matrix. [Section: 12.2]

a. Redo the general controller design covered in Section 12.2, assuming that the plant is represented in controller canonical form rather than phase-variable form.

b. Apply your derivation to Example 12.1 if the uncompensated plant is represented in controller canonical form.

3. Repeat Problem I-1 assuming that the plant is represented in the cascade form. Do not convert to phase-variable form. [Section: 12.4]

4. In Section 12.4, we discussed how to design a controller for systems not represented in phase-variable form with its typical lower companion matrix. We described how to convert the system to phase-variable form, design the controller, and convert back to the original representation. This technique can be applied just as easily if the original representation is converted to controller canonical form with its typical upper companion matrix. Redo Example 12.4 in the text by designing the controller after converting the uncompensated plant to controller canonical form. [Section: 12.4]

5. Repeat Problem 10 in the text problems assuming that the plant is represented in parallel form. [Section: 12.4]

6. The open-loop system of Problem 10 in the text problems is represented as shown in Figure I-12.1. If the output of each block is assigned to be a state variable, design the controller gains for feedback from these state variables. [Section: 12.4]

7. For a specific individual, the linear time-invariant model of the hypothalamic-pituitary-adrenal axis of the endocrine system with five state variables has been found to be (*Kyrylov, 2005*)

MATLAB ML

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -0.014 & 0 & -1.4 & 0 & 0 \\ 0.023 & -0.023 & -0.023 & 0 & 0 \\ 0.134 & 0.67 & -0.67 & 0.38 & 0.003264 \\ 0 & 0 & 0.06 & -0.06 & 0 \\ 0 & 0 & 0.0017 & 0 & -0.001 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} d_0$$

The state-variable definitions were given in Problem I-4, Chapter 3.

a. Use MATLAB to determine if the system is controllable.

b. Use MATLAB to express the matrices A and B in phase-variable form.

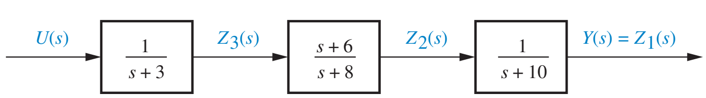

FIGURE I-12.1

8. Repeat Problem 13 in the text problems assuming that the plant is represented in phase-variable form. Do not convert to observer canonical form. [Section: 12.7]

9. a. Given the plant of Figure I-12.2, what relationship must exist between c_1 and c_2 in order for the system to be unobservable?

b. What values of c_1 will make the system uncontrollable if $c_2 = 1$? [Section: 12.6]

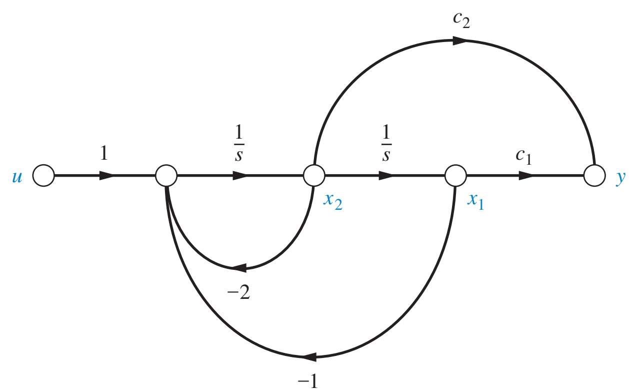

FIGURE I-12.2

10. Repeat Problem 18 in the text problems assuming that the plant is represented in parallel form. [Section: 12.7]

11. Repeat Problem 22 in the text problems for the following plant: [Section: 12.8]

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 \\ 0 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}; \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix}\mathbf{x}$$

12. Problem I-3 in Chapter 3 introduced the model for patients treated under a regimen of a single day of Glargine insulin (*Tarín, 2005*). The model to find the response for a specific patient to medication can be expressed in phase-variable form with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -501.6 \times 10^{-6} & -128.8 \times 10^{-3} & -854 \times 10^{-3} \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 0.78 \times 10^{-4} & 41.4 \times 10^{-4} & 0.01 \end{bmatrix};$$

$$\mathbf{D} = 0$$

The state variables will take on a different significance in this expression, but the input and the output remain the same. Recall that u = external insulin flow, and y = plasma insulin concentration.

a. Obtain a state-feedback gain matrix so that the closed-loop system will have two of its poles placed at $-1/15$ and the third pole at $-1/2$.

b. Use MATLAB to verify that the poles appear at the positions specified in Part **a**. MATLAB **ML**

13. In the dc–dc converter of Problem I-18, Chapter 4 (*Van Dijk, 1995*), with $L = 6$ mH, $C = 1$ mF, $R = 100\,\Omega$, a 50% PWM duty cycle, and assuming the system's output is the voltage across the capacitor, the model can be expressed as

$$\begin{bmatrix} \dot{i}_L \\ \dot{u}_C \end{bmatrix} = \begin{bmatrix} 0 & -83.33 \\ 500 & -10 \end{bmatrix}\begin{bmatrix} i_L \\ u_C \end{bmatrix} + \begin{bmatrix} 166.67 \\ 0 \end{bmatrix}E_s$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} i_L \\ u_C \end{bmatrix}$$

a. Find the system's transfer function.

b. Express the system's state equations in phase-variable form.

c. Find a set of state-feedback gains to obtain 20% overshoot and a settling time of 0.5 second in the phase-variable system.

d. Obtain the corresponding set of state-feedback gains in the original system.

e. Verify that the set of gains in Part d places the closed-loop poles at the desired positions.

f. Simulate the unit-step response of the system using MATLAB. MATLAB **ML**

14. a. Design an observer for the dc–dc converter of Problem I-13. The observer should have time constants 10 times smaller than those of the original system.

b. Simulate your system and observer for a unit-step input using Simulink. Assume that the initial conditions for the original system are $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. The observer should have initial conditions $\hat{\mathbf{x}}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Simulink **SL**

Bibliography

Kyrylov, V., Severyanova, L. A., and Vieira, A. Modeling Robust Oscillatory Behavior of the Hypothalamic-Pituitary-Adrenal Axis. *IEEE Transactions on Biomedical Engineering*, vol. 52, no. 12, 2005, pp. 1977–1983.

Tarín, C., Teufel, E., Picó, J., Bondia, J., and Pfleiderer, H. J. Comprehensive Pharmacokinetic Model of Insulin Glargine and Other Insulin Formulations. *IEEE Transactions on Biomedical Engineering*, vol. 52, no. 12, 2005, pp. 1994–2005.

Van Dijk, E., Spruijt, J. N., O'Sullivan, D. M., and Klaasens, J. B. PWM-Switch Modeling of DC-DC Converters. *IEEE Transactions on Power Electronics*, vol. 10, 1995, pp. 659–665.