

# Instructor's Supplement Problems

## Chapter 4

1. Plot the step responses for Problem 2 in the text problems using MATLAB. MATLAB  
ML
2. Plot the step response for Problem 5 in the text problems using MATLAB. From your plots, find the time constant, rise time, and settling time. Use  $M = 1$  and  $M = 2$ . MATLAB  
ML
3. Write the general form of the capacitor voltage for the electrical network shown in Figure I-4.1. [Section: 4.4]

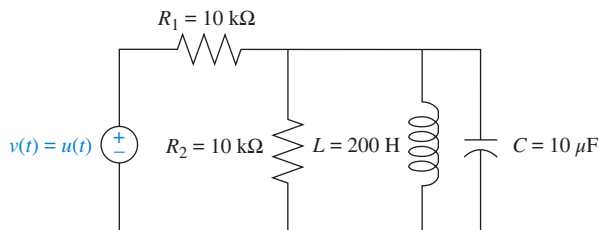


FIGURE I-4.1

4. Use MATLAB to plot the capacitor voltage in Problem I-3. MATLAB  
ML [Section: 4.4]
5. The system shown in Figure I-4.2 has a unit step input. Find the output response as a function of time. Assume the system is underdamped. Notice that the result will be Eq. (13.28). [Section: 4.6]

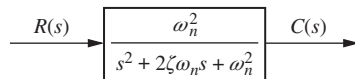


FIGURE I-4.2

6. Derive the relationship for damping ratio as a function of percent overshoot, Eq. (4.39). [Section: 4.6]
7. Calculate the exact response of each system of Problem 6 in the text problems using Laplace transform techniques, and compare the results to those obtained in that problem. [Sections: 4.3, 4.4]

8. Find the transfer function of a second-order system that yields a 15% overshoot and a settling time of 0.7 second. [Section: 4.6]
9. An autonomous robot to pick asparagus (*Dong, 2011*) capable of following planting rows has an orientation system with transfer function

$$\frac{\theta}{\theta_{\text{ref}}} = \frac{53.176}{4.6s^2 + 31.281s + 53.176}$$

Make a sketch of  $\theta(t)$  in response to  $\theta_{\text{ref}}(t) = 3u(t)$ . Indicate in your plot  $C_{\text{final}}$ ,  $C_{\text{max}}$ ,  $T_p$ , and  $T_s$ . (Hint: You may use the result of Problem 19c in the text problems).

10. Figure I-4.3 shows five step responses of an automatic voltage regulation system as one of the system parameters varies (*Gozde, 2011*). Assume for all five responses that they are those of a second-order system with an overshoot of 20%. Make a sketch of the positions of the poles in the complex plane for each one of the responses. Label the curves A through E from left to right.

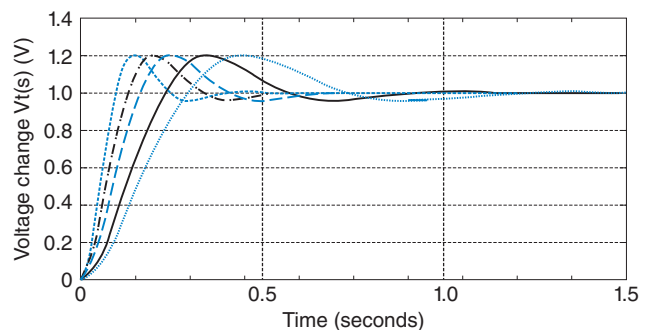


FIGURE I-4.3 Time responses for an automatic voltage regulation system

11. Derive the unit step response for each transfer function in Example 4.8. [Section: 4.7]

## 2 Instructor's Supplement Problems

12. For each of the following transfer functions with zeros, find the component parts of the unit step response: (1) the derivative of the response without a zero and (2) the response without a zero, scaled to the negative of the zero value. Also, find and plot the total response. Describe any nonminimum-phase behavior. [Section: 4.8]

a.  $G(s) = \frac{s+2}{s^2+3s+36}$

b.  $G(s) = \frac{s-2}{s^2+3s+36}$

13. Use MATLAB's Simulink to obtain the step response of a system,

Simulink  
**SL**

$$G(s) = \frac{1}{s^2 + 3s + 10}$$

under the following conditions: [Section: 4.9]

- a. The system is linear and driven by an amplifier whose gain is 10.
- b. An amplifier whose gain is 10 drives the system. The amplifier saturates at  $\pm 0.25$  volts. Describe the effect of the saturation on the system's output.
- c. An amplifier whose gain is 10 drives the system. The amplifier saturates at  $\pm 0.25$  volts. The system drives a 1:1 gear train that has backlash. The dead-band width of the backlash is 0.02 rad. Describe the effect of saturation and backlash on the system's output.

14. Using classical (not Laplace) methods only, solve for the state-transition matrix, the state vector, and the output of the system represented here, where  $u(t)$  is the unit step: [Section: 4.11]

State Space  
**SS**

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = [3 \quad 4] \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

15. Use MATLAB to simulate the following system and plot the output,  $y(t)$ , for a step input. Mark on the plot the steady-state value, percent overshoot, and the rise time, peak time, and settling time.

State Space  
**SS**

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -12 & -8 & 1 \\ 0 & 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 1 \quad 0] \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

16. A MOEMS (optical MEMS) is a MEMS (Micro Electromechanical Systems) with an optical fiber channel that takes light generated from a laser diode. It also has a photodetector that measures light intensity variations and outputs voltage variations proportional to small mechanical device deflections. Additionally, a voltage input is capable of deflecting the device. The apparatus can be used as an optical switch or as a variable optical attenuator, and it does not exceed 2000  $\mu\text{m}$  in any dimension. Figure I-4.4 shows input-output signal pairs used to identify the parameters of the system. Assume a second-order transfer function and find the system's transfer function (Borovic, 2005).

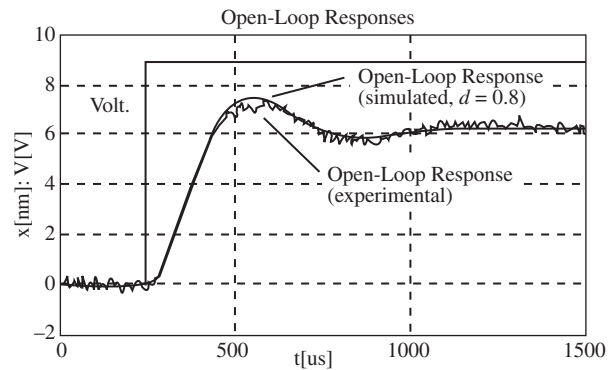


FIGURE I-4.4<sup>1</sup>

17. Using wind tunnel tests, insect flight dynamics can be studied in a very similar fashion to that of man-made aircraft. Linearized longitudinal flight equations for a bumblebee have been found in the unforced case to be

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -8.792 \times 10^{-3} & 0.56 \times 10^{-3} & -1.0 \times 10^{-3} & -13.79 \times 10^{-3} \\ -0.347 \times 10^{-3} & -11.7 \times 10^{-3} & -0.347 \times 10^{-3} & 0 \\ 0.261 & -20.8 \times 10^{-3} & -96.6 \times 10^{-3} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

where  $u$  = forward velocity;  $w$  = vertical velocity,  $q$  = angular pitch rate at center of mass, and  $\theta$  = pitch angle between the flight direction and the horizontal (Sun, 2005).

- a. Use MATLAB to obtain the system's eigenvalues.

MATLAB  
**ML**

<sup>1</sup> Borovic B., Liu A.Q., Popa D., Lewis F.L. Open-loop versus closed-loop control of MEMS devices: choices and issues *J. Micromech. Microeng.* Vol. 15, 2005. Figure 4, p. 1919.

- b. Write the general form of the state-transition matrix. How many constants would have to be found?

State Space

SS

18. A dc–dc converter is a device that takes as an input an unregulated dc voltage and provides a regulated dc voltage as its output. The output voltage may be lower (buck converter), higher (boost converter), or the same as the input voltage. Switching dc–dc converters have a semiconductor active switch (BJT or FET) that is closed periodically with a duty cycle  $d$  in a pulse width modulated (PWM) manner. For a boost converter, averaging techniques can be used to arrive at the following state equations (Van Dijk, 1995):

State Space

SS

$$L \frac{di_L}{dt} = -(1-d)u_c + E_s$$

$$C \frac{du_c}{dt} = (1-d)i_L - \frac{u_c}{R}$$

where  $L$  and  $C$  are, respectively, the values of internal inductance and capacitance;  $i_L$  is the current through the internal inductor;  $R$  is the resistive load connected to the converter;  $E_s$  is the dc input voltage; and the capacitor voltage,  $u_c$ , is the converter's output.

- a. Write the converter's equations in the form

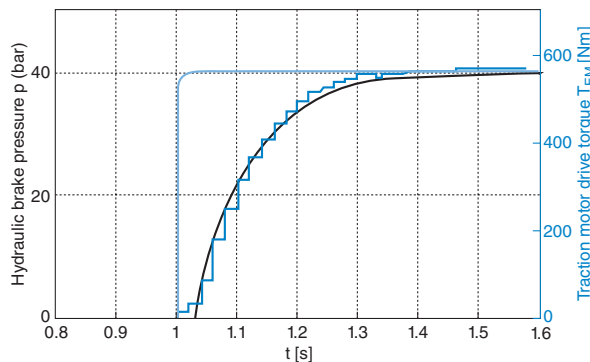
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

assuming  $d$  is a constant.

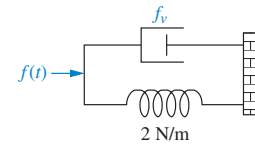
- b. Using the  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  matrices of Part a, obtain the converter's transfer function  $\frac{U_C(s)}{E_s(s)}$ .

19. Figure I-4.5 shows the step response of an electric vehicle's mechanical brakes when the input is the drive torque (N-m) and the output is the hydraulic brake pressure (bar) (Ringdorfer, 2011).



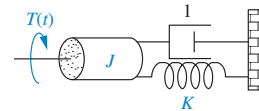
**FIGURE I-4.5** Step response of an electric vehicle's mechanical brake<sup>2</sup>

- a. Find the transfer function of the system.
- b. Use the values of the parameters for the transfer function obtained in Part a to find an expression for the brake pressure as a function of time.
- c. Find the output in bars of the system 0.2 sec after the input is applied. Check your result against Figure I-4.5.
20. Find an equation that relates 2% settling time to the value of  $f_v$  for the translational mechanical system shown in Figure I-4.6. Neglect the mass of all components. [Section: 4.6]



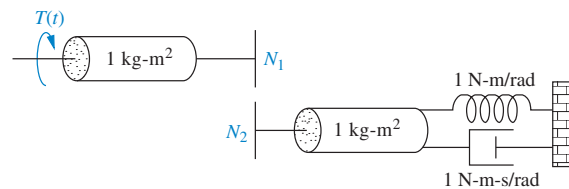
**FIGURE I-4.6**

21. Find  $J$  and  $K$  in the rotational system shown in Figure I-4.7 to yield a 30% overshoot and a settling time of 3 seconds for a step input in torque. [Section: 4.6]



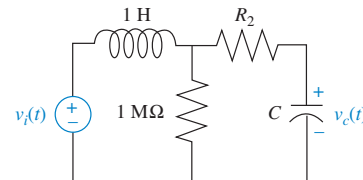
**FIGURE I-4.7**

22. For the system shown in Figure I-4.8, find  $N_1/N_2$  so that the settling time for a step torque input is 16 seconds. [Section: 4.6]



**FIGURE I-4.8**

23. For the circuit shown in Figure I-4.9, find the values of  $R_2$  and  $C$  to yield 8% overshoot with a settling time of 1 ms for the voltage across the capacitor, with  $v_i(t)$  as a step input. [Section: 4.6]



**FIGURE I-4.9**

<sup>2</sup> Ringdorfer M., and Horn M. Development of a Wheel Slip Actuator Controller for Electric Vehicles using Energy Recuperation and Hydraulic Brake Control, 2011 IEEE International Conference on Control Applications (CCA), Denver, CO, USA. September 28–30, 2011, pp. 313–318. Figure 4, p. 315. Modelling Symposium (AMS), 2012 Sixth Asia by IEEE. Reproduced with permission of IEEE in the format Republish in a book via Copyright Clearance Center.

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