Instructor's Supplement Problems

Chapter 08

1. Sketch the root locus for the unity-feedback system shown in Figure P8.3 in the text problems for the following transfer functions: [Section: 8.4]

a. $G(s) = \dfrac{K(s+2)(s+6)}{s^2+8s+25}$

b. $G(s) = \dfrac{K(s^2+4)}{(s^2+1)}$

c. $G(s) = \dfrac{K(s^2+1)}{s^2}$

d. $G(s) = \dfrac{K}{(s+1)^3(s+4)}$

2. Let

$$G(s) = \frac{-K(s+1)^2}{s^2+2s+2}$$

with $K > 0$ in Figure P8.3 in the text problems. [Sections: 8.5, 8.9]

a. Find the range of K for closed-loop stability.

b. Sketch the system's root locus.

c. Find the position of the closed-loop poles when $K = 1$ and $K = 2$.

3. Figure I-8.1 shows open-loop poles and zeros. There are two possibilities for the sketch of the root locus. Sketch each of the two possibilities. Be aware that only one can be the *real* locus for specific open-loop pole and zero values. [Section: 8.4]

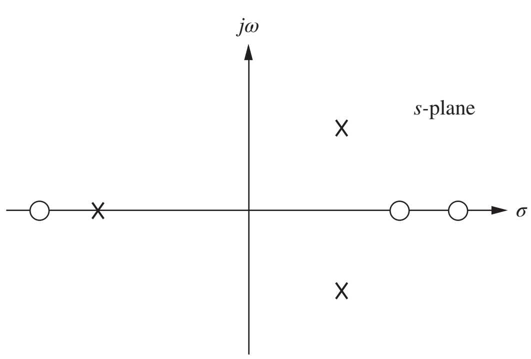

FIGURE I-8.1

4. Sketch the root locus for the unity-feedback system shown in Figure P8.3 in the text problems, where

$$G(s) = \frac{K(s^2+2)}{(s+3)(s+4)}$$

Give the values for all critical points of interest. Is the system ever unstable? If so, for what range of K? [Section: 8.5]

5. Given the unity-feedback system of Figure P8.3 in the text problems, make an accurate plot of the root locus for the following:

a. $G(s) = \dfrac{K(s^2-2s+2)}{(s+1)(s+2)}$

b. $G(s) = \dfrac{K(s-1)(s-2)}{(s+1)(s+2)}$

Also, do the following:
Calibrate the gain for at least four points for each case. Also find the breakaway points, the $j\omega$-axis crossing, and the range of gain for stability for each case. Find the angles of arrival for Part **a**. [Section: 8.5]

6. For the unity-feedback system shown in Figure P8.3 in the text problems, where

$$G(s) = \frac{K(s+10)(s+20)}{(s+30)(s^2-10s+100)}$$

do the following: [Section: 8.7]

a. Sketch the root locus.

b. Find the range of gain, K, that makes the system stable.

c. Find the value of K that yields a damping ratio of 0.707 for the system's closed-loop dominant poles.

d. Find the value of K that yields closed-loop critically damped dominant poles.

7. Root loci are usually plotted for variations in the gain. Sometimes we are interested in the variation of the closed-loop poles as other parameters are changed. For the system shown in Figure I-8.2, sketch the root locus as α is varied. [Section: 8.8]

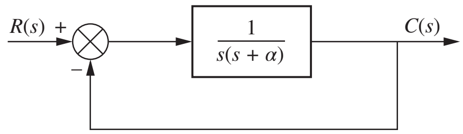

FIGURE I-8.2

8. For the unity-feedback system shown in Figure P8.3 in the text problems, where

$$G(s) = \frac{K(s+2)}{s(s+6)(s+10)}$$

find K to yield closed-loop complex poles with a damping ratio of 0.55. Does your solution require a justification of a second-order approximation? Explain. [Section: 8.7]

9. `Repeat Problem 24 in the text problems using MATLAB. Use one program to do the following:` MATLAB **ML**

a. `Display a root locus and pause.`

b. `Draw a close-up of the root locus where the axes go from −2 to 0 on the real axis and −2 to 2 on the imaginary axis.`

c. `Overlay the 10% overshoot line on the close-up root locus.`

d. `Select interactively the point where the root locus crosses the 10% overshoot line, and respond with the gain at that point as well as all of the closed-loop poles at that gain.`

e. `Generate the step response at the gain for 10% overshoot.`

10. For the unity-feedback system shown in Figure P8.3, in the text problems where

$$G(s) = \frac{K}{(s+3)(s^2+4s+5)}$$

do the following: [Section: 8.7]

a. Find the location of the closed-loop dominant poles if the system is operating with 15% overshoot.

b. Find the gain for Part **a.**

c. Find all other closed-loop poles.

d. Evaluate the accuracy of your second-order approximation.

11. Given the unity-feedback system shown in Figure P8.3 in the text problems, where

$$G(s) = \frac{K}{s(s+1.25)(s+8)}$$

evaluate the pole sensitivity of the closed-loop system if the second-order, underdamped closed-loop poles are set for [Section: 8.10]

a. $\zeta = 0.591$

b. $\zeta = 0.456$

c. Which of the two previous cases has more desirable sensitivity?

12. Repeat Problem I-1 but sketch your root loci for negative values of K. [Section: 8.9]

13. It is known that mammals have hormonal regulation mechanisms that help maintain almost constant calcium plasma levels (0.08–0.1 g/L in dairy cows). This control is necessary to maintain healthy functions, as calcium is responsible for diverse physiological functions, such as bone formation, intracellular communications, and blood clotting. It has been postulated that the mechanism of calcium control resembles that of a PI (proportional-plus-integral) controller. PI controllers (discussed in detail in Chapter 9) are placed in cascade with the plant and used to improve steady-state error. Assume that the PI controller has the form $G_c(s) = \left[K_P + \frac{K_I}{S}\right]$ where K_P and K_I are constants. Also assume that the mammal's system accumulates calcium in an integrator-like fashion, namely $P(s) = \frac{1}{V_s}$, where V is the plasma volume. The closed-loop model is similar to that of Figure P8.3 in the text problems, where $G(s) = G_c(s)P(s)$ (*Khammash, 2004*).

a. Sketch the system's root locus as a function of K_P, assuming $K_I > 0$ is constant.

b. Sketch the system's root locus as a function of K_I, assuming $K_P > 0$ is constant.

14. Problem I-17 in Chapter 7 introduced the model of a TCP/IP router whose packet-drop probability is controlled by using a random early detection (RED) algorithm (*Hollot, 2001*). Using Figure P8.3 in the text problems as a model, a specific router queue's open-loop transfer function is

$$G(s) = \frac{7031250Le^{-0.2s}}{(s+0.667)(s+5)(s+50)}$$

The function $e^{-0.2s}$ represents delay. To apply the root locus method, the delay function must be replaced with a rational function approximation. A first-order Padé approximation can be used for this purpose. Let $e^{-sD} \approx 1 - Ds$. Using this approximation, plot the root locus of the system as a function of L.

15. A robotic manipulator together with a cascade PI controller (used to improve steady-state response and discussed in Chapter 9) has a transfer function (*Low, 2005*)

$$G(s) = \left(K_p + \frac{K_1}{s}\right)\frac{48500}{s^2 + 2.89s}$$

Assume the robot's joint will be controlled in the configuration shown in Figure P8.3 in the text problems.

a. Find the value of K_I that will result in $e_{ss} = 2\%$ for a parabolic input.

b. Using the value of K_I found in Part **a**, plot the root locus of the system as a function of K_P.

c. Find the value of K_P that will result in a real pole at -1. Find the location of the other two poles.

16. An active system for the elimination of floor vibrations due to human presence is presented in (*Nyawako, 2009*). The system consists of a sensor that measures the floor's vertical acceleration and an actuator that changes the floor characteristics. The open-loop transmission of the particular setup used can be described by $G(s) = KG_a(s)F(s)G_m(s)$, where the actuator's transfer function is

GUI Tool
GUIT

$$G_a(s) = \frac{10.26}{s^2 + 11.31s + 127.9}$$

The floor's dynamic characteristics can be modeled by

$$F(s) = \frac{6.667 \times 10^{-5}s^2}{s^2 + 0.2287s + 817.3}$$

The sensor's transfer function is

$$G_m(s) = \frac{s}{s^2 + 5.181s + 22.18}$$

and K is the gain of the controller. The system operations can be described by the unity-gain feedback loop of Figure P8.3 in the text problems.

a. Use MATLAB's Control System Designer to obtain the root locus of the system in terms of K.

b. Find the range of K for closed-loop stability.

c. Find, if possible, a value of K that will yield a closed-loop overdamped response.

17. It is important to precisely control the amount of organic fertilizer applied to a specific crop area in order to provide specific nutrient quantities and to avoid unnecessary environmental pollution. A precise delivery liquid manure machine has been developed for this purpose (*Saeys, 2008*). The system consists of a pressurized tank, a valve, and a rheological flow sensor. After simplification, the system can be modeled as a closed-loop negative-feedback system with a forward-path transfer function

MATLAB
ML
GUI Tool
GUIT

$$G(s) = \frac{2057.38K(s^2 - 120s + 4800)}{s(s + 13.17)(s^2 + 120s + 4800)}$$

consisting of an electrohydraulic system in cascade with the gain of the manure flow valve and a variable gain, K. The feedback path is comprised of

$$H(s) = \frac{10(s^2 - 4s + 5.333)}{(s + 10)(s^2 + 4s + 5.333)}$$

a. Use the Control System Designer in MATLAB to obtain the root locus of the system.

b. Use the Control System Designer to find the range of K for closed-loop stability.

c. Find the value of K that will result in the smallest settling time for this system.

d. Calculate the expected settling time for a step input with the value of K obtained in Part **c**.

e. Check your result through a step-response simulation.

Bibliography

Hollot, C. V., Misra, V., Towsley, D., and Gong, W. A Control Theoretic Analysis of RED. *Proceedings of IEEE INFOCOM*, 2001, pp. 1510–1519.

Khammash, M., and El-Samad, H. Systems Biology: From Physiology to Gene Regulation. *IEEE Control Systems*, August 2004, pp. 62–76.

Low, K. H., Wang, H., Liew, K. M., and Cai, Y. Modeling and Motion Control of Robotic Hand for Telemanipulation Application. *International Journal of Software Engineering and Knowledge Engineering*, vol. 15, 2005, pp. 147–152.

Nyawako, D., and Reynolds, P. Response-Dependent Velocity Feedback Control for Mitigation of Human-Induced Floor Vibrations. *Smart Materials and Structures*, vol. 18, IOP Publishing Ltd., UK, 2009, pp. 1–13.

Saeys, W., Deblander, J., Ramon, H., and Anthonis, J. High-Performance Flow Control for Site-Specific Application of Liquid Manure. *Biosystems Engineering*, vol. 99, 2008, pp. 22–34.