Section 2.3:

2. Let A and B be events with $P(A) = 0.5$ and $P(A \cap B^c) = 0.4$. For what value of $P(B)$ will A and B be independent?

6. The article "Integrating Risk Assessment and Life Cycle Assessment: A Case Study of Insulation" (Y. Nishioka, J. Levy, et al., *Risk Analysis*, 2002: 1003–1017) estimates that 5.6% of a certain population has asthma, and that an asthmatic has probability 0.027 of suffering an asthma attack on a given day. A person is chosen at random from this population. What is the probability that this person has an asthma attack on that day?

10. At a certain college, 30% of the students major in engineering, 20% play club sports, and 10% both major in engineering and play club sports. A student is selected at random.

a. What is the probability that the student is majoring in engineering?

b. What is the probability that the student plays club sports?

c. Given that the student is majoring in engineering, what is the probability that the student plays club sports?

d. Given that the student plays club sports, what is the probability that the student is majoring in engineering?

e. Given that the student is majoring in engineering, what is the probability that the student does not play club sports?

f. Given that the student plays club sports, what is the probability that the student is not majoring in engineering?

14. Lorez and Felipe each fire one shot at a target. Lorez has probability 0.5 of hitting the target, and Felipe has probability 0.3. The shots are independent.

a. Find the probability that the target is hit.

b. Find the probability that the target is hit by exactly one shot.

c. Given that the target was hit by exactly one shot, find the probability that Lorez hit the target.

18. A car dealer sold 750 automobiles last year. The following table categorizes the cars sold by size and color and presents the number of cars in each category. A car is to be chosen at random from the 750 for which the owner will win a lifetime of free oil changes.

	Color			
Size	**White**	**Black**	**Red**	**Grey**
Small	102	71	33	134
Midsize	86	63	36	105
Large	26	32	22	40

a. If the car is small, what is the probability that it is black?

b. If the car is white, what is the probability that it is midsize?

c. If the car is large, what is the probability that it is red?

d. If the car is red, what is the probability that it is large?

e. If the car is not small, what is the probability that it is not grey?

20. An automobile insurance company divides customers into three categories, good risks, medium risks, and poor risks. Assume that 70% of the customers are good risks, 20% are medium risks, and 10% are poor risks. Assume that during the course of a year, a good risk customer has probability 0.005 of filing an accident claim, a medium risk customer has probability 0.01, and a poor risk customer has probability 0.025. A customer is chosen at random.

a. What is the probability that the customer is a good risk and has filed a claim?

b. What is the probability that the customer has filed a claim?

c. Given that the customer has filed a claim, what is the probability that the customer is a good risk?

24. A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. Find $P(A)$.

b. Find $P(B|A)$.

c. Find $P(A \cap B)$.

d. Find $P(A^c \cap B)$.

e. Find $P(B)$.

f. Find $P(A|B)$.

g. Are A and B independent? Is it reasonable to treat A and B as though they were independent? Explain.

26. A certain delivery service offers both express and standard delivery. Seventy-five percent of parcels are sent by standard delivery, and 25% are sent by express. Of those sent standard, 80% arrive the next day, and of those sent express, 95% arrive the next day. A record of a parcel delivery is chosen at random from the company's files.

a. What is the probability that the parcel was shipped express and arrived the next day?

b. What is the probability that it arrived the next day?

c. Given that the package arrived the next day, what is the probability that it was sent express?

32. A quality-control program at a plastic bottle production line involves inspecting finished bottles for flaws such as microscopic holes. The proportion of bottles that actually have such a flaw is only 0.0002. If a bottle has a flaw, the probability is 0.995 that it will fail the inspection. If a bottle does not have a flaw, the probability is 0.99 that it will pass the inspection.

a. If a bottle fails inspection, what is the probability that it has a flaw?

b. Which of the following is the more correct interpretation of the answer to part (a)?

i. Most bottles that fail inspection do not have a flaw.

ii. Most bottles that pass inspection do have a flaw.

c. If a bottle passes inspection, what is the probability that it does not have a flaw?

d. Which of the following is the more correct interpretation of the answer to part (c)?

i. Most bottles that fail inspection do have a flaw.

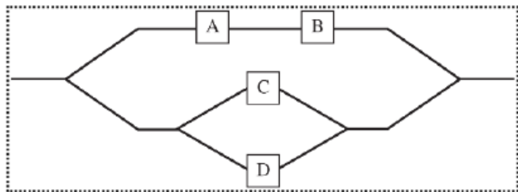
ii. Most bottles that pass inspection do not have a flaw.

e. Explain why a small probability in part (a) is not a problem, so long as the probability in part (c) is large.

34 Is a challenging one! The system functions if a signal can get from the left to the right along any path. Notice that in order for a signal to get through the top path, both A and B would have to work. In order for the signal to move along the bottom path either C or D has to work. For

example, if A, B and D fail, the system is fine because the signal can go along the path with C. However, if A, C and D fail, the whole system fails since all possible paths are blocked. Do do this problem, you want to carefully think about how you calculate probabilities of “A and B” happening vs “A or B” happening.

34. A system consists of four components connected as shown in the following diagram:



Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.10, 0.05, 0.10, and 0.20, respectively, what is the probability that the system functions?

Section 2.4:

2. Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

a. Find $P(X \leq 2)$.

b. Find $P(X > 1)$.

c. Find μ_X.

d. Find σ_X^2.

4. Let X represent the number of tires with low air pressure on a randomly chosen car.

a. Which of the three functions below is a possible probability mass function of X? Explain.

	x				
	0	**1**	**2**	**3**	**4**
$p_1(x)$	0.2	0.2	0.3	0.1	0.1
$p_2(x)$	0.1	0.3	0.3	0.2	0.2
$p_3(x)$	0.1	0.2	0.4	0.2	0.1

b. For the possible probability mass function, compute μ_X and σ_X^2.

In 10d: In theory, they may have to interview arbitrarily many candidates. Your probability mass function should be a function that you can use to compute the probability that they interview any number of candidates. (i.e. the probability that they have to interview 500 candidates before finding someone is pretty small, but you should be able to find p(x) to determine how small.) Use your calculations from parts b and c to see if you can find a pattern.

10. Candidates for a job are interviewed one by one until a qualified candidate is found. Thirty percent of the candidates are qualified.

a. What is the probability that the first candidate is qualified?

b. What is the probability that the first candidate is unqualified and the second candidate is qualified?

c. Let X represent the number of candidates interviewed up to and including the first qualified candidate. Find $P(X = 4)$.

d. Find the probability mass function of X.

4.1:

2. A certain brand of dinnerware set comes in three colors: red, white, and blue. Twenty percent of customers order the red set, 45% order the white, and 35% order the blue. Let $X = 1$ if a randomly chosen order is for a red set, let $X = 0$ otherwise; let $Y = 1$ if the order is for a white set, let $Y = 0$ otherwise; let $Z = 1$ if it is for either a red or white set, and let $Z = 0$ otherwise.

a. Let p_X denote the success probability for X. Find p_X.

b. Let p_Y denote the success probability for Y. Find p_Y.

c. Let p_Z denote the success probability for Z. Find p_Z.

d. Is it possible for both X and Y to equal 1?

e. Does $p_Z = p_X + p_Y$?

f. Does $Z = X + Y$? Explain.

6. Two dice are rolled. Let $X = 1$ if the dice come up doubles and let $X = 0$ otherwise. Let $Y = 1$ if the sum is 6, and let $Y = 0$ otherwise. Let $Z = 1$ if the dice come up both doubles and with a sum of 6 (that is, double 3), and let $Z = 0$ otherwise.

a. Let p_X denote the success probability for X. Find p_X.

b. Let p_Y denote the success probability for Y. Find p_Y.

c. Let p_Z denote the success probability for Z. Find p_Z.

d. Are X and Y independent?

e. Does $p_Z = p_X p_Y$?

f. Does $Z = XY$? Explain.

4.2:

2. Let $X \sim \text{Bin}(9, 0.4)$. Find

a. $P(X > 6)$

b. $P(X \geq 2)$

c. $P(2 \leq X < 5)$

d. $P(2 < X \leq 5)$

e. $P(X = 0)$

f. $P(X = 7)$

g. μ_X

h. σ_X^2

4. At a certain airport, 75% of the flights arrive on time. A sample of 10 flights is studied.

a. Find the probability that all 10 of the flights were on time.

b. Find the probability that exactly eight of the flights were on time.

c. Find the probability that eight or more of the flights were on time.

14. Gears produced by a grinding process are categorized as either conforming (suitable for their intended purpose), downgraded (unsuitable for the intended purpose but usable for another purpose), or scrap (not usable). Suppose that 80% of the gears produced are conforming, 15% are downgraded, and 5% are scrap. Ten gears are selected at random.

a. What is the probability that one or more is scrap?

b. What is the probability that eight or more are not scrap?

c. What is the probability that more than two are either downgraded or scrap?

d. What is the probability that exactly nine are either conforming or downgraded?

20. An insurance company offers a discount to homeowners who install smoke detectors in their homes. A company representative claims that 80% or more of policyholders have smoke detectors. You draw a random sample of eight policyholders. Let X be the number of policyholders in the sample who have smoke detectors.

a. If exactly 80% of the policyholders have smoke detectors (so the representative's claim is true, but just barely), what is $P(X \leq 1)$?

b. Based on the answer to part (a), if 80% of the policyholders have smoke detectors, would one policyholder with a smoke detector in a sample of size 8 be an unusually small number?

c. If you found that one of the eight sample policyholders had a smoke detector, would this be convincing evidence that the claim is false? Explain.

d. If exactly 80% of the policyholders have smoke detectors, what is $P(X \leq 6)$?

e. Based on the answer to part (d), if 80% of the policyholders have smoke detectors, would six policyholders with smoke detectors in a sample of size 8 be an unusually small number?

f. If you found that six of the eight sample policyholders had smoke detectors, would this be convincing evidence that the claim is false? Explain.

4.3:

4. Geologists estimate the time since the most recent cooling of a mineral by counting the number of uranium fission tracks on the surface of the mineral. A certain mineral specimen is of such an age that there should be an average of 6 tracks per cm^2 of surface area. Assume the number of tracks in an area follows a Poisson distribution. Let X represent the number of tracks counted in 1 cm^2 of surface area. Find

a. $P(X = 7)$

b. $P(X \geq 3)$

c. $P(2 < X < 7)$

d. μ_X

e. σ_X

6. One out of every 5000 individuals in a population carries a certain defective gene. A random sample of 1000 individuals is studied.

a. What is the probability that exactly one of the sample individuals carries the gene?

b. What is the probability that none of the sample individuals carries the gene?

c. What is the probability that more than two of the sample individuals carry the gene?

d. What is the mean of the number of sample individuals that carry the gene?

e. What is the standard deviation of the number of sample individuals that carry the gene?

8. The number of cars arriving at a given intersection follows a Poisson distribution with a mean rate of 4 per second.

a. What is the probability that 3 cars arrive in a given second?

b. What is the probability that 8 cars arrive in three seconds?

c. What is the probability that more than 3 cars arrive in a period of two seconds?

10. To estimate the concentration of particles in a certain suspension, a chemist withdraws 3 mL of the suspension and counts 48 particles. Estimate the concentration in particles per mL, and find the uncertainty in the estimate.

18. You have received a radioactive mass that is claimed to have a mean decay rate of at least 1 particle per second. If the mean decay rate is less than 1 per second, you may return the product for a refund. Let X be the number of decay events counted in 10 seconds.

a. If the mean decay rate is exactly 1 per second (so that the claim is true, but just barely), what is $P(X \leq 1)$?

b. Based on the answer to part (a), if the mean decay rate is 1 particle per second, would one event in 10 seconds be an unusually small number?

c. If you counted one decay event in 10 seconds, would this be convincing evidence that the product should be returned? Explain.

d. If the mean decay rate is exactly 1 per second, what is $P(X \leq 8)$?

e. Based on the answer to part (d), if the mean decay rate is 1 particle per second, would eight events in 10 seconds be an unusually small number?

f. If you counted eight decay events in 10 seconds, would this be convincing evidence that the product should be returned? Explain.