Instructor's Supplement Problems

Chapter 05

1. Find the closed-loop transfer function, $T(s) = C(s)/R(s)$ for the system shown in Figure I-5.1, using block diagram reduction. [Section: 13.2]

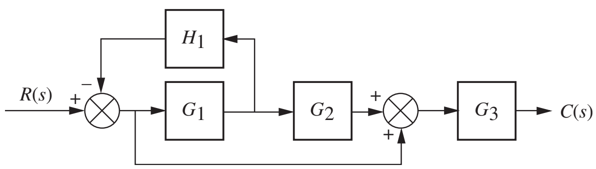

FIGURE I-5.1

2. Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure I-5.2. [Section: 13.2]

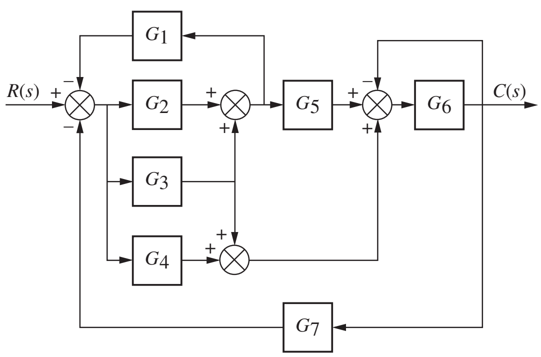

FIGURE I-5.2

3. Find the transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure I-5.3. Use the following methods:

a. Block diagram reduction [Section: 13.2]

b. MATLAB. Use the following transfer functions:

MATLAB
ML

$G_1(s) = 1/(s+7)$, $G_2(s) = 1/(s^2+2s+3)$, $G_3(s) = 1/(s+4)$, $G_4(s) = 1/s$, $G_5(s) = 5/(s+7)$, $G_6(s) = 1/(s^2+5s+10)$, $G_7(s) = 3/(s+2)$, $G_8(s) = 1/(s+6)$.
Hint: Use the **append** and **connect** commands in MATLAB's Control System Toolbox.

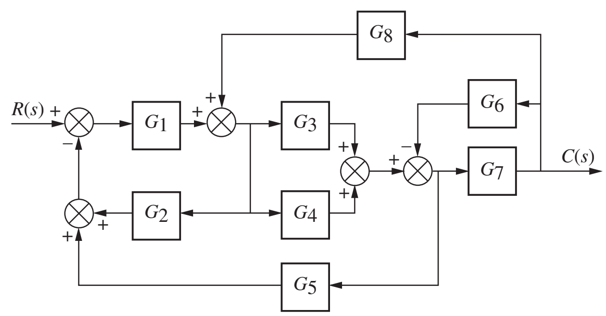

FIGURE I-5.3

4. Given the block diagram of a system shown in Figure I-5.4, find the transfer function $G(s) = \theta_{22}(s)/\theta_{11}(s)$. [Section: 13.2]

5. Reduce the block diagram shown in Figure I-5.5 to a single block representing the transfer function, $T(s) = C(s)/R(s)$. [Section: 13.2]

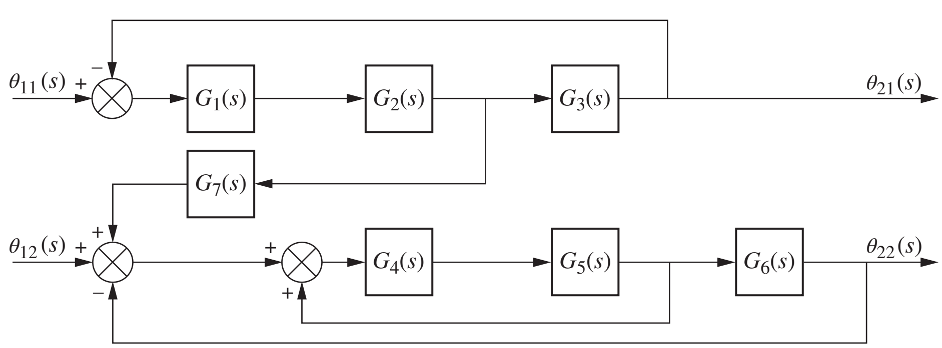

FIGURE I-5.4

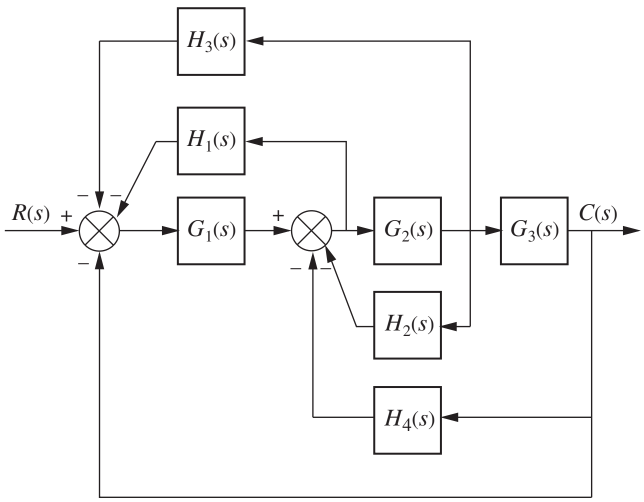

FIGURE I-5.5

6. Find the transfer function $G(s) = E_o(s)/T(s)$ for the system shown in Figure I-5.6.

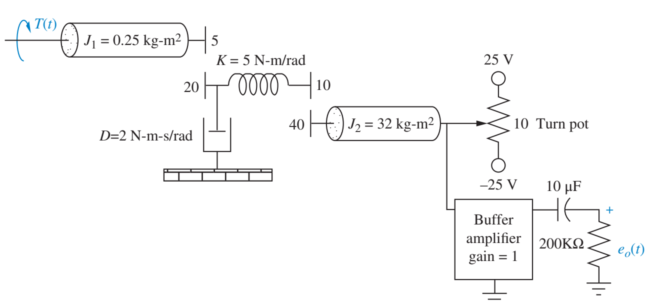

FIGURE I-5.6

7. Draw a signal-flow graph for each of the following state equations: [Section: 13.6]

State Space
SS

a. $$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}\mathbf{x}$$

b. $$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}\mathbf{x}$$

c. $$\dot{\mathbf{x}} = \begin{bmatrix} 7 & 1 & 0 \\ -3 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}\mathbf{x}$$

8. Using Mason's rule, find the transfer function, $T(s) = C(s)/R(s)$, for the system represented by Figure I-5.7. [Section: 13.5]

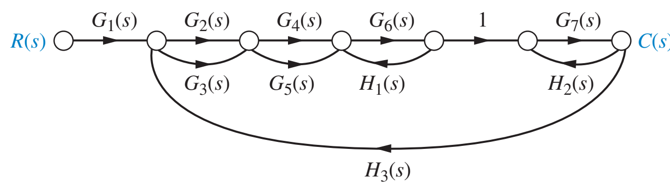

FIGURE I-5.7

9. Use block diagram reduction to find the transfer function of Figure 13.21 in the text, and compare your answer with that obtained by Mason's rule. [Section: 13.5]

10. Given a unity feedback system with the forward-path transfer function

MATLAB
ML
State Space
SS

$$G(s) = \frac{8}{s(s+8)(s+10)}$$

use MATLAB to represent the closed-loop system in state space in

a. phase-variable form;

b. parallel form.

11. Consider the parallel subsystems shown in Figure I-5.8. If $G_1(s)$ is represented in state space as

State Space
SS

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 r \\ y_1 &= \mathbf{C}_1\mathbf{x}_1 \end{aligned}$$

and $G_2(s)$ is represented in state space as

$$\begin{aligned} \dot{\mathbf{x}}_2 &= \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 r \\ y_2 &= \mathbf{C}_2\mathbf{x}_2 \end{aligned}$$

show that the entire system can be represented in state space as

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \cdots \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \cdots \\ \mathbf{B}_2 \end{bmatrix} r$$
$$y = \begin{bmatrix} \mathbf{C}_1 & \vdots & \mathbf{C}_2 \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix}$$

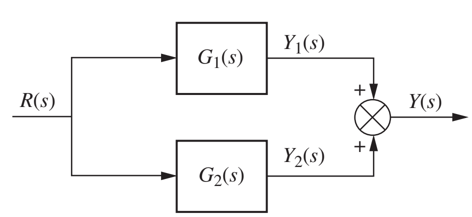

FIGURE I-5.8

12. Repeat Problem 32 in the text problems for the following system: [Section: 5.8]

State Space **SS**

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 1 & 1 \\ 9 & -9 & -9 \\ -9 & -1 & 8 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 9 \\ -4 \\ 0 \end{bmatrix} r$$

$$y = [-2 \quad -4 \quad 1]\mathbf{x}$$

and the following state-vector transformation:

$$\mathbf{z} = \begin{bmatrix} 5 & -4 & 9 \\ 6 & -7 & 6 \\ 6 & -5 & -3 \end{bmatrix}\mathbf{x}$$

13. Diagonalize the following system: [Section: 5.8]

State Space **SS**

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & -5 & 4 \\ 2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} r$$

$$y = [-1 \quad 1 \quad 2]\mathbf{x}$$

14. `Repeat Problem 35 in the text problems using MATLAB.`

MATLAB **ML**

15. Problem I-3 in Chapter 1 describes a high-speed proportional solenoid valve. A subsystem of the valve is the solenoid coil shown in Figure I-5.9. Current through the coil, L, generates a magnetic field that produces a force to operate the valve. Figure I-5.9 can be represented as a block diagram (*Vaughan, 1996*).

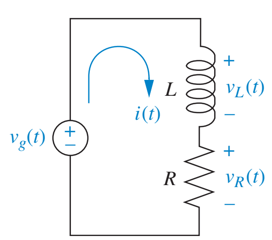

FIGURE I-5.9 Solenoid coil circuit

a. Derive a block diagram of a feedback system that represents the coil circuit, where the applied voltage, $v_g(t)$, is the input, the coil voltage, $v_L(t)$, is the error voltage, and the current, $i(t)$, is the output.

b. For the block diagram found in Part **a**, find the Laplace transform of the output current, $I(s)$.

c. Solve the circuit of Figure I-5.9 for $I(s)$, and compare to your result in Part **b**.

16. Figure I-5.10 shows the diagram of an inverting operational amplifier.

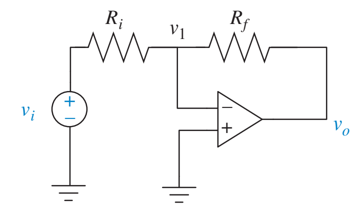

FIGURE I-5.10 Inverting operational amplifier

a. Assuming an ideal operational amplifier, use a similar procedure to the one outlined in Problem 37 in the text problems to find the system equations.

b. Draw a corresponding block diagram and obtain the transfer function $\dfrac{V_o(s)}{V_i(s)}$.

c. Show that when $A \to \infty$, $\dfrac{V_o(s)}{V_i(s)} = -\dfrac{R_f}{R_i}$.

17. `Use LabVIEW's Control Design and Simulation Module to obtain the controller and the observer canonical forms for:`

State Space **SS**
LabVIEW **LV**

$$G(s) = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

18. A hybrid solar cell and diesel power distribution system has been proposed and tested (*Lee, 2007*). The system has been shown to have a very good uninterruptible power supply as well as line voltage regulation capabilities. Figure I-5.11 shows a signal-flow diagram of the system. The output, V_{Load}, is the voltage across the load. The two inputs are I_{Cf}, the reference current, and I_{Dist}, the disturbance representing current changes in the supply.

a. Refer to Figure I-5.11 and find the transfer function $\dfrac{V_{Load}(s)}{I_{Cf}(s)}$.

b. Find the transfer function $\dfrac{V_{Load}(s)}{I_{Dist}(s)}$.

19. A simplified second-order transfer function model for bicycle dynamics is given by

State Space **SS**

$$\frac{\varphi(s)}{\delta(s)} = \frac{aV}{bh}\,\frac{\left(s + \dfrac{V}{a}\right)}{\left(s^2 - \dfrac{g}{h}\right)}$$

The input is $\delta(s)$, the steering angle, and the output is $\varphi(s)$, the tilt angle (between the floor and the bicycle longitudinal plane). In the model, parameter a is the horizontal distance from the center of the back wheel to the bicycle center of mass; b is the horizontal distance between the centers of both wheels; h is the vertical distance from the center of mass to the floor; V is the rear wheel velocity (assumed constant); and g is the gravity constant. It is also assumed that the rider remains at a fixed position

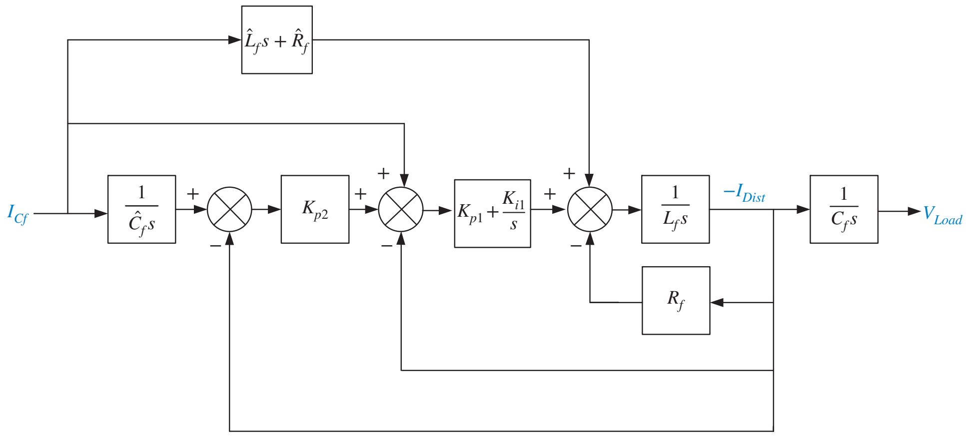

FIGURE I-5.11

with respect to the bicycle so that the steer axis is vertical and that all angle deviations are small (*Åstrom, 2005*).

a. Obtain a state-space representation for the bicycle model in phase-variable form.

b. Find system eigenvalues and eigenvectors.

c. Find an appropriate similarity transformation matrix to diagonalize the system and obtain the state-space system's diagonal representation.

20. Assume that the motor whose transfer function is shown in Figure I-5.12(*a*) is used as the forward path of a closed-loop, unity feedback system.

a. Calculate the percent overshoot and settling time that could be expected.

b. You want to improve the response found in Part **a**. Since the motor and the motor constants cannot be changed, an amplifier and a tachometer (voltage generator) are inserted into the loop, as shown in Figure I-5.12. Find the values of K_1 and K_2 to yield a 20% overshoot and a settling time of 0.25 second.

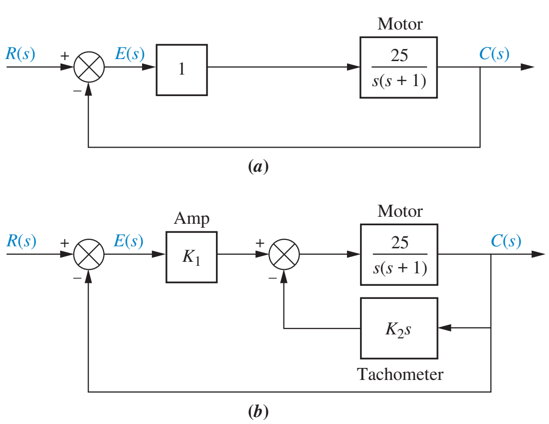

FIGURE I-5.12 **a.** Position control; **b.** position control with tachometer

21. Given the rotational system shown in Figure I-4.8, do the following:

a. Using the transfer function you derived for that system, $G(s) = \Theta_1(s)/T(s)$, where $\Theta_1(s)$ is the angular displacement of the first shaft, find the value of $n = N_1/N_2$ that yields a settling time of 10 seconds for a step input in torque.

b. If this rotational system is the controlled unit, $G(s)$, in the feedback system of Figure I-5.13, find the values of ζ, ω_n, $\%O.S.$, and T_s for a controller gain $K = 4$ N-m/rad and $r(t) = u(t)$ radians.

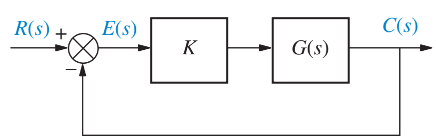

FIGURE I-5.13

22. A process is simulated by the second-order passive circuit, shown in Figure I-5.14, where the feedback amplifier, controller, and final control element are represented by op-amp circuits.

a. Denoting the input and output as $R(s) = V_i(s)$ and $C(s) = V_o(s)$, with $R(s) - C(s) = E(s)$, and noting that the feedback amplifier has a unity gain, draw a block diagram for this feedback control system, where $G_C(s)$, $G_F(s)$, and $G_P(s)$ are the transfer functions of the controller, final control element, and the process, respectively.

b. Find the value of R_P that makes the circuit representing the process critically damped.

c. Noting that the proportional controller is simply an amplifier, $G_C(s) = K_P$, find the value of its gain K_P that results in dominant closed-loop poles with a damping ratio, $\zeta = 0.5$, and a settling time, $T_s = 4$ ms. Verify that the other pole is nondominant. What would be the appropriate value of the controller potentiometer, R_F, given that its tolerance is $\pm 10\%$?

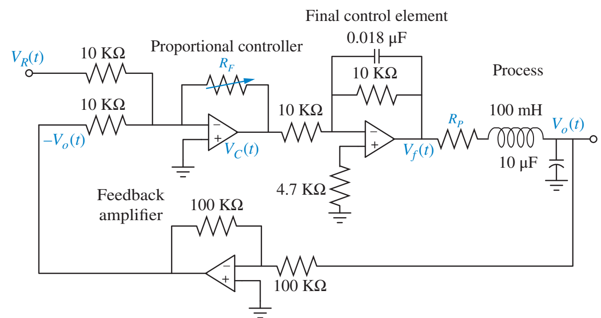

FIGURE I-5.14

Bibliography

Åstrom, K., Klein, R. E., and Lennartsson, A. Bicycle Dynamics and Control. *IEEE Control Systems*, August 2005, pp. 26–47.

Lee, S.-R., Ko, S.-H., Dehbonei, H., Jeon, C.-H., and Kwon, O.-S. Operational Characteristics of PV/Diesel Hybrid Distributed Generation System Using Dual Voltage Source Inverter for Weak Grid, *ISIS 2007 Proceedings*, *Eighth Symposium on Advanced Intelligent Systems*. 2007.

Vaughan, N. D., and Gamble, J. B. The Modeling and Simulation of a Proportional Solenoid Valve. *Journal of Dynamic Systems, Measurements, and Control*, vol. 118, March 1996, pp. 120–125.