**Chapter 7 – Solutions to Instructor Reserve Problems**

**I-1 Instructor**

. Therefore, *e*(∞) = 0.563.

**I-2 Instructor**

. Thus,  =   
= 0.0110.

**I-3 Instructor**

**a.**  Thus, .

**b.** Kv = 10.

**c.** The minimum error will occur for the maximum gain before instability. Using the Routh-Hurwitz Criterion along with :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| s4 | 1 | 196 | 7*K* | For Stability |
| s3 | 25 | 480+*K* |  |  |
| s2 | 4420-*K* | 175*K* |  | *K* < 4420 |
| s1 |  |  |  | -1690.2 < *K* < 1255.2 |
| s0 | 175*K* |  |  | *K > 0* |

Thus, for stability and minimum error *K* = 1255.2*.* Thus,  and .

**I-4 Instructor**



**I-5 Instructor**

.

**I-6 Instructor**

The system is stable for 0 < K < 2000. Since the maximum Kv is Kv = = = 6.25, the minimum steady-state error is = = 0.16.

**I-7 Instructor**

System Type = 1. T(s) = = . From G(s), Kv = = 110. For 12% overshoot,  = 0.56. Therefore, 2n = a , and n2 = K. Hence, a = 1.12 .   
Also, a = . Solving simultaneously,   
K = 1.52 x 104, and a = 1.38 x 102.

**I-8 Instructor**

**a.** For the inner loop:

G1(s) = =

Ge(s) =

T(s) = =

**b.** From Ge(s), system is Type 1.

**c.** Since system is Type 1, ess = 0

**d.** From Ge(s), Kv =  = = . Therefore, ess = = 8.

**e.** Poles of T(s) = -5.4755, -0.7622 ± j1.7526, -1. Therefore, system is stable and results of parts c and d are valid.

**I-9 Instructor**

**Program:**

numg1=[1 9];deng1=poly([0 -6 -12 -14]);

'G1(s)='

G1=tf(numg1,deng1)

numg2=6\*poly([-9 -17]);deng2=poly([-12 -32 -68]);

'G2(s)='

G2=tf(numg2,deng2)

numh1=13;denh1=1;

'H1(s)='

H1=tf(numh1,denh1)

numh2=1;denh2=[1 7];

'H2(s)='

H2=tf(numh2,denh2)

%Close loop with H1 and form G3

'G3(s)=G2(s)/(1+G2(s)H1(s)'

G3=feedback(G2,H1)

%Form G4=G1G3

'G4(s)=G1(s)G3(s)'

G4=series(G1,G3)

%Form Ge=G4/1+G4H2

'Ge(s)=G4(s)/(1+G4(s)H2(s))'

Ge=feedback(G4,H2)

%Form T(s)=Ge(s)/(1+Ge(s)) to test stability

'T(s)=Ge(s)/(1+Ge(s))'

T=feedback(Ge,1)

'Poles of T(s)'

pole(T)

%Computer response shows that system is stable. Now find error specs.

Kp=dcgain(Ge)

'sGe(s)='

sGe=tf([1 0],1)\*Ge;

'sGe(s)'

sGe=minreal(sGe)

Kv=dcgain(sGe)

's^2Ge(s)='

s2Ge=tf([1 0],1)\*sGe;

's^2Ge(s)'

s2Ge=minreal(s2Ge)

Ka=dcgain(s2Ge)

essstep=100/(1+Kp)

essramp=100/Kv

essparabola=200/Ka

**Computer response:**

ans =

G1(s)=

Transfer function:

s + 9

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s^4 + 32 s^3 + 324 s^2 + 1008 s

ans =

G2(s)=

Transfer function:

6 s^2 + 156 s + 918

------------------------------

s^3 + 112 s^2 + 3376 s + 26112

ans =

H1(s)=

Transfer function:

13

ans =

H2(s)=

Transfer function:

1

-----

s + 7

ans =

G3(s)=G2(s)/(1+G2(s)H1(s)

Transfer function:

6 s^2 + 156 s + 918

------------------------------

s^3 + 190 s^2 + 5404 s + 38046

Solutions to Problems 7-13

ans =

G4(s)=G1(s)G3(s)

Transfer function:

6 s^3 + 210 s^2 + 2322 s + 8262

------------------------------------------------------

s^7 + 222 s^6 + 11808 s^5 + 273542 s^4 + 3.16e006 s^3

+ 1.777e007 s^2 + 3.835e007 s

ans =

Ge(s)=G4(s)/(1+G4(s)H2(s))

Transfer function:

6 s^4 + 252 s^3 + 3792 s^2 + 24516 s + 57834

-------------------------------------------------------

s^8 + 229 s^7 + 13362 s^6 + 356198 s^5 + 5.075e006 s^4

+ 3.989e007 s^3 + 1.628e008 s^2 + 2.685e008 s

+ 8262

ans =

T(s)=Ge(s)/(1+Ge(s))

Transfer function:

6 s^4 + 252 s^3 + 3792 s^2 + 24516 s + 57834

-------------------------------------------------------

s^8 + 229 s^7 + 13362 s^6 + 356198 s^5 + 5.075e006 s^4

+ 3.989e007 s^3 + 1.628e008 s^2 + 2.685e008 s

+ 66096

ans =

Poles of T(s)

ans =

-157.1538

-21.6791

-14.0006

-11.9987

-11.1678

-7.0001

-5.9997

-0.0002

Kp =

7

ans =

sGe(s)=

ans =

sGe(s)

Transfer function:

6 s^5 + 252 s^4 + 3792 s^3 + 2.452e004 s^2

+ 5.783e004 s

--------------------------------------------------------

s^8 + 229 s^7 + 1.336e004 s^6 + 3.562e005 s^5

+ 5.075e006 s^4 + 3.989e007 s^3 + 1.628e008 s^2

+ 2.685e008 s + 8262

Kv =

0

ans =

s^2Ge(s)=

ans =

s^2Ge(s)

Transfer function:

6 s^6 + 252 s^5 + 3792 s^4 + 2.452e004 s^3

+ 5.783e004 s^2

--------------------------------------------------------

s^8 + 229 s^7 + 1.336e004 s^6 + 3.562e005 s^5

+ 5.075e006 s^4 + 3.989e007 s^3 + 1.628e008 s^2

+ 2.685e008 s + 8262

Solutions to Problems 7-15

Ka =

0

essstep =

12.5000

essramp =

Inf

essparabola =

Inf

**I-10 Instructor**

We calculate the Velocity Error Constant,



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For a unit ramp input the steady state error is . The input slope is 

**I-11 Instructor**

e(∞) = lim, where G1(s) = and G2 = . From the problem statement,

R(s) = D(s) = . Hence, e(∞) = lim= - .

**I-12 Instructor**

**a.** Produce a unity-feedback system:





Thus, . System is Type 0.

**b.** Since Type 0, appropriate static error constant is*Kp*.

**c.** 

**d.** 

Check stability: Using original block diagram,

.



Making a Routh table:

|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | *K*+8 | 5*K* |
| s3 | 6 | 6*K* | 0 |
| s2 | 8 | 5*K* | 0 |
| s1 |  | 0 | 0 |
| s0 | 5*K* | 0 | 0 |

Therefore, system is stable for *K* > 0 and steady-state error calculations are valid.

**I-13 Instructor**

**a.**

**b.** .

**I-14 Instructor**

**a.**  so and . Also , so .

**b.** The system is type 1, so for it is required that . So .

**I-15 Instructor**

**a.** Using Eq. (7.89) with



yields e(∞) = 1.09756 for a step input and e(∞) = ∞ for a ramp input. The same results are obtained using



and Eq. (7.96) for a step input and Eq. (7.103) for a ramp input.

**b.** Using Eq. (7.89) with



yields e(∞) = 0 for a step input and e(∞) = for a ramp input. The same results are obtained using



and Eq. (12.123) for a step input and Eq. (12.130) for a ramp input.

**c.** Using Eq. (7.89) with



yields e(∞) = 6 for a step input and e(∞) = ∞ for a ramp input. The same results are obtained using



and Eq. (7.96) for a step input and Eq. (7.103) for a ramp input.

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**I-16 Instructor**

**a.** Letting , , we get two equations:

Solving the first equation for , substituting into the second and simplifying we get

**b.** The systems characteristic equation is calculated from

The Routh array is

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
|  |  |  |
|  |  |  |

For closed loop stability the second row requires or .

The third row requires . The intersection of both requirements is

1. From part a. zero steady state error requires , however this value is limited by stability requirements as shown in part b. Therefore zero steady state error is not possible.

**SOLUTIONS TO DESIGN PROBLEMS**

**I-17 Instructor**

Substituting values we have , 

The proportional error constant 

 which gives .